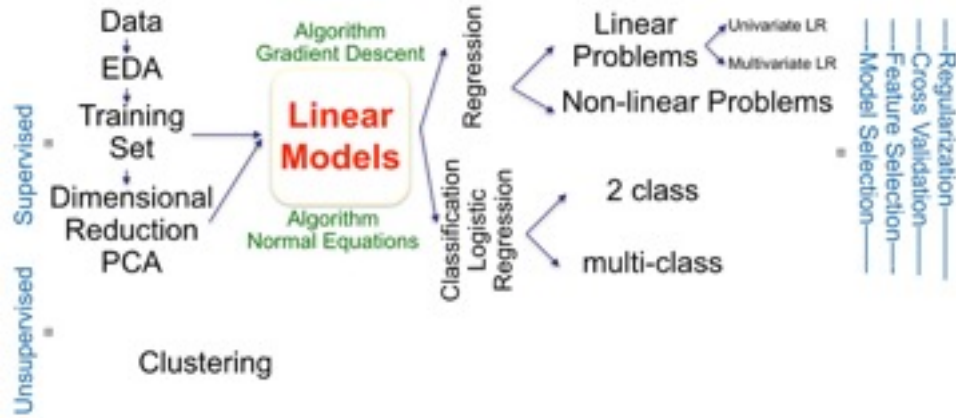


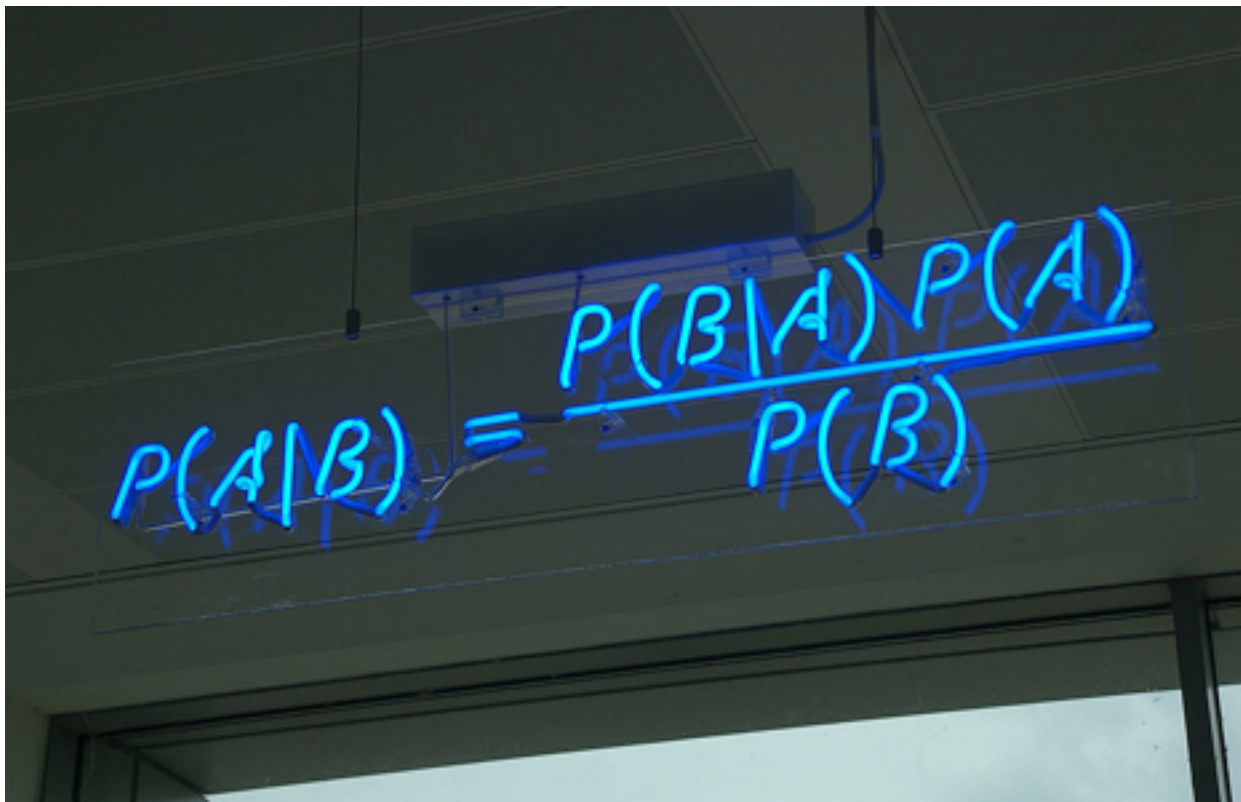
INTRO TO DATA SCIENCE

LECTURE 13: BAYES THEOREM & NAIVE BAYES CLASSIFIERS

WHERE ARE WE ON THE DATA SCIENCE ROAD-MAP?



KEY CONCEPTS



A photograph of a blue neon sign mounted on a ceiling, displaying the formula for Bayes' theorem. The sign is illuminated and shows the equation $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$ in a stylized, handwritten font. The background is dark, and the sign is the primary source of light in the image.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

KEY CONCEPTS - FREQUENTIST OR BAYESIAN?

Frequentist

- the classical view of statistics
- probability is the long-run frequency of an event
- returns a point estimate (a single number)

Example: what is the probability of a plane crash?

Bayesian

- probability measures your belief based on evidence (data)
- individuals may differ in their probability estimates (i.e. their beliefs) based upon their data
- as new information arrives this may change the belief
- returns a probability distribution or probabilities, from which a point estimate may be made

KEY CONCEPTS - FREQUENTIST OR BAYESIAN?

An imaginary frequentist function

IN: my coin has returned heads in the last 4 flips; is my coin biased?

OUT: 'Yes'

An imaginary bayesian function

IN: my coin has returned heads in the last 4 flips; I believe it is a fair coin; is my coin biased?

OUT: 'Yes' with $p = 0.9$, 'No' with $p = 0.1$

KEY CONCEPTS - FREQUENTIST OR BAYESIAN?

- The effect of prior information - I believe my coin is fair, i.e. has a 50/50 chance of coming up heads or tails
- As more evidence comes in this prior belief will get 'washed out'
- As the number of examples increases and approaches infinity so the Bayesian estimate will align with the frequentist estimate

KEY CONCEPTS - FREQUENTIST OR BAYESIAN?

Andrew Gelman (2005):

Sample sizes are never large. If N is too small to get a sufficiently-precise estimate, you need to get more data (or make more assumptions). But once N is "large enough," you can start subdividing the data to learn more (for example, in a public opinion poll, once you have a good estimate for the entire country, you can estimate among men and women, northerners and southerners, different age groups, etc.). N is never enough because if it were "enough" you'd already be on to the next problem for which you need more data.

DATA SCIENCE

PROBABILITY THEORY

KEY CONCEPTS - PROBABILITY THEORY

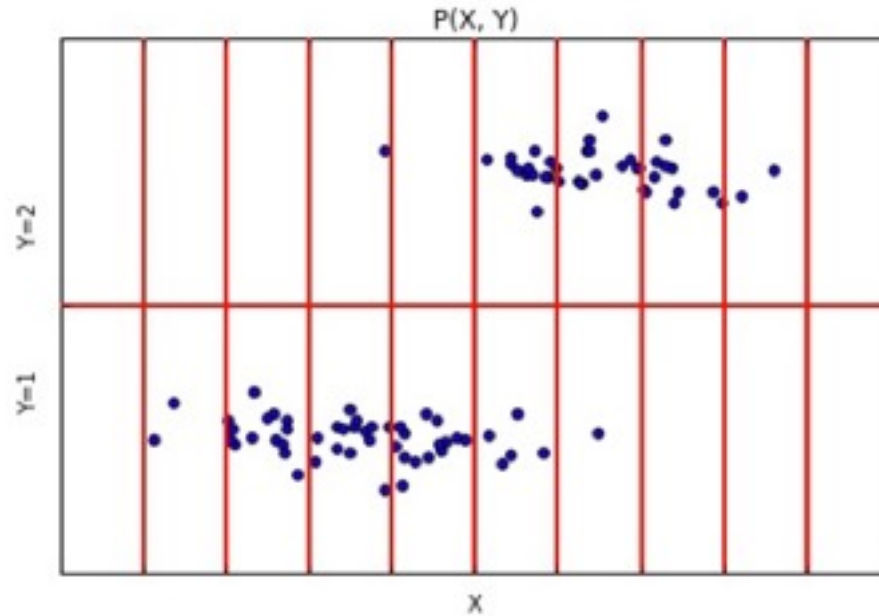
- A central foundation for pattern recognition
- Provides a consistent framework for handling uncertainty
 - Noise in data
 - Finite size of data sets
- Combined with decision theory allows for optimal predictions

KEY CONCEPTS - PROBABILITY THEORY

- Probability (of an event) = the fraction of times that the event occurs out of the total number of events
- The set of all events is called the sample space
- By definition probabilities lie in the interval $[0, 1]$
- If events are mutually exclusive then the probabilities of all events in the sample space must sum to 1

KEY CONCEPTS - PROBABILITY THEORY

For example, take N instances of 2 random variables, X and Y

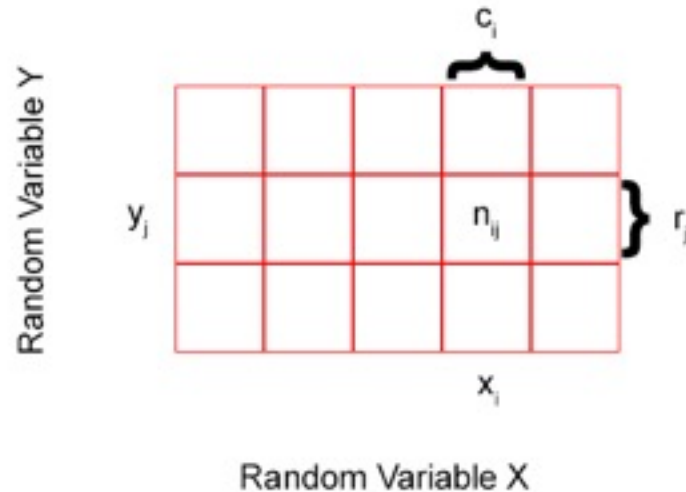


KEY CONCEPTS - JOINT PROBABILITY

The number of instances of $X = x_i$ AND $Y = y_j$ is n_{ij} , which equals the number of points in the intersecting cell

The number of instances or points in column i , corresponding to $X = x_i$ is c_i

The number of points in row j , corresponding to $Y = y_j$ is r_j



X takes the value x_i , where $i = 1, \dots, 5$

Y takes the value y_j , where $j = 1, \dots, 3$

KEY CONCEPTS - JOINT PROBABILITY

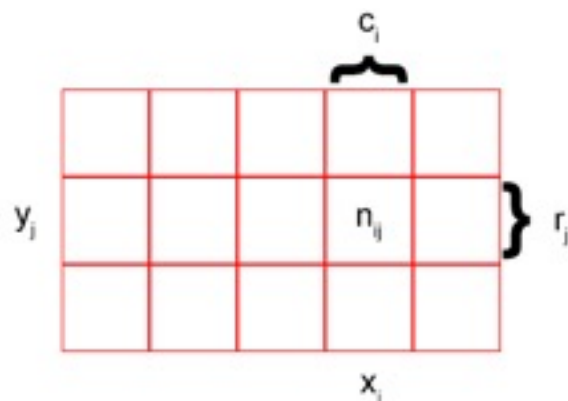
Joint Probability

The probability that X takes the value x_i AND Y takes the value y_j is denoted:

$$p(X = x_i, Y = y_j) \text{ or } P(X, Y)$$

$$p(X, Y) = \frac{n_{ij}}{N}$$

and called the joint probability of X and Y



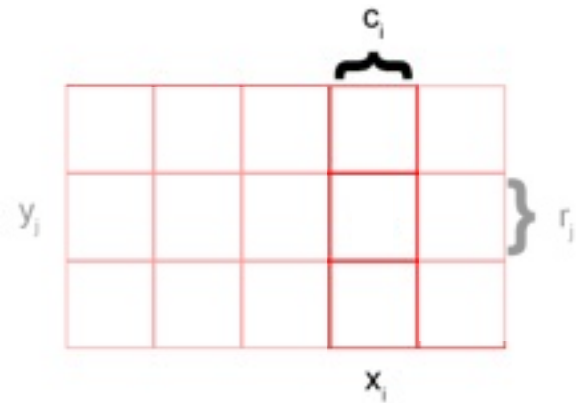
KEY CONCEPTS - MARGINAL PROBABILITY

The probability that X takes the value x_i irrespective of the value of Y is denoted:

$$p(X = x_i) \text{ or } P(X)$$

$$p(X) = \frac{c_i}{N}$$

Y has been ‘marginalized’ out...

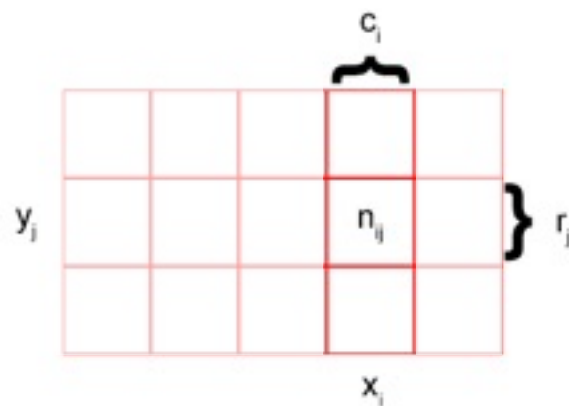


KEY CONCEPTS - MARGINAL PROBABILITY

c_i may be expressed as the sum of the values of n_{ij} summed over all the values of j

$$c_i = \sum_j n_{ij}$$

$$\therefore p(X) = \frac{c_i}{N} = \frac{\sum_j n_{ij}}{N} = \sum_j \frac{n_{ij}}{N} = \sum_Y p(X, Y)$$



KEY CONCEPTS - SUM RULE

The SUM RULE:

$$p(X) = \sum_Y p(X, Y)$$

$p(X)$ is called the MARGINAL PROBABILITY

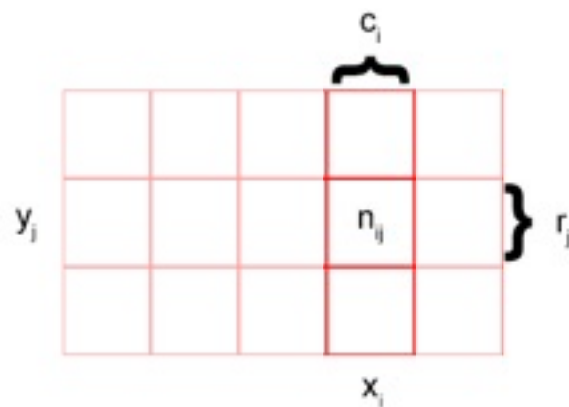
Other variables, Y here, are being summed out or marginalized

KEY CONCEPTS - CONDITIONAL PROBABILITY

Consider those instances for which $X = x_i$, then the fraction of such instances for which $Y = y_j$ is written $p(Y = y_j | X = x_i)$

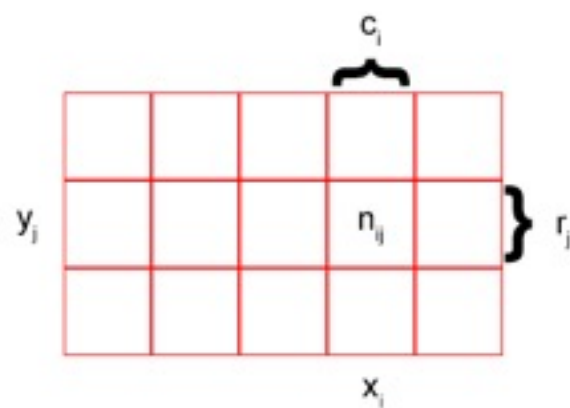
$p(Y|X)$ is called the **CONDITIONAL PROBABILITY**.

$$p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i}$$



KEY CONCEPTS - RULES OF PROBABILITY

$$p(X, Y) = \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \times \frac{c_i}{N} = p(Y|X)p(X)$$



The PRODUCT RULE:

$$p(X, Y) = p(Y|X)p(X)$$

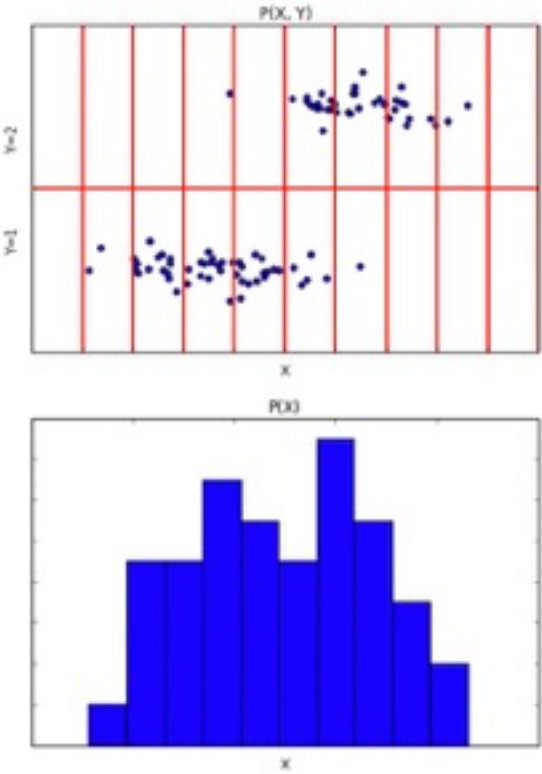
RULES OF PROBABILITY:

Sum Rule: $P(X) = \sum_Y P(X, Y)$

Product Rule: $P(X, Y) = P(Y|X)p(X)$

KEY CONCEPTS - MARGINAL AND CONDITIONAL DISTRIBUTIONS

Marginal Distribution



Marginal Distribution

Conditional Distribution

BAYES' THEOREM:

$$P(X, Y) = P(Y, X)$$

$$P(X, Y) = P(Y|X)P(X) \text{ and } P(Y, X) = P(X|Y)P(Y)$$

$$P(Y|X)P(X) = P(X|Y)P(Y)$$

$$\therefore P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

KEY CONCEPTS - BAYES' THEOREM

Bayes' Theorem

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

OR

$$\text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Normalization}}$$

KEY CONCEPTS - INDEPENDENCE

INDEPENDENCE

If $p(X, Y) = p(X)p(Y)$ then X and Y are said to be independent

This means that

$p(Y|X) = P(Y)$ So the conditional distribution Y given X , is indeed independent of X

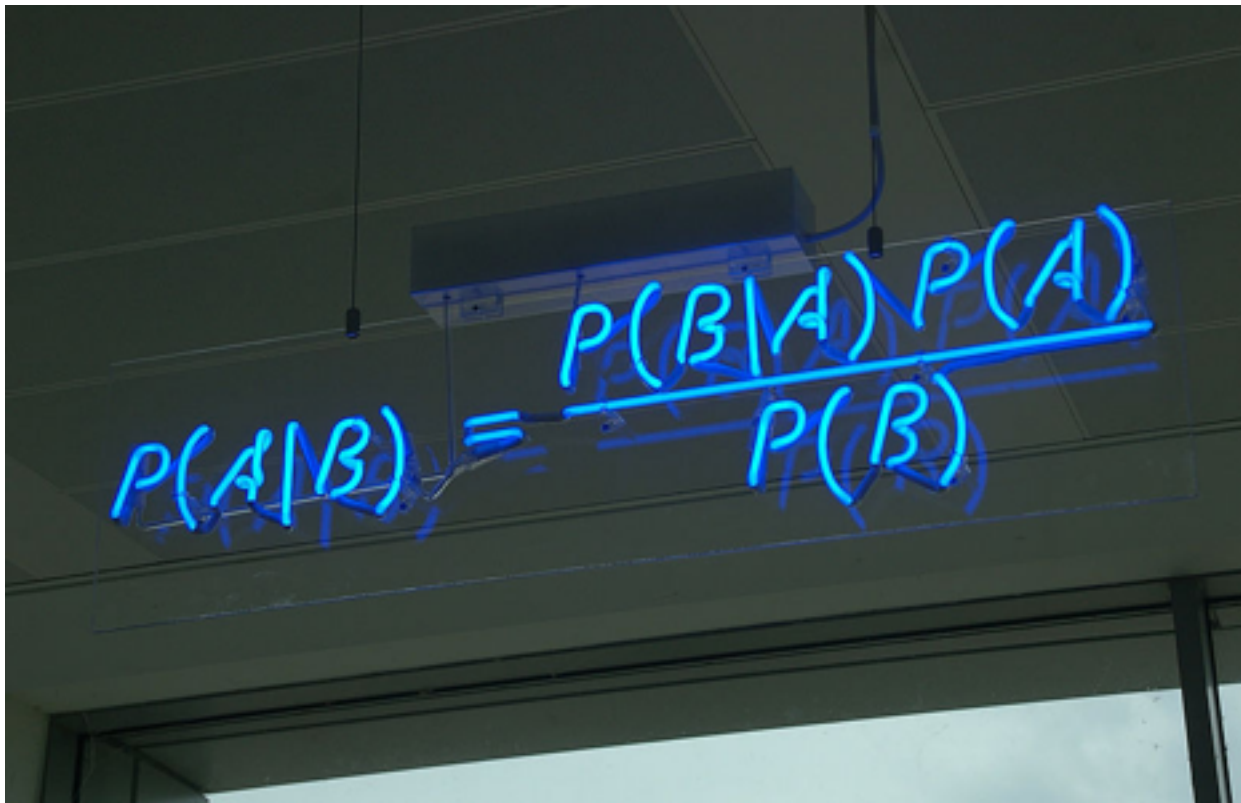
KEY CONCEPTS - THOMAS BAYES 1701 - 1761



English clergyman, amateur scientist and mathematician. One problem of his time concerned 'inverse probability', to which he proposed a solution in a paper called 'Essay towards solving a problem in the doctrine of chances'. This was published 3 years after his death.

Bayes only formulated his theory for the case of the uniform prior. Pierre-Simon Laplace who independently rediscovered the theory in general form and demonstrated its broad applicability.

KEY CONCEPTS - BAYES



A photograph of a blue neon sign mounted on a ceiling, displaying the formula for Bayes' Theorem. The sign is illuminated with a bright blue light, and the background is dark. The formula is written in a stylized, handwritten font. The sign is slightly tilted and has some visible wear and tear, including a small hole in the neon tube.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

KEY CONCEPTS - BAYES THEOREM

- The overall goal of Bayesian computation is to determine the posterior distribution of a particular variable given some data
- From this distribution you can derive point estimates
- It is important to realize the importance of the prior distribution. Often, in real world problems, we do not know the prior. A 'default' assumption is equal priors
- The Likelihood is derived from the (labeled) data

KEY CONCEPTS - BAYES' THEOREM

Comments about Bayes:

- It is a relatively simple algebraic relationship
- It is extremely powerful as a computational tool
- It can be unbelievably confusing
- If it all sounds crazy don't worry!

KEY CONCEPTS - PROBABILITY - A SIMPLE EXAMPLE

Suppose you pick a box, blindfolded, and over the course of many trials determine that you pick the blue box 60% of the time

1 Red Box - containing 2 apples and 6 oranges

1 Blue Box - containing 3 apples and 1 orange

Having chosen a box to pick fruit from you randomly select an item of fruit

Each piece of fruit in a box is equally likely to be chosen

You replace the fruit between trials



KEY CONCEPTS - PROBABILITY - A SIMPLE EXAMPLE

- Probability box chosen is blue,
 - $P(\text{Box}=\text{blue}) = 0.6$, and
 - $P(\text{Box}=\text{red}) = 1.0 - 0.6 = 0.4$
- Choosing a box is mutually exclusive, you cannot choose both boxes at the same time, it is either one or the other

KEY CONCEPTS - PROBABILITY - A SIMPLE EXAMPLE

1. Conditional Probability:

What is $P(F=\text{apple} \mid B=\text{blue})$?

2. Marginal Probability:

What is $P(\text{apple})$?

3. Bayes' Theorem:

Given you have selected an apple what is the probability it came from the blue box?



KEY CONCEPTS - CLASSIFICATION USING BAYES

- Family of algorithms based upon a common algorithmic structure
- Highly applicable where frequency counts are the features
- Highly scalable
- Very competitive, in terms of performance
- Only requires a relatively small amount of data in order to make estimates

KEY CONCEPTS - NAIVE BAYES CLASSIFIERS

$P(C_k|\vec{x})$ where $x = (x_1, x_2, \dots, x_N)$

using Bayes:

$$P(C_k|\vec{x}) = \frac{P(\vec{x}|C_k) \times P(C_k)}{P(\vec{x})}$$

Because we will be comparing 2 probabilities (for at least a 2 class problem) we can ignore the denominator. The relatively magnitude of $P(C_k|\vec{x})$ will be the same whether or not we divide by $P(\vec{x})$

KEY CONCEPTS - NAIVE BAYES CLASSIFIERS

Let $N = 3$

$$P(C_k|\vec{x}) = P(C_k|x_1, x_2, x_3)$$

Expand using Bayes and forget the denominator

$$= P(x_1, x_2, x_3|C_k) \times p(C_k)$$

Now use the product rule to expand out the first term

$$= P(x_1|C_k) \times P(x_2, x_3|C_k, x_1) \times p(C_k)$$

$$= P(x_2|C_k, x_1) \times P(x_3|C_k, x_1, x_2) \times P(x_1|C_k) \times p(C_k)$$

RULES OF PROBABILITY:

Sum Rule: $P(X) = \sum_Y P(X, Y)$

Product Rule: $P(X, Y) = P(Y|X)p(X)$

KEY CONCEPTS - ESTIMATING THE LIKELIHOOD FUNCTION

$$\begin{aligned}P(x_1, x_2, x_3|C_k) &= P(x_1|C_k) \times P(x_2, x_3|C_k, x_1) \\&= P(x_2|C_k, x_1) \times P(x_3|C_k, x_1, x_2) \times P(x_1|C_k)\end{aligned}$$

To estimate these conditional probabilities would require data on all possible combinations!

It is simply impractical

KEY CONCEPTS - NAIVE BAYES CLASSIFIERS

If x_1, x_2 , and x_3 are independent then

$$P(x_2|C_k, x_1) = P(x_2|C_k) \text{ and}$$

$$P(x_3|C_k, x_1, x_2) = P(x_3|C_k)$$

INDEPENDENCE

If $p(X, Y) = p(X)p(Y)$ then X and Y are said to be independent

This means that

$p(Y|X) = P(Y)$ So the conditional distribution Y given X, is indeed independent of X

$$P(C_k|\vec{x}) = P(C_k, x_1, x_2, x_3)$$

$$= P(C_k) \times P(x_1|C_k) \times P(x_2|C_k) \times P(x_3|C_k)$$

KEY CONCEPTS - NAIVE BAYES CLASSIFIERS

$$\hat{y} = \operatorname{argmax}_{k \in 1, \dots, K} P(C_k) \prod_{i=1}^N P(x_i | C_k)$$

KEY CONCEPTS - THE 'NAIVE' IN NAIVE BAYES

- It is called naive, because we choose to ignore an important assumption
- We assume that the features in x are independent of each other
- Naive Bayes classifiers do not output accurate probabilities!, they are meant to classify purely based upon which class output has greatest magnitude.
- The independence assumption avoids the curse of dimensionality problem, hence the reason why these models scale so well

KEY CONCEPTS - NAIVE BAYES CLASSIFIERS

- Naive Bayes is for classification only, there is no corresponding or regression version

KEY CONCEPTS - SKLEARN - NAIVE BAYES CLASSIFIERS

1. Gaussian. For each class the features of x are the mean and std deviation
2. Multinomial. For each class the features of x are a histogram representing the frequency of occurrence
3. Bernoulli. For each class the features of x are the occurrence or not

NGRAMS COUNT VECTORIZATION

KEY CONCEPTS - SKLEARN - COUNTVECTORIZER

- A count vectorizer simply breaks apart text into words and counts them, putting the word frequencies into a matrix
- An n-gram is a contiguous sequence of n items from a given sequence of text