Problem 1

$$P\{H\} = 0.05$$
$$P\{pos.|H\} = 0.98$$
$$P\{pos.|\overline{H}\} = 0.03$$

$$P\{H|pos.\} = \frac{P\{H \cap pos.\}}{P\{pos.\}} = \frac{P\{pos.|H\} \times P\{H\}}{P\{pos.|H\} \times P\{H\} + P\{pos.|\overline{H}\} \times (1 - P\{H\})}$$

$$= \frac{0.05 \times 0.98}{0.98 \times 0.05 + 0.03 \times 0.95} = 0.632$$

Problem 2

$$PDF: \; f = \frac{1}{\sigma\sqrt{2\pi}}e^{\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}$$

$$Z = \frac{x - \mu}{\sigma}$$

$$\rightarrow E[Z] = \int_{-\infty}^{+\infty} \frac{t}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2}dt = \frac{-1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}t^2}d\left(-\frac{1}{2}t^2\right) = \frac{-1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}t^2}|_{-\infty}^{+\infty} = 0$$

$$\rightarrow Var(Z) = E[Z^2] - E[Z]^2 = \int_{-\infty}^{+\infty} \frac{t^2}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2}dt = \frac{-1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} td\left(e^{-\frac{1}{2}t^2}\right)$$

$$= \frac{-1}{\sqrt{2\pi}}\left(te^{-\frac{1}{2}t^2}|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} e^{-\frac{1}{2}t^2}dt\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}t^2}d(t) = 1$$

$$E[X] = \mu + \sigma E[Z] = \mu$$
$$Var(X) = \sigma^2 Var(Z) = \sigma^2$$