Basic Maths for ICCP Simulation Tool

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Steel Pipeline of length $l = 1000m$,with $d = 0.9144m$ and wall thickness $0.0159m$. The area of the pipe is $A = d\pi l = 2872.672m^2$. Assume that the pipeline resistivity $\rho = 10^{-7}\Omega.m$.

C1 : Current density with one rectifier

$$R = \frac{\rho L}{A} = 3.48 \times 10^{-8}\Omega$$

The rectifier output is $10A$. By Ohm's law $U = R \times I = 3.48 \times 10^{-7}V$

$$U = R \times I \to I = \frac{U}{R}$$

with $J = \frac{I}{A}$,we end with

$$J = \frac{I}{A} = \frac{10}{2872.672} = 0.00348A.m^{-2} = 3.48mA.m^{-2}$$

Okay, so this is the current density over ALL the pipeline. If we divide the pipe into 100 hypothetical pieces, where each piece is equal to 10 m, and we put the rectifier at the position 0 (the start of the pipe).

Then for the cell number 1 :

$$A_1 = \frac{A}{100} = 28,72m^2 \to J_1 = \frac{I}{A_1} = 0.348A.m^{-2}$$

For the cell number 2 :

$$A_2 = A_1 \times 2 = 57.44m^2 \to J_2 = \frac{I}{A_2} = 0.174A.m^{-2}$$

For the cell number 3 :

$$A_3 = A_1 \times 3 = 86.16m^2 \to J_3 = \frac{I}{A_3} = 0.116A.m^{-2}$$

A recursive formula can be obtained for the n-th cell

$$A_n = A_1 \times n \to J_n = \frac{I}{A_n}$$

If we put the rectifier at the middle of the pipeline (say position 50 at $l = 500m$), the current density magnitude will be symmetric to the position of the rectifier. So at cells 50 and 51 the current density is

$$J_1 = \frac{I}{A_1} = 0.348A.m^{-2}$$

at the celles 49 and 52

$$J_2 = \frac{I}{A_2} = 0.174A.m^{-2}$$

until the cells 1 and 100

$$J_{50} = \frac{I}{A_{50}} = \frac{I}{A_1 \times 50} = \frac{10}{1436} = 6.96mA.m^{-2}.$$

C2 : Add more rectifiers.

We take the same example as for $C1$ for a rectifier A in position 50 (with a current I_A), and we add another rectifier B at the position 0 with $I_B = 15A$.

Assuming that each cell contain the same pipe surface area $A_1 = 28,72m^2$.

The current densities in each cell is the addition of the current from both rectifiers, we denote $J_A$the current density from rectifier A and J_B for the rectifier B.

At the cell 1.

$$J_{A50} = \frac{I_1}{A_{50}} = 6.96mA.m^{-2}$$
$$J_{B1} = \frac{I_2}{A_1} = 0.522A.m^{-2}$$
$$J_{T1} = J_{A50} + J_{B1} = 0.529A.m^{-2}$$

At the cell 2

$$J_{A49} = \frac{I_1}{A_{49}} = 7.1mA.m^{-2}$$
$$J_{B1} = \frac{I_2}{A_2} = 0.261A.m^{-2}$$
$$J_{T2} = J_{A49} + J_{B2} = 0.268A.m^{-2}$$

Thus the generalization for n-th cell give the total current density ;

$$J_{Tn} = \frac{I_1}{A_1 \times ((50 - n) + 1)} + \frac{I_2}{A_1 \times n}$$

A more general formula for The current density measured at n cell, with M rectifiers (each with I_M current produced at p_M position)

$$J_{Tn} = \sum_{1}^{M} \frac{I_M}{A_1 \times (|p_M - n| + 1)} \tag{1}$$

For our case with 2 rectifiers with $I_A = 10A$ and $I_B = 15A$

$$\begin{aligned} J_{Tn} &= \sum_{1}^{2} \frac{I_M}{A_1 \times (|p_M - n| + 1)} \\ &= \frac{I_A}{A_1 \times (|p_A - n| + 1)} + \frac{I_B}{A_1 \times (|p_B - n| + 1)} \end{aligned} \tag{2}$$

Current Density Direction

Since we want to simplify the mathematical framework first, we assume that the current density $\vec{J}$ is one 1 dimension Ox where Ox exists on the pipeline. $\vec{J} = nq\vec{v}_d$, with $\vec{v}_d$ is the drift velocity of the charged particles. In our case the charged particles are electrons e, therefore the most important part is the sign. We don't need to compute the elements of the vector $\vec{J}$, since $\vec{v}_d$ is opposite to direction of the charges and $q = -e$, we end with a positive current density $\vec{J}$ in respect for the choosen axis.

C3 : Basic Circuit with Anodes

We assume we have an anode with the following data : S12 Zinc anode with 20kg weight, 0.762m length, 0.152m diameter , Y factor equal 1 and F factor equal 1.06, $h = 5m$. The soil resistivity is $1270\Omega.cm$. For an uncoated pipeline the coating efficiency is 0%.

The current output of the anode is

$$I_{Zn} = \frac{5 \times 10^4 \times f \times Y}{\rho} = 4.73mA$$

The resistance of the anode is

$$\begin{aligned} R_h &= \frac{0.00521\rho}{L} \left[2.303 \left(\log \frac{4L}{D} + \log \frac{L}{H} \right) + \frac{2h}{L} - 2 \right] \\ &= \frac{0.00521 \times 1270 \times 10^{-2}}{0.762} \left[2.303 \left(\log \frac{4 \times 0.762}{0.152} + \log \frac{0.762}{5} \right) + \frac{2 \times 5}{0.762} - 2 \right] \\ &= 1.19\,\Omega \end{aligned}$$

The wire resistance $R_W = resistance\,of\,wire\,ohms.m^{-1} \times (l + l\,10\%) = 0.259 \times 10^{-3} \times 1000 = 0.259\Omega$

Let's say that after 30 hours of applying the currents I (that can be $I_1$and I_2) the original potential before applying cathodic protection was for instance $E = -0.557V$. Voltage drop is $\Delta E = -0.85 - E = -0.293$.

Total current requirement $I = AJ\,(1 - C_E) = AJ$. Let's assume that the required current density is $J = 5 \times 10^{-3} A.m^{-2}$. Then $I = 2872.672 \times 5 \times 10^{-3} = 14.36A$

Pipe to soil resistance $R_{PS} = \frac{\Delta E}{I} = \frac{0.293}{14.36} = 0.021\,\Omega$

Total Circuit Resistance $R_T = R_a + R_W + R_{PS}$

- Rectifier Output $\begin{cases} V_{rec} = IR_T \times (150\%) \\ I_{Req} \end{cases}$

C4 : Coating Type, Current Requirement and Pipe Potential

The pipe resistance

$$R_p = \frac{\rho}{\pi \times L \times (OD - WT)}$$

where OD is the outside diameter, WT the wall thickness, L the length of the pipe and ρ the pipe resistivity.

The coating conductance is given by

$$G = \frac{\pi \times OD}{\omega}$$

where ω is the spectific coating resistance.

The attenuation constant is

$$\alpha = \sqrt{R_p \times G}$$

Pipe Characteristic Resistance

$$R_{CR} = \sqrt{\frac{R_p}{G}}$$

Pipe potential shifting

$$\Delta E = \cos\left(\alpha \times L\right)$$

Required protective current

$$I_0 \approx \frac{\Delta E}{R_{CR}} \tan\left(\alpha \times L\right)$$

Pipe to soil potential

$$J = \sigma E = \frac{E}{\rho}$$

$$E = \rho J$$

The electrons moves on the Ox plan, therefore

$$\vec{E}_x = \rho \vec{J}_x$$
$$E_x = \rho J_x$$

On the pipe surface of cell 6 $E_x = 1270 \times 10^{-2} \times 0.09681868 = 1.229596 V.m^{-1}$.