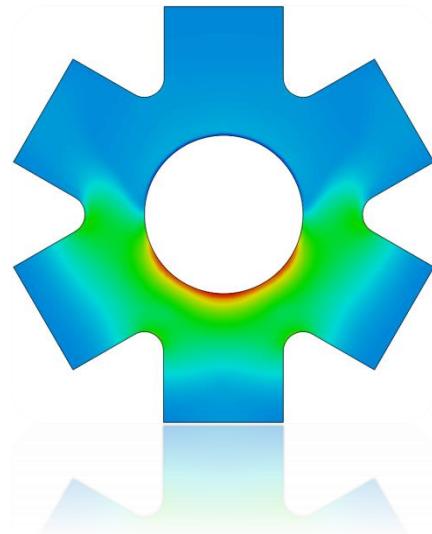


# QUICK FATIGUE TOOL FOR MATLAB®

Multiaxial Fatigue Analysis Code for Finite Element Models

## User Guide



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# Version Information

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## 1. Introduction

### 1.1 Overview

Quick Fatigue Tool for MATLAB is an experimental fatigue analysis code. The application includes:

- A general stress-life and strain-life fatigue analysis framework, configured via a text-based interface;
- Material Manager, a material database and MATLAB application which allows the user to create and store materials for fatigue analysis (Section 5);
- Multiaxial Gauge Fatigue, a strain-life code and MATLAB application which allows the user to perform fatigue analysis from measured strain gauge histories (Appendix III); and
- Export Tool, an ODB interface which allows the user to export fatigue results to an Output Database (.odb) file for visualization in SIMULIA Abaqus/Viewer (Section 10.4).

Quick Fatigue Tool runs entirely within the MATLAB environment, making it a highly customizable code which is free from external dependencies.

The general analysis framework allows the user to analyse elastic stresses from Finite Element Analysis (FEA) results. One of the main advantages of calculating fatigue lives from FEA is that it eliminates the requirement to manually compute stress concentration and notch sensitivity factors. The program is optimised for reading field output from Abaqus field report files. However, field output can be specified in any ASCII format provided the data structure in Section 3 is observed.

The stress-life program requires the following inputs from the user:

1. A material definition
2. A loading definition consisting of:
  - a. Stress datasets
  - b. Load histories

The above input is specified by means of a *job* file. This is an .m script or text file containing a list of options which completely define the analysis. Analyses are performed by running the job file. Basic fatigue result output is written to the command window, and extensive output is written to a set of individual data files.

## 1.2 The stress-life methodology

The stress-life methodology is used for calculating fatigue damage where the expected lives are long and the stresses are elastic. The method is also well-suited to infinite life design where a pass/fail criterion based on the fatigue limit is sufficient. The stress-life approach ignores local plasticity and provides a “total life” estimate of fatigue life [1] [2] [3]. This is illustrated by Figure 1.1. If the analyst wishes to gain insights into the life up to crack initiation ( $N_i$ ), or wishes to find the number of cycles required to cause crack growth ( $N_i + N_p$ ), strain and fracture mechanics-based methods should be explored instead [4].



Figure 1.1: Illustration of the stress-life method where the total life,  $N_f$ , is the sum of the life to crack initiation,  $N_i$ , and life to final crack propagation,  $N_p$ .

## 1.3 The strain-life methodology

The strain-life methodology is used for calculating fatigue damage where the cycles are dominated by local plasticity. Although the majority of engineering structures are designed such that the operational stresses do not exceed the elastic limit, unavoidable design features such as notches can result in local plastic strains.

The strain-life methodology correlates the local plastic deformation in the vicinity of a stress concentration to the far-field elastic stresses and strains using the constitutive response determined from displacement-controlled fatigue tests on simple (smooth) laboratory specimens.

Fatigue analysis using the strain-life methodology is capable of accurate predictions of crack initiation down to a few hundred cycles. Depending on the strain-life data, failure is usually assumed as a surface crack with a length of approximately 2mm.

## 1.4 Why fatigue from FEA?

Modern design workflows demand a complex and multidisciplinary mind set from the analyst [5]. The combination of complex geometry and service loading can make the determination of the most important stresses an insurmountable task in the absence of powerful computer software.

The finite element method is a popular tool which allows the analyst to determine the stresses acting on a component with a high degree of accuracy. However, selection of the correct stress is often still not obvious. Take Figure 1.2 as an example.



Figure 1.2: Uniaxial load applied to a fillet joint

A simple fillet joint is loaded in bending by a unidirectional pressure force. The load is applied to  $18\text{ MPa}$  and then removed, resulting in a single pulsating loading event. Figure 1.3 shows the result of the finite element analysis. The simplest way to relate the stress to fatigue life is by the Wöhler stress-life curve [6]:

$$S = \sigma_f' N_f^b \quad [1.1]$$

The damage parameter,  $S$ , is related to the fatigue life in repeats,  $N_f$  by the material constants  $\sigma_f'$  and  $b$ . Considering the results from Figure 1.3, it is not obvious which stress should be chosen to take the place of the parameter  $S$ . There are several approaches for the evaluation of the fatigue life.



**Figure 1.3: FEA stresses on the fillet joint due to bending load**

A common approach is to take the node with the maximum principal stress and substitute this value into the stress-life equation. Alternatively, the model can be viewed in terms of effective stress (for example, von Mises), and using this parameter for the fatigue calculation. Both of these approaches have serious drawbacks in that they do not correctly account for the presence of multiaxiality and non-proportionality which commonly arises in fatigue loadings. Fatigue results obtained using these techniques can be in significant error and even miss the location of fatigue crack initiation.

The best practise is to employ multiaxial algorithms which correctly identify the stresses on the most damaging planes. Even unidirectional loads, such as those in the above example, can result in multiaxial stress fields. Therefore, multiaxial analysis algorithms are always recommended over uniaxial and effective stress methods.

The fillet joint is analysed using Quick Fatigue Tool with several fatigue criteria, the results of which are summarised in Figure 1.4 and the tabulated data. Algorithms with “(CP)” indicate that they are multiaxial (critical plane) methods.

The Uniaxial Stress-Life method underestimates the fatigue life, whereas the von Mises stress overestimates. By considering the maximum principal stress, the uniaxial method assumes that fatigue failure will occur on a plane perpendicular to the material surface where the shear stress is zero. In reality, most metals experience crack initiation on shear planes where there is no normal stress. For this reason, both the uniaxial and normal stress methods produce highly conservative life predictions.

The Stress-based Brown-Miller and Findley’s Method produce the most accurate results, since they consider the action of both the normal and shear stress acting on several planes.



**Figure 1.4: Fatigue analysis results showing (logarithmic) life using the Stress-based Brown-Miller multiaxial algorithm**

Analysis algorithm	Fatigue life (repeats)
Uniaxial Stress-Life	462,000
Stress Invariant Parameter (von Mises)	1,570,000
Normal Stress (CP)	263,000
Stress-based Brown-Miller (CP)	800,000
Findley's Method (CP)	1,040,000

By combining results from FEA with a multiaxial analysis technique, the most accurate life prediction can be obtained. Due to the fact that multiple planes have to be searched in order to take into account multiaxial stress states, the multiaxial algorithms are very time-consuming compared to the uniaxial and effective stress methods. Confining the analysis to the location of maximum stress is not guaranteed to be successful since the location of crack initiation is not guaranteed to coincide with the location of maximum stress.

## 1.5 Limitations

### FEA Models

If the model contains plane stress elements, set PLANE\_STRESS = 1.0 in the job file. Two-dimensional elements such as beams, pipes and wires are not supported. Stress tensors read from FE models must use a Cartesian coordinate system.

Quick Fatigue Tool is a stress-based calculator and assumes that all of the FEA stress datasets are elastic. As such, the program cannot offer realistic fatigue life predictions below approximately one million cycles. In cases where plasticity correction is used for low-cycle fatigue calculations, the results should be taken as a rough approximation only.

### Loading

Quick Fatigue Tool does not directly support multiple block loading. A workaround involves splitting the load spectrum into several jobs and allowing Quick Fatigue Tool to automatically overlay the fatigue damage onto the previous results to give the total damage due to all blocks. Analysis continuation is discussed in Section 4.8.

### Materials

It is assumed that stress relaxation does not occur during the loading and that the material is cyclically stable. This expedites the fatigue calculation by allowing analysis of each node without considering the effects of neighbouring nodes, but precludes the effect of global plasticity being accurately taken into account. However, for the majority of cases this should not be an issue since the stress-life method is elastic, and the strain-life method is intended for components experiencing relatively small amounts of local plasticity.

Quick Fatigue Tool is applicable to metals and some engineering plastics where the stresses and temperatures are sufficiently low that viscoelastic effects are negligible. Nonlinear interactions such as creep, cyclic hardening and stress relaxation are not taken into account.

Treatment of local notch plasticity requires the use of strain-based fatigue methods. Treatment of crack propagation requires the use of crack growth methods such as VCCT, CTOD and Paris Law LCF.

Analysis of viscoelastic, hyperelastic, anisotropic and quasi-brittle materials is not supported. Materials whose fatigue behaviour cannot reasonably be modelled by linear elastic stresses and stress-life curves are not supported.

## **Performance**

The MATLAB programming language is very convenient in terms of the ease and speed of development it offers. However, in runtime the code is slow in comparison to other languages. Therefore, stress datasets from even a modest finite element model can result in cumbersome analyses. The user is advised to consult *Section 10: Modelling considerations* for assistance on how to minimise analysis time without compromising on the accuracy of the solution.

## **GUI appearance**

It is recommended that you set your monitor DPI scaling to 125% and the resolution to 1920x1080. On Windows 7, the DPI settings are found at *Control Panel\Appearance and Personalization\Display*. On Windows 10 the settings are at the same location, but you must select *set a custom scaling level* under the “Change size of items” section.

If the above settings are not used, Material Manager, Export Tool and the Gauge Fatigue Toolbox apps may display incorrectly.

An alternative to using the Material Manager app is to import material data from a text file. For instructions on creating material text files, consult Section 5.6 “Creating a material from a text file”.

## **Validation**

Quick Fatigue Tool has not been validated against any official standard. The author does not take any responsibility for the accuracy or reliability of the code. Fatigue analysis results calculated by the code should be treated as supplementary and further investigation is strongly recommended.

## **Before reading further**

- a) It is recommended that you consult the file *README.txt* in the *\bin* directory before proceeding to the next section of the guide
- b) Modifying the file structure (e.g. renaming folders) may prevent the program from working.
- c) Known issues and the change log for the latest version can be found in *known\_issues.txt* and *change\_log.txt*, respectively
- d) Quick Fatigue Tool is free for distribution without license, provided that the author information is retained in each source file

To submit a bug report or request an enhancement, please contact the author:

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## 2. Getting started

### 2.1 Preparing the application

Preparing Quick Fatigue Tool requires minimal intervention from the user, although it is important to follow a few simple steps before running an analysis:

Make sure the working directory is set to the root of the Quick Fatigue Tool directory, e.g. `\..\Quick Fatigue Tool\6.x-yy`. The directory structure is shown in Figure 2.1. All folders and sub folders should be added to the MATLAB search path using the function `addpath()`, or by selecting the folders `Application_Files` and `Project`, right-clicking and selecting **Add to Path → Selected Folders and Subfolders**.

If the MATLAB working directory is not configured exactly as described above (e.g. the user enters the job directory before running a job file), **the application will not run**.

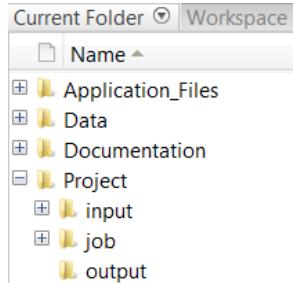


Figure 2.1: Quick Fatigue Tool file structure

Before running a fatigue analysis, it is recommended that you first run the job `tutorial_intro`. This is because the initial run of a MATLAB function requires some additional computational overhead which slows down the first analysis.

### 2.2 How the application handles variables

Quick Fatigue Tool does not store variables in the base workspace, nor does it modify or delete existing workspace variables. During analysis, variables are stored either in the function workspaces or as application-defined data using the `setappdata()` and `getappdata()` methods.

The application data is utilised for convenience, since variables can easily be accessed throughout the code without having to pass variables between many functions. In order to prevent unwarranted data loss, Quick Fatigue Tool does not modify existing application data by default. However, this means that variables from previous analyses will remain in the application data and could cause unexpected behaviour in subsequent analyses, such as incorrect fatigue results and spurious crashes.

In order to eliminate the possibility of such conflicts, the user is strongly advised to restart MATLAB between each analysis so that the application data is cleared. If the user does not wish to restart MATLAB for each analysis and is not concerned about Quick Fatigue Tool modifying the application data, the following environment variable may be set:

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
cleanAppData	3.0;

This ensures that the application data is completely cleared before and after each analysis. This has the same effect of restarting MATLAB.

The environment file and all of the available user settings are discussed in the document *Quick Fatigue Tool User Settings Reference Guide*.

## 2.3 File structure

Quick Fatigue Tool separates various components of the code into folders. Below is a brief description of what each folder contains:

<b><i>Application_Files</i></b>	Source code and application-specific settings. There is usually no need to modify the contents of this directory
<b><i>Data</i></b>	User-specific data (models, surface finish curves, material data, etc.)
<b><i>Documentation</i></b>	README file and User Guide
<b><i>input</i></b>	Required location for stress datasets and load histories
<b><i>job</i></b>	Job files defining each analysis
<b><i>output</i></b>	Fatigue results directory. If this folder does not exist, it will automatically be created during the analysis

## 2.4 Configuring and running an analysis

### 2.4.1 Configuring a standard analysis

Standard analyses are configured and submitted from an *.m* file.

In this example, a simple fatigue analysis is configured by combining a stress dataset with a load history.

1. **Define a stress dataset:** In the *Project\input* directory, open the file *stress\_uni.dat*. A simple stress definition consists of six components defining the Cauchy stress tensor. The components are defined in the following order:

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6
$S_{11}$	$S_{22}$	$S_{33}$	$S_{12}$	$S_{13}$	$S_{23}$

The file *stress\_uni.dat* contains a stress tensor at a single material point in a state of uniaxial tension ( $S_{11} = 200MPa$ ).

2. **Define a load history:** In the *Project\input* directory, open the file *history\_fully\_reversed.dat*. A simple load history consists of two loading points. Below is a list of common load definitions:

Load type	Definition
Fully-reversed (push-pull)	[1, -1]
Pure tension	[0, 1]
Pure compression	[0, -1]

The file *history\_fully\_reversed.dat* defines the fully-reversed loading event: ( $S_{11} = +200MPa \rightarrow -200MPa$ ).

3. **Create the job file:** In the *Project\job* directory, open the job file *tutorial\_intro.m*. The file contains a set of options specifying all the information necessary for fatigue analysis. Options can be strings or numeric depending on the meaning of the option. Not all options require a user setting. Below is a summary of each option in the job file. The most important settings in the file are highlighted in **bold**. The user need not worry about the number of definitions; all of the job file options are explained in the document *Quick Fatigue Tool User Settings Reference Guide* and in the tutorials later in this guide.

Option	Meaning	Additional notes
JOB_NAME	The name of the job	
JOB_DESCRIPTION	A description of the job	The job name and description are printed to the log file for reference
CONTINUE_FROM	Superimpose results onto a previous job	This feature is useful for block loading, or specifying the analysis algorithm based on model regions

DATA_CHECK	Runs the job up to the beginning of the analysis	Useful for checking the message file for initial notes and warnings, without having to run the full fatigue analysis
<b>MATERIAL</b>	Material used for analysis	'SAE-950C.mat' references a file containing the material properties. Materials are stored in <i>Data\material\local</i> . Materials are defined using the Material Manager app. Usage of the app is discussed in Section 5
USE_SN	Stress-life data	1.0; A flag indicating that S-N data should be used if available
SN_SCALE	Stress-life data scale factor	1.0; A linear scale factor applied to each S-N data point
SN_KNOCK_DOWN	S-N knock-down factors	Knock-down factors are not used in this analysis. Knock-down factors are discussed in Section 4.8
<b>DATASET</b>	Stress data	'stress_uni.dat' references the stress dataset file. Stress datasets should be saved in <i>Project\input</i>
<b>HISTORY</b>	Load history	'history_fully_reversed.dat' references the load history file. Load histories should be saved in <i>Project\input</i>
UNITS	Stress units	3.0; A flag with the definition of MPa
CONV	Conversion factor for stress units	
LOAD_EQ	Load equivalency	The default loading equivalence is 1 repeat. If the loading represents another dimension, the fatigue results can be expressed in a more appropriate unit (e.g. 1000 hours)
SCALE	Stress scale	0.8285; A linear scale factor applied to the entire loading
OFFSET	Offset value for stress history	Loading offsets are discussed in Section 3.4
REPEATS	Number of repetitions of the loading	
HF_DATASET	Dataset(s) for high frequency loads	
HF_HISTORY	Load history for high frequency loads	

HF_TIME	Time compression for high frequency loads	
HF_SCALE	Scale factor for high frequency loads	High frequency loads are discussed in Section 3.5
PLANE_STRESS	Element type (3D stress or planar)	0.0; A flag indicating that a 3D element type should be assumed. The distinction that Quick Fatigue Tool makes about element types is discussed in Sections 3.2.4 and 3.6.1
OUTPUT_DATABASE	Model output database (.odb) file from an Abaqus FE analysis	
EXPLICIT_FEA	FEA procedure type	
PART_INSTANCE	FEA part instance name	
STEP_NAME	FEA step name	
RESULT_POSITION	FEA result position	Associating a job with an Abaqus .odb file is discussed in Sections 4.6 and 9.5. This job is not associated with an .odb file
<b>ALGORITHM</b>	Analysis algorithm	0.0; A flag indicating that the default analysis algorithm should be used (Stress-based Brown-Miller). Analysis algorithms are discussed in Section 6
<b>MS_CORRECTION</b>	Mean stress correction	2.0; A flag indicating that the Goodman mean stress correction will be used. Mean stress corrections are discussed in Section 7
ITEMS	List of items for analysis	'ALL' indicates that all items in the model (1) should be analysed. Selecting analysis items is discussed in Section 4.5.3
DESIGN_LIFE	The target life of the system	'CAEL' indicates that the target life should be set to the material's constant amplitude endurance limit
<b>KT_DEF</b>	Surface finish definition	
<b>KT_CURVE</b>	Surface finish type	This analysis assumes a surface finish factor of 1. Surface finish definition is discussed in Section 4.3

NOTCH_CONSTANT	Notch sensitivity constant	
NOTCH_RADIUS	Notch root radius	
GAUGE_LOCATION	Virtual strain gauge definition	
GAUGE_ORIENTATION	Virtual strain gauge orientation	
RESIDUAL	Residual stress	0.0; A residual stress value which is added to the fatigue cycle during the damage calculation. Residual stress is discussed in Section 4.4
FACTOR_OF_STRENGTH	Factor of strength calculation	0.0; A flag indicating that a factor of strength calculation will not be performed. Factor of strength is discussed in Section 8.3
FATIGUE_RESERVE_FACTOR		2.0; A flag indicating that the Goodman B envelope will be used for Fatigue Reserve Factor calculations. The Fatigue Reserve Factor is discussed in Section 8.2
HOTSPOT	Hotspot calculation	0.0; A flag indicating that a hotspot calculation will not be performed. Factor of strength is discussed in Section 4.5.3
<b>OUTPUT_FIELD</b>	Request for field output	0.0; A flag indicating that field output will not be written
<b>OUTPUT_HISTORY</b>	Request for history output	0.0; A flag indicating that history output will not be written
<b>OUTPUT FIGURE</b>	Request for MATLAB figures	0.0; A flag indicating that MATLAB figures will not be written. Analysis output is discussed in Section 10
WELD_CLASS	Weld classification for BS 7608 analysis	
YIELD_STRENGTH	Yield strength for BS 7608 analyses	
UTS	Ultimate tensile strength for BS 7608 analyses	
DEVIATIONS_BELOW_MEAN	Degree of uncertainty for BS7608 analyses	

FAILURE_MODE	Failure mode for BS 7608 analyses	
CHARACTERISTIC_LENGTH	Characteristic dimension for BS 7608 analyses	
SEA_WATER	Environmental effects factor for BS 7608 analyses	This analysis does not require a weld definition. The BS 7608 algorithm is discussed in Section 6.6
B2	Fatigue strength exponent after knee point	
B2_NF	Knee point definition (as life)	
UCS	Ultimate compressive strength	This analysis does not require additional material data

4. **Select the material and analysis type:** This analysis uses SAE-950C Manten steel as the material. The materials available for analysis are located in *Data\material\user\_materials*. Stress units are in MPa. The analysis algorithm is the default algorithm (Stress-based Brown-Miller), the Goodman mean stress correction is used, as well as user-defined stress-life data points.
5. **Run the job:** Before running the analysis, check that the input stresses and load histories defined in the job file are located in the *Project\input* folder. The files *stress\_uni.dat* and *history\_fully\_reversed.dat* have already been copied into this directory. To execute the analysis, right-click on *tutorial\_intro.m* in the *Project\job* directory and click *run*.
6. A summary of the analysis progress is written to the command window. When the analysis is complete, the command window should look like that of Figure 2.1.2.

```

FATIGUE RESULTS SUMMARY:
=====
Worst Life-Repeats : 5.3e+06
at item 1.1

Analysis time       : 0:00:0.044

=====
Job tutorial_intro completed with warnings. See message file for details.

```

**Figure 2.1.2: Fatigue results summary**

7. The fatigue results summary reports a life of 5.3 million cycles to failure at location 1.1. This is the default location when a stress dataset is provided without position labels.

8. In the `\output` directory, a folder with the name of the job is created which contains all of the requested output. In this analysis, extensive output was not requested, so only the following three basic files are written:

File	Contents
<code>&lt;job_name&gt;.log</code>	<ul style="list-style-type: none"> <li>• Input summary</li> <li>• Analysis groups</li> <li>• Critical plane summary</li> <li>• Factor of Strength diagnostics</li> <li>• Fatigue results summary</li> </ul>
<code>&lt;job_name&gt;.msg</code>	<ul style="list-style-type: none"> <li>• Pre and post analysis messages <ul style="list-style-type: none"> <li>◦ Analysis-specific notes offering useful information to the user</li> <li>◦ Analysis-specific warnings explaining potential issues with the analysis</li> </ul> </li> </ul>
<code>&lt;job_name&gt;.sta</code>	<ul style="list-style-type: none"> <li>• An item-by-item summary of the analysis progress</li> </ul>

9. Open the log and status files and examine their contents. According to the command window summary, the analysis completed with warnings. Examine the contents of the message file:
- a. Extensive output was not requested by the user
  - b. The damage at design life (10 million cycles by default) is over unity, corresponding to failure
  - c. There is a warning that Quick Fatigue Tool encountered an ambiguity whilst determining the element (stress tensor) type for analysis. A 3D stress tensor was assumed as the input stress dataset. Since this assumption is correct, the warning can be ignored
10. To run another analysis, it is recommended that you first restart MATLAB to ensure that all the application data from the previous analysis is cleared.

### 2.4.2 Configuring a data check analysis

A data check runs the job file through the analysis pre-processor, without performing the fatigue analysis.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATA_CHECK	[0.0   1.0];

Data checks are useful for ensuring that the analysis definitions are valid, allowing the user to correct errors which may otherwise only become apparent after a long analysis run. The data check feature checks the following for consistency:

- ODB interface settings
- Results directory
- Analysis continuation settings
- Material definitions and S-N interpolation
- Algorithm and mean stress correction settings
- Dataset and history definitions
- Principal stress histories
- Custom mean stress, FRF and surface finish data
- Yield analysis
- Nodal elimination
- Load proportionality
- Virtual strain gauge definition
- Duplicate analysis item IDs

Pertinent information regarding the data check run can be found in the message file in *Project\output\<jobName>*.

If the user requested field output from the job file, the worst tensor and principal stress per node for the whole model are written to the files *datacheck\_tensor.dat* and *datacheck\_principal.dat*, respectively.

### 2.4.3 Configuring an analysis from a text file

Quick Fatigue Tool allows the user to submit a job from a text file containing only the options which are required to define the analysis. This results in job files which are less cumbersome than the standard .m file which must contain every option regardless of whether or not it is required.

To define a job from a text file, options are specified as keywords.

*Job file usage:*

<i>Option</i>	<i>Value</i>
*<keyword> =	<value>

Keywords are exactly the same as job file options, but they always begin with an asterisk (\*). For example, the job file option DATASET is declared in the text file as \*DATASET. Entries which do not begin with an asterisk, or are not proceeded with an equal sign (=) followed by a value, are ignored by the input file processor.

Job file options containing underscores are specified in the text file with spaces. For example, the option JOB\_NAME is specified in the text file as \*JOB NAME.

The following should be noted when defining jobs from a text file:

- it is not necessary to end the definition with a semi-colon;
- it is not necessary to enclose strings with apostrophes;
- white spaces are ignored; and
- mathematical expressions are not supported.

The user must adhere to the following syntax when defining cells in the text file.

<b>Cell type</b>	<b>Text input</b>
Strings	$\{<string_1>, <string_2>, \dots, <string_n>\}$
Numeric arrays	$\{[a_{11}, a_{12}, \dots, a_{1n}], [a_{21}, a_{22}, \dots, a_{2n}], \dots, [a_{n1}, a_{n2}, \dots, a_{nn}]\}$
Mixture of strings and numeric arrays	$\{<string_1>, [a_{11}, a_{12}, \dots, a_{1n}]\}$

Any combination of strings and numerical inputs are supported, provided each element is separated by a comma.

Jobs defined as text files are submitted from the command line.

*Command line usage:*

```
>> job <jobFile>  
>> job <jobFile> 'option'
```

The parameter **option** has two mutually-inclusive values:

**interactive** – prints an echo of the message (.msg) file to the MATLAB command window.

**datacheck** – submits the analysis job as a data check analysis.

Any file extension is accepted provided the contents is ASCII text. Job files with the extension *.inp* can be specified without appending *.inp* on the command line. For all other file types, the extension must be specified. Apostrophes are not required when specifying the input file name.

## 2.5 The analysis method

1. A loading definition is created by combining elastic stress datasets with load histories to produce a scaled history of stresses for each item in the model
2. If high frequency datasets are provided, these are interpolated and superimposed onto the original load history
3. If requested, the load histories are pre-gated before the analysis which aims to remove small cycles from the loading
4. The principal stress history is calculated for each analysis item
5. The load history at each point in the model is assessed for proportionality. The critical plane step size may automatically be increased if the load is considered to be proportional
6. If requested, nodal elimination is performed which removes analysis items whose maximum stress range is less than the fatigue limit of the material
7. User stress-life data is interpolated to find the endurance curve for a fully-reversed cycle
8. Stresses are resolved onto planes in a spherical coordinate space to find the plane on which the most damaging stresses occur<sup>1</sup>
9. Stresses on this plane are counted using the rainflow cycle counting method<sup>2</sup>. If requested, the stress tensors on this plane are gated prior to cycle counting
10. If requested, the stress cycles are corrected for material non-linearity
11. The stress cycles are corrected for the effect of mean stress
12. A damage calculation is performed for each cycle using Miner's Rule of linear damage accumulation [7]. The endurance limit may be reduced to 25% of its original value if the cycle stress amplitudes are damaging
13. Steps 8-12 are repeated for each analysis item
14. If requested, the item with the worst life is analysed once more to calculate extensive output
15. If requested, Factor of Strength (FOS) iterations are performed. The damage is recalculated for each analysis item to obtain the linear loading scale factor which, if applied to the original loading, would result in the user-defined design life

---

<sup>1</sup> Only if the fatigue analysis algorithm is multiaxial

<sup>2</sup> Only if the load history contains more than two data points

## 3. Loading methods

### 3.1 Background

The loading definition forms the basis of the analysis, and describes the stress history at each point in the model. Loadings usually consist of stress datasets (user-defined or from FEA) and load histories. The stress datasets contain the static stress state at each location in the model. This can be at a node, integration point, centroid or otherwise. The load history defines the variation of the stresses through time. However, Quick Fatigue tool does not distinguish elapsed time between loading points and hence the load history is treated as being rate-independent.

Quick Fatigue Tool offers four methods for creating loading definitions:

1. Uniaxial history
2. Simple loading
3. Multiple load history (scale and combine)
4. Dataset sequence

Stress datasets are specified as ASCII text files:

```
'dataset-file-name.*' | {'dataset-file-name-1.*', 'dataset-file-name-2.*', ..., 'dataset-file-name-n.*'}
```

Load histories are specified as ASCII text files, or directly as one or more vectors:

```
'load-history-file-name.*' | {'history-file-name-1.*', 'history-file-name-2.*', ..., 'history-file-name-n.*'}
```

```
[h1, h2, ..., hn] | {[h11, h12, ..., h1n]1, [h21, h22, ..., h2n]2, ..., [hm1, hm2, ..., hmn]m}
```

#### Uniaxial history

A single load history is supplied without a stress dataset.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATASET	[ ];
HISTORY	'history-file-name.*'   [h <sub>1</sub> , h <sub>2</sub> , ..., h <sub>n</sub> ];

The load history is analysed without respect to a particular model, and is only valid for uniaxial states of stress. Uniaxial histories can only be used with the Uniaxial Stress-Life and Uniaxial Strain-Life algorithms.

*Job file usage:*

<i>Option</i>	<i>Value</i>
ALGORITHM	[3.0   'UNIAXIAL'];

## Simple Loading

A simple loading consists of a single stress dataset multiplied by a load history.

*Job file usage:*

Option	Value
DATASET	'dataset-file-name.*';
HISTORY	'history-file-name.*'   [h <sub>1</sub> , h <sub>2</sub> ,..., h <sub>n</sub> ];

An example of a simple fatigue loading is given by Figure 3.1.

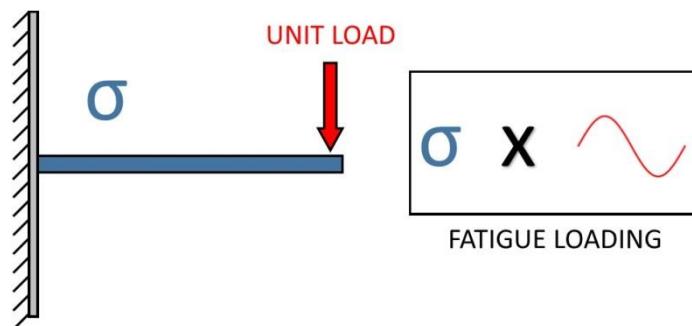


Figure 3.1: Demonstration of a simple loading. The stresses,  $\sigma$ , due to a unit load are multiplied by a load history

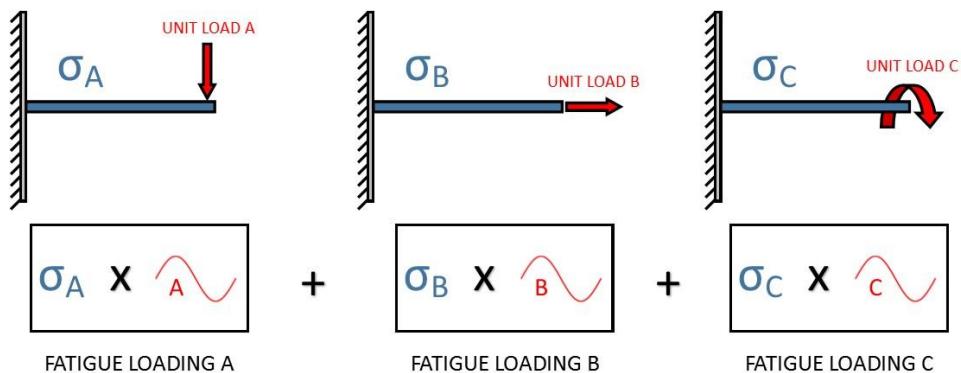
## Multiple load history (scale and combine)

A multiple load history consists of several stress datasets multiplied by the same number of histories. At each analysis item, the stress tensor from each dataset is scaled with its respective load history and combined into a single stress history. The number of stress datasets and load histories must be the same, although the number of history points in each load history need not be the same.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATASET	<code>{'dataset-file-name-1.*', 'dataset-file-name-2.*', ..., 'dataset-file-name-n.*'};</code>
HISTORY	<code>{'history-file-name-1.*', 'history-file-name-2.*', ..., 'history-file-name-n.*'};</code>

An example of a scale and combine loading is given by Figure 3.2.



**Figure 3.2: Demonstration of a multiple load history (scale and combine).** The stresses,  $\sigma_A$ ,  $\sigma_B$  and  $\sigma_C$  due to unit loads A, B and C are multiplied by their respective load histories and summed to produce the resultant fatigue loading

Scale and combine loading is only physically meaningful for elastic stresses. Each channel can be scaled by its respective load history since the load is directly proportional to the elastic FEA stress. The scale and combine method assumes that each loading channel is occurring simultaneously.

The load history may be defined by any combination of load history files and vectors.

*Job file usage:*

<i>Option</i>	<i>Value</i>
HISTORY	<code>{'history-file-name.*', [h<sub>1</sub>, h<sub>2</sub>, ..., h<sub>n</sub>]};</code>

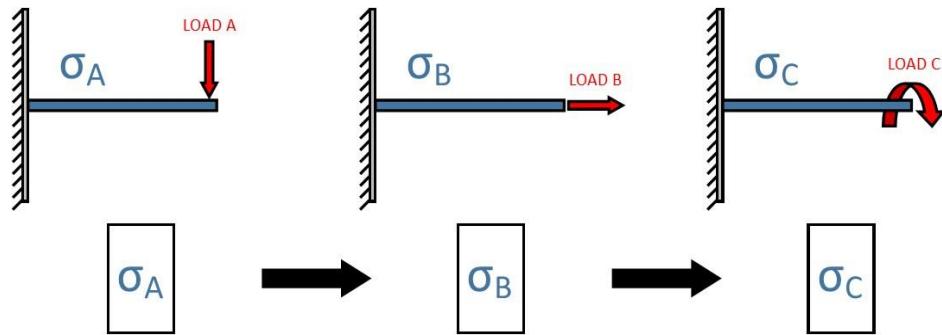
## Dataset sequence

A dataset sequence loading consists of several stress datasets.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATASET	<code>{'dataset-file-name-1.*', 'dataset-file-name-2.*', ..., 'dataset-file-name-n.*'};</code>
HISTORY	[ ];

An example of a dataset sequence loading is given by Figure 3.3.



**Figure 3.3: Demonstration of a stress dataset sequence loading. The fatigue loading is formed by the sequence of stress solutions  $\sigma_A$ ,  $\sigma_B$  and  $\sigma_C$ , due to the applied loads A, B and C, respectively**

Since the fatigue loading is 'completely described by the variation of stresses between each dataset, specification of load histories is not required.'

## 3.2 Creating a stress dataset file

### 3.2.1 Dataset structure

Stress datasets are text files containing a list of stress tensors. The simplest way to create a stress dataset file is to specify the tensor components as follows:

Line 1:	$\sigma_{11}, \sigma_{122}, \sigma_{133}, \sigma_{12}, \sigma_{13}, \sigma_{23}$
Line 2:	$\sigma_{211}, \sigma_{222}, \sigma_{233}, \sigma_{212}, \sigma_{213}, \sigma_{223}$
.	.
.	.
.	.
Line $n$	$\sigma_{n11}, \sigma_{n22}, \sigma_{n33}, \sigma_{n12}, \sigma_{n13}, \sigma_{n23}$

Each line defines the stress tensor at each location in the model.

### 3.2.2 Creating a dataset from Abaqus/Viewer

Stress datasets may be generated from finite element analysis (FEA). To create a stress dataset file from Abaqus/Viewer, complete the following steps:

1. In the Visualization module, from the main menu, select **Result → Options...**
2. Under “Averaging”, uncheck “Average element output at nodes”.
3. From the main menu, select **Report → Field Output...**
4. Under “Step/Frame”, select the step and the frame in the analysis from which the stresses will be written.
5. In the “Variable” tab, under “Output Variables”, select the position of the output (Integration Point, Centroid, Element Nodal or Unique Nodal).
6. Expand the variable “S: Stress components” and select all the available Cauchy tensor variables (S11, S22, S33, S12, S13 and S23). If plane stress elements are used, select (S11, S22, S33 and S12) and set PLANE\_STRESS = 1.0 in the job file. This informs Quick Fatigue Tool that out-of-plane stresses are not being written to the .rpt file
7. In the “Setup” tab, set the file path to `\6.x-xx\Project\input\<filename>.rpt`
8. Under “Data”, uncheck “Column totals” and “Column min/max”. Make sure “Field output” is checked.
9. Click **OK**

Stress dataset files must be stored in the `Project\input` folder so that Quick Fatigue Tool can locate the data.

### 3.2.3 Creating datasets from other FEA packages

If the user wishes to create a stress dataset from an FEA package other than Abaqus, the following standard data format must be observed for 3D stress elements:

MAIN POSITION ID (OPTIONAL)	SUB POSITION ID (OPTIONAL)	Sxx	Syy	Szz	Sxy	Sxz	Syz
--------------------------------	-------------------------------	-----	-----	-----	-----	-----	-----

For example, a particular stress dataset may look like the following:

2	2	113.924E-03	-3.09283	77.0076E-03	-948.772E-03	-34.9769E-03	-1.08771
2	3	117.622E-03	-4.64155	-41.7606E-03	-390.323E-03	-31.3847E-03	-1.95848
2	89	142.139E-03	-2.54017	-57.3497E-03	-122.545E-03	179.476E-03	-1.97810
2	121	193.577E-03	800.952E-03	63.2928E-03	-889.262E-03	82.9093E-03	-1.12254
2	817	222.935E-03	-5.01846	9.43557E-03	-207.232E-03	-59.9510E-03	-1.93998
2	841	293.969E-03	-4.13284	50.9750E-03	-400.696E-03	-87.5143E-03	-1.40172
2	3029	200.802E-03	-3.41846	19.5431E-06	143.706E-03	178.775E-03	-2.00069
2	3053	275.228E-03	-1.49917	56.2520E-03	-145.282E-03	108.319E-03	-1.49560
3	57	12.4055	-6.48471	59.9500E-03	-2.88770	-1.17872	-1.87426
3	58	16.5065	-4.70019	-165.896E-03	-4.86937	-1.48686	-2.28258
3	820	13.1704	-5.57775	-208.927E-03	-3.59939	-2.71752	-3.96404
3	827	9.55963	-6.77472	-24.9396E-03	-1.75746	-2.11207	-3.14150
3	1017	11.4619	-13.3344	46.7205E-03	-2.41033	-774.306E-03	-2.23137
3	1018	15.2980	-12.7558	-121.544E-03	-4.58736	-1.05162	-2.75391
3	3032	12.2131	-11.4543	-144.094E-03	-3.12038	-1.93731	-4.75567
3	3039	8.75289	-11.2319	-34.1504E-03	-1.11163	-1.44957	-3.73494

The position labels can be arbitrary, but usually represent the location on the finite element model (e.g. *element.node*). Quick Fatigue Tool will quote the position of the shortest fatigue life. Position labels are not compulsory: The stress dataset can be specified with tensor information only. Furthermore, it is not compulsory to include both *main* and *sub* IDs. For example, if the stress data is unique nodal (nodal averaged), there is only one position ID which is the node number.

### 3.2.4 Creating datasets with different element types

Quick Fatigue Tool automatically recognises stress datasets from Abaqus containing multiple element types. Such files are split into regions, each of which defines the stress tensors for a specific element type. If the stress dataset file is user-defined, the following conventions must be observed:

*3D stress elements:*

MAIN POSITION ID (OPTIONAL)	SUB POSITION ID (OPTIONAL)	Sxx	Syy	Szz	Sxy	Sxz	Syz
--------------------------------	-------------------------------	-----	-----	-----	-----	-----	-----

*Plane stress elements without shell face information:*

MAIN POSITION ID (OPTIONAL)	SUB POSITION ID (OPTIONAL)	Sxx	Syy	Szz	Sxy
--------------------------------	-------------------------------	-----	-----	-----	-----

*Plane stress elements with shell face information:*

MAIN POSITION ID (OPTIONAL)	SUB POSITION ID (OPTIONAL)	Sxx (+ve face)	Sxx (-ve face)	Syy (+ve face)	Syy (-ve face)	Szz (+ve face)	Szz (-ve face)	Sxy (+ve face)	Sxy (-ve face)
--------------------------------	-------------------------------	-------------------	-------------------	-------------------	-------------------	-------------------	-------------------	-------------------	-------------------

Below is an example of a user-defined stress dataset file containing two elements.

```
#REGION_1:
    1   1   297.850   31.6377   70.1334   -4.77388E-09   -921.773   -255.103
    1   2   70.1306   31.6377   297.843   255.077E-06   -1.55842   2.28914
    1   3   297.849   31.6377   70.1337   -4.06311E-09   -1.73031   255.097
#REGION_2:
    2   1   329.355   338.458   31.6551   31.6885   0.   0.   3.86495   4.83387
    2   2   391.347   400.629   31.5931   31.6263   0.   0.   -2.39578   -2.17509
    2   3   329.352   338.455   31.6551   31.6886   0.   0.   2.87967   3.30657
```

Each element region must be declared by a text header in order to be recognised, and the header must start with a non-numeric character. In the above example, *REGION\_1* defines a 3D element and *REGION\_2* defines a 2D element with results at both shell faces. Both elements are defined with element-nodal (nodal un-averaged) position labels. All the datasets in the loading must be defined with the same position labels otherwise the analysis will not run. To check whether or not the dataset definition was processed correctly, Quick Fatigue Tool prints the number of detected regions to the message file. This can be found in *Project\output\<jobName>\<jobName>.msg*.

Quick Fatigue Tool will automatically detect the element type based on the number of columns in the dataset file. For example, if there are five columns, this will be interpreted as plane stress (four columns define the tensor) with one column defining the element position (either unique nodal or centroidal). If the dataset file contains six columns, this could either be interpreted as 3D stress (all six columns define the stress tensor) with no position labels, or plane stress (four columns define the tensor) with two columns defining the element position (either element-nodal or integration point). This ambiguity is resolved with the use of the following job file option:

*Job file usage:*

<i>Option</i>	<i>Value</i>
PLANE_STRESS	[0.0   1.0];

If the value of PLANE\_STRESS is set equal to 1.0, Quick Fatigue Tool will assume that the element definition is plane stress if it encounters a data region with six data columns.

A complete description of how dataset files are interpreted is provided in Section 3.6.

### 3.3 Creating a load history

Load histories can be defined in three ways:

1. From a text file
2. As a direct definition
3. As a workspace variable

#### Create a load history from a text file

If the load history is defined from a text file, it must contain a single  $1 \times n$  or  $n \times 1$  vector of loading points, as follows:

$$\begin{array}{ll} P_1 & \leftarrow \text{First loading point} \\ P_2 & \\ \cdot & \\ \cdot & \\ P_n & \leftarrow \text{Last loading point} \end{array}$$

For example, a fully-reversed load history would look like that of Figure 3.4.

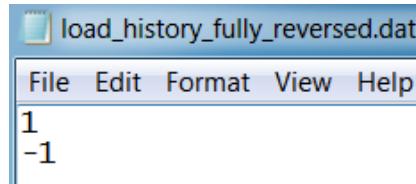


Figure 3.4: Fully-reversed load history

*Job file usage:*

<i>Option</i>	<i>Value</i>
HISTORY	'history-file-name.*';

Load history files must be stored in the *Project\input* folder in order for Quick Fatigue Tool to locate the data.

### Create a load history as a direct definition

Load histories can be specified in the job file as a  $1 \times n$  vector of scale factors.

*Job file usage:*

<i>Option</i>	<i>Value</i>
HISTORY	$[h_1, h_2, \dots, h_n];$

Alternatively, the load history can be defined as a function.

*Job file usage:*

<i>Option</i>	<i>Value</i>
HISTORY	$A * sind(1:n);$

where  $A$  is a stress amplitude scale factor.

### Create a load history as a workspace variable

If the load history is defined as a workspace variable, it must be a  $1 \times n$  or  $n \times 1$  numerical array.

*Job file usage:*

<i>Option</i>	<i>Value</i>
HISTORY	$\{var_1, var_2, \dots, var_n\};$

In addition, the variables declared in HISTORY must also be specified as inputs to the function declaration and the function call.

*Job file usage:*

```
function [ ] = <jobName>(var1, var2, ..., varn)
```

The job is then submitted by executing the job file from the command line.

*Command line usage:*

```
>> <jobName>(var1, var2, ..., varn)
```

For scale and combine loadings, it is possible to define a load history using a combination of text files, direct definitions and workspace variables.

*Job file usage:*

<i>Option</i>	<i>Value</i>
HISTORY	{ <i>history-file-name.*</i> , [ $x_1, x_2$ ], <i>var</i> };

### Treatment of multiple load histories

The load histories do not have to be the same length. Before the analysis, all the load histories will be modified to have the same length by appending zeroes to the shorter histories. However, in order to maximise the reliability and performance of the cycle counting algorithm, it is strongly recommended that the loadings have a similar length.

Note that multiple load histories are not supported for uniaxial analysis.

### 3.4 Load modulation

The fatigue loading is scaled and offset using the SCALE and OFFSET job file options, respectively.

The scaled and offset fatigue load,  $S^*$ , is given by the product of the scale factors and the stress datasets,  $S$ , with the sum of the load histories,  $H$ , and the offset factors, according to Equation 3.1.

$$S^* = \text{scale} \times (H + \text{offset}) \times S \quad [3.1]$$

#### Defining load scale factors

Load scale factors are defined as follows:

*Job file usage:*

<i>Option</i>	<i>Value</i>
SCALE	$[F_{s1}, F_{s2}, \dots, F_{sn}]$ ;

If the analysis is a scale and combine loading,  $n$  is the number of dataset-history pairs; each load scale factor is multiplied by its respective dataset-history pair. If the analysis is a dataset sequence,  $n$  is the number of datasets; each load scale factor is multiplied by its respective dataset in the sequence.

If the user specifies the Uniaxial Stress-Life algorithm, a single scale factor may be specified. Load history scales can be used with any loading methods.

#### Defining load offset values

Load offset values are defined as follows:

*Job file usage:*

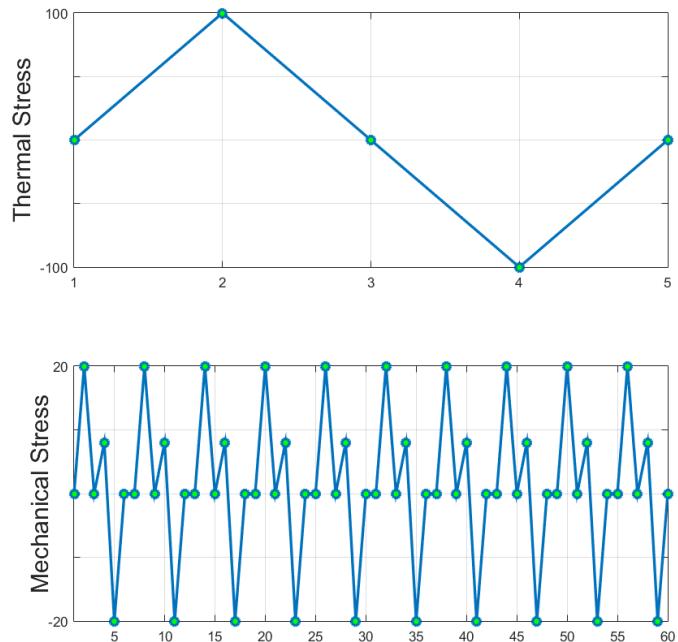
<i>Option</i>	<i>Value</i>
OFFSET	$[F_{o1}, F_{o2}, \dots, F_{on}]$ ;

where  $n$  is the number of dataset-history pairs; each load offset value is summed with its respective dataset-history pair.

Since load offset values are applied to the load history points only, they may not be used with dataset sequence loadings. Load offsets may be used with all other loading methods.

### 3.5 High frequency datasets

The scale and combine method outlined in Section 3.1 does not distinguish between elapsed time and can produce physically incorrect load histories if two load signals with very different time periods are analysed together. Take the example of a piston which experiences combined thermal and mechanical stresses shown in Figure 3.5.



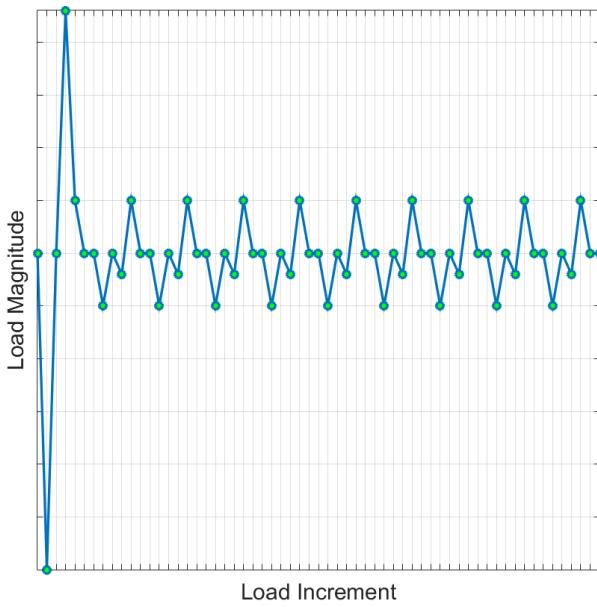
**Figure 3.5: Thermal and mechanical load signals occurring over the same time period**

For a scale and combine loading, the two signals are defined by the following load histories:

Normalized thermal load [0, 1, 0, -1, 0]

Normalized mechanical load  
1,0,0,1,0,0.4,-1,0,0,1,0,0.4,-1,0,0,1,0,0.4,-  
1,0,0,1,0,0.4,-1,0,0,1,0,0.4,-1,0,0,1,0,0.4,-  
1,0,0,1,0,0.4,-1,0]

A problem arises if the two loads occur over the same time interval. Since the mechanical load has many more time points than the thermal load, Quick Fatigue Tool will append most of the mechanical load onto the end of the load history. Using a standard scale and combine, the resulting load history would be that of Figure 3.6.



**Figure 3.6: Result of using the scale and combine technique for the thermal-mechanical load**

This loading definition is physically incorrect because it does not allow for the fact that the two loads occur simultaneously over the same time period. The solution is to define the mechanical load as a high frequency dataset. The modified load histories are as follows:

Normalized thermal load [0, 1, 0, -1, 0]

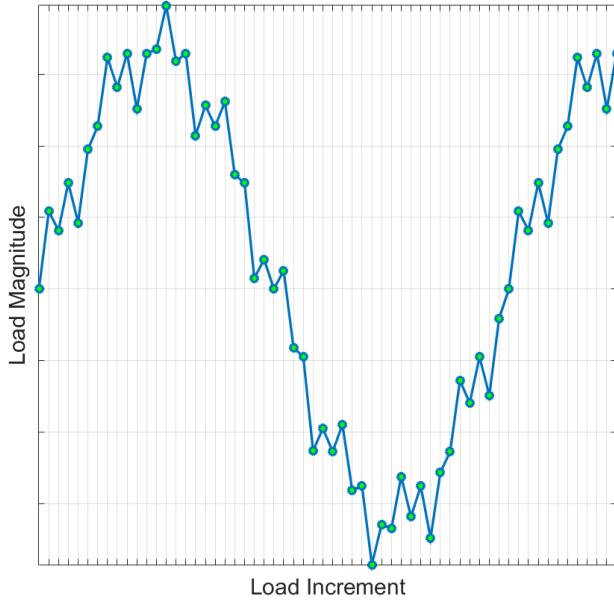
Normalized mechanical load [0, 1, 0, 0.4, -1, 0]

In this case, the high frequency data is specified as a single repeat of the mechanical load.

*Job file usage:*

<i>Option</i>	<i>Value</i>
HF_DATASET	'mechanical-dataset-file-name.*';
HF_HISTORY	'mechanical-history-file-name.*';
HF_TIME	{100.0, 10.0};

High frequency datasets and load histories are specified in the same manner as standard datasets and load histories. In order for Quick Fatigue Tool to correctly superimpose the high frequency dataset(s), it must know the time period for both loadings. In this example, the period of the low frequency data is 100 seconds and the period of a single repeat of the high frequency data is 10 seconds. This means that the high frequency dataset will be superimposed  $100/10 = 10$  times into the low frequency data.



**Figure 3.7: Result of using high frequency datasets for the thermal-mechanical load**

The resulting load history is shown in Figure 3.7.

Quick Fatigue Tool interpolates the thermal load so that it contains the correct number of data points for the mechanical load to be superimposed, without resulting in trailing data. In using this technique, the mechanical data is correctly represented as occurring over the same time period as the thermal data.

High frequency datasets can be defined in four ways:

- Simple loading
- Complex (scale and combine) loading
- Dataset sequence
- Single load history (Uniaxial Stress-Life and Strain-Life analysis only)

The same loading techniques apply as those outlined in Section 3.1. When using the Uniaxial Stress-Life and Strain-Life analysis algorithms, high frequency data is defined in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
HF_DATASET	[ ];
HF_HISTORY	' <i>history-file-name.*</i> ';
HF_TIME	{ $T_{lf}, T_{hf}$ };

The user should take into account the following points when using high frequency datasets:

- high frequency datasets should be used if two or more load histories occur over the same time interval, where one or more of the load histories is a repetitive load at a much higher frequency;
- the number of analysis items in the high frequency datasets must be the same as the number of items in all other stress datasets. If specific items are listed in the job file, those same items will be used in the high frequency datasets;
- if the original datasets contain stresses at shell faces, the high frequency data must also contain shell face data;
- the main load history should contain at least three data points, otherwise the high frequency datasets may not be interpolated properly. If the original load history contains only two data points, a zero value will be appended to the end of the history;
- when defining the high frequency load history, only a single cycle needs to be defined, along with the time period for that cycle. If the entire load history is provided, the resulting load history will be incorrect;
- if load history pre-gating is enabled, the original datasets may be modified prior to the high frequency datasets being added. This may result in an unexpected load history;
- if the high frequency data is not in the form of a peak-valley sequence (the loading contains intermediate data between turning points), this data will not be considered by the selected gating criterion;
- the units of the high and low frequency data must be the same; and
- using high frequency datasets can increase the analysis time dramatically.

## 3.6 The dataset processor

### 3.6.1 Determining the element type

Quick Fatigue Tool determines the type of dataset based on the number of columns in the dataset file. The following table describes how datasets are processed.

Number of columns in dataset	Assumption
< 4	Not applicable. Quick Fatigue Tool will exit with an error.
4	<b>Element type:</b> Plane stress <b>Position:</b> Unknown
5	<b>Element type:</b> Plane stress <b>Position:</b> Unique nodal or centroidal
6	IF PLANE_STRESS = 0.0 in the job file:  <b>Element type:</b> 3D stress <b>Position:</b> Unknown  ELSEIF PLANE_STRESS = 1.0 in the job file:  <b>Element type:</b> Plane stress <b>Position:</b> Element-nodal or integration point
7	<b>Element type:</b> 3D stress <b>Position:</b> Unique nodal or centroidal
8	<b>Element type:</b> 3D stress <b>Position:</b> Element-nodal or integration point
9	<b>Element type:</b> Plane stress with shell face data <b>Position:</b> Unique nodal or centroidal
10	<b>Element type:</b> Plane stress with shell face data <b>Position:</b> Element-nodal or integration point
> 10	Not applicable. Quick Fatigue Tool will exit with an error.

### 3.6.2 Plane stress elements

If the stress datasets originate from an FE model consisting of plane stress elements, the *.rpt* file can contain results on both faces, as shown by Figure 3.8.

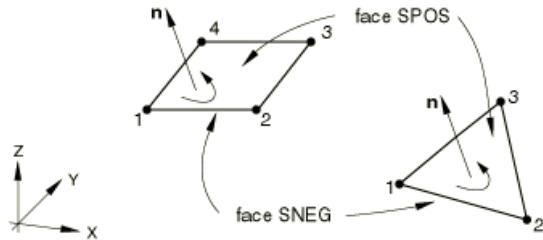


Figure 3.8: Positive normals for three-dimensional conventional shell elements

Quick Fatigue Tool can read the stresses from either the positive or negative face. The default face is defined as a variable in the environment file.

*Environment file usage:*

Variable	Value
shellLocation	[1.0   2.0];

Values of 1.0 and 2.0 correspond to the negative (SNEG) and positive (SPOS) element faces, respectively.

### **3.6.3 Multiple element groups**

If the stress dataset file was written from an Abaqus output database, it is possible for the data to be separated into multiple regions. This can happen if there are multiple part instances in the model, or if the model contains a mixture of element types. In such cases, Quick Fatigue Tool will automatically process each region and concatenate the stress tensors after all the datasets have been read.

Common examples of when the dataset can contain multiple regions are the analysis of surfaces with in-plane residual stress and/or surface finish. The recommended practice is to create a skin on the surface of the FE model and apply the residual stress and surface finish definitions on the skin elements using the GROUP option in the job file (see Sections 4.3 and 4.4 for more detailed information). Because of the combination of plane stress elements forming the skin of the component and the underlying 3D elements, Abaqus (and possibly other FEA packages) may assign duplicate node numbers between the skin and the solid bulk.

Although the fatigue calculation is not affected by the presence of duplicate node numbers, Quick Fatigue Tool may report the results at incorrect locations. Problems may also arise when writing results back to an Abaqus *.odb* file because Quick Fatigue Tool is unable to resolve the correct location of the node on the finite element mesh. Thus, the visualization in Abaqus/Viewer could be incorrect.

The workaround in such cases is to use stresses at the element nodes. This ensures that each node in the model has a unique identifier even in the presence of multiple element regions.

## 4. Analysis settings

### 4.1 Background

The user can control analysis parameters in addition to the analysis algorithm and mean stress correction. Each setting is described in this section.

For guidance on specifying analysis settings in the job and environment files, consult the document *Quick Fatigue Tool User Settings Reference Guide*.

### 4.2 Material approximation model

For stress-life applications, material nonlinearity can usually be ignored. If the predicted life of the component is calculated below 1 million cycles, then it is likely that the real system is experiencing nonlinearity, and the fatigue results will become increasingly conservative.

The user can enable the Ramberg-Osgood nonlinear elastic material model, which approximates the measured stresses for the endurance curve from elastic stresses [8] [9]. If the analysis is being performed with nonlinear FEA stresses, then the Ramberg-Osgood model is not required.

Elastic stresses are converted to nonlinear elastic stresses using Equation 4.1.

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n} \quad [4.1]$$

where  $K$  and  $n$  are the cyclic strain hardening coefficient and exponent, respectively. The difference in the monotonic response between the Hooke and the Ramberg-Osgood model is shown in Figure 4.1.

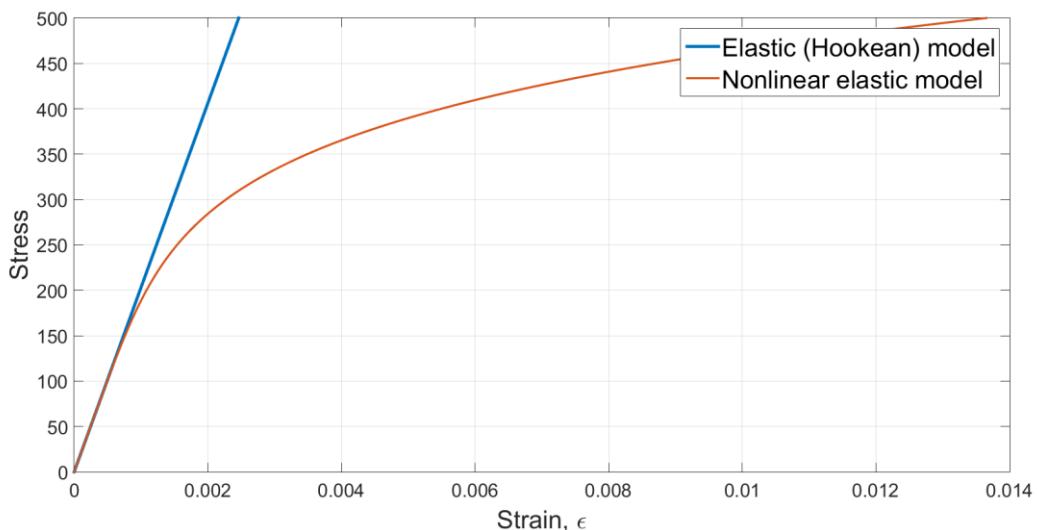


Figure 4.1: Comparison between linear (Hookean) and nonlinear elastic (Ramberg-Osgood) material models

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
nlMaterial	[0.0   1.0];
cssTolerance	$k_{tol}$ ;
cssMaxIterations	$I_{max}$ ;

### **Limitations of the nonlinear model**

In order to convert between linear and nonlinear quantities, Quick Fatigue Tool uses Neuber's Rule, which stipulates that the accumulated strain energy is the same at a notch as it would be in an elastic stress field far away.

The correction therefore is only valid for local notch plasticity. For smooth specimens, care must be taken. If the stress at the notch is below the yield strength, stress redistribution occurs, resulting in a larger plastic zone. Neuber's Rule does not work as well in these cases because it assumes that the peak stress is localised and that the plastic zone is surrounded by a comparatively large elastic zone, forcing the plastic zone to behave similarly to the nearby elastic stress field.

The Stress-Life methodology is based on the elastic stress at a point on the component which is not affected significantly by local stress concentrations. Traditionally, the elastic stress is used with an S-N curve which has been corrected with a notch factor to account for the presence of the stress concentration. As such, the Stress-Life methodology is not intended to be used with true stress quantities. This causes problems when performing fatigue estimates from elastic FE analyses, because the stresses at the notch are typically over-estimated. This can result in highly conservative life predictions compared to Strain-Life methods. The plasticity correction in Quick Fatigue Tool approximates the nonlinear elastic stress on a cycle-by-cycle basis and does not consider the effect of hysteresis or material memory. For cases where the stress concentration is judged to be significant, the following is recommended in lieu of the plasticity correction:

1. Limit the fatigue analysis to the elastic stress a small distance away from the notch and apply a correction factor using the *SN\_SCALE* option in the job file. Notch factors for specific geometries are readily available in the literature
2. Use Strain-Life methods

## 4.3 Surface finish and notch sensitivity

### 4.3.1 Surface finish

Surface roughness has a strong influence on the component's resistance to crack initiation [10]. While FEA is able to account for stress concentrations which arise from geometric complexity, the effect of surface finish cannot be modelled directly. Instead, a surface stress concentration factor,  $K_t$ , may be used to scale the endurance curve so that it corresponds more accurately to the surface strength of the material.

The result of applying a stress concentration factor to the endurance curve is shown in Figure 4.2.

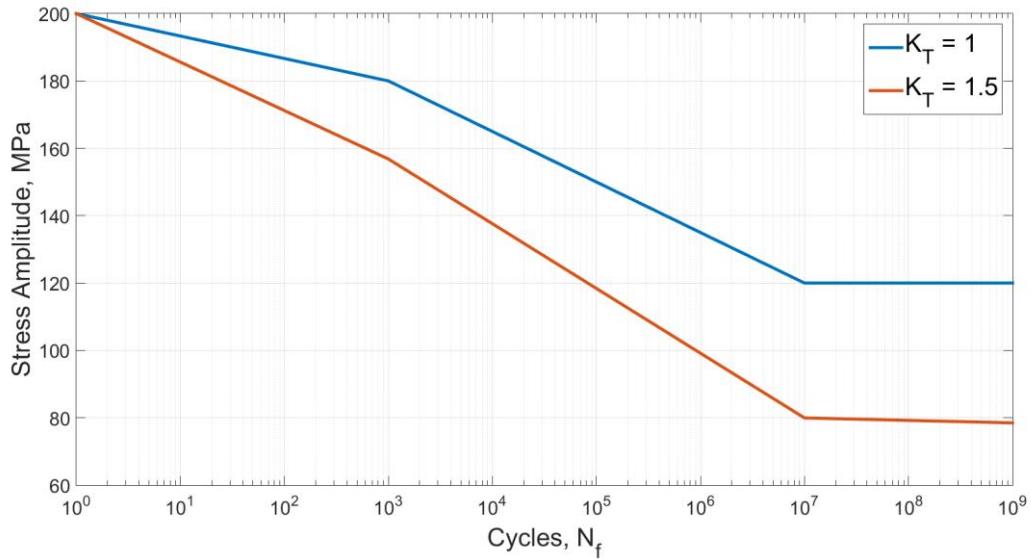


Figure 4.2: Reference endurance curve for  $K_t = 1$  and after applying a stress concentration factor of  $K_T = 1.5$

Quick Fatigue tool allows the surface finish to be defined in three ways:

1. As a surface stress concentration factor ( $K_t$  value)
2. From a list of surface finish types ( $R_a$  curve)
3. As a surface roughness value ( $R_z$  value)

#### Define the surface finish as a $K_t$ value

*Job file usage:*

<i>Option</i>	<i>Value</i>
KT_DEF	$n;$
KT_CURVE	[ ];

where  $n$  is a value for the surface stress concentration factor,  $K_t$ .

## Define the surface finish as an $R_a$ curve

To specify the surface finish from a list of surface finish types, a surface finish .kt file from the *Data\kt* directory must be specified. The surface finish definition files contain pre-defined curves for various surface finishes, as a function of the material's ultimate tensile strength.

*Job file usage:*

<i>Option</i>	<i>Value</i>
KT_DEF	<code>'surface-finish-file-name.kt';</code>
KT_CURVE	<code>n;</code>

where '`surface-finish-file-name.kt`' is the name of the .kt file containing a list of surface finish definitions, and  $n$  is the curve number.

The file '*default.kt*' is plotted in Figure 4.3 as an example. Based on the chosen curve and the ultimate tensile strength of the material, Quick Fatigue Tool linearly interpolates to find the corresponding value of  $K_t$ . If the material's ultimate tensile strength exceeds the range specified by the  $K_t$  curve, the last value of  $K_t$  is used.

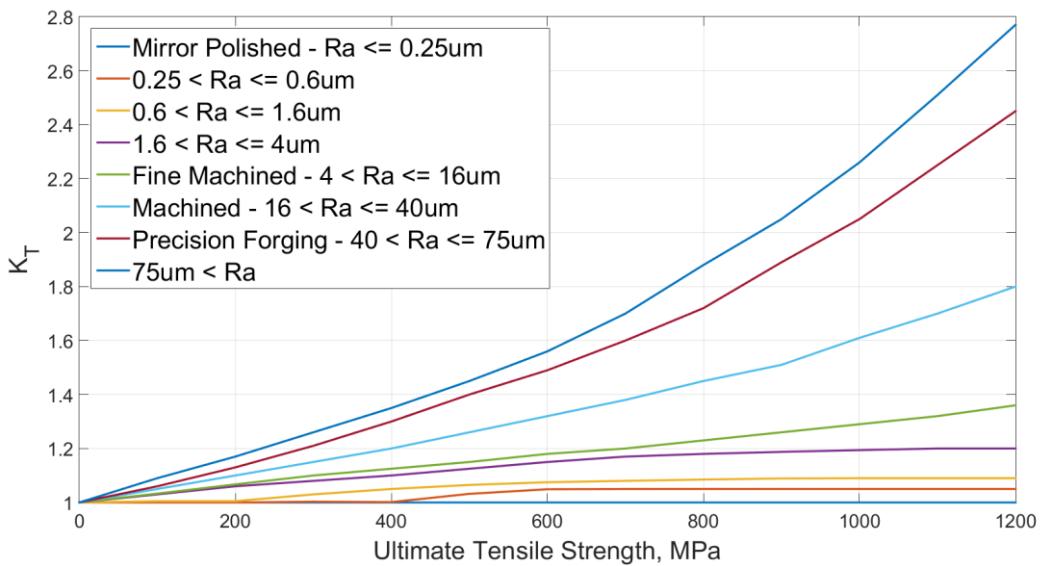


Figure 4.3:  $K_t$  curves for various surface finishes, from the file '*default.kt*'

The following .kt files and the available curves are shown below:

<b>'default.kt'</b>	<b>Surface Finish</b>
1	Mirror Polished – Ra <= 0.25um
2	0.25 < Ra <= 0.6um
3	0.6 < Ra <= 1.6um
4	1.6 < Ra <= 4um
5	Fine Machined – 4 < Ra <= 16um
6	Machined – 16 < Ra <= 40um
7	Precision Forging – 40 < Ra <= 75um
8	75um < Ra

<b>'juvinall-1967.kt'</b>	<b>Surface finish</b>
1	Mirror Polished
2	Fine-ground or commercially polished
3	Machined
4	Hot-rolled
5	As forged
6	Corroded in tap water
7	Corroded in salt water

<b>'rcjohnson-1973.kt'</b>	<b>Surface finish</b>
1	AA = 1uins
2	AA = 2uins
3	AA = 4uins
4	AA = 8uins
5	AA = 16uins
6	AA = 32uins
7	AA = 83uins
8	AA = 125uins
9	AA = 250uins
10	AA = 500uins
11	AA = 1000uins
12	AA = 2000uins

It is possible to specify the surface finish from a user-defined .kt file. The following file format must be obeyed:

*First column:* Range of UTS values over which  $K_t$  is defined

*Second column:*  $K_t$  values for the first curve

*Third column:*  $K_t$  values for the second curve

*Nth column:*  $K_t$  values for the (N-1)th curve

#### Define the surface finish as an $R_z$ value

To specify the surface finish as a surface roughness ( $R_z$ ) value, a surface roughness .ktx file from the Data\kt directory must be specified. The surface roughness files contain  $K_t$  curves over a range of roughness values, as a function of the material's ultimate tensile strength.

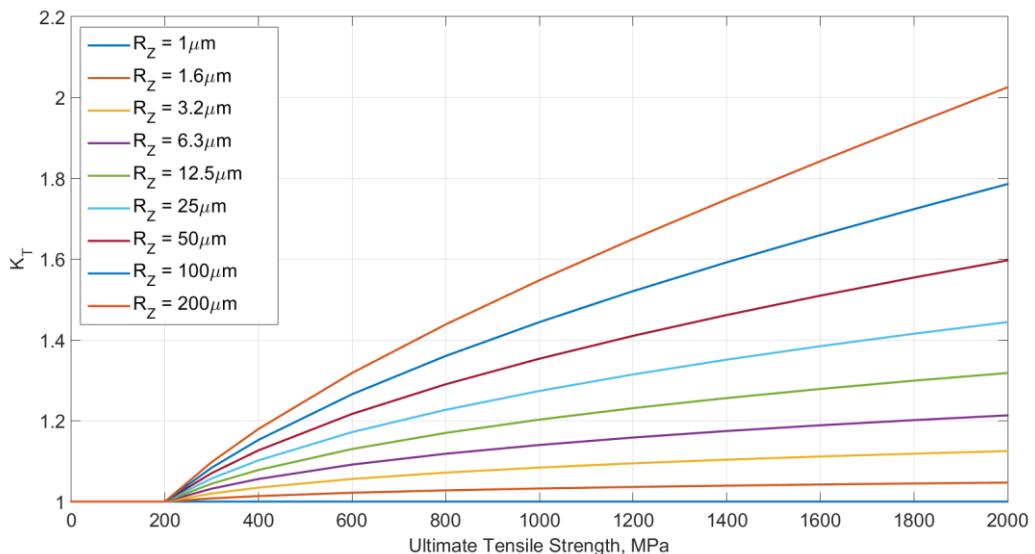
*Job file usage:*

<i>Option</i>	<i>Value</i>
KT_DEF	'surface-roughness-file-name.ktx';
KT_CURVE	$n$ ;

where <filename> is the name of the .ktx file containing the  $K_t$  curves and  $n$  is the surface roughness,  $R_z$ .

The 'Niemann-Winter-Rolled-Steel.ktx' file is plotted in Figure 4.4 as an example. Quick fatigue tool linearly interpolates to find the  $K_t$  curve which corresponds to the user-specified  $R_z$  value. If the surface roughness value exceeds the maximum surface roughness defined in the data, the last set of  $K_t$  values are used.

Based on the ultimate tensile strength of the material, Quick Fatigue Tool linearly interpolates once more to find the corresponding value of  $K_t$ . If the material's ultimate tensile strength exceeds the range specified by the  $K_t$  curve, the last value of  $K_t$  is used.



**Figure 4.4:  $K_t$  curves for various surface roughness values, from the file ‘Niemann-Winter-Rolled-Steel.ktx’**

The following .ktx files and surface roughness ranges are shown below:

<b>‘Niemann-Winter-Cast-Iron-Lamellar-Graphite.ktx’</b>	
UTS Range	0 – 2000Mpa
$R_Z$ Range	1 – 200um

<b>‘Niemann-Winter-Cast-Iron-Nodular-Graphite.ktx’</b>	
UTS Range	0 – 2000Mpa
$R_Z$ Range	1 – 200um

<b>‘Niemann-Winter-Cast-Steel.ktx’</b>	
UTS Range	0 – 2000Mpa
$R_Z$ Range	1 – 200um

<b>‘Niemann-Winter-Malleable-Cast-Iron.ktx’</b>	
UTS Range	0 – 2000Mpa
$R_Z$ Range	1 – 200um

<b>‘Niemann-Winter-Rolled-Steel.ktx’</b>	
UTS Range	0 – 2000Mpa
$R_Z$ Range	1 – 200um

It is possible to specify the surface finish from a user-defined *.ktx* file. The following file format must be obeyed:

*First row:* Range of  $R_z$  values over which  $K_t$  values are defined

*Second row:* First UTS value, followed by corresponding  $K_t$  values for each  $R_z$  value

*Third row:* Second UTS value, followed by corresponding  $K_t$  values for each  $R_z$  value

*Nth row:* ( $N - 1$ )*th* UTS value, followed by corresponding  $K_t$  values for each  $R_z$  value

#### 4.3.2 Effect of notch sensitivity

If the component contains a notch, the stress-life curve may require modification to account for the notch sensitivity of the material, since the stresses which contribute to fatigue of a notched component are not on the notch root surface, but a small distance into the subsurface. As such, for notch-insensitive materials, the stress concentration factor in fatigue is different to the elastic stress concentration factor,  $K_t$ . This is termed the fatigue notch factor,  $K_f$ .

The value of  $K_f$  can be approximated in several ways, and is set in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
notchFactorEstimation	<i>n;</i>

The value of *n* dictates the following:

1. Peterson (default)
2. Peterson B
3. Neuber
4. Harris
5. Heywood
6. Notch sensitivity

#### Peterson (default)

Quick Fatigue Tool is optimized to use stresses from finite element analysis. Therefore, the effect of  $K_t$  is implicit in the stress solution. By default, the reduction in fatigue strength due to elastic stress concentration is calculated using results obtained by Peterson. Equation 4.4 is used to scale the value of  $K_t$  as a function of endurance [11].

$$K_t(N) = 1 + \frac{K_t - 1}{0.915 + \frac{200}{(\log_{10} N)^4}} \quad [4.4]$$

The endurance curve is scaled by  $K_t(N)^{-1}$  at each value of *N*. The Peterson relationship is visualized by Figure 4.5, for values of  $K_t = 2, 3, 4$ .

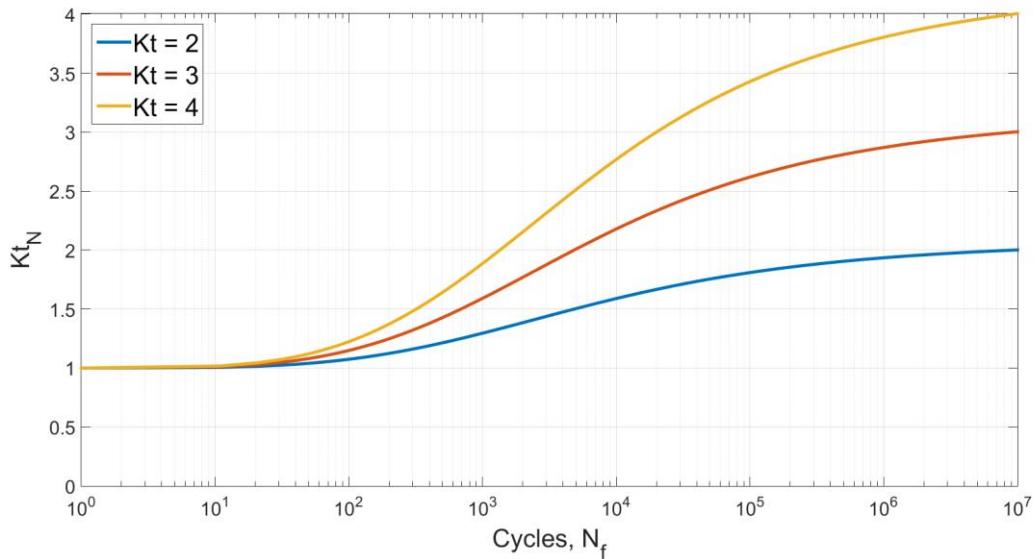


Figure 4.5:  $K_t(N)$  as a function of endurance, for  $K_t = 2, 3, 4$ .

### Peterson B

Peterson observed that, in general, good approximations for  $R = -1$  loading can be obtained using Equation 4.5 [12].

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{a}{r}} \quad [4.5]$$

where  $a$  is a characteristic length and  $r$  is the notch root radius. The value of  $a$  can be determined empirically as a function of the ultimate tensile strength,  $\sigma_u$ :

Material	$a$
Steel	$0.0254 \left( \frac{2070}{\sigma_u [MPa]} \right)^{1.8} mm$
Aluminium Alloy	$0.635mm$

### Neuber

For parallel side grooves, Neuber developed the following approximate formula for the notch factor for  $R = -1$  loading [13]:

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{\rho}{r}}} \quad [4.6]$$

where  $\rho$  is a characteristic length.

## Harris

Harris proposed the relationship in Equation 4.7 [14].

$$K_f = e^{-\frac{r}{\rho H}} + K_t \left( 1 - e^{-\frac{r}{\rho H}} \right) \quad [4.7]$$

where  $\rho H$  is a characteristic length. Suggested values of  $\rho H$  are shown in the table below

Material	$\rho H$
Steel	$10^{-2} \left( \frac{84.3}{\sigma_u [\text{ksi}]} \right)^2 \text{ in}$
Aluminium Alloy	0.004 in

## Heywood

Heywood proposed the relationship in Equation 4.8 [15].

$$K_f = \frac{K_t}{1 + 2\sqrt{\frac{a_H}{r}}} \quad [4.8]$$

where  $a_H$  is a characteristic length. The value of  $a_H$  varies depending on the notch type. Typical values proposed for steel are shown in the table below. The values assume that the notch root radius is measured in inches.

Notch Type	$a_H$
Hole	$\left( \frac{5}{\sigma_u [\text{ksi}]} \right)^2$
Shoulder	$\left( \frac{4}{\sigma_u [\text{ksi}]} \right)^2$
Groove	$\left( \frac{3}{\sigma_u [\text{ksi}]} \right)^2$

## Notch sensitivity

The fatigue notch factor can be defined in terms of the notch sensitivity,  $q$ .

$$K_f = 1 + q(K_t - 1) \quad [4.9]$$

Typical values of  $q$  for steels and aluminium alloys are shown in Figures 4.6-7 for bending and torsion, respectively [16].

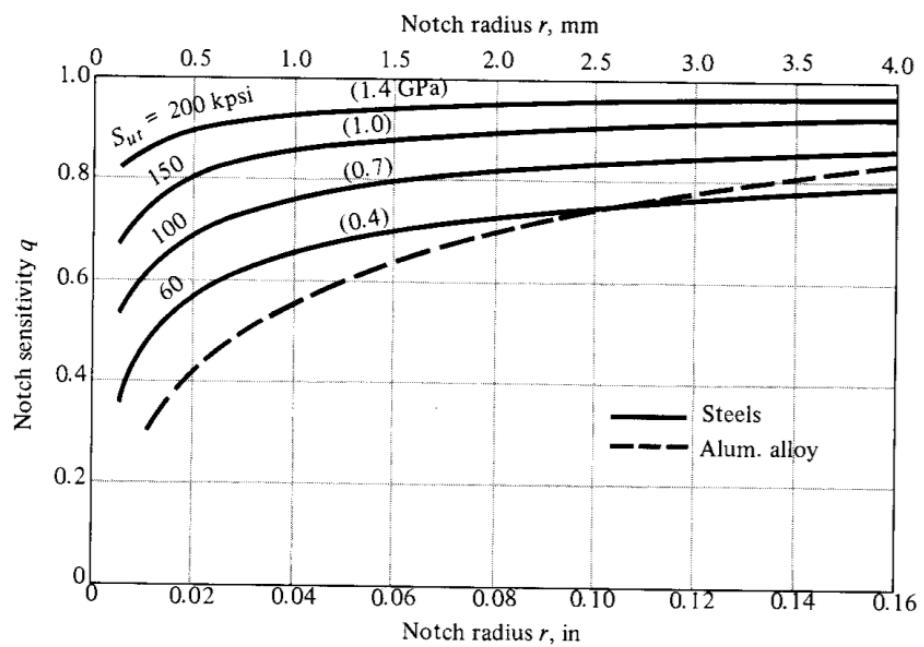


Figure 4.6: Notch sensitivity factors for steels and aluminium alloys (bending)

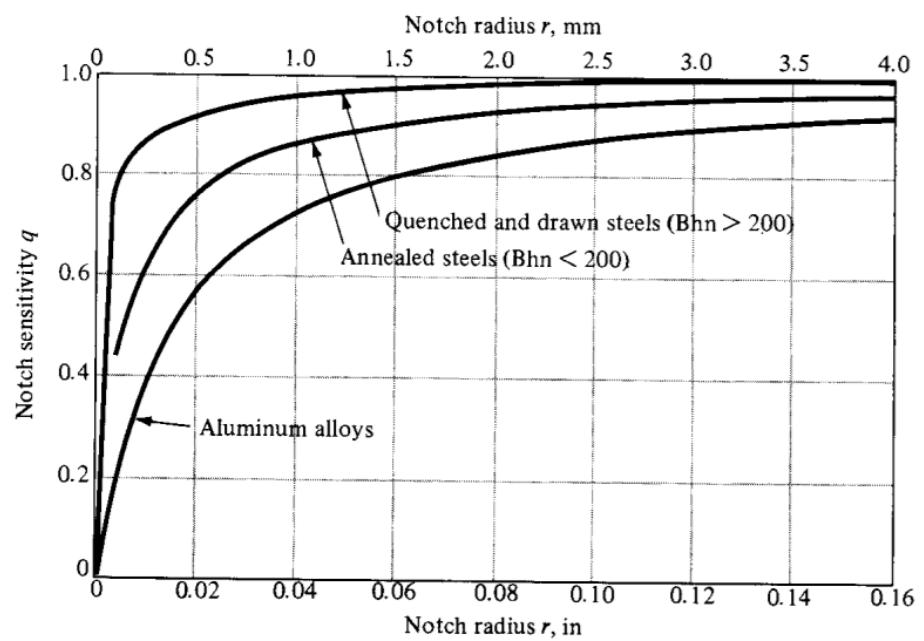


Figure 4.7: Notch sensitivity factors for steels and aluminium alloys (torsion)

## Defining notch parameters

The notch characteristic length and the notch root radius are defined in the job file.

*Job file usage:*

Option	Value
NOTCH_CONSTANT	$C_n;$
NOTCH_RADIUS	$C_r;$

The notch constant is defined according to the table below.

Notch factor estimation method	Notch constant
Peterson (default)	<i>Not applicable</i>
Peterson B	$a$
Neuber	$\rho$
Harris	$\rho H$
Heywood	$a_H$
Notch sensitivity	$q$

The notch root radius defines the parameter  $r$  in Equations 4.5-4.8.

## Specifying notch factors for FEA stresses

Since Quick Fatigue Tool assumes that the FEA stresses account for the effects of geometry, the default meaning of  $K_t$  is that of a supplementary factor which describes surface finish effects which cannot easily be modelled in finite elements. To that end, the user must be careful when considering the inclusion of the fatigue notch factor if the stresses originate from FEA.

If the S-N data is produced from smooth (un-notched) specimens then the nominal stress is equal to the local stress, as there is no stress gradient effect. In this case, the fatigue notch factor is not required since the FE solution provides the local stress at the notch surface. However, if the S-N data originated from a notched test, then the FEA stresses at the notch will produce excessively conservative fatigue life predictions. In such cases, the user must follow the procedure outlined by their chosen stress-life guideline in order to estimate the pseudo nominal stress a certain distance away from the notch tip, and use this stress on the notched S-N curve instead.

The fatigue notch factor is estimated from the value of  $K_t$ .

*Job file usage:*

<i>Option</i>	<i>Value</i>
KT_DEF	$n;$

where  $n$  is not the surface finish factor, but the elastic stress concentration factor. By defining the notch sensitivity constant and the notch root radius, the corresponding notch sensitivity is estimated. The stresses (or the S-N curve) will be scaled in the same way as is done with a standard surface finish definition (Figure 4.2).

#### 4.3.3 Modelling guidance

##### Specifying $K_t$ on a material surface

Unless analysis groups are defined, the surface finish definition is applied to every analysis item in the model. This behaviour is incorrect if the model contains subsurface nodes. If subsurface nodes are being analysed, the recommended practice is to define a skin on the finite element model and apply the surface finish definition to the skin using a separate analysis group.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GROUP	{'skin-group-file-name.*', 'DEFAULT'};

The procedure for creating analysis groups is described in Section 4.7. The surface finish is then defined in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
KT_DEF	{'surface-finish/roughness-file-name.kt/ktx', []};
KT_CURVE	$n;$

## Specifying $K_f$ directly

If the fatigue test data was measured for a smooth specimen but the component contains a notch, the user can specify the fatigue notch factor directly if it is already known. By assuming that the material is fully notch-sensitive ( $q = 1$ ), then  $K_t = K_f$ . This means that the surface finish factor can be used as the notch sensitivity factor.

*Environment file usage:*

Variable	Value
notchFactorEstimation	6.0;

*Job file usage:*

Option	Value
KT_DEF	$K_f$
KT_CURVE	[ ];
NOTCH_CONSTANT	1.0;

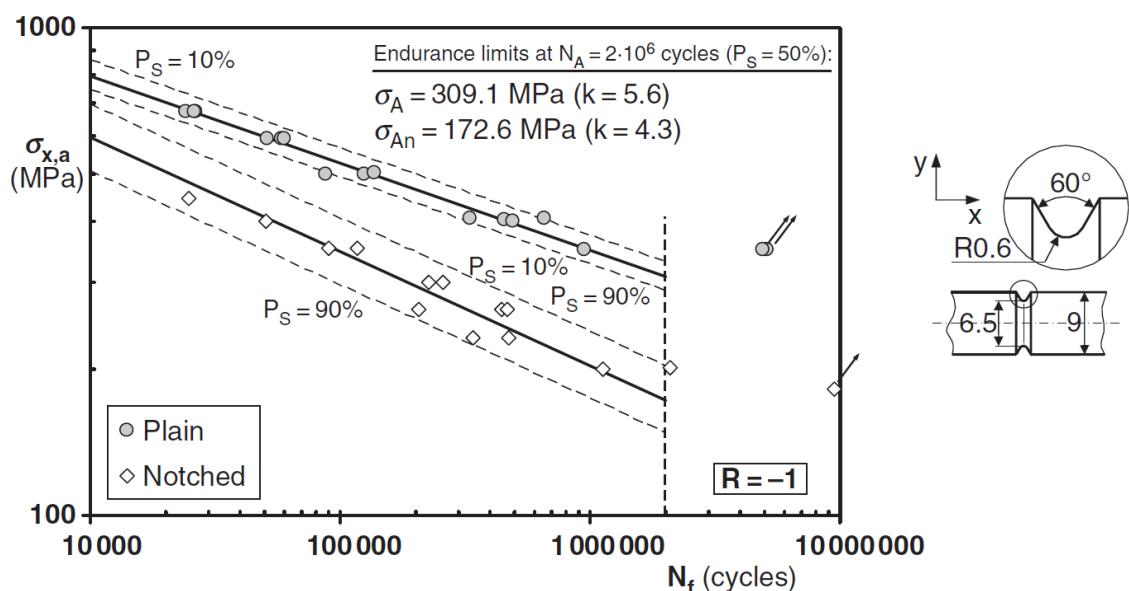


Figure 4.8: Fatigue results generated by testing plain and V-notched cylindrical specimens of S690 steel under rotating bending [17]

Take Figure 4.8 as an example. This data was obtained by Susmel [17], and shows the S-N curves for a notched and a smooth specimen. Using the data at the endurance limit (two million cycles), the value of  $K_f$  is estimated to be  $\sigma_A/\sigma_{An} = 1.791$  at 50% probability of survival. For a load ratio of  $R = -1$ , a cycle with a stress amplitude of  $309.1\text{ MPa}$  will produce the same life on a smooth specimen as a cycle with a stress amplitude of  $172.6\text{ MPa}$  on the notched specimen.

This can be verified in Quick Fatigue Tool by defining the smooth specimen material data with the following S-N data points:

<b>N – Values</b>	<b>S – Values</b>
10000	800
2000000	309.1

The two definitions below should result in a fatigue life of two million cycles. The Uniaxial Stress-Life algorithm is used for both definitions. For the second definition, the notch sensitivity is used as the fatigue notch factor estimation method.

### **Definition A**

*Job file usage:*

<i>Option</i>	<i>Value</i>
HISTORY	[309.1, -309.1];
KT_DEF	1.0;
NOTCH_CONSTANT	[ ];

### **Definition A**

*Job file usage:*

<i>Option</i>	<i>Value</i>
HISTORY	[172.6, -172.6];
KT_DEF	1.791;
NOTCH_CONSTANT	1.0;

## 4.4 In-plane residual stress

### 4.4.1 Overview

Many engineering components exhibit surface residual stresses due to their manufacturing process. Compressive residual stresses are often introduced by design. However, tensile residual stresses can accelerate fatigue damage accumulation.

An in-plane residual stress component can be specified directly in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
RESIDUAL	$\sigma_R$ ;

The stress is assumed to act uniformly in all directions and is added to the mean stress of each cycle.

Unless analysis groups are defined, the residual stress definition is applied to every analysis item in the model. This behaviour is incorrect if the model contains subsurface nodes. If subsurface nodes are being analysed, the recommended practice is to define a skin on the finite element model and apply the residual stress to the skin using a separate analysis group.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GROUP	{'skin-group-file-name.*', 'DEFAULT'};

The procedure for creating analysis groups is described in Section 4.7. The residual stress is then defined in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
RESIDUAL	$[\sigma_R, 0.0]$ ;

Since the residual stress is applied directly to the fatigue cycle and assumes the orientation of the critical plane, the quantity cannot be visualized with field output. For example, the variable SMAX does not include the effect of residual stress. Furthermore, the residual stress is not considered by nodal elimination.

### 4.4.2 Limitations

Definition of residual stress is not compatible with the BS 7608 algorithm. If residual stress is defined with Findley's Method, the stress is added to the cycle during the damage calculation instead of the mean stress.

## 4.5 Analysis speed control

### 4.5.1 Pre-processing time histories

Load histories often contain data which does not contribute to fatigue and can have a spurious effect on the fatigue calculation. Take the signal in Figure 4.9 as an example.

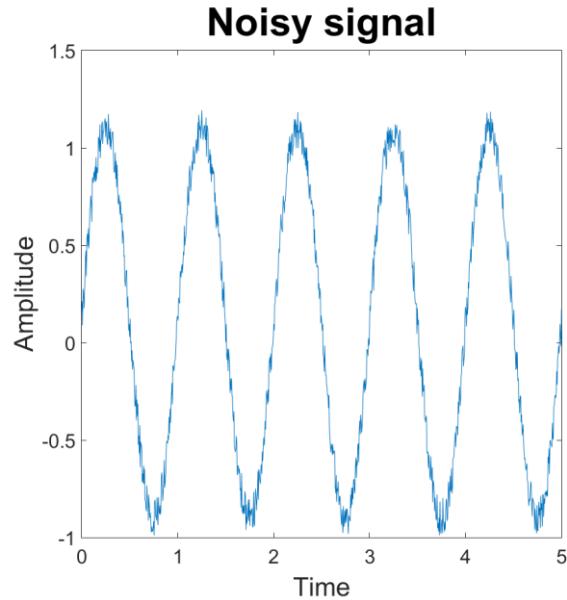
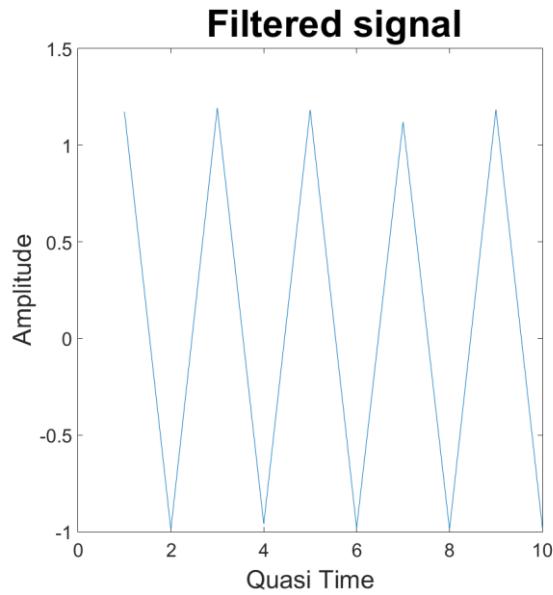


Figure 4.9: Noisy test signal

Analysing signals which contain small cycles puts additional computational load on the cycle counting algorithm and can impact the analysis time significantly, without improving the estimate of the fatigue damage. Quick Fatigue Tool offers three procedures for removing redundant cycles.

1. Pre-gate load histories
2. Gate tensors
3. Noise reduction

Quick Fatigue Tool assumes that there is no correlation between the phase of the load histories and the damage parameter on the critical plane. Therefore, the default behaviour is to gate the stress tensors as this is the most reliable method.



**Figure 4.10: Filtered signal**

### Pre-gate load histories

When load history pre-gating is enabled, the load history from each channel is converted into a separate peak-valley sequence before being combined with the other channels.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
gateHistories	[0.0   1.0];
historyGate	[ $g_1, g_2, \dots, g_n$ ];

where  $n$  is the number of loading channels. The gating values are defined as the percentage of the maximum component in the load history.

Figure 4.10 shows the result of gating the noisy signal with a 15% gating criterion. The pros and cons of load history gating are listed below.

Pros:

- load history gating is performed once for each loading channel before the start of the analysis, so it is very fast; and
- for simple loadings, load history gating has a similar accuracy to tensor gating.

Cons:

- fatigue results may be in error if the loading consists of multiple histories. The method is applied separately to each history so there is no guarantee that the phase relationship between the loading channels is maintained.

When using pre-gated load histories with multiple load channels, the user should compare the change in life values between the gated and the original signal. If the difference is significant, load history pre-gating should be disabled.

When load history pre-gating is enabled and Quick Fatigue Tool detects a constant amplitude loading condition, then only the first cycle is analysed. This avoids invoking the rainflow cycle counting algorithm unnecessarily. The number of repeats is automatically adjusted to account for the additional cycles, according to Equation 4.10.

$$R_T = R_{job} \times R_{load} \quad [4.10]$$

The total number of repeats,  $R_T$ , is the product of the number of repeats defined by the REPEATS job file option,  $R_{job}$ , and the number of repeats in the load history,  $R_{load}$ .

History point  $h_i$  is considered to be constant amplitude with respect to the point  $h_{i-2}$  provided that the two points lie within the user-defined tolerance specified by the `historyGate` environment variable. If multiple gating values are specified, then the first value will be used.

### Gate tensors

When tensor gating is enabled, the original load history is used to determine the principal stress history and the damage parameter on the critical plane. The damage parameter is then converted into a peak-valley sequence prior to cycle counting.

*Environment file usage:*

Variable	Value
<code>gateTensors</code>	[0.0   1.0];
<code>tensorGate</code>	[ $g_1, g_2, \dots, g_n$ ];

where  $n$  is the number of loading channels. The gating values are defined as the percentage of the maximum component in the stress tensor which defines the damage parameter.

The pros and cons of tensor gating are listed below.

Pros:

- The most accurate method of gating. Since the damage parameter accounts for the combination of the fatigue loading, the phase relationship between the loading channels is always maintained

Cons:

- The damage parameter is gated per plane, per node, thus tensor gating is much slower than pre-gated load histories

If it is necessary to remove intermediate data (points which lie between peaks and valleys), but additional gating is not required, the gate value can be set to zero. Quick Fatigue Tool will use a zero derivative method and all peak-valley pairs will be retained.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
tensorGate	0.0;
historyGate	0.0;

Quick Fatigue Tool includes an alternative peak-valley analysis algorithm, written by Adam Nielsony. This may be used in cases where the gating criterion is not known. The recommended practice is to use a gating method.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
gateHistories	2.0;
gateTensors	2.0;

## Noise reduction

It is possible to apply noise reduction to the load histories prior to analysis. Quick Fatigue Tool uses a low-pass filter which removes spikes in the data.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
noiseReduction	[0.0   1.0];
numberOfWindows	$n$ ;

where  $n$  is the number of averaging segments used to filter the signal.

Care should be taken when using the low-pass filter, since the stress amplitude is always reduced. This may result in excessively optimistic fatigue life results. Noise reduction should not be used as an alternative to gating, and its use is not recommended in general unless the load signal is highly affected by measurement noise.

#### 4.5.2 Determining load multiaxiality

It is possible for the stress state in certain parts of the model to be uniaxial, even if the global stress state is multiaxial. Furthermore, some multiaxial stresses may show little or no variation in the orientation of the principal stress. This type of loading is termed “proportional” and analysis items with such stresses do not require thorough critical plane searching.

Quick Fatigue Tool can automatically assess the model for proportional loading before the start of the analysis and relax the critical plane search increment for these areas.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
checkLoadProportionality	[0.0   1.0];
proportionalityTolerance	$k_{tol};$

The principal stress history of the loading is calculated at each location in the model, along with the orientation of the first principal stress. If the largest change of the angle of the first principal stress does not exceed the specified tolerance, the loading is assumed to be proportional. Quick Fatigue Tool will then take the following action:

- If the critical plane step size is greater than 45 degrees, the step size is not changed
- If the critical plane step size is smaller than 45 degrees, the step size is increased to 45 degrees

Note that load proportionality checking is not compatible with Findley's Method.

### 4.5.3 Custom analysis items

It is possible to restrict the analysis to one or more specific items from the stress dataset file(s). For very large models, it may be economical to perform the analysis only at the locations where fatigue failure is likely to occur. Alternatively, if the location of fatigue failure has already been determined by a previous analysis, an additional analysis may be performed at this location with additional output and/or more rigorous critical plane searching.

Custom analysis items can be defined in four ways:

1. Whole model
2. Peek
3. Hotspot
4. User-defined

By default, Quick Fatigue Tool performs a whole model analysis:

*Job file usage:*

<i>Option</i>	<i>Value</i>
ITEMS	'ALL';

Sometimes it can be useful to preview the analysis at the item which has the largest principal stress range in the model. This allows the user to quickly “peek” at the item which is likely to reside in one of the damage hotspots in the model.

*Job file usage:*

<i>Option</i>	<i>Value</i>
ITEMS	'PEEK';

This setting can be useful for very large models where even a simple fatigue analysis (such as Stress Invariant Parameter) could take a long time to complete. Using the peek option allows the user to select a more rigorous analysis algorithm on a single analysis item without first having to run a whole model analysis. This option is not intended to replace a complete analysis, however; the location of maximum stress range does not necessarily coincide with the location of maximum fatigue damage. Therefore, the peek option should be used with great care if the loading is non-proportional.

Even if the job contains non-default group definitions, Quick Fatigue Tool will still search the whole model as defined by the DATASET option. If the item with the largest principal stress range exists outside any of the groups, the analysis will be aborted. This is because the code cannot determine valid properties for an item which does not have any material or analysis data associated with it.

Custom analysis items may be used in conjunction with an automatically generated hotspot file:

*Job file usage:*

<i>Option</i>	<i>Value</i>
HOTSPOT	[0.0   1.0];

Quick Fatigue Tool will save a list of items whose lives fall below the design life (specified by the DESIGN\_LIFE job file option) to a separate file. The hotspot file is written to the *Project\input* folder and has the name *hotspots\_<jobName>.dat*. The hotspot file is specified in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
ITEMS	'hotspots-file-name.*';

Quick Fatigue Tool will automatically extract the analysis item IDs from the stress dataset file(s) to restrict the analysis to regions of the model where the life is less than the design life, according to the preceding hotspot analysis.

User-defined analysis items can be specified directly as a list of item IDs.

*Job file usage:*

<i>Option</i>	<i>Value</i>
ITEMS	[ID <sub>1</sub> , ID <sub>2</sub> ,..., ID <sub>n</sub> ];

Consider the FEA definition in Figure 4.11. If a setting of ITEMS=16 is specified, the analysis will only consider the 16<sup>th</sup> item in the definition. Note that if the definition file contains a header, the value of ITEMS will not correspond to the line number in the file.

After each analysis, Quick Fatigue Tool writes to the message file the ID(s) of the item(s) in the stress dataset(s) with the worst life. This value can be used in conjunction with the ITEMS option to re-run the analysis at the worst location.

20	Node	Label	S.S11 @Loc 1	S.S22 @Loc 1	S.S33 @Loc 1	S.S12 @Loc 1	S.S13 @Loc 1	S.S23 @Loc 1
21								
22								
23								
24		1	-795.763E-03	-4.48699	-2.95655	6.92934	-3.93393	1.82896
25		2	4.90945	1.50557	-4.52454	5.76533	-3.86309	3.06441
26		3	415.521E-03	726.238E-03	640.271E-03	592.421E-03	1.13344	1.97704
27		4	9.63988	-8.78137	49.1831E-03	5.92665	834.660E-03	1.07187
28		5	2.33473	3.17949	514.433E-03	2.61164	1.37854	1.49722
29		6	-465.532E-03	-14.7080	-423.602E-03	954.029E-03	484.061E-03	3.20978
30		7	-6.34151	2.17191	-5.48330	-4.71242	6.09594	5.78890
31		8	-1.61290	3.09481	-1.66385	985.416E-03	1.58106	-842.751E-03
32		9	-20.6121	-12.0342	-12.6017E-03	-15.7107	118.645E-03	40.7449E-03
33		10	-4.48167	-4.84691	6.92384E-03	-4.60756	29.1547E-03	33.7680E-03
34		11	4.41018	4.17798	342.322E-03	4.38294	1.23794	1.21576
35		12	4.38159	4.72635	348.698E-03	4.66588	1.23816	1.30647
36		13	49.8089	-12.6640	-1.03326	6.16651	-2.52934	-1.17407
37		14	19.4927	7.00084	3.45965	7.36442	3.71231	176.971E-03
38		15	17.9602	9.06355	4.16721	6.33369	2.33977	-593.006E-03
39		16	308.628E-03	-3.64435	653.559E-03	548.751E-03	-481.142E-03	-1.08482
40		17	9.56567	-838.844E-03	1.00148	865.819E-03	2.04524	-170.132E-03
41		18	6.68310	-6.61508	-13.8474E-03	-1.64620	-735.383E-03	-395.798E-03
42		19	7.48860	-3.53067	356.557E-03	-1.50177	-1.02732	-202.813E-03
		20	3.53093	-858.371E-03	176.520E-03	2.74948	-934.752E-03	-789.951E-03

Figure 4.11: Example FEA definition file

#### 4.5.4 Nodal elimination

##### Introduction

The analysis time can be reduced by ignoring analysis items whose maximum stress is unlikely to cause damage. When the nodal elimination algorithm is enabled, Quick Fatigue Tool checks the maximum principal stress range at each item before beginning the analysis. If the stress is below a certain percentage of the conditional stress,  $\sigma_{COND}$ , then the item is not included for analysis. The value of  $\sigma_{COND}$  is calculated as a function of the target life,  $N_T$ .

Nodal elimination is enabled from the environment file.

*Environment file usage:*

Variable	Value
nodalElimination	$n;$

If  $n = 1$ ,  $N_T$  is taken as the material's constant amplitude endurance limit (*CAEL*) by default. The value of  $\sigma_{COND}$  is the stress amplitude which will result in a life of  $N_T$  cycles.

The value of  $\sigma_{COND}$  may be specified directly if a user-defined endurance limit is specified in the environment file.

*Environment file usage:*

Variable	Value
enduranceLimitSource	3.0;
userEnduranceLimit	$\sigma_{COND};$

If  $n = 2$ ,  $N_T$  is taken as the life defined by the option DESIGN\_LIFE in the job file. By default, DESIGN\_LIFE = 'CAEL', in which case  $N_T$  is taken from the value of *CAEL* defined in the material.

For example, if an analysis is run with the Stress-based Brown-Miller algorithm and nodalElimination = 1.0, the conditional stress is obtained from Equation 4.11:

$$\sigma_{COND} = E \left( 1.65 \frac{\sigma_f'}{E} (N_T)^b + 1.75 \varepsilon_f' (N_T)^c \right) \quad [4.11]$$

where  $\sigma_{COND}$  is the conditional stress at which the life is equal to the constant amplitude endurance limit,  $N_T = CAEL$ . The analysis item is then removed if the following inequality is satisfied:

$$f_{thr} \cdot \sigma_{COND} > 0.5 \cdot (\sigma_1 - \sigma_3)|_{max} \quad [4.12]$$

where  $(\sigma_1 - \sigma_3)|_{max}$  is the maximum difference between the first and third principal stresses in the loading and  $f_{thr}$  is the elimination threshold scale factor.

### Scaling the conditional stress with $f_{thr}$

If the fatigue loading contains only one cycle, then the conditional stress can be determined directly from the fatigue limit. However, if the loading contains multiple cycles then it is possible for finite life even if the majority of the cycles are below the fatigue limit. Consequently, it is necessary to use a reduced value of the conditional stress such that the nodal elimination algorithm is effective for complex loads.

The elimination threshold scale factor is set in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>thresholdScalingFactor</code>	$f_{thr};$

The default value of  $f_{thr}$  is 0.8 (80%).

## 4.6 Analysis groups

### 4.6.1 Overview

Analysis groups are used to define regions in the model having distinct properties. Analysis groups can have their own definitions for the following job file options:

Property	Job file option
Material properties	MATERIAL
S-N data scale factor	SN_SCALE
S-N knock-down curves	SN_KNOCK_DOWN
Fatigue Reserve Factor envelope definition	FATIGUE_RESERVE_FACTOR
Surface finish definition	KT_DEF
Surface finish curve	KT_CURVE
Residual stress	RESIDUAL
Fatigue strength exponent above knee point	B2
Life at knee point	B2_NF
Ultimate compressive strength	UCS
Notch sensitivity constant	NOTCH_CONSTANT
Notch root radius	NOTCH_RADIUS

Analysis groups can have their own definitions for the following environment variables:

<b>Property</b>	<b>Environment variable</b>
Goodman envelope definition	modifiedGoodman
Goodman mean stress limit	goodmanMeanStressLimit
User-defined Walker $\gamma$ parameter	userWalkerGamma
User-defined fatigue limit	userFatigueLimit
User FRF tensile mean stress normalization parameter	frfNormParamMeanT
User FRF compressive mean stress normalization parameter	frfNormParamMeanC
User FRF stress amplitude normalization parameter	frfNormParamAmp

Analysis groups can be defined in two ways:

1. An item ID list
2. An FEA subset

An item ID list is a list of indexes corresponding to the row numbers in the stress dataset(s) as they are defined in the file. An FEA subset is a list of position IDs referencing the location of the elements, nodes, centroids or integration points in the model.

Analysis groups are declared in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GROUP	{'group-file-name-1.*', ..., 'group-file-name-n.*'};

Groups are defined as text files and must be located in the *Project\input* folder of the Quick Fatigue Tool directory.

#### 4.6.2 Defining analysis groups as an item ID list

Consider the stress dataset file in Figure 4.12.

Element Label	Node Label	S.S11 @Loc 1	S.S22 @Loc 1	S.S33 @Loc 1	S.S12 @Loc 1	S.S13 @Loc 1	S.S23 @Loc 1
1	43	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	44	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	537	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	538	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	591	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	592	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	1996	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	2014	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
2	44	96.7538	7.40922	-1.47880	28.9776	18.0028	674.289E-03
2	45	96.7538	7.40922	-1.47880	28.9776	18.0028	674.289E-03
2	538	96.7538	7.40922	-1.47880	28.9776	18.0028	674.289E-03
2	539	96.7538	7.40922	-1.47880	28.9776	18.0028	674.289E-03
2	592	96.7538	7.40922	-1.47880	28.9776	18.0028	674.289E-03
2	593	96.7538	7.40922	-1.47880	28.9776	18.0028	674.289E-03
2	2014	96.7538	7.40922	-1.47880	28.9776	18.0028	674.289E-03
2	2015	96.7538	7.40922	-1.47880	28.9776	18.0028	674.289E-03

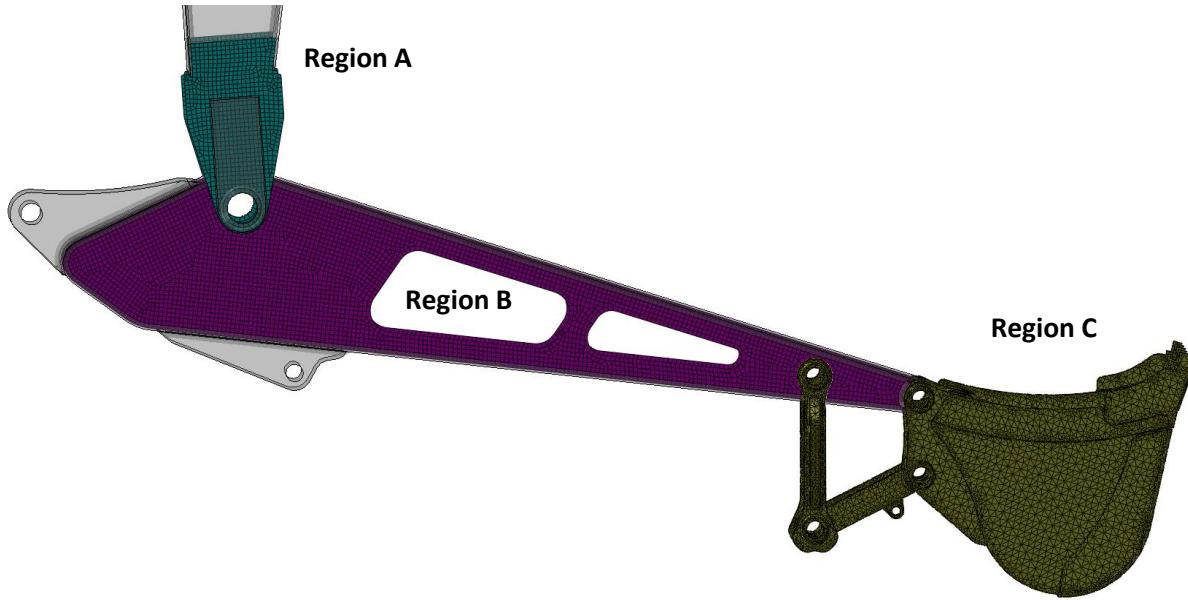
Figure 4.12: Example stress dataset

The dataset consists of two quadratic elements (eight-node hexahedrons). The dataset can be split between two analysis groups, each defining one of the elements in the dataset. This can be achieved by creating an item ID list defining each element. For example, the ID list for the first element is the integer series from 1 to 8, while the second element is defined as the integer series from 9 to 16. These are simply the row numbers corresponding to the nodes of the two elements. Example text file contents for each group are given below.

‘element 1.txt’	‘element 2.txt’
1	9
2	10
3	11
4	12
5	13
6	14
7	15
8	16

### 4.6.3 Defining analysis groups as an FEA subset

Item ID lists are a direct and relatively simple way of defining analysis groups for small models. However, consider the model of an excavator arm shown by Figure 4.13.



**Figure 4.13: Finite element model of an excavator arm with two regions of interest**

In this instance, creating groups for regions A, B and C with an item ID list would be a very cumbersome task. Instead, FEA subsets can be used which define the groups by their element labels. In Abaqus, this is easily achieved by creating a display group of the region of interest, then exporting the element labels as an *.rpt* file. The process of generating such a file is exactly the same as for generating FEA stress datasets, and is discussed in detail in Section 3.2.

For example, consider again the dataset in Figure 4.12. An FEA subset may then resemble that shown in Figure 4.14.

Element Label	Node Label	S.S11 @Loc 1	S.S22 @Loc 1	S.S33 @Loc 1	S.S12 @Loc 1	S.S13 @Loc 1	S.S23 @Loc 1
1	43	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	44	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	537	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	538	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	591	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	592	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	1996	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029
1	2014	89.1674	12.8745	429.077E-03	40.4575	20.3763	2.23029

**Figure 4.14: FEA subset of the dataset in Figure 4.9**

The group is literally a subset of the element labels from the original dataset. When using FEA subsets, the following guidance should be observed:

1. The results position between the FEA subset and the original dataset must agree, i.e. if the original dataset uses element-nodal position labels, then so should the FEA subset
2. Including field data with the FEA subset is not compulsory. However, since Abaqus requires at least one field to be exported, FEA subsets written by Abaqus/Viewer will always contain at least one column of field data

It is quite possible that a group defined as an FEA subset will reference more than one element with the same position label. In order for the group definition to be correct, Quick Fatigue Tool must somehow match the ID to the correct location in the FE model. In the event that duplicate IDs are found in a group, field data in the group definition can be used to check the stress tensor of the duplicate IDs against the tensors in the original stress dataset. Therefore, it is recommended that FEA subsets retain the same field information as the original datasets.

Quick Fatigue Tool automatically attempts to resolve duplicate IDs based on the availability of the field data; no additional intervention is required by the user. However, if the loading is defined as either a multiple scale and combine or as a dataset sequence (the DATASET option was specified with more than one argument), then the field data from the group file must agree with the last dataset in the loading.

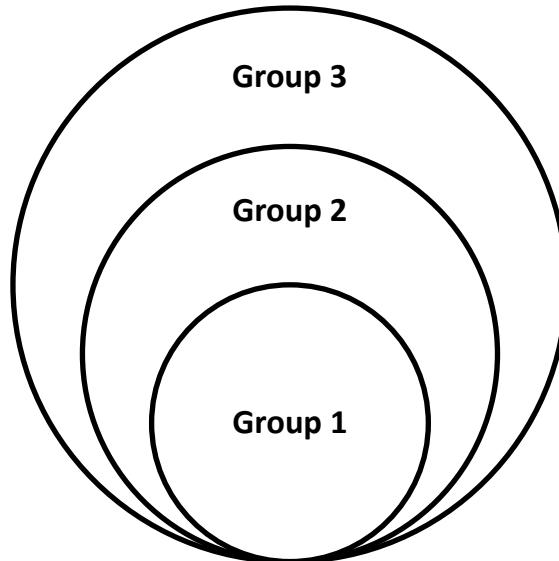
#### 4.6.4 Arranging groups in the job file

Both item ID lists and FEA subsets can be used together for an analysis, and the groups do not have to be mutually exclusive. Quick Fatigue Tool will process each group as they are encountered in the GROUP option and will exclude the items so that they will not be read a second time if they appear in subsequent groups. Therefore, the order in which groups are defined will affect which group the items are assigned to if any of the definitions are inclusive of each other. The hierarchical nature of the group reading process is illustrated by Figure 4.15.

Group 1 is a subset of Group 2, which in turn is a subset of Group 3. Consider the case where the groups are defined in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GROUP	{'group-1.*', 'group-2.*', 'group-3.*'};



**Figure 4.15: Group hierarchy**

Quick Fatigue Tool will first read Group 1. All items from Group 1 will be read into the group definition, and excluded from re-definition in later groups. Hence, Group 2 will include all of its items *except* those already belonging to Group 1. The same logic will apply to Group 3, which will have all items from Group 2 and Group 1 excluded from its definition.

The groups defined in the excavator model from Figure 4.10 could be defined in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GROUP	{'region-A.*', 'region-B.*'};

Quick Fatigue Tool will analyse all of Region A and Region B. None of the items from Region C will be analysed. In this case, the order does not matter since the groups are mutually exclusive.

If all three regions are to be analysed, but only Region C requires individual properties, the following group definition may be used.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GROUP	{'region-C.*', 'DEFAULT'};

In this instance, Quick Fatigue Tool will analyse Region C with individual properties for that group, followed by all other items in the model. In general, use of the '**DEFAULT**' parameter instructs the program to analyse all remaining items in the model (all other items in the original dataset which do not belong to any preceding groups). The '**DEFAULT**' parameter may only be used as the last argument in the GROUP option.

The '`DEFAULT`' parameter is used by its self to analyse the whole model, with no group definitions.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GROUP	{' <code>DEFAULT</code> '};

Quick Fatigue Tool determines whether a group definition is an item ID list or an FEA subset based on the contents of the file, and depending on the user setting in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>groupDefinition</code>	[0.0   1.0];

Using this default value, the application will assume that the group definition is an item ID list if there is only one column of data in the file, and an FEA subset if there is more than one column. By setting `groupDefinition` to a value of 1.0, an FEA subset will always be assumed.

#### 4.6.5 Assigning properties to groups

Group properties (job file options and environment variables) are specified by assigning multiple definitions according to the number of groups in the analysis.

Usually, the number of property definitions should match the number of groups in the analysis. If the analysis contains  $n$  groups where  $n > 1$ , and the user assigns a single definition to a property, Quick Fatigue Tool automatically propagates that definition to all of the analysis groups. Otherwise, the number of property definitions must exactly match the number of group definitions.

Most properties require  $n$  valid definitions for  $n$  number of analysis groups.

*Job file usage:*

<i>Option</i>	<i>Value</i>
MATERIAL	{'material-file-1.mat', ..., 'material-file-n.mat'};

This requirement applies to the following job file options and environment variables:

<b>Job file option</b>	<b>Environment variable</b>
MATERIAL	frfNormParamMeanT
SN_SCALE	frfNormParamMeanC
NOTCH_SENSITIVITY_CONSTANT	frfNormParamAmp
FATIGUE_RESERVE_FACTOR	userWalkerGamma
RESIDUAL	userEnduranceLimit
B2	midifiedGoodman
B2_NF	goodmanMeanStressLimit
UCS	

S-N knock-down factors may not be required for every analysis group.

*Job file usage:*

<i>Option</i>	<i>Value</i>
SN_KNOCK_DOWN	{'knock-down-file-1.kd', [], 'knock-down-file-n.kd'};

In this example, knock-down factors have been specified in groups 1 and  $n$ , and none is define for group 2. In cases where some groups do not require a definition, this must be indicated by an empty assignment ([ ]), otherwise the group definition will be processed incorrectly.

A combination of surface finish definition files and  $K_T$  values can be specified over several groups.

*Job file usage:*

<i>Option</i>	<i>Value</i>
KT_DEF	{'surface-finish-file-1.kt', 1.2, 'surface-finish-file-n.kt'};

In such cases, the KT\_CURVE option needs only to be specified according to the number of surface finish files defined by KT\_DEF.

*Job file usage:*

<i>Option</i>	<i>Value</i>
KT_CURVE	$[C_1, C_n];$

In this example, the curve numbers  $C_1$  and  $C_n$  correspond to the surface finish files '*surface-finish-file-1.kt*' and '*surface-finish-file-n.kt*', respectively; a curve number for the  $K_T$  value is not required since the surface finish is defined directly.

A list of all properties eligible for group definitions, along with typical syntax is provided in the table below.

<b>Job file option/Environment variable</b>	<b>Example definition</b>
MATERIAL	{'material-file-1.mat', ..., 'material-file-n.mat'};
SN_SCALE	$[F_{s1}, \dots, F_{sn}];$
SN_KNOCK_DOWN	{'knock-down-file-1.kd', 'knock-down-file-n.kd'};
FATIGUE_RESERVE_FACTOR	{ $E_1, \dots, E_n$ ; 'msc-file-name-n.msc'};
KT_DEF	{ $K_{T1}, \dots, K_{Tn}$ ; 'surface-finish.file-2.kd', 'surface-finish.file-3.kd'};
KT_CURVE	$[value_1, \dots, value_n];$
RESIDUAL	$[value_1, \dots, value_n];$
B2	$[value_1, \dots, value_n];$
B2_NF	$[value_1, \dots, value_n];$
UCS	$[value_1, \dots, value_n];$
NOTCH_SENSITIVITY_CONSTANT	$[value_1, \dots, value_n];$
NOTCH_ROOT_RADIUS	$[value_1, \dots, value_n];$
frfNormParamMeanT	{ $value_1, \dots, value_n$ }
frfNormParamMeanC	{ $value_1, \dots, value_n$ }
frfNormParamAmp	{ $value_1, \dots, value_n$ }
userWalkerGamma	$[value_1, \dots, value_n];$
userEnduranceLimit	$[value_1, \dots, value_n];$
midifiedGoodman	$[value_1, \dots, value_n];$
goodmanMeanStressLimit	{ $value_1, \dots, value_n$ }

The following caveats should be noted for the use of analysis groups:

- materials must be defined as a cell;
- values for SN\_SCALE are only required if USE\_SN = 1.0;
- a combination of surface finish .kt/.ktx files and/or surface finish values can be used within the same KT\_DEF statement;
- the number of values in KT\_CURVE need only reflect the number of .kt/.ktx files defined in KT\_DEF;
- the number of values in `frfNormParamMeanT`, `frfNormParamMeanC` and `frfNormParamAmp` need only reflect the number of custom FRF envelope definitions in FATIGUE\_RESERVE\_FACTOR;
- the number of values in `goodmanMeanStressLimit` need only reflect the number of zero-valued entries in `modifiedGoodman` (the Goodman limit stress can only be defined for the standard Goodman envelope);
- if the '`DEFAULT`' parameter is used as the last argument in a group definition, the default group is included in the total number of groups;
- if the default analysis algorithm or mean stress correction is specified, the algorithm and mean stress correction used for analysis is chosen from the last argument of the MATERIAL option; and
- if the DESIGN\_LIFE option is set to the material's constant amplitude endurance limit ('`CAEL`'), the endurance value is chosen from the last argument of the MATERIAL option.

#### 4.6.6 Limitations

Analysis groups in Quick Fatigue Tool currently do not support multiple definitions of the following:

- mean stress corrections/analysis algorithms; and
- BS 7608 material properties.

If a different algorithm or mean stress correction is required for each analysis group, the workaround is to split the analysis into multiple jobs and superimpose the fatigue results onto a single field output file using the option CONTINUE\_FROM. This technique is discussed in Section 4.8.

## 4.7 S-N knock-down factors

### 4.7.1 Overview

A set of S-N scale factors can be applied to the material S-N data in the form of knock-down factors, which scale the stress data points for each specified life value. Knock-down factors are defined in a separate *.kd* file and specified in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
SN_KNOCK_DOWN	{'knock-down-file-name.kd'};
USE_SN	1.0;

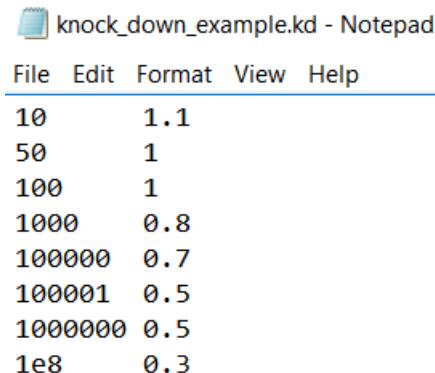
### 4.7.2 Defining a knock-down curve file

The knock-down curve file is defined as follows:

*First column:* Life values ( $N_f$ ) at which knock-down factors are to be applied

*Second column:* Knock-down factors corresponding to each life value

An example *.kd* file is given by Figure 4.16.



File	Edit	Format	View	Help
10	1.1			
50	1			
100	1			
1000	0.8			
100000	0.7			
100001	0.5			
1000000	0.5			
1e8	0.3			

**Figure 4.16: Example *.kd* file containing life values in the first column and knock-down factors in the second column**

The knock-down factors are applied to the S-N data before the analysis to produce the modified endurance curve. The life values in the *.kd* file are treated as sample points and as such, they do not have to match the position of the S-N data points. Quick Fatigue Tool will automatically interpolate and extrapolate the S-N data before scaling each data point. An example of this process is shown by the following table. Note that bracketed values in bold have been interpolated or scaled.

Original S-N curve		Knock-down curve		Scaled S-N curve	
S	N	Factor	N	S	N
(800)	(10)	1.1	10	(880)	(10)
(1725)	(50)	1	50	(1725)	(50)
(1400)	(100)	1	100	(1400)	(100)
700	1000	0.8	1000	(560)	(1000)
350	10000	(0.75)	(10000)	(263)	(10000)
(313)	(100000)	0.7	100000	(219)	(100000)
(313)	(100001)	0.5	100001	(157)	(100001)
(280)	(1000000)	0.5	1000000	(140)	(1000000)
250	10000000	(0.4)	(10000000)	(100)	(10000000)
-	-	0.3	100000000	-	-

Knock-down data specified below the minimum life of the original S-N data is extrapolated. However, Quick Fatigue Tool assumes that the last data point in the original S-N data represents the material's endurance limit and thus knock-down data provided beyond this point is not extrapolated.

The original S-N, knock-down and modified S-N curves are illustrated by Figures 4.17-19. If the material contains multiple S-N curves, each curve is scaled by the knock-down curve before the beginning of the analysis.

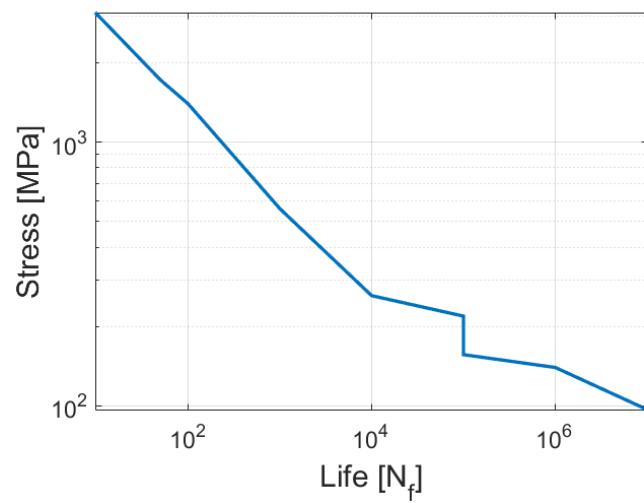
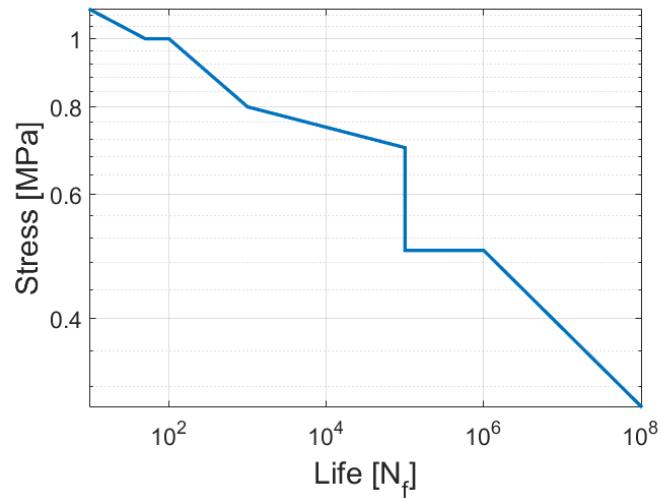
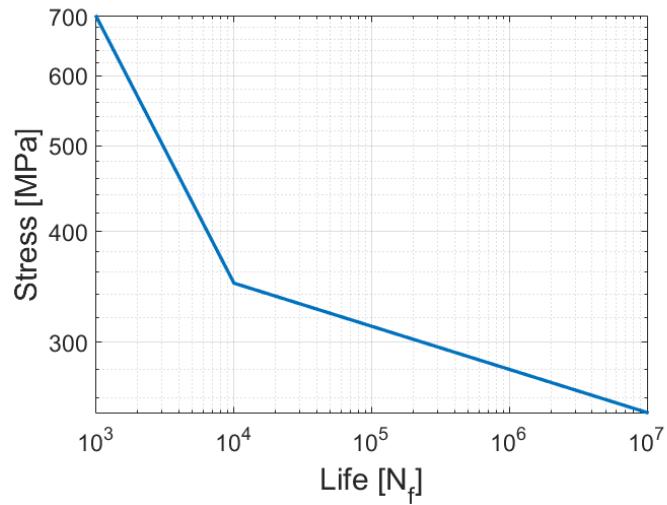


Figure 4.17-19: Original S-N curve (top), knock-down factors (middle) and modified S-N curve (bottom)

#### 4.7.3 Example applications

Knock-down curves can be used in addition to the surface finish definition in order to account for additional manufacturing effects. Imperfections of the manufacturing process can lead to defects, which reduce the endurance of the material.

Knock-down curves may be used with cast metals where inclusions result in a local loss of fatigue performance. Another application is in the injection moulding of plastic components. During this process multiple flow regions can meet, causing a weld line at the interface of the two melts. During cooling of the newly formed part, voids may develop wherein large thermal gradients exist. Both of these phenomena can be accounted for via the use of S-N knock-down factors.

One possible approach to using knock-down factors would be to identify the nodes on the FE model which represent the manufacturing defect, and export a dataset file containing the node numbers and field data, the process of which is described in Section 3.2. This dataset can then be used to define a group in the job file with an individual knock-down curve applied. Such a configuration could be achieved by using the following options.

*Job file usage:*

<i>Option</i>	<i>Value</i>
MATERIAL	{'knock_down.mat', 'default.mat'};
USE_SN	1.0;
SN_KNOCK_DOWN	{'knock_down_curve.kd', [ ]};
GROUP	{'defect_group.rpt', 'DEFAULT'};

#### 4.7.4 Exporting knock-down curves

The knock-down curves can be exported to MATLAB figures from the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
figure_KDSN	[0.0   1.0];

S-N knock-down curves are exported for groups which have knock-down factors specified. MATLAB figures must be requested in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
OUTPUT_PICTURE	1.0;

## 4.8 Analysis continuation techniques

### 4.8.1 Overview

Quick Fatigue Tool provides the capability to perform an analysis as a continuation of a previous job. Field output from the current job is written onto the field output from a previous job. An analysis which uses the continuation feature:

- can be used to model block loading, where each job defines a distinct loading event;
- can be used to assign different analysis algorithms to multiple regions in a model; and
- allows the user to specify completely new definitions for any job file option for each job.

When used in conjunction with the ODB interface, analysis continuation:

- can be used to superimpose field data onto the same mesh;
- can be used to append field data onto a mesh at locations different to those which were analysed in the previous job; and
- a combination of the above.

A job which uses analysis continuation runs in the usual way. At the end of the analysis, field data is superimposed onto the field output file from a specified job. The rules for combining field data depend on the variable type, and are listed in the table below.

Variables	Rule
D	$D^* = D_1 + D_2$
L; LL; DDL	Derived from the value of $D^*$
FOS; SFA; FRFR; FFH; FRFV	$F^* = \min(F_1, F_2)$
FRFW	Derived from the values of $FRFR^*$ , $FRFH^*$ and $FRFV^*$
SMAX; SMXP; SMXU; TRF; WCM; WCA; WCDP; YIELD	$F^* = \max(F_1, F_2)$
WCATAN	Derived from the values of $WCM^*$ and $WCA^*$

The subscripts 1 and 2 correspond to the first and second jobs, respectively. The superscript \* corresponds to the superimposed field variable.

#### 4.8.2 Referencing the previous job

The previous analysis is specified in the job file of the current analysis.

*Job file usage:*

<i>Option</i>	<i>Value</i>
CONTINUE_FROM	' <i>previous-job-name</i> ';

The name of the previous job is the name given by the option JOB\_NAME in the previous job file. Field output must be requested for the first job. Field output is written automatically for the second job, provided that a valid definition of CONTINUE\_FROM is specified.

*Job file usage:*

<i>Option</i>	<i>Value</i>
OUTPUT_FIELD	1.0;

#### 4.8.3 Example applications

##### Multiple analysis algorithms

Analysis continuation can be used to circumvent the limitation of analysis groups, which does not support multiple definitions for the analysis algorithm or mean stress correction. Analysis continuation allows the user to define the algorithm and mean stress correction for multiple regions in the model, and analyse each region as a separate job. Field data is superimposed onto the previous field data file in a chained fashion by specifying the name of the previous job for each analysis. Finally, the cumulative results of all analyses may be written to an .odb file using the ODB interface, which is discussed in Section 10.4.

##### Multiple block loading

The definition of multiple loading blocks is not directly supported by Quick Fatigue Tool. However, analysis continuation can be used to separate the load spectrum across several job files, where each analysis represents a particular loading block, or event, in the component's operational duty.

Consider the loading profile for a given component:

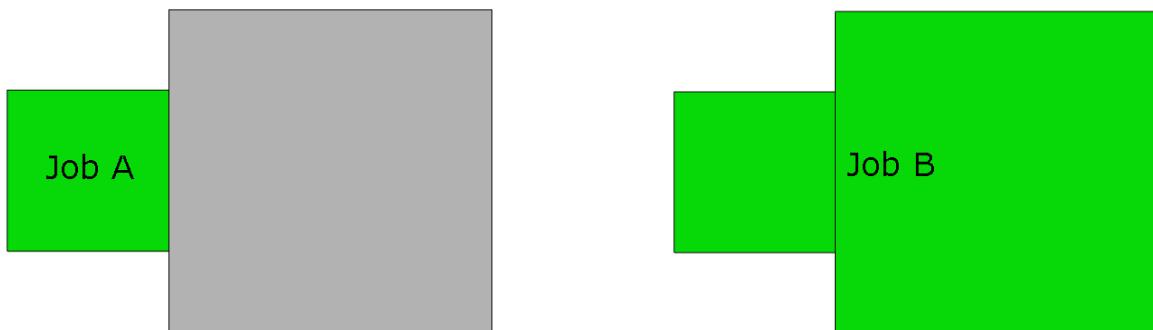
Block #	Descriptor
1	Applied load in 1-direction; fully-reversed [1, -1] load history; 3 repeats
2	Applied load in 2-direction; pulsating [0, 1] load history; 10 repeats
3	Applied load in 1-direction; mixed [1, -1, 2, -1] load history; 1 repeat

Each block is defined as a separate job file, each its own definitions for the DATASET, HISTORY and REPEATS options.

#### **4.8.4 Additional guidance**

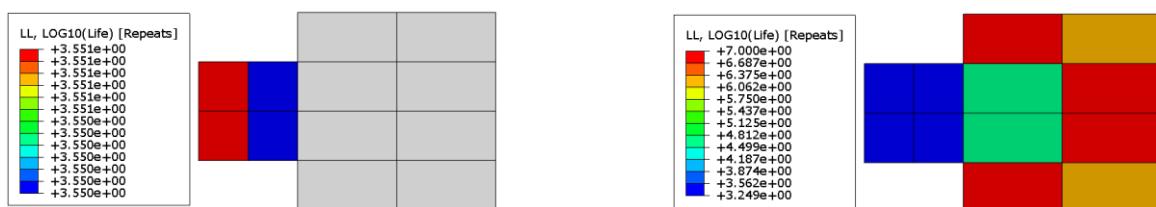
When superimposing results onto a previous analysis, Quick Fatigue Tool searches for matching item IDs between the two field output files. Field data at items which match with the previous file are superimposed to create the field data for the combined result. Items which do not match with the previous job are appended onto the end of the field output file. This is advantageous, since the loading in each block does not have to be applied to the same region between analyses; the definition of DATASET does not have to be consistent.

Such a scenario is illustrated by Figure 4.20.



**Figure 4.20:** FE model split into two analysis regions (green). The first loading block is applied to the smaller region, while the second loading block is applied to the whole model.

After running *Job A*, fatigue results are written to the left-hand portion of the model. After running *Job B* with `CONTINUE_FROM='Job A'`, fatigue results common to the analysis region from *Job A* are superimposed onto the previous data. Fatigue results corresponding to the right-hand portion of the model are appended to the field data without superimposition. This is illustrated by Figure 4.21. Note how the results at the left-hand side have changed to account for the second loading block from *Job B*.



**Figure 4.21: Fatigue results for the model depicted in Figure 4.20. Common nodes are superimposed; new nodes are appended as additional field data**

#### 4.8.5 Limitations

Analysis continuation is subject to the following limitations:

- the load equivalency (LOAD\_EQ) must be the same for all jobs;
- the design life (DESIGN\_LIFE) must be the same for all jobs;
- the results position must be the same for all jobs;
- the number of cycles quoted in the log file only applies to the most recent analysis;
- the field output file must be located in the default directory according to the job name. Renaming of files or folders will prevent Quick Fatigue Tool from locating the necessary files;
- the analysis will crash if the previous field output file is opened by an external process during the fatigue analysis;
- load transitions are not supported. The cycle counting algorithm will treat each block as a separate event, therefore the effect of large cycles in previous blocks will not be taken into effect in subsequent blocks. If the yield calculation is active over multiple blocks, the effect of hardening is not imported to the next job, rather, the stress-strain state will be reset to zero at the beginning of each block;
- the values of **L** and **LL** are capped at the constant amplitude endurance limit of the material corresponding to the second analysis;
- virtual strain gauge definitions are not carried forward to subsequent analyses;
- if ODB element/node sets are written to the *.odb* file after each analysis, the set names must be unique between jobs, otherwise the ODB interface will exit with an error; and
- while the ODB interface can handle collapsed elements, results from these elements cannot be superimposed onto previous field data, since Quick Fatigue Tool is unable to resolve the ambiguity caused by duplicate position labels.

## 4.9 Virtual strain gauges

### 4.9.1 Overview

Virtual strain gauges are used to assess the behaviour of calculated load histories compared to measured strain data, as a means to validate the input stresses. In FEA, strain gauges can be difficult to model and usually require the definition of axial connector elements positioned at strategic locations on the mesh surface which correspond to the laboratory test. Quick Fatigue Tool offers the specification of a virtual strain gauge, which is a location on the model (integration point, node, etc.) at which the strain history is measured in a particular direction.

### 4.9.2 Gauge definition

The virtual strain gauge has a rectangular rosette format, an example of which is shown in Figure 4.22.

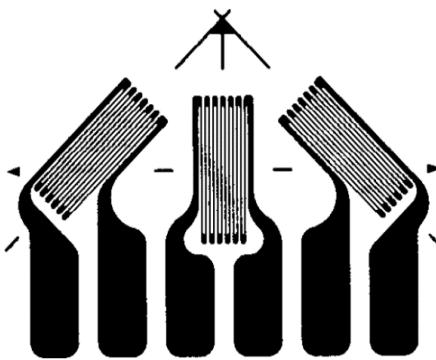


Figure 4.22: Typical layout of a rectangular rosette gauge

The gauge is represented in Cartesian space in Figure 4.23, with the rosette arms *A*, *B* and *C* orientated according to the angles  $\alpha$ ,  $\beta$  and  $\gamma$ , counter clockwise from the positive x-direction.

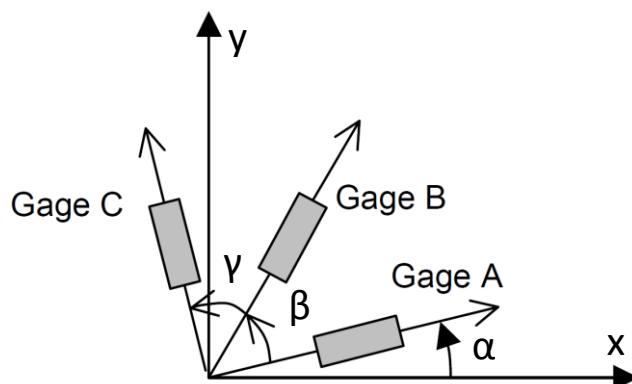


Figure 4.23: Rosette gauge orientation relative to Cartesian axes

### 4.9.3 Technical background

The calculation of the strain histories is based on the linear elastic stresses defined in the stress data set file. If the material contains values of the cyclic strain hardening coefficient and exponent ( $K'$  and  $n'$ , respectively) then the elastic stresses are first converted to elasto-plastic strain histories. These histories are then resolved onto the directions of the gauge arms according to Equations 4.13-15.

$$\varepsilon_A = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2\alpha + \varepsilon_{xy} \sin 2\alpha \quad [4.13]$$

$$\varepsilon_B = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2(\alpha + \beta) + \varepsilon_{xy} \sin 2(\alpha + \beta) \quad [4.14]$$

$$\varepsilon_C = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2(\alpha + \beta + \gamma) + \varepsilon_{xy} \sin 2(\alpha + \beta + \gamma) \quad [4.15]$$

The plasticity correction uses the same algorithm as the Multiaxial Gauge Fatigue app and is described in Appendix A3.2.4.

If cyclic data is not provided, then the strains are calculated from the stresses elastically, according to Equations 4.16-18.

$$\varepsilon_{xx} = \sigma_{xx} E^{-1} \quad [4.16]$$

$$\varepsilon_{yy} = \sigma_{yy} E^{-1} \quad [4.17]$$

$$\varepsilon_{xy} = 2\tau_{xy}(1 - \nu)E^{-1} \quad [4.18]$$

### 4.9.4 Specifying the position of the strain gauge

Virtual strain gauges are defined by specifying the position IDs identifying the location of the gauges 1 to  $n$  on the model.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GAUGE_LOCATION	{'<mainID>.<subID> <sub>1</sub> ', ..., '<mainID>.<subID> <sub>n</sub> '};

For uniaxial analyses, the gauge location is simply '[1.1](#)'. The user may specify multiple strain gauges by listing the position IDs as separate strings.

#### 4.9.5 Specifying the orientation of the strain gauge

The strain gauge orientation is defined by providing the values of  $\alpha$ ,  $\beta$  and  $\gamma$  for gauges 1 to  $n$ , according to Figure 4.23.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GAUGE_ORIENTATION	$\{[\alpha, \beta, \gamma]_1, \dots, [\alpha, \beta, \gamma]_n\};$

Alternatively, the user may use the flags '**RECTANGULAR**' or '**DELTA**' to indicate that the gauge has a rectangular [ $0^\circ, 45^\circ, 45^\circ$ ] or delta [ $30^\circ, 60^\circ, 60^\circ$ ] layout, respectively.

*Job file usage:*

<i>Option</i>	<i>Value</i>
GAUGE_ORIENTATION	<b>{'RECTANGULAR'   'DELTA'}</b> ;

#### 4.9.6 Example usage

Consider the model in Figure 4.24. A gauge is to be defined at element 657, node 7. The gauge is aligned with the global x-axis, giving an orientation of  $\alpha = 0^\circ$ ,  $\beta = 45^\circ$  and  $\gamma = 45^\circ$ . The virtual gauge is defined in the job file.

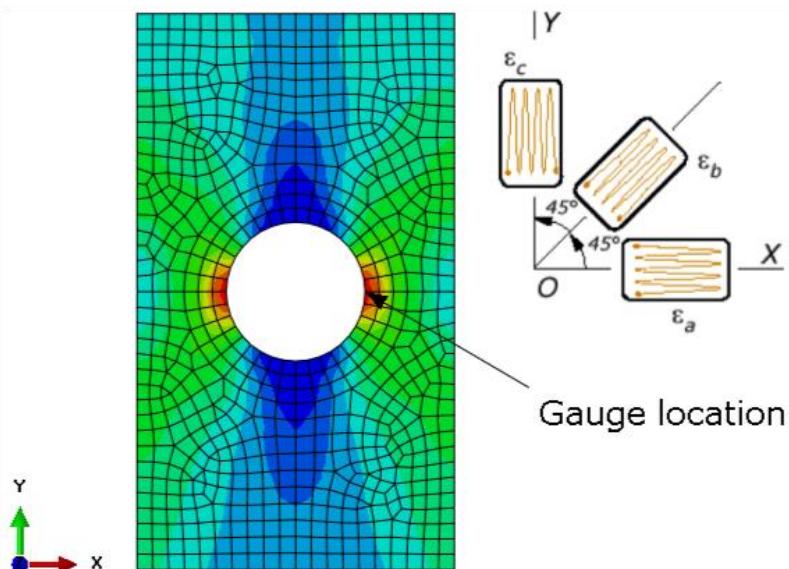


Figure 4.24: Virtual gauge location on FE model

*Job file usage:*

<i>Option</i>	<i>Value</i>
GAUGE_LOCATION	{'657.7'};
GAUGE_ORIENTATION	{[0.0, 45.0, 45.0]};

Additional gauges can be added by appending position IDs and orientations as necessary.

Results for each gauge are written to a text file and stored in *Project\output\<jobName>\Data Files*.

#### 4.9.7 Modelling guidance

Virtual strain gauges are intended for components in a state of plane strain. The gauge will only detect the two-dimensional state of strain relative to the global x-y plane. The user is therefore advised to specify the gauges on plane stress elements to facilitate the definition. If the model contains solid elements, a skin should be applied over the surface (a layer of shell elements), and the gauges defined on these elements.

The position IDs used to define the gauges should be consistent with the element position used to generate the stress data set file. For example, if the stresses were exported at integration points or element nodes, both the main and sub IDs are required. For centroidal and unique nodal data, the item is defined by the main ID; the sub ID always has a value of 1.0.

Output from virtual strain gauges can be used as input to the Multiaxial Gauge Fatigue application (Appendix A3.2). The user must ensure that the orientations specified by GAUGE\_ORIENTATION match those defined in the application.

If the user wishes only to extract virtual strain gauge histories, the fatigue analysis can be omitted by setting DATA\_CHECK=1 in the job file. Alternatively, the Virtual Strain Gauge application (Appendix A3.4) can be used to generate strain gauge histories from a strain tensor definition; both methods use the same underlying analysis technique discussed in this section.

#### 4.9.8 Limitations

Virtual strain gauges convert the linear elastic stresses to elasto-plastic strains using a simple multilinear hardening rule. The correction is applied separately to each strain component and as such, results will be inaccurate where a large amount of plasticity is present in the material. The virtual strain gauge should be used as a rough estimate of the local nonlinear strain history only; in cases where a more thorough treatment is required, the user is advised to define the gauge directly on the finite element model.

## 5. Materials

### 5.1 Background

#### 5.1.1 Overview

This section describes how to define and use material properties for analysis in Quick Fatigue Tool. In general, this involves:

- creating material properties interactively with the Material Manager application, or from a text file; and
- specifying the material in the job file.

Material data is stored as a MATLAB binary (*.mat*) file and located in the *Data\material\local* directory.

#### 5.1.2 Accessing the Material Manager

The Material Manager application is used to create and edit material data. It can be accessed either from the command line or by installing the Material Manager GUI application.

To install the Material Manager, double-click the file *Material Manager.mlappinstall* in the *Application\_Files\toolbox* directory. The app will appear in the apps bar in MATLAB.

Figure 5.1 shows the Material Manager GUI.

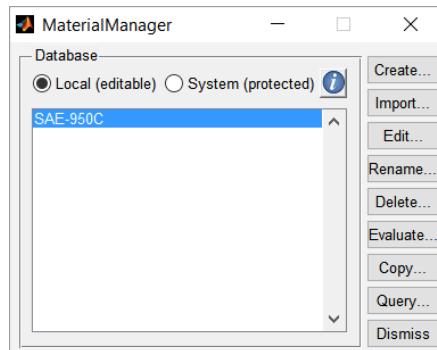


Figure 5.1: Material Manager GUI

The Material Manager GUI is launched by doing one of the following:

- Select the Material Manager launch icon from the APPS ribbon; or
- Execute the command *material.manage* from the MATLAB command line.

## 5.2 Material databases

Material Manager separates material data into two databases:

### Local

- Local copies of materials are stored here
- Materials in this database can be modified
- Materials in this database can be used for analysis

### System

- Database containing materials included with the Quick Fatigue Tool application
- Materials in this database cannot be modified
- Materials in this database must be fetched in order to be used for analysis

By default, user-defined materials are stored in the local database. Unless the full path to the material .mat file is specified in the job file, Quick Fatigue Tool always searches *Data\material\local* first.

## 5.3 Using material data for analysis

### 5.3.1 Specifying the material in the job file

The analysis material is specified in the job file by providing the *.mat* file containing the material data.

*Job file usage (M-file):*

Option	Value
MATERIAL	'material-file-name.mat';

where '*material-file-name.mat*' is a material in the local database.

*Job file usage (text file):*

Option	Value
*MATERIAL =	'material-file-name.mat'

Detailed guidance on specifying job file options in a text file is provided in Section 2.4.3.

### 5.3.2 Fetching materials from the system database

To use a material from the system database, the material is fetched from the file *mat.mat* in *Data\material\system* and a copy is stored in the local database.

*Material Manager usage:* Select **System (protected)** from the Database region. Select a material from the list of system materials. Select **Fetch...** Verify the name of the material. Select **OK**.

*Command line usage:*

```
>> material.fetch()  
>> <database number>  
>> <material number>
```

The fetched material appears in the *Local* database list in the Material Manager GUI. A list of materials in the local database is shown using the following command:

*Command line usage:*

```
>> material.list()
```

## 5.4 Creating materials using the Material Manager GUI

Material properties are defined by launching the material property editor.

*Material Manager usage:* To create a new material, select **Create....** To modify an existing material, select **Edit....**

*Command line usage:*

```
>> material.create()  
  
>> material.edit('material-name')
```

The material property editor is shown in Figure 5.2.

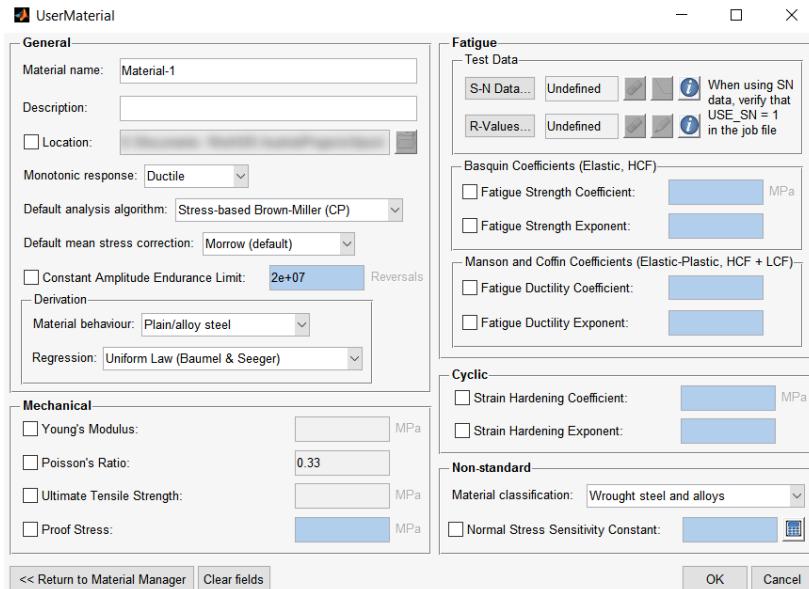


Figure 5.2: Material property editor for a user-defined material

The majority of analyses only require elastic fatigue properties (fatigue strength coefficients and exponents). However, certain output variables require additional material properties.

Property values are specified by first checking the property to indicate that it is user-defined. When a property is unchecked, its input field turns blue, meaning that the property is not defined by the user and will be derived automatically if applicable.

The fatigue analysis algorithm and mean stress correction are considered a material property, and are used by default unless specified otherwise in the job file.

## 5.5 Creating materials from a text file

### 5.5.1 Overview

Material data defined in a text file can be used for

- importing materials into the local database using Material Manager; and
- defining material data directly in the job file for text-based job submission.

Material properties are declared using keywords in combination with parameters and data lines (if applicable).

### 5.5.2 Material keyword syntax

Materials defined from text files must adhere to strict syntax rules which allow Quick Fatigue Tool to recognize the data. Each material definition must begin with the following keyword:

*Text file usage:*

<i>Keyword</i>	<i>Parameter</i>
*USER MATERIAL	name

This keyword declares the material definition and assigns a material name using the *name* parameter.

An example usage of this keyword is given below:

```
*USER MATERIAL, manten steel
```

There are no data lines associated with this keyword; only the keyword itself and the material name are required.

Some keywords require a data line to complete their definition:

*Text file usage:*

<i>Keyword</i>	<i>Parameter</i>	<i>Data line(s)</i>
*CAEL	(none)	<i>cael, cael<sup>(a)</sup></i>

This keyword defines the constant amplitude endurance limit. There is no associated parameter. The first (and only) data line defines the constant amplitude endurance limit value, *cael*, and an optional flag, *cael<sup>(a)</sup>*, indicating whether this value is active in the material definition. Several material parameters have an associated ...<sup>(a)</sup> flag; a value of 1.0 is equivalent to the action of checking the respective property box in the Material Manager GUI.

An example usage of this keyword is given below:

```
*CAEL  
2e7, 1.0
```

Some keywords have optional data lines:

*Text file usage:*

<i>Keyword</i>	<i>Parameter</i>	<i>Data line(s)</i>
*MECHANICAL	(none)	First line: $E, \nu, \sigma_U, \sigma_y$ Second line: $E^{(a)}, \nu^{(a)}, \sigma_U^{(a)}, \sigma_y^{(a)}$

This keyword defines the mechanical constants for the material. The data line entries  $E, \nu, \sigma_U$  and  $\sigma_y$  specify the Young's Modulus, Poisson's ratio, the ultimate tensile strength and the yield strength, respectively.

In this case, only the first data line is compulsory. The second data line may be used to specify whether the properties are active in the definition. It is not necessary to define all the properties on the first data line, and consequently the user is only required to specify the ...<sup>(a)</sup> flags corresponding to the defined properties.

An example usage of this keyword where all properties are defined is given below:

```
*MECHANICAL  
200e3, 0.3, 400, 325
```

In this case, all four mechanical properties have been specified. The ...<sup>(a)</sup> flag has a default value of 1.0 for any defined properties, so all of the properties are active in the material.

An example usage of this keyword where only some properties are defined is given below:

```
*MECHANICAL  
200e3, , 400,,  
1.0, , 0.0,,
```

In this case, only the Young's Modulus and the ultimate tensile strength are defined. The ...<sup>(a)</sup> flags are specified such that the Young's Modulus is active and the ultimate tensile strength is inactive.

If a parameter is left undefined, this must be indicated by an empty assignment (two consecutive commas), otherwise the definition may be processed incorrectly.

The user indicates the end of a material definition by specifying \*END MATERIAL as the last keyword in the definition. This instructs Quick Fatigue Tool to stop processing the material text file.

### 5.5.3 Importing materials from a text file

Materials are imported into the local database by using the *Import* function in Material Manager. The material text file is read through a text file processor and the definitions are saved as a MATLAB binary (.mat) file in *Data\material\local*.

Material data is imported into the local database using the Material Manager GUI or via the command line.

*Material Manager usage:* Select **Import....** Change the file selection filter to **Normal text file (\*.txt)**. Select the text file containing material data and select **Open**.

*Command line usage:*

```
>> material.import('material-file-name.*')
```

### 5.5.4 Specifying material properties in a job file

Material data may be defined as part of a text-based job file. Job submission from text files is discussed in Section 2.4.3. The material definition may be placed anywhere in the job file provided that it begins and ends with the keywords \*USER MATERIAL and \*END MATERIAL, respectively. Failing to do so may result in an error.

An example job file containing material data is given below:

```
*JOB NAME = holePlate
*MATERIAL = steel
*DATASET = stressData.dat
*HISTORY = [1, -1]
beginning of material definitions:
*USER MATERIAL, steel
*MECHANICAL
200e3, , 400, ,
*FATIGUE, constants
930, -0.095, , ,
1, 1, ,
*END MATERIAL
*USER MATERIAL, aluminium
*MECHANICAL
79e3, , 110, ,
*FATIGUE, test data
10000, 62.7, 51.6
1e6, 38.3, 32.7
*R RATIOS
-1, 0
*END MATERIAL
additional options to define the fatigue analysis:
*OUTPUT FIELD = 1
*FATIGUE RESERVE FACTOR = 1
```

Note that the job file contains two material definitions; although only one of the materials is referenced by the MATERIAL job file option, both materials are copied to the local material database as *.mat* files.

### **5.5.5 Material keyword reference**

This section has been released in the document *Quick Fatigue Tool User Settings Reference Guide*. It contains a complete list of the material keywords and their usage.

## 5.6 Custom stress-life data

### 5.6.1 Overview

It is often the case that only stress-life (S-N) data points are available in place of fatigue coefficients. Quick Fatigue Tool can perform the fatigue life calculation based on custom S-N data by interpolating logarithmically between adjacent points on the curve.

*Job file usage:*

<i>Option</i>	<i>Value</i>
USE_SN	[0.0   1.0];

Quick Fatigue Tool checks the value of *USE\_SN* before the analysis. If the value is set to 1.0, custom S-N data will be used if it is available; otherwise fatigue coefficients will be used instead. If neither is available, Quick Fatigue Tool will attempt to derive the fatigue coefficients from the mechanical properties. If the required mechanical properties are unavailable, the analysis will be aborted due to insufficient materials data.

### 5.6.2 Defining user S-N data

S-N data is read into the property editor via a text file. The data can be tab, space or comma separated, and has the following format:

<b><i>N – Values</i></b>	<b><i>S – Values at R1</i></b>	<b><i>S – Values at R2</i></b>	<b><i>...</i></b>	<b><i>S – Values at R<sub>j</sub></i></b>
<i>N1</i>	<i>S<sub>1,1</sub></i>	<i>S<sub>1,2</sub></i>	.	<i>S<sub>1,j</sub></i>
<i>N2</i>	<i>S<sub>2,1</sub></i>	<i>S<sub>2,2</sub></i>	.	<i>S<sub>2,j</sub></i>
.	.	.	.	.
<i>N<sub>i</sub></i>	<i>S<sub>i,1</sub></i>	<i>S<sub>i,2</sub></i>	.	<i>S<sub>i,j</sub></i>

The first data column is always cycles, and the number of cycles must be increasing down the column. Each subsequent data column is the stress amplitude for each measured load ratio ( $R_1$  to  $R_j$ ). The stress datasets should be ordered so that the load ratios are monotonically increasing. The S-N data can also be specified as the transpose of the above (row-wise).

If S-N data is provided for one load ratio, the stress amplitude will be assumed to represent a fully-reversed cycle, irrespective of the R-ratio provided.

S-N data can be plotted using the plot button. An example of S-N data for multiple R-ratios is given in Figure 5.3.

S-N data stress values must be specified in MPa.

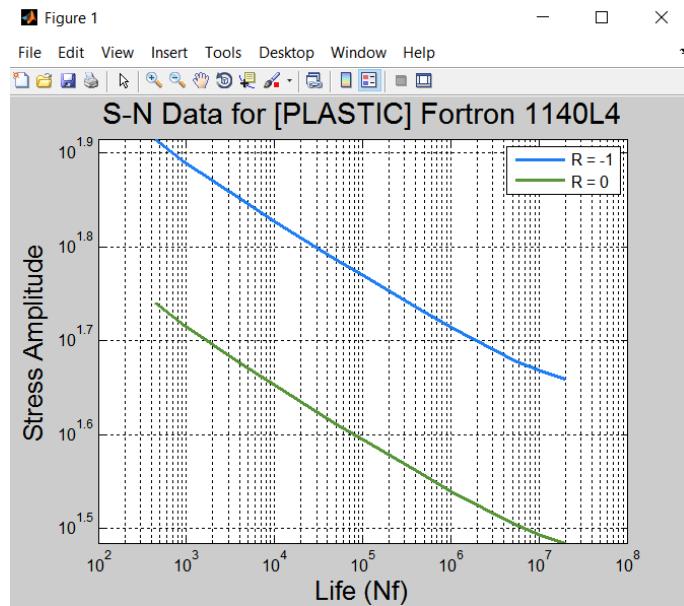


Figure 5.3: S-N data for Fortron 1140L4

### 5.6.3 Specifying user S-N data in the material

User S-N data is specified using the material property editor or from a text file.

*Material Manager usage:* In the material property editor, from the Test Data region, select **S-N Data**. Navigate to the S-N data file and select **Open**.

*Text file usage:*

Keyword	Parameter	Data line(s)
*FATIGUE	TEST DATA	First line: $N_1, S_{1,1}, S_{1,2}, \dots, S_{1,j}$ Second line: $N_2, S_{2,1}, S_{2,2}, \dots, S_{2,j}$ $i$ th line: $N_i, S_{i,1}, S_{i,2}, \dots, S_{i,j}$

If the S-N data is defined over multiple R-ratios, these must be specified separately.

*Material Manager usage:* In the material property editor, from the Test Data region, select **R-Values**. Enter the list of R-values in order of lowest to highest. Select **OK**.

*Text file usage:*

Keyword	Parameter	Data line(s)
*R RATIOS	(none)	First (and only) line: $R_1, R_2, \dots, R_j$

#### 5.6.4 Defining the fatigue limit for user S-N data

If the stress cycle is below the fatigue limit, Quick Fatigue Tool may assume zero damage. The fatigue limit is usually calculated from the value of the Constant Amplitude Endurance Limit (defined in Material Manager). However, it is possible to use the fatigue limit directly from the S-N data. When USE\_SN=1 in the job file Quick Fatigue Tool determines the fatigue and endurance limits based on the following logic:

Status in Material Manager	Text file usage	Fatigue limit definition	Endurance limit definition
n	*CAEL n 1.0	<b>USE_SN=[1 0]:</b> Derived from $n$ using the definition of the <code>fatigueLimitSource</code> environment variable (unless fatigue limit is user-defined)	<b>USE_SN=[1 0]:</b> $n$
2e+07	*CAEL 2e+07 0.0	<b>USE_SN=1:</b> The S-value corresponding to the last N-value on the user S-N curve  <b>USE_SN=0:</b> Derived from $2 \times 10^7$ reversals using the definition of the <code>fatigueLimitSource</code> environment variable (unless fatigue limit is user-defined)	<b>USE_SN=1:</b> The N-value corresponding to the last S-value on the user S-N curve  <b>USE_SN=0:</b> $2 \times 10^7$ reversals

If the fatigue limit is derived from the S-N data and multiple S-N curves are defined, Quick Fatigue Tool uses the  $R = -1$  curve (interpolated if necessary).

Not all materials exhibit an endurance limit. Therefore, Quick Fatigue Tool only enforces the limit in certain conditions. Settings related to the endurance limit are described in Appendix I.

## 5.7 Estimating material properties

### 5.7.1 Overview

Fatigue material properties are difficult to find, and often only partial data is available. The Material Manager uses an “opt-out” logic whereby Quick Fatigue Tool will automatically attempt to approximate any material property which is not user-defined, using a specified regression algorithm. These properties are distinguished by light blue and grey input fields. If a property is approximated, the method selected in the *Regression* drop-down menu will be used. For detailed information on each regression method, consult *Appendix III: Materials data generation*.

A material property has one of four statuses: **Undefined**, **user-defined**, **approximated** or **default**.

The material estimation logic is described in the table below.

	<ul style="list-style-type: none"><li>• The property is unspecified</li><li>• The property cannot be approximated</li><li>• The property is <b>undefined</b></li></ul>
2e+07	<ul style="list-style-type: none"><li>• The property is specified indirectly</li><li>• The property cannot be approximated</li><li>• Quick Fatigue Tool will use the <b>default</b> value</li></ul>
	<ul style="list-style-type: none"><li>• The property is unspecified</li><li>• Quick Fatigue Tool will attempt to <b>approximate</b> the property</li><li>• If the property cannot be approximated, it will be <b>undefined</b></li></ul>
930	<ul style="list-style-type: none"><li>• The property is specified indirectly</li><li>• Quick Fatigue Tool will attempt to <b>approximate</b> the property first</li><li>• If the property cannot be approximated, the <b>user-defined</b> value will be used</li></ul>
930	<ul style="list-style-type: none"><li>• The property is specified directly</li><li>• Quick Fatigue Tool will not attempt to approximate the property</li><li>• The property is <b>user-defined</b></li></ul>

### 5.7.2 Disabling material approximation

The aforementioned logic of automatically approximating undefined material properties can be disabled.

*Material Manager usage:* In the material property editor, from the Derivation region, select **None** from the Regression drop-down box.

*Text file usage:*

<i>Keyword</i>	<i>Parameter</i>	<i>Data line(s)</i>
*REGRESSION	NONE	(none)

## 6. Analysis algorithms

### 6.1 Background

The choice of fatigue analysis algorithm is very important for obtaining a good correlation between the applied stresses and the fatigue life. This section explains the algorithms available in Quick Fatigue Tool and recommendations for how they could be applied.

Below is a summary of the available algorithms and their applications.

Algorithm	Application	Job file option
Uniaxial Strain-Life	<ul style="list-style-type: none"><li>Uniaxial strains <b>only</b></li></ul>	ALGORITHM = [3.0   'uniaxial strain'];
Stress-based Brown-Miller	<ul style="list-style-type: none"><li>General</li><li>Ductile metals</li></ul>	ALGORITHM = [4.0   'sbbm'];
Normal Stress	<ul style="list-style-type: none"><li>General</li><li>Brittle metals</li><li>Engineering plastics</li></ul>	ALGORITHM = [5.0   'normal'];
Findley's Method	<ul style="list-style-type: none"><li>Compliance - Marine/Automotive</li><li>Ductile and brittle metals, crankshafts</li></ul>	ALGORITHM = [6.0   'findley'];
Stress Invariant Parameter	<ul style="list-style-type: none"><li>General, compliance</li></ul>	ALGORITHM = [7.0   'invariant'];
BS 7608	<ul style="list-style-type: none"><li>Compliance - Offshore</li><li>Welded steel joints</li><li>Axially loaded bolts</li></ul>	ALGORITHM = [8.0   'weld'];
NASALIFE	<ul style="list-style-type: none"><li>Compliance – Aerospace</li><li>Aero engine components</li></ul>	ALGORITHM = [9.0   'nasalife'];
Uniaxial Stress-Life	<ul style="list-style-type: none"><li>Uniaxial stresses <b>only</b></li></ul>	ALGORITHM = [10.0   'uniaxial stress'];
User-defined	<ul style="list-style-type: none"><li>N/A</li></ul>	ALGORITHM = [11.0   'user'];

## 6.2 Stress-based Brown-Miller

### 6.2.1 Overview

The Brown-Miller algorithm postulates that the fatigue damage is dominated by the combination of shear and normal strain [18] [19] [20]:

$$\frac{\Delta\gamma_{max}}{2} + \frac{\Delta\varepsilon_N}{2} = 1.65 \frac{\sigma_f'}{E} (2N_f)^b + 1.75 \varepsilon_f' (2N_f)^c \quad [6.2.1]$$

where  $\frac{\Delta\gamma_{max}}{2}$  is the maximum shear strain amplitude,  $\frac{\Delta\varepsilon_N}{2}$  is the normal strain amplitude,  $\sigma_f'$  is the tensile fatigue strength coefficient,  $E$  is Young's Modulus,  $b$  is Basquin's exponent,  $\varepsilon_f'$  is the fatigue ductility coefficient,  $c$  is the fatigue ductility exponent and  $N_f$  is the life in repeats (cycles).

The Stress-based Brown-Miller is the same algorithm, but the damage parameter is stress-based. Thus, Equation 6.2.1 becomes:

$$\frac{\Delta\tau_{max}}{2} + \frac{\Delta\sigma_N}{2} = E \left\{ 1.65 \frac{\sigma_f'}{E} (2N_f)^b + 1.75 \varepsilon_f' (2N_f)^c \right\} \quad [6.2.2]$$

Elastic
Plastic

where  $\frac{\Delta\tau_{max}}{2}$  is the maximum shear stress amplitude and  $\frac{\Delta\sigma_N}{2}$  is the normal stress amplitude. The right-hand side of Equation 6.2.2 is multiplied by Young's Modulus to retain homogeneity.

The Brown-Miller strain-life curve is shown in Figure 6.1.1:

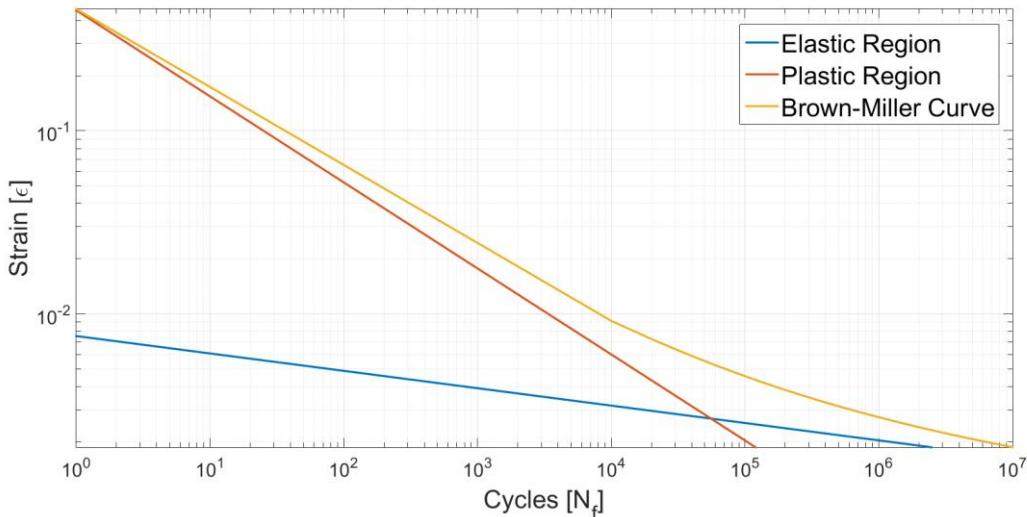


Figure 6.1.1: Brown-Miller strain-life curve, with separate elastic and plastic regions shown.

For lives greater than one million cycles, the Brown-Miller curve closely resembles its elastic constituent. Therefore, for HCF applications it is usually sufficient to assume that plasticity effects are sufficiently small so that the plastic portion of the equation can be neglected. This is the default behaviour in Quick Fatigue Tool; however, the plastic portion of the equation can be activated with

the environment variable `plasticSN`. This invokes one-dimensional interpolation and can cause the analysis time to increase significantly.

The Stress-based Brown-Miller algorithm gives the best results for ductile metals. Using the algorithm for brittle materials can result in non-conservative fatigue life predictions.

The following material properties are required to perform an analysis with the Stress-based Brown-Miller algorithm:

Property	Symbol	Importance
Tensile fatigue strength coefficient	$\sigma_f'$	REQUIRED
Tensile fatigue strength exponent	$b$	REQUIRED
Young's Modulus	$E$	REQUIRED
Fatigue ductility coefficient	$\varepsilon_f'$	OPTIONAL
Fatigue ductility exponent	$c$	OPTIONAL

### 6.2.2 Using stress-life data

Due to the nature of the Stress-based Brown-Miller equation, results obtained from stress-life data can differ significantly compared to the use of material coefficients. When using stress-life data with `USE_SN = 1.0` in the job file, Quick Fatigue Tool uses the damage parameter to interpolate the endurance curve. Since the endurance curve typically arises from stress-based testing and the Stress-based Brown-Miller equation take sit form from its strain-life counterpart, the corresponding life values are not guaranteed to be the same.

### 6.2.3 Cycle counting

The Stress-based Brown-Miller algorithm uses the normal and shear stress amplitude to define the total damage parameter. This poses an additional challenge for the cycle counting process, because the formulation in Equation 6.2.2 suggests that the damage parameter is the sum of the cycle counted normal and shear stress. However, if these two quantities are cycle counted before being summed, there is no guarantee that the counted histories will still have the same length, and matrix addition may not be immediately possible.

The alternative is to combine the normal and the shear stress beforehand, and cycle count the single combined parameter. This circumvents the issue of matrix addition, but may lead to incorrect fatigue results. For example, consider the normal and shear histories [100, 50] and [20, 50], respectively. Their individual amplitudes are 25 and 15, respectively, meaning that the sum of their amplitudes is 40. If the parameters are combined first to give the history [120, 100], the resulting amplitude is 10. Therefore, cycle counting the combined history may lead to a totally different value of fatigue damage.

The user can control the order of operations to suit their needs.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
rainflowAlgorithm	<i>n;</i>

The value of *n* dictates the following:

1. Combine the normal and shear parameters, then count the resulting history (default)
2. Count the normal and shear parameters separately, then combine the resulting histories

The second method is considered by the author to be the most physically correct approach, although it is significantly more time-consuming. Before the cycle counted normal and shear stress histories are combined, Quick Fatigue Tool checks the length of each history and resamples the shorter parameter in order to allow matrix addition. If significant resampling is required, the combined stress parameter may no longer be accurate. In such cases, the user should compare the fatigue life results for both cycle counting methods.

Testing reveals that for the majority of simple load cases, there is little or no difference in the fatigue result between the two methods. For loads where there is a significant phase difference between the normal and shear stresses, the user should compare the fatigue life results for both cycle counting methods and elect the method which offers the favourable accuracy-to-time ratio. The default setting is the first approach.

## 6.3 Normal Stress

### 6.3.1 Overview

The Normal Stress algorithm uses the normal stress amplitude as the damage parameter in the stress-life equation:

$$\frac{\Delta\sigma_{max}}{2} = \sigma_f'(2N_f)^b \quad [6.3.1]$$

where  $\frac{\Delta\sigma_{max}}{2}$  is the maximum normal stress amplitude on the critical plane and the other symbols have their usual meaning. The algorithm predicts that the fatigue strength in torsion and tension is the same. In reality, the allowable normal stress in torsion is approximately 60% of the allowable axial stress. Therefore, the Normal Stress algorithm only provides accurate fatigue life estimates for brittle materials whose crack initiation is dominated by normal stresses. The algorithm is highly non-conservative for ductile metals where the fatigue life is dominated by shear stresses [21].

The following material properties are required to perform an analysis with the Normal Stress algorithm:

Property	Symbol	Importance
Tensile fatigue strength coefficient	$\sigma_f'$	REQUIRED
Tensile fatigue strength exponent	$b$	REQUIRED

### 6.3.2 Critical plane searching

The principal stress is often used as an effective stress parameter; since it is an invariant quantity, it is tempting to bypass critical plane searching. However, even for simple loadings, relying on the value of the maximum principal stress can yield unexpected results. Take the simple stress tensor given by Equation 6.3.2:

$$\sigma_{ij} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [6.3.2]$$

The resulting principal stress state when the above tensor is subjected to a fully-reversed loading event,  $[1, -1]$ , is given by Figure 6.3.1.

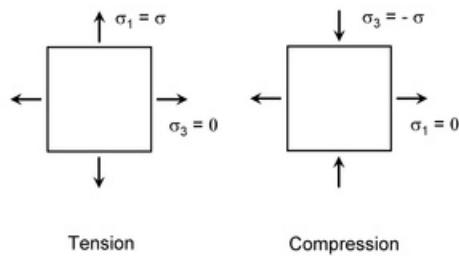
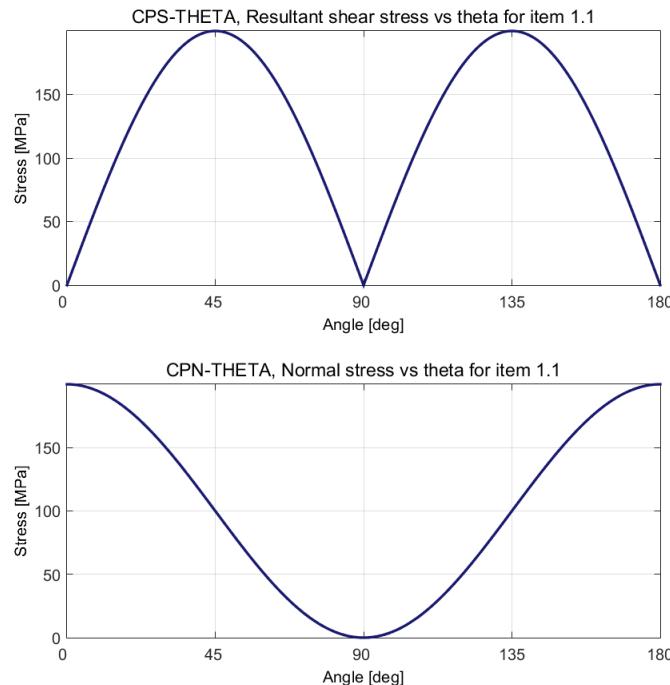


Figure 6.3.1: Principal stress state due to fully-reversed load

Since the principal stresses are ordered such that  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ , a simple tension-compression event causes the principal direction to rotate by 90 degrees. This problem is avoided if the normal tensile stresses are calculated over a series of planes using critical plane searching [22]. The critical plane is defined as the plane which experiences the largest combination of normal stress range and mean stress.

The Normal Stress algorithm posits that a component subjected to a uniaxial stress cycle,  $[\sigma_{11}, -\sigma_{11}]$ , will fail on a plane where the shear stress is zero. The normal and shear stress on the critical plane using the Normal Stress algorithm are illustrated in Figure 6.3.2.



**Figure 6.3.2: Normal and resultant shear stress on the critical plane for uniaxial tension**

The critical plane occurs when theta is zero, which corresponds to one of the principal planes.

## 6.4 Findley's Method

### 6.4.1 Overview

Findley's Method proposes that crack initiation is due to the combined effect of the average normal stress and the alternating shear stress on the critical plane [23] [24] [25]:

$$\frac{\Delta\tau}{2} + k\sigma_n \Big|_{max} = \tau_f^*(N_f)^b \quad [6.4.1]$$

where  $\frac{\Delta\tau}{2}$  is the maximum shear stress amplitude on the critical plane,  $\sigma_n$  is the normal stress,  $k$  is a material constant describing the sensitivity of the material to normal stresses and  $\tau_f^*$  is a function of the torsional fatigue strength coefficient.

Findley's Method was originally presented in the form of a safety factor; however, by introducing the stress-life curve defined by  $N_f$  and  $b$ , the algorithm is also well-suited to finite, HCF life prediction. The value of  $k$  is determined experimentally from the tensile and torsional fatigue limit, and determines the influence of the normal stress on the calculated fatigue damage. Consider the example where a specimen under torsional loading experiences a very large mean normal stress. Even if the normal stress amplitude is small (close to static), the predicted fatigue damage from Findley's Method can be very conservative. The normal stress sensitivity constant acts to attenuate the effect of  $\sigma_n$  on the loading and can be considered a form of mean stress correction.

Another advantage of Findley's Method is that it is well-suited to both brittle and ductile metals. The tensile and torsional fatigue limit can be used to "tune" a value of  $k$  which accurately characterises the material response.

Work by Kallmeyer et al. has shown that the Findley critical plane method provides the best representation for smooth bar data, which gives the method significance in applications involving shafts under shear loads [26] [27].

The following material properties are required to perform an analysis with the Findley algorithm:

Property	Symbol	Importance
Tensile fatigue strength coefficient	$\sigma_f'$	REQUIRED
Tensile fatigue strength exponent	$b$	REQUIRED
Normal stress sensitivity constant	$k$	REQUIRED <sup>3</sup>
Modified fatigue shear strength coefficient	$\tau_f^*$	REQUIRED <sup>4</sup>
Poisson's Ratio	$\nu$	OPTIONAL
Tensile Fatigue Strength Limit	$f$	OPTIONAL
Torsional Fatigue Strength Limit	$t$	OPTIONAL
Ultimate Tensile Strength	$\sigma_U$	OPTIONAL

<sup>3</sup> If no value is specified, a default value is used.

<sup>4</sup> This parameter is computed automatically.

#### 6.4.2 Determining the value of $\tau_f^*$

The modified fatigue shear strength coefficient is calculated from the standard equation:

$$\tau_f^* = \sqrt{1 + k^2} \tau_f' \quad [6.4.2]$$

where  $\tau_f'$  is the fatigue shear strength coefficient.  $\tau_f'$  is obtained from the following table:

Material	Fatigue Shear Strength Coefficient
Wrought steel and alloys	$\tau_f' \approx 0.75\sigma_f'$
Ductile iron	$\tau_f' \approx 0.90\sigma_f'$
Malleable iron – pearlitic structure	$\tau_f' \approx 1.00\sigma_f'$
Wrought iron	$\tau_f' \approx 0.83\sigma_f'$
Cast iron	$\tau_f' \approx 1.30\sigma_f'$
Aluminium/copper and alloys	$\tau_f' \approx 0.65\sigma_f'$
Other	$\tau_f' \approx \frac{\sigma_f'}{2(1+v)}$

#### 6.4.3 Determining the value of $k$

The normal stress sensitivity constant is determined by comparing the fatigue limit of a material under tension and torsion fatigue tests. The value of  $k$  is specified in the *Non-standard* region of the material editor (Figure 6.4.1).

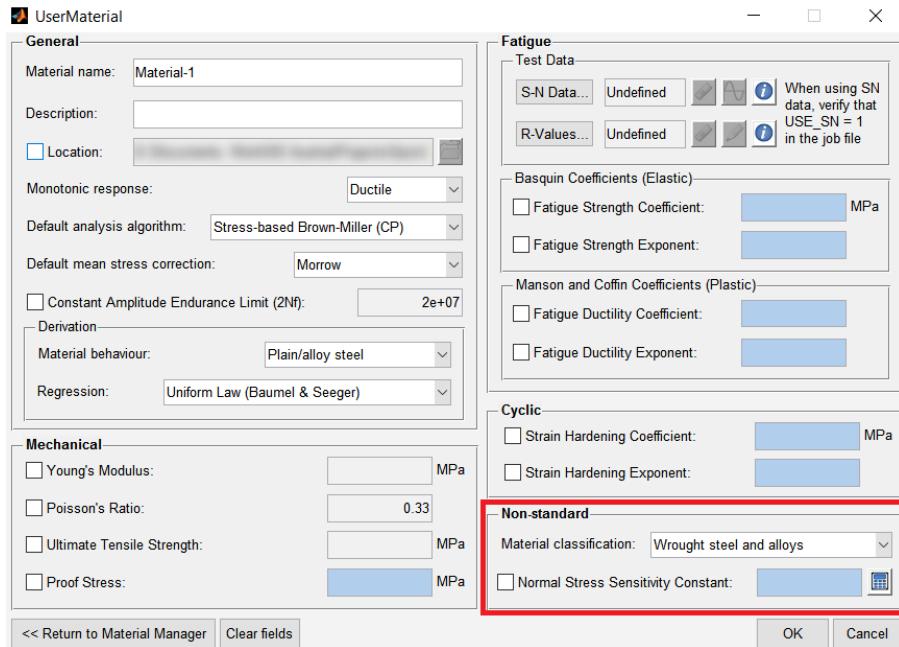
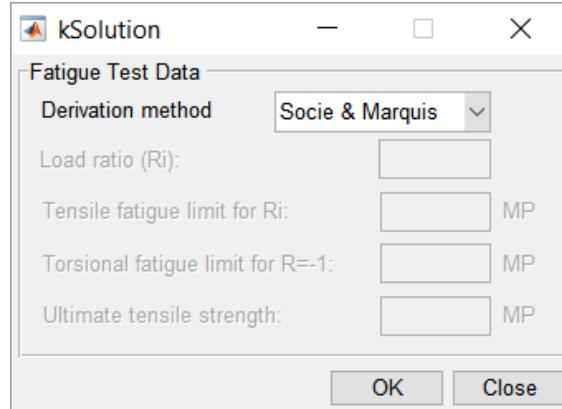


Figure 6.4.1: Definition of the normal stress sensitivity constant in the material editor

The value can either be specified directly by checking the box next to the input box, or a value may be calculated based on the fatigue limit. This is done by clicking on the calculator button. The resulting dialogue box is shown in Figure 6.4.2.



**Figure 6.4.2: Calculator tool to estimate the value of  $k$**

The value of  $k$  is calculated according to the table below. Note that if the input field is left blank, the Socie & Marquis value of 0.2857 will be assumed [28]. For ductile materials  $k$  has a solution in the range of 0.2 – 0.3 [29].

Derivation Method	Solution
Socie & Marquis	$k = 0.2857$
General formula	$\frac{f_R}{t_{-1}} = \frac{2 \cdot \sqrt{1 + k^2}}{\sqrt{\left(\frac{2k}{1-R}\right)^2 + 1} + \frac{2k}{1-R}}$
Dang van	$k = \frac{3t_{-1}}{f_{-1}} - \frac{3}{2}$
Sines	$k = \frac{3t_{-1}(\sigma_U + f_{-1})}{\sigma_U \cdot f_{-1}} - \sqrt{3}$
Crossland	$k = \frac{3t_{-1}}{f_{-1}} - \sqrt{3}$

Where  $f_R$  is the tensile fatigue limit for a load ratio  $R$ ,  $f_{-1}$  is the fully-reversed tensile fatigue limit and  $t_{-1}$  is fully-reversed torsional fatigue limit. Since the derivation models attempt to approximate  $k$  based on the fatigue limit, they are not guaranteed to find a solution. In such cases, the Socie & Marquis value may be used.

Note that defining  $k$  as zero makes Findley's Method a maximum shear stress criterion.

#### 6.4.4 Critical plane searching

Findley's Method uses critical plane searching to determine the value of  $\frac{\Delta\tau}{2} + k\sigma_n|_{max}$ . The stress tensor on the critical plane is split into one normal and two shear components:

$$\sigma_n = \sigma'(1,1)$$

$$\tau_{xy} = \sigma'(1,2) \quad [6.4.3]$$

$$\tau_{xz} = \sigma'(1,3)$$

The variable  $\sigma'$  is described in Appendix I. The normal and shear stress history is illustrated for a plane  $\Delta$  of an arbitrary orientation by Figure 6.4.3.

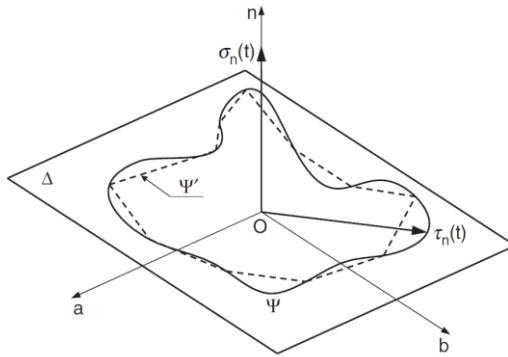


Figure 6.4.3: Normal and shear stress relative to plane  $\Delta$

On these planes, the  $n$  direction is always perpendicular to the  $ab$  plane on which the shear stresses act. The quantity  $\tau_n(t)$  is the resultant shear stress history, which scribes the path  $\Psi$ . The value of the maximum normal stress is simply the maximum value of the normal stress history,  $\sigma_n(t)$ . Determining the value of the maximum shear stress is less trivial.

Several methods have been proposed for determining the maximum shear stress. These include, but are not limited to, the longest chord [30], the longest projection [31], the minimum circumscribed circle [32], the minimum circumscribed ellipse [33] [34] and the maximum variance [35] [36] of the path  $\Psi$ . It has been noted by Susmel [25] that the longest chord method is not only simple but also very effective when applied in conjunction with the critical plane concept. Quick Fatigue Tool uses the longest chord method to determine the maximum shear stress history on the critical plane. This is given by Equation 6.4.4.

$$\tau_{n,a} = \frac{1}{2} \max_{t_1 \in T} \left[ \max_{t_2 \in T} |\tau_n(t_1) - \tau_n(t_2)| \right] \quad [6.4.4]$$

where  $t_1$  and  $t_2$  are two instants of the cyclic load history having period equal to  $T$ . The maximum chord method requires every shear pair along  $\Psi$  to be compared. Therefore, for large histories the analysis may be slowed down significantly. In cases where the calculation time of the maximum shear stress is unreasonable, the maximum resultant shear stress may be used instead:

$$\tau_{n,a} = \sqrt{\tau_{xy}^2 + \tau_{xz}^2} \quad [6.4.5]$$

This option is specified by in the environment file

*Environment file usage:*

Variable	Value
cpShearStress	[1.0   2.0];

The maximum chord method suffers from a theoretical set-back. Figure 6.4.4 illustrates how the mean value of the shear stress cannot be defined with certainty when two or more reference chords having the same length can be defined. However, Susmel states that from a practical point of view, this ambiguity should not affect the accuracy in estimating high-cycle fatigue since the torsional mean stress can be neglected provided that the maximum shear stress in the loading does not exceed the material torsional yield strength.

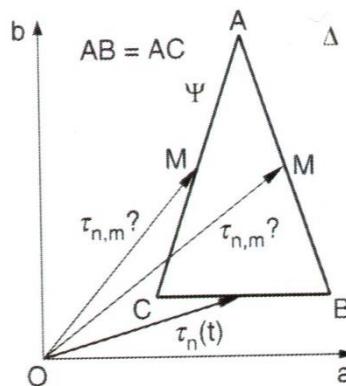
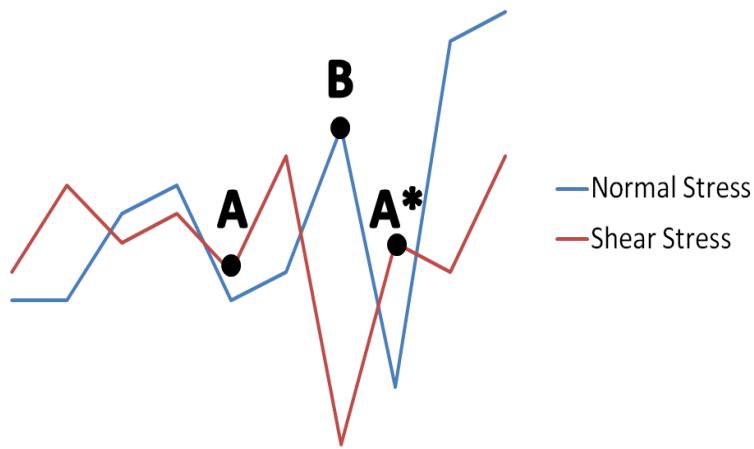


Figure 6.4.4: Limitation of the longest chord method

The maximum chord method is only applied to the calculation of the maximum shear stress history on the critical plane (output variable **CS**). For the critical plane analysis, the cycle counted shear quantity is the resultant shear stress given by Equation 6.4.5.



**Figure 6.4.5:** A shear stress cycle with start and end points A and A\*, respectively. By default, the normal stress associated with the shear cycle A-A\* is the maximum normal stress occurring over the period of the shear cycle

The maximum shear stress history on each plane is cycle counted using the Rainflow method described in Appendix I. It is not obvious how the maximum normal stress should be combined with the shear stress cycles. This issue is illustrated by Figure 6.4.5.

By default, Quick Fatigue Tool uses the maximum normal stress which occurs in the interval of each shear cycle. This behaviour can be changed to use the average normal stress in each shear cycle interval or the maximum normal stress over the entire loading. The latter is the most conservative approach. Treatment of the normal stress is specified in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>findleyNormalStress</code>	[1.0   2.0   3.0];

#### 6.4.5 Output

The variables WCM and SFA have a slightly different meaning when using Findley's Method:

Variable	Usual meaning	Findley's Method
<i>WCM</i>	Mean value of the damage parameter on the critical plane (algorithm-dependent)	Mean value of the resultant shear stress on the critical plane
<i>SFA</i>	Ratio between the material fatigue limit and the maximum stress in the loading	Ratio between the material fatigue limit and the Findley parameter

#### 6.4.6 Limitations

The critical plane search algorithm used by Findley's Method is found to be very sensitive to the search increment when compared to other algorithms which use critical plane searching. Therefore, Findley's Method is not compatible with load proportionality checking. The step size will always be the value defined in the environment file.

Literature sources reference the shear stress in conjunction with Rainflow cycle counting. However, it is not clear which shear stress quantity should be cycle counted. Currently, Quick Fatigue Tool uses the resultant shear stress history as the cycle counting parameter. This has the drawback that the resultant shear stress is always positive. In order to circumvent this problem, a scheme similar to that used by the Stress Invariant Parameter algorithm is used, whereby the shear stress history is multiplied by a factor based on a pre-determined sign convention (Section 6.5.3).

## 6.5 Stress Invariant Parameter

### 6.5.1 Overview

The Stress Invariant Parameter analysis algorithm uses a Cauchy stress invariant term as the damage parameter in the stress-life relationship:

$$\frac{\Delta\sigma_{eff}}{2} = \sigma_f'(2N_f)^b \quad [6.5.1]$$

where  $\frac{\Delta\sigma_{eff}}{2}$  is the effective stress amplitude. The user can specify one of the following stress invariant parameters as the effective stress amplitude:

0. Program controlled
1. von Mises
2. Principal
3. Hydrostatic
4. Tresca

The parameter is specified in the environment file.

*Environment file usage:*

Variable	Value
stressInvariantParameter	<i>n;</i>

The value of *n* takes its meaning from the list above.

The following material properties are required to perform an analysis with the Stress Invariant Parameter algorithm:

Property	Symbol	Importance
Tensile fatigue strength coefficient	$\sigma_f'$	REQUIRED
Tensile fatigue strength exponent	<i>b</i>	REQUIRED

## 6.5.2 Effective stress parameters

### von Mises

The von Mises stress is based on the second invariant stress, and provides an estimate of the onset of yielding. The von Mises stress is given by Equation 6.5.2:

$$\sigma_{eff} = \sqrt{\frac{3}{2} s_{ij} s_{ij}} \quad [6.5.2]$$

where  $s_{ij}$  are the components of the stress deviator tensor  $\sigma^{dev}$ :

$$\sigma^{dev} = \sigma - \frac{1}{3} (tr \sigma) I \quad [6.5.3]$$

### Principal

The principal stress parameter defines the load history as the largest (positive or negative) principal stress at each point in the loading. For example, if the absolute value of the minimum principal stress is larger than the value of the maximum principal stress at a given loading point, then the minimum principal stress is used for that point. The following load history illustrates the use of the principal stress as the invariant parameter.

<b>S1</b>	349	294	174	441
<b>S3</b>	-294	-349	-147	-523
<b>Load history</b>	349	-349	174	-523

The principal stress is valid for uniaxial test data. Loadings which exhibit a high degree of biaxiality do not correlate well to the principal stress, since failure is not guaranteed to occur at the locations of maximum stress. For loadings which exhibit a high degree of non-proportionality, the direction of the principal stress will change throughout the history; in such cases, the Normal Stress algorithm is recommended instead.

### Hydrostatic

The hydrostatic stress defines the load history in terms of the equivalent pressure stress.

$$\sigma_{eff} = -\frac{1}{3} (tr \sigma) = -\frac{1}{3} \sigma_{ii} \quad [6.5.4]$$

The hydrostatic stress is an isotropic parameter given by the average of the direct pressure forces acting on a body. Deformation states dominated by expansion correlate well with a hydrostatic criterion. The hydrostatic stress parameter is less conservative than the principal stress parameter.

## Tresca

The Tresca stress is defined as the maximum difference between principal stresses.

$$\sigma_{eff} = \sigma_1 - \sigma_3 \quad [6.5.5]$$

The Tresca stress is the maximal shear stress and is used as a yield criterion for ductile metals. As a fatigue criterion, the Tresca stress assumes that crack initiation is driven by states of pure shear. This can provide reasonable estimates for shear-dominated loads with a high degree of proportionality. In all other cases, a balanced shear-normal biaxial criterion such as the Stress-based Brown-Miller algorithm or Findley's method is recommended instead.

## Program controlled

Quick Fatigue Tool can attempt to choose a suitable stress invariant parameter. The applicability of a given stress invariant parameter depends on the biaxiality ratio,  $\alpha$ :

$$\alpha = \frac{\sigma_2}{\sigma_1} \quad [6.5.6]$$

Stress invariants are applicable to uniaxial ( $\alpha = 0$ ) and equibiaxial ( $\alpha = 1$ ) loads, as well as proportional biaxial loads ( $-1 \leq \alpha \leq 1$ ) [37]. The table below shows the applicable range of the biaxiality ratio for each stress invariant parameter.

Invariant	Range of $\alpha$
von Mises	$\alpha = 0$ and $\alpha = 1$
Principal	$-1 \leq \alpha \leq 0$
Hydrostatic	$-1 \leq \alpha \leq 0$
Tresca	$0 \leq \alpha \leq 1$

The stress invariant parameter is chosen on the basis of the applicable range of  $\alpha$ . If no suitable parameter can be found, the principal stress is used by default.

The range of  $\alpha$  over the loading is printed in the message file at the item with the largest principal stress, for each analysis group.

### 6.5.3 Specifying a sign convention

The von Mises and Tresca stresses are always positive, meaning that damage in compression is neglected. A material element in a state of pure hydrostatic compression appears to experience zero effective stress, even if the volumetric deformation is large enough to cause fatigue damage. The solution is to correct the stresses by using a criterion which determines the correct sign of the effective stress parameter. Quick Fatigue Tool corrects the stresses based either on the sign of the hydrostatic stress or the largest principal stress. The sign convention is set from the environment file.

*Environment file usage:*

Variable	Value
signConvention	[1.0   2.0   3.0];

### 6.5.4 Additional guidance

The stress invariant parameters do not correlate well to multiaxial stress states. Although the algorithm can be used to quickly locate the region of expected maximum stress, the location of maximum damage can often be elsewhere due to the fact that the in-plane principal directions can change during the loading. Thus, the Stress Invariant Parameter analysis algorithm is included for completeness only; none of the invariants are recommended as a damage parameter except for the simplest cases.

The user should check the validity of the selected stress invariant parameter before the analysis. This can be done by setting DATA\_CHECK=1 in the job file and inspecting the message file for feedback regarding the selected parameter. The following points should be observed:

- for  $-1 \leq \alpha < 0$ , the Tresca stress is very conservative; the von Mises stress is conservative; the principal stress is acceptable;
- for  $\alpha = 0$ , the Tresca, von Mises and principal stresses are all acceptable;
- for  $0 < \alpha < 1$ , the Tresca stress is acceptable; the von Mises stress is non-conservative; and
- for  $\alpha = 1$ , both the Tresca and von Mises stresses are acceptable.

Critical plane searching is not required, and rough estimates of life can be obtained very quickly. However, the Stress Invariant Parameter algorithm is not considered to be a valid durability assessment criterion for general fatigue analysis problems [6].

## 6.6 BS 7608 Fatigue of Welded Steel Joints

### 6.6.1 Overview

Quick Fatigue Tool includes an implementation of the British Standard BS 7608:1993 code of practice for fatigue design and assessment of steel structures [38]. The standard is applicable to the following:

- a) Parent material remote from joints
- b) Welded joints (in air or sea water) in such material
- c) Bolted or riveted joints in such material
- d) Shear connectors between concrete slabs and steel girders acting compositely in flexure

The standard offers a family of  $S_r - N$  curves based on weld geometry criteria, spanning ten weld classifications. The damage parameter is the stress range acting on the critical plane. The code stipulates that the stress range is whichever of the two in-plane principal stresses lie within +/- 45 degrees of an axis perpendicular to the weld toe. However, Quick Fatigue Tool performs a full critical plane search in a spherical coordinate space. The user may choose between the normal and the shear stress as the damage parameter.

The standard is based on the assumption that the exact stresses at the weld toe cannot be determined analytically. The provided  $S_r - N$  curves account for the effect of the stress concentration, thus the analyst need only compute the stresses as if the weld feature did not exist. If the stress solution is obtained from finite element analysis and the weld detail is modelled, the calculated stress range may be greatly overestimated and could result in highly conservative fatigue life predictions. Therefore, in such cases the analyst should choose the stress a short distance away from the weld toe, an example of which is given in Figure 6.6.1.

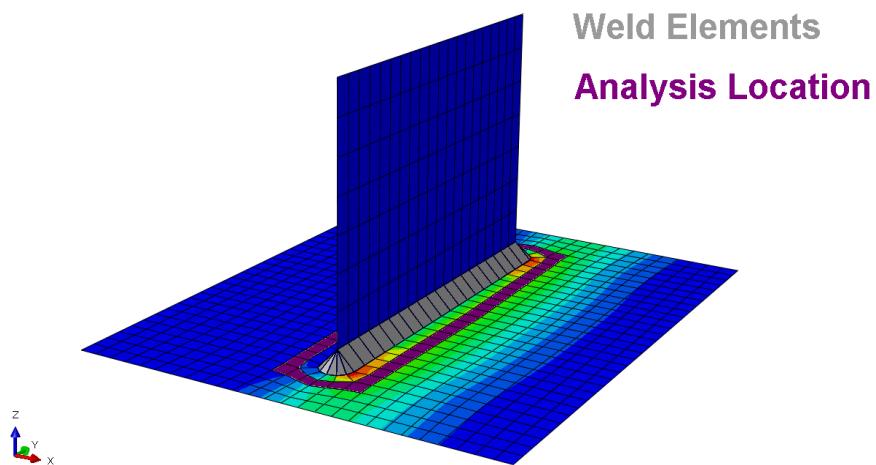


Figure 6.6.1: Example analysis location for BS 7608

### 6.6.2 Derivation of the $S_r - N$ curve

The  $S_r - N$  curve is defined explicitly as Equation 6.6.1

$$S_r^m N = C_d \quad [6.6.1]$$

where  $S_r$  is the stress range,  $m$  is the Paris Law exponent related to the energy release rate of a crack,  $N$  is the number of cycles to failure and  $C_d$  is a constant relating to the weld classification. The resulting  $S_r - N$  curves are shown in Figure 6.6.2. The curves are material-independent and as such, no material needs to be specified in the job file.

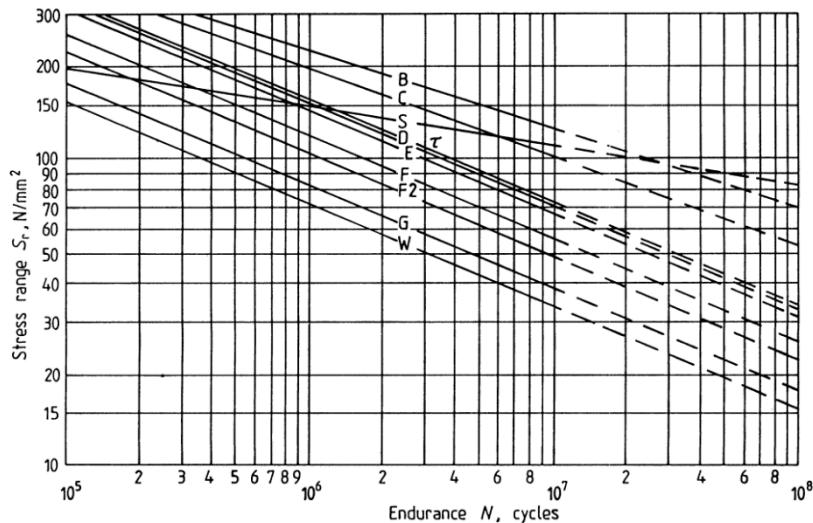


Figure 6.6.2:  $S_r - N$  curve family for BS 7608

The weld classification constant,  $C_d$ , is related to the probability of failure by the number of standard deviations from the mean  $S_r - N$  curve. The number of deviations,  $d$ , can be specified in the job file. For example, a value of 2.0 means that there is a 97.7% probability that the component will fail before the predicted life. Some values of  $d$  and their corresponding probabilities of failure are given below.

Probability of failure (%)	$d$
50	0.0
69	0.5
84	1.0
97.7	2.0
99.86	3.0

A value of  $d = 0$  corresponds to the mean-line  $S_r - N$  curve, while a value of  $d = 2$  corresponds to the standard design curve.

### 6.6.3 Analysis of axially loaded bolts

In addition to welded joints, BS 7608 also offers a set of  $\frac{S_r}{UTS} - N$  curves for axially loaded bolts with cut, ground or rolled threads up to 25mm in diameter. These curves belong to class X. The curves are defined by Equations 6.6.2-3 and illustrated by Figure 6.6.3.

$$\text{Mean curve: } \left( \frac{S_r}{UTS} \right)^3 N = 800 \quad [6.6.2]$$

$$\text{Mean} - 2SD: \left( \frac{S_r}{UTS} \right)^3 N = 400 \quad [6.6.3]$$

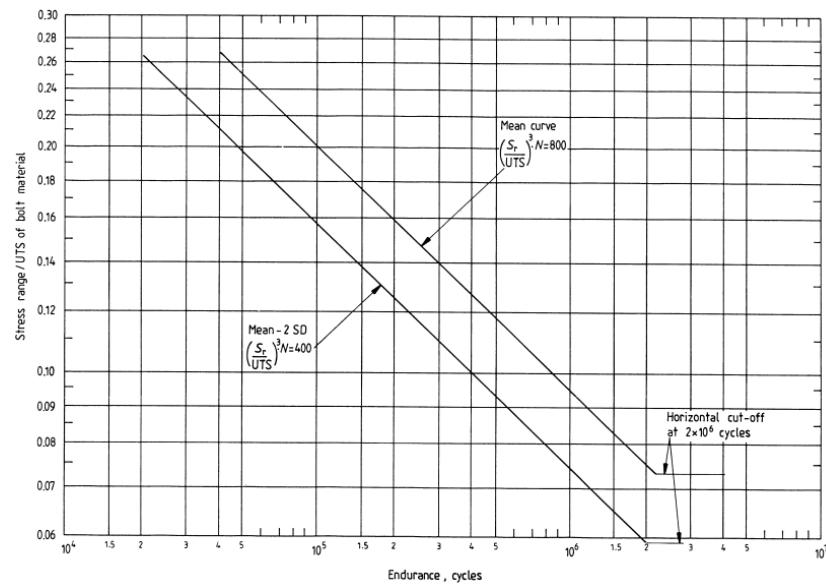


Figure 6.6.3:  $\frac{S_r}{UTS} - N$  curve family for axially loaded bolts

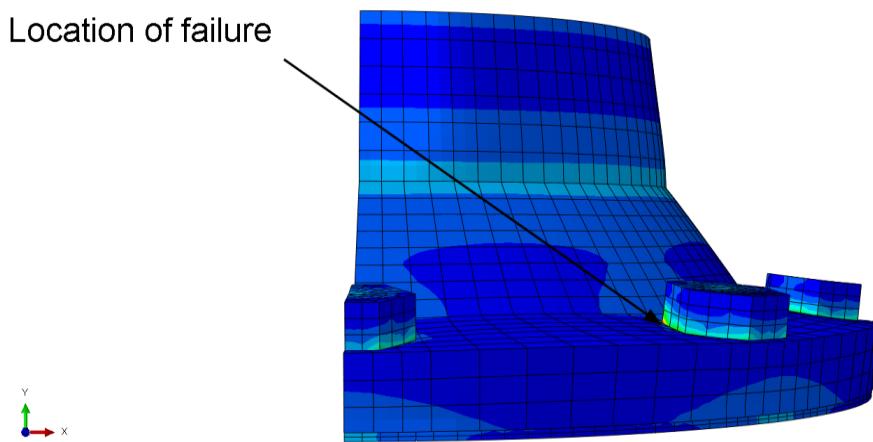


Figure 6.6.4: Example of a bolt failure in FEA

The  $\frac{S_r}{UTS} - N$  curves in this classification are only defined for the mean line or two standard deviations from the mean. The ultimate tensile strength of the bolt feature must be provided. The  $\frac{S_r}{UTS} - N$  curves are valid only for values of the ultimate tensile strength up to 785MPa.

If the bolt is modelled in FEA such as the one shown in Figure 6.6.4, the effective stress should be taken a small distance away from the stress concentration at the location of crack initiation otherwise the fatigue analysis will produce conservative results.

*Job file usage:*

<i>Option</i>	<i>Value</i>
WELD_CLASS	'X';

#### 6.6.4 Effect of the characteristic length

For welded joints, the characteristic length is the plate thickness. For class X (axially loaded bolts), the bolt diameter is used instead.

The fatigue life of welded joints and bolts reduces with increasing characteristic length. If a value for the weld length is specified in the job file, the  $S_r - N$  curve may be scaled according to      Equation 6.6.4

$$S_F = \frac{S}{S_B \left( \frac{16}{t} \right)^{0.25}} \quad [6.6.4]$$

where  $S$  is the fatigue strength of a weld (or bolt) of thickness (or diameter)  $t$  and  $S_B$  is the fatigue strength of the weld without considering the effect of thickness (or diameter). The characteristic length is given in units of mm. The  $S_r - N$  curves are already valid for the lengths given by the table below. Thus, the correction is only performed if the specified length lies outside the pre-defined range.

<b>Classification</b>	<b>Range of characteristic length</b>
Nodal joints (Class T)	16mm only
Non-nodal joints (Classes B to G)	Up to 16mm
Bolts (Class X)	Up to 25mm diameter
All other weld classes	16mm only

#### 6.6.5 Effect of stress relief

BS 7608 assumes that the stress range is the sum of the tensile and 60% of the compressive component of the cycle. For example, if a cycle has a range of -100 to 50, the effective range will be calculated as  $abs(0.6 \times -100) + 50 = 110$ .

#### 6.6.6 Effect of small cycles

BS 7608 stipulates that earlier fatigue failure could be predicted if it is assumed that all stress ranges below the fatigue limit are non-damaging. Thus, the Paris Law exponent is changed from  $m$  to  $(m + 2)$  for cycles below the fatigue limit, at which the calculated life is 1e7 cycles.

### 6.6.7 Effect of large cycles

BS 7608 stipulates that the  $S_r - N$  curves may be extrapolated no further than twice the material's yield strength. Cycles exceeding this value will result in non-fatigue failure.

### 6.6.8 Effect of exposure to sea water

Unprotected welds situated in sea water accumulate fatigue damage faster than the same weld in fresh air. If the effect of sea water is specified in the job file, the correction for small cycles is ignored and the fatigue strength of the weld is reduced by a factor of two.

### 6.6.9 Failure mode

According to BS 7608, the damage parameter is taken as the principal stress acting on the critical plane. However, the implementation in Quick Fatigue Tool allows the user to choose between a pure normal, pure shear and combined normal-shear stress criterion, depending on how the crack is expected to propagate.

*Job file usage:*

<i>Option</i>	<i>Value</i>
FAILURE_MODE	['NORMAL'   'SHEAR'   'COMBINED'];

The damage parameter quoted in the field output corresponds to either the normal, shear or combined (normal + shear) stress on the critical plane, according to the definition of the above option.

### 6.6.10 Specifying the $S_r - N$ curve as a BS 7608 weld class

The  $S_r - N$  curve may be defined by one of the following BS 7608 weld classes: 'B', 'C', 'D', 'E', 'F', 'F2', 'G', 'W', 'S' or 'T'. Choice of weld class requires knowledge of the weld features and access to *Section 2. Classification of details* from document BS 7608:1993.

*Job file usage:*

<i>Option</i>	<i>Value</i>
WELD_CLASS	'<weldClass_ID>';

where '<weldClass\_ID>' is one of the above listed string values.

### 6.6.11 Specifying the $S_r - N$ curve as user data

The  $S_r - N$  curve may be defined as tabulated data.

*Job file usage:*

<i>Option</i>	<i>Value</i>
WELD_CLASS	'user-weld-curve-file-name.sn';

The format of the user  $S_r - N$  data file should meet the requirements explained in Section 5.4. The  $S_r$  values in the data file must be provided in terms of the stress range.  $S_r$  values are assumed to be

provided at a load ratio of  $R = -1$ ;  $S_r$  values defined over multiple load ratios are not accepted. The fatigue limit is taken as the last  $S_r$  value in the data.

By default, Quick Fatigue Tool processes the user  $S_r - N$  in a column-wise fashion:

<b><math>N - Values</math></b>	<b><math>S_r - Values at R = -1</math></b>
$N_1$	$S_1$
$N_2$	$S_2$
.	.
$N_i$	$S_i$

However, both column-wise and row-wise  $S_r - N$  data may be provided by indicating this in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
WELD_CLASS	{'user-weld-curve-file-name.sn', ['ROW'   'COL']};

### 6.6.12 Compatibility with other features

The BS 7608 method is not compatible with the following analysis features:

- Mean stress correction
- Nodal elimination
- Plasticity correction
- S-N data (USE\_SN)
- S-N Scale factors (SN\_SCALE)
- S-N knockdown curves (SN\_KNOCK\_DOWN)
- Fatigue notch factors

The following output variables are not available:

- FOS, SFA, FRFH, FRFV, FRFR, FRFW

### 6.6.13 Configuring the analysis parameters

Material properties are not required to perform analyses with BS 7608. However, the user can configure algorithm-specific settings from the job file in the “Algorithm Specific Settings” section. The available options are shown below. For a description of each option, consult the document *Quick Fatigue Tool User Settings Reference Guide*.

Option	Meaning	Importance
WELD_CLASS	$S_r - N$ curve for analysis	REQUIRED
YIELD_STRENGTH	Used to set extrapolation limit	OPTIONAL
UTS	Used to define $\frac{S_r}{UTS} - N$ curve	REQUIRED <sup>5</sup>
DEVIATIONS_BELOW_MEAN	Standard deviations below mean $S_r - N$ curve	REQUIRED
FAILURE_MODE	Failure criterion (normal or shear)	OPTIONAL
CHARACTERISTIC_LENGTH	Plate thickness or bolt diameter	OPTIONAL
SEA_WATER	Fatigue strength correction for sea water exposure	OPTIONAL

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<sup>5</sup> For Class X welds only.

## 6.7 NASALIFE

### 6.7.1 Overview

NASALIFE is a fatigue life prediction software developed by General Electric Aircraft Engines and the NASA Enabling Propulsion Materials program, to assess the durability of ceramic matrix composites (CMCs) subject to varying thermo-mechanical loads. The methodology is required by some regulatory bodies in the aviation sector for the validation of aero engine components [39].

### 6.7.2 Methodology

The NASALIFE method has been partially implemented in Quick Fatigue Tool as a stress-based, HCF fatigue analysis algorithm. The analysis procedure is as follows:

1. Explore all possible stress pair (cycle) combinations in the load history
2. For each cycle, calculate the effective mean stress and stress amplitude based on the effective stress parameter
3. Using the Walker mean stress correction, find the cycle pair with the largest damage. This is the most damaging major cycle (MDMC)
4. For the MDMC, find the principal directions and orientation of the octahedral shear plane
5. Align the stress tensor history with the octahedral plane of the MDMC and convert the stress tensor history into the octahedral shear stress history
6. Rainflow cycle count the shear stress history and record the position index of each cycle in the stress history
7. Convert the effective stress history into a matrix of cycles based on the indexes from step 6
8. Repeat steps 2 and 3 to calculate the damage of each cycle

The stress tensor history is organized into all possible pairs using the combination formula given by Equation 6.7.1:

$$C_r = \frac{n!}{2!(n-2)!} \quad [6.7.1]$$

where  $C_r$  is the number of stress tensor pair combinations from a loading consisting of  $n$  history points.

Each cycle is calculated from an effective stress parameter. This parameter is explained in more detail in Section 6.7.3. For the case of the Manson McKnight parameter, the equivalent mean stress and stress amplitude are calculated by Equations 6.7.2-3, respectively.

$$\sigma_m = SIGN(I_1) \times \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xm} - \sigma_{ym})^2 + (\sigma_{ym} - \sigma_{zm})^2 + (\sigma_{xm} - \sigma_{zm})^2 + 6(\tau_{xym}^2 + \tau_{yzm}^2 + \tau_{xzm}^2)} \quad [6.7.2]$$

$$\sigma_a = \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya} - \sigma_{za})^2 + (\sigma_{xa} - \sigma_{za})^2 + 6(\tau_{xya}^2 + \tau_{yza}^2 + \tau_{xza}^2)} \quad [6.7.3]$$

Where  $I_1$  is the first stress invariant (hydrostatic stress).

Each stress cycle is corrected for the effect of mean stress using the Walker mean stress correction in Equation 6.7.4:

$$\sigma_{aw} = \sigma_a \left( \frac{2}{1 - R} \right)^{1-\gamma} \quad [6.7.4]$$

where  $\sigma_{aw}$  is the effective stress amplitude due to cycle  $\sigma_a$ , the load ratio  $R$  and the Walker parameter  $\gamma$ . The method of calculating  $\gamma$  is discussed in Section 7.7. The  $A$ -ratio is calculated as Equation 6.7.5:

$$A = \frac{\sigma_a}{\sigma_m} \quad [6.7.5]$$

If the  $A$ -ratio is less than 0 or greater than  $10^7$ , the Walker mean stress correction is modified to Equation 6.7.6:

$$\sigma_{aw} = \sigma_a [0.5(1 - R)]^{\gamma-1} \quad [6.7.6]$$

The mean stress correction is limited to positive mean stress only; negative mean stress will not increase the fatigue life of the component.

The MDMC is resolved onto octahedral planes, shown by Figure 6.7.1. The octahedral shear stress,  $\tau_{oct}$ , is given by Equation 6.7.7.

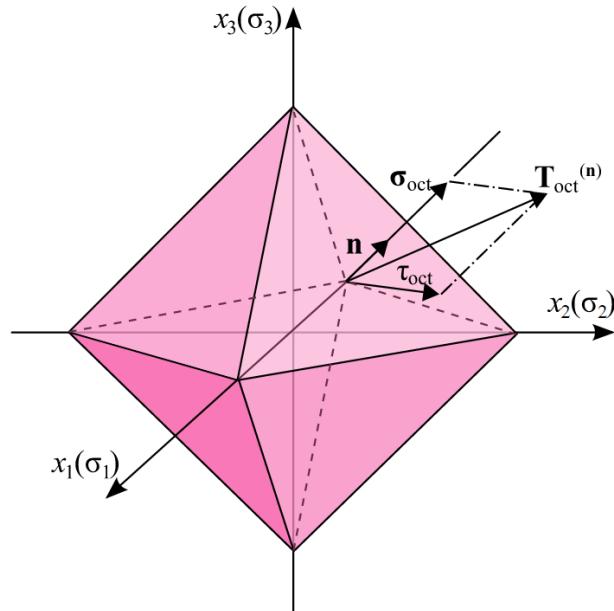


Figure 6.7.1: Octahedral planes showing the unit normal and shear directions

$$\tau_{oct} = \frac{1}{3} \sqrt{2I_1^2 - 6I_2^2} \quad [6.7.7]$$

where  $I_2$  is the second stress invariant, and is related to the von Mises stress,  $\sigma_e$ , by Equation 6.7.8.

$$\sigma_e = \sqrt{3I_2} \quad [6.7.8]$$

After identifying the octahedral shear plane, the original stress history is transformed to the principal directions of the MDMC using the rotation matrix and tensor transform given by Equations 6.7.9-10, respectively:

$$R = \begin{bmatrix} \sqrt{\frac{\sigma_1}{\sigma_{11}}} & 0 & 0 \\ 0 & \sqrt{\frac{\sigma_2}{\sigma_{22}}} & 0 \\ 0 & 0 & \sqrt{\frac{\sigma_3}{\sigma_{33}}} \end{bmatrix} \quad [6.7.9]$$

where  $\sigma_{i,j,k}$  are the principal and  $\sigma_{ii,jj,kk}$  are the normal stresses of the MDMC tensor.

$$\sigma' = R^T \sigma R \quad [6.7.10]$$

Where  $\sigma'$  and  $\sigma$  are the rotated and un-rotated stress tensors for each point in the load history, respectively.

The transformed load history is resolved into its octahedral shear component and then cycle counted. The indices of the octahedral shear cycles are then used with the original tensor history to locate stress cycles from the effective stress. The damage per cycle is calculated using Equation 6.7.11:

$$\sigma_{aw} = \sigma_f' (2N_f)^b \quad [6.7.11]$$

### 6.7.3 The effective stress parameter

The NASALIFE algorithm uses one of five effective stress parameters for the calculation of the cycle combinations. The effective stress parameter is set in the environment file.

*Environment file usage:*

Variable	Value
nasalifeParameter	[1.0   2.0   3.0   4.0   5.0];

#### 1. Manson-McKnight

This is the default effective stress parameter. The effective mean stress and stress amplitude are based on the concept of a signed von Mises stress, given by Equations 6.7.12-13. The sign is taken from the hydrostatic stress of the current cycle.

$$\sigma_m = \text{SIGN}(\sigma_{xm} + \sigma_{ym} + \sigma_{zm}) \times \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xm} - \sigma_{ym})^2 + (\sigma_{ym} - \sigma_{zm})^2 + (\sigma_{xm} - \sigma_{zm})^2 + 6(\tau_{xym}^2 + \tau_{yzm}^2 + \tau_{xzm}^2)} \quad [6.7.12]$$

$$\sigma_a = \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya} - \sigma_{za})^2 + (\sigma_{xa} - \sigma_{za})^2 + 6(\tau_{xya}^2 + \tau_{yza}^2 + \tau_{xza}^2)} \quad [6.7.13]$$

For situations where the loading is shear-dominated, the sign of the mean stress can be unreliable. Therefore, if the signs of the maximum and the minimum principal stresses differ, a modified version of the Manson-McKnight method is used instead, given by Equation 6.7.14:

$$\sigma_m = \text{SIGN}\left(\frac{\sigma_1 + \sigma_3}{\sigma_1 - \sigma_3}\right) \times \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xm} - \sigma_{ym})^2 + (\sigma_{ym} - \sigma_{zm})^2 + (\sigma_{xm} - \sigma_{zm})^2 + 6(\tau_{xym}^2 + \tau_{yzm}^2 + \tau_{xzm}^2)} \quad [6.7.14]$$

#### 2. Sines

The Sines method determines the effective mean stress as the hydrostatic stress [40]. The effective stress amplitude is then modified by the mean stress. These are given by Equations 6.7.15-16:

$$\sigma_m = \sigma_{xm} + \sigma_{ym} + \sigma_{zm} \quad [6.7.15]$$

$$\sigma_a = \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya} - \sigma_{za})^2 + (\sigma_{xa} - \sigma_{za})^2 + 6(\tau_{xya}^2 + \tau_{yza}^2 + \tau_{xza}^2)} - C\sigma_m \quad [6.7.16]$$

The constant  $C$  is intended to have a value of 0.5 for uniaxial loads and 1.0 for multiaxial loads. However, the loading is assumed to be multiaxial and a value of 1.0 is always used.

The  $A$ -ratio is assumed to be infinite, hence the Walker mean stress correction takes the modified form of Equation 6.7.6.

### 3. Smith-Watson-Topper

The Smith-Watson-Topper method assumes that the effective mean stress is zero, therefore for loadings with a load ratio of  $R \neq -1$ , use of this parameter will produce highly non-conservative results. The effective stress amplitude is a function of the maximum and the minimum value of the first principal stress of the current cycle [41]. These are given by Equations 6.7.17-18:

$$\sigma_m = 0 \quad [6.7.17]$$

$$\sigma_a = \frac{1}{2} \sqrt{\sigma_{1,max}(\sigma_{1,max} - \sigma_{1,min})} \quad [6.7.18]$$

### 4. R-Ratio Sines

The R-Ratio Sines Method uses the hydrostatic stress as the effective mean stress, but keeps the original definition of the effective stress amplitude. These are given by Equations 6.7.19-20:

$$\sigma_m = \sigma_{xm} + \sigma_{ym} + \sigma_{zm} \quad [6.7.19]$$

$$\sigma_a = \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya} - \sigma_{za})^2 + (\sigma_{xa} - \sigma_{za})^2 + 6(\tau_{xya}^2 + \tau_{yza}^2 + \tau_{xza}^2)} \quad [6.7.20]$$

### 5. Effective Method

The Effective method defines the effective mean stress as twice the distortion energy minus the effective stress amplitude. The effective stress amplitude takes the same form as that from the Manson McKnight method. These are given by Equations 6.7.21-22:

$$\sigma_m = \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xm} - \sigma_{ym})^2 + (\sigma_{ym} - \sigma_{zm})^2 + (\sigma_{xm} - \sigma_{zm})^2 + 6(\tau_{xym}^2 + \tau_{yzm}^2 + \tau_{xzm}^2)} - \sigma_a \quad [6.7.21]$$

$$\sigma_a = \frac{\sqrt{2}}{2} \sqrt{(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya} - \sigma_{za})^2 + (\sigma_{xa} - \sigma_{za})^2 + 6(\tau_{xya}^2 + \tau_{yza}^2 + \tau_{xza}^2)} \quad [6.7.22]$$

#### 6.7.4 Defining a NASALIFE analysis

NASALIFE analyses require the following material parameters:

Property	Symbol	Importance
Tensile fatigue strength coefficient	$\sigma_f'$	REQUIRED
Tensile fatigue strength exponent	$b$	REQUIRED
Ultimate tensile strength	$S_u$	OPTIONAL
Walker gamma parameter	$\gamma$	REQUIRED <sup>6</sup>

#### 6.7.5 Guidance for load history gating

The NASALIFE algorithm locates the MDMC by considering every stress tensor combination in the load history. If tensor gating is enabled then the original, un-gated load history is used, and the resulting analysis time can become very large. If the load history contains many data points, it may be expedient to pre-gate the load histories.

*Environment file usage:*

Variable	Value
gateTensors	0.0;
gateHistories	1.0;

Care should be taken when pre-gating multiple load histories, as the phase relationship between the loading channels may be lost and the accuracy of the fatigue result may be adversely affected.

---

<sup>6</sup> This parameter can be user-defined or computed automatically. Consult Section 7.7 for detailed information about the Walker gamma parameter.

## 6.8 Uniaxial Stress-Life

### 6.8.1 Overview

Uniaxial Stress-Life is the most basic fatigue analysis technique. The method is ideal for simple loading conditions where fatigue damage is caused primarily by stresses in a single direction. The algorithm is especially useful for measured stress data from plane stress specimens. Since the algorithm does not require critical plane searching, it is computationally much less expensive than the biaxial methods such as the Stress-based Brown-Miller fatigue algorithm.

The Uniaxial Stress-Life algorithm is defined by Equation 6.8.1:

$$\frac{\Delta\sigma}{2} = \sigma_f' (2N_f)^b \quad [6.8.1]$$

where  $\frac{\Delta\sigma}{2}$  is the uniaxial stress amplitude.

### 6.8.2 Defining a uniaxial stress-life analysis

The Uniaxial Stress-Life algorithm only requires a single stress history. Stress datasets are not recognised by the program.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATASET	[ ];
HISTORY	'history-file-name.*';

The following material properties are required to perform an analysis with the Uniaxial Stress-Life algorithm:

Property	Symbol	Importance
Fatigue strength coefficient	$\sigma_f'$	REQUIRED
Fatigue strength exponent	$b$	REQUIRED

## 6.9 Uniaxial Strain-Life

### 6.9.1 Overview

Uniaxial Strain-Life is a method for analysing simple unidirectional strain histories. The uniaxial elastic stress history is converted into an inelastic strain history using the Ramberg Osgood nonlinear elastic strain-hardening model. The same model is used to convert the elastic principal stress histories into inelastic principal strain histories.

The Uniaxial Strain-Life algorithm is defined by Equation 6.9.1:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E} (2N_f)^b + \varepsilon_f' (2N_f)^c \quad [6.9.1]$$

where  $\frac{\Delta\varepsilon}{2}$  is the uniaxial strain amplitude.

### 6.9.2 Defining a uniaxial strain-life analysis

The Uniaxial Strain-Life algorithm only requires a single elastic stress history. Stress datasets are not recognised by the program.

*Job file usage:*

Option	Value
DATASET	[ ];
HISTORY	'history-file-name.*';

The following material properties are required to perform an analysis with the Uniaxial Strain-Life algorithm:

Property	Symbol	Importance
Fatigue strength coefficient	$\sigma_f'$	REQUIRED
Fatigue strength exponent	$b$	REQUIRED
Fatigue ductility coefficient	$\varepsilon_f'$	REQUIRED
Fatigue ductility exponent	$c$	REQUIRED
Young's Modulus	$E$	REQUIRED
Cyclic strain-hardening coefficient	$K'$	REQUIRED
Cyclic strain-hardening exponent	$n'$	REQUIRED

## 6.10 User-defined algorithms

### 6.10.1 Overview

The Quick Fatigue Tool framework allows the user to create their own fatigue analysis algorithm. Information is passed into the class *algorithm\_user*, which is used to evaluate the fatigue damage at each analysis item.

The user algorithm class file *algorithm\_user.m* is located in *Application\_Files\code\main*. The file contains a single function called *main*. This is the function which Quick Fatigue Tool calls iteratively for each analysis item.

The class *algorithm\_user* can be expanded so that the function *main* calls other functions within *algorithm\_user*.

### 6.10.2 Variables passed in for information

Quick Fatigue Tool passes the following arguments into *algorithm\_user.main*:

Argument	Description	Notes
S11	Stress tensor history in the normal Cartesian 1-direction.	
S22	Stress tensor history in the normal Cartesian 2-direction.	
S33	Stress tensor history in the normal Cartesian 3-direction.	
S12	Stress tensor history in the shear Cartesian 12-direction.	
S23	Stress tensor history in the shear Cartesian 23-direction.	
S13	Stress tensor history in the shear Cartesian 13-direction.	
N	Current analysis item number.	N = 1 unless the stress dataset contains more than one analysis item.
MSC	Identifier defining the selected mean stress correction.	Consult Section 7.1 for a table relating the value of MSC to the mean stress correction.

### 6.10.3 Variables to be defined

The user must define the following output arguments:

Argument	Description	Format	Example usage (per analysis item)
DPARAMI	Maximum damage parameter at the current analysis item.	$1 \times L$ numeric array.	$\text{DPARAMI}(N) = X;$ $X$ is the maximum damage parameter.
AMPI	Stress amplitude of each cycle for the loading at the current analysis item.	$1 \times L$ cell array.	$\text{AMPI}\{N\} = [A_1, A_2, \dots, A_N];$ $A_1$ to $A_N$ are the amplitudes over the load history.
PAIRI	Cycle pairs for the loading at the current analysis item.	$C \times 2$ cell array.	$\text{PAIRI}\{N\} = [P_{1,min}, P_{1,max}; \dots; P_{C,min}, P_{C,max}];$ $P_{min}$ and $P_{max}$ are the minimum and maximum cycle values for each pair in the loading.
DAMI	Total damage for the loading at the current analysis item.	$1 \times L$ numeric array.	$\text{DAMI}(N) = D;$ $D$ is the total damage over the load history.

$L$  is the number of analysis items;  $C$  is the number of cycle in the load history;  $N$  is the current analysis item.

### 6.10.4 Material properties

Material properties are accessed using the *getappdata()* method. For example, the fatigue strength coefficient is requested as follows:

```
Sf = getappdata(0, 'Sf');
```

A complete list of material properties and their identifiers is given below:

Property	Identifier	Property	Identifier
Young's modulus	E	Default analysis algorithm	defaultAlgorithm
Poisson's ratio	poisson	Default mean stress correction	defaultMSC
Ultimate tensile strength	uts	Constant amplitude endurance limit	cael
Proof stress	twops	S-N data points: S-values (interpolated at R=-1)	s_values_reduced
S-N data points: S-values	s_values	S-N data points: Number of curves	nSNDatasets
S-N data points: N-values	n_values	Fatigue strength exponent at knee-point	b2
S-N data points: R-values	r_values	Life at knee-point	b2Nf
Fatigue strength coefficient	Sf	Fatigue limit	fatigueLimit
Fatigue strength exponent	b	Residual stress	residualStress
Fatigue ductility coefficient	Ef	Surface finish	kt
Fatigue Ductility exponent	c	Notch sensitivity constant	notchRootRadius

Cyclic strain hardening coefficient	<code>kp</code>	Notch root radius	<code>notchSensitivityConstant</code>
Cyclic strain hardening exponent	<code>np</code>		
Normal stress sensitivity constant	<code>k</code>		

If analysis groups are used, material properties will depend on which group the current analysis item belongs to.

### 6.10.5 Utility functions

User-defined algorithms can call to other Quick Fatigue Tool functions to facilitate the analysis.

#### Rainflow cycle counting

A stress history can be cycle counted using the following function:

*M-file usage:*

```
rfData = analysis.rainflow(history);
```

The variable *rfData* is a  $C \times 4$  matrix where  $C$  is the number of counted cycles. The first two columns are the cycle points. The last two columns are the indexes in the stress history corresponding to the cycle.

The amplitude of each cycle is extracted using the following function:

*M-file usage:*

```
pairs = rfData(:, 1:0:2.0);
amplitudes = analysis.getAmps(pairs);
```

#### Mean stress correction

The variables *pairs* and *amplitudes*, as defined above, can be corrected for the effect of mean stress.

*M-file usage:*

```
[mscAmplitudes, ~, ~] = analysis.msc(amplitudes, pairs, MSC);
```

The variable *MSC* is the identifier which is passed into *algorithm\_user.main*.

### 6.10.6 Additional stress histories

Before the analysis, Quick Fatigue Tool calculates the principal stress and von Mises stress history. These can be accessed at the current analysis item using `getappdata()`:

History variable	Definition
First principal stress	<code>getappdata(0, 'S1');</code>
Second principal stress	<code>getappdata(0, 'S2');</code>
Third principal stress	<code>getappdata(0, 'S3');</code>
Von Mises stress	<code>getappdata(0, 'VM');</code>

## 7. Mean stress corrections

### 7.1 Background

Tensile mean stresses tend to reduce the fatigue life of components, so a mean stress correction is necessary in order to obtain accurate life predictions.

Mean stress corrections are usually represented by Haigh diagrams, which gives the allowable stress amplitude as a function of the mean stress (Figure 7.1). Each line on the Haigh diagram represents the allowable combinations of stress amplitude and mean stress for a given fatigue life.

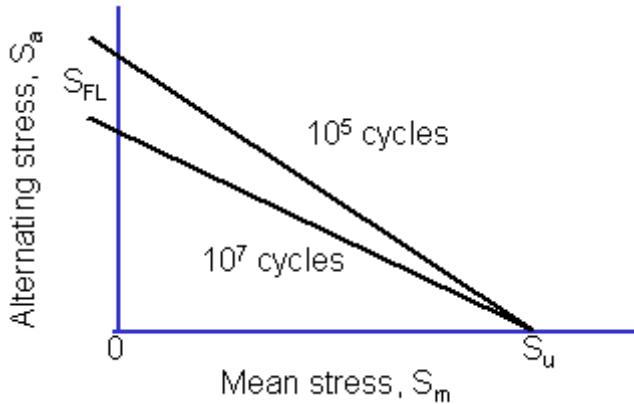


Figure 7.1: Haigh diagram showing contours of constant life.  
Image courtesy of eFatigue.

Quick Fatigue Tool offers several mean stress corrections, depending on the selected algorithm:

Algorithm	Available Mean Stress Corrections
Stress-based Brown Miller	<ul style="list-style-type: none"><li>• Morrow</li><li>• Goodman</li><li>• Gerber</li><li>• Soderberg</li><li>• R-Ratio S-N Curves</li><li>• User-defined</li></ul>
Normal Stress	<ul style="list-style-type: none"><li>• Morrow</li><li>• Goodman</li><li>• Gerber</li><li>• Walker</li><li>• R-Ratio S-N Curves</li><li>• Smith-Watson Topper</li><li>• User-defined</li></ul>
Findley's Method	<ul style="list-style-type: none"><li>• None (built-in)</li></ul>

Stress Invariant Parameter	<ul style="list-style-type: none"> <li>• Goodman</li> <li>• Gerber</li> <li>• Walker</li> <li>• R-Ratio S-N Curves</li> <li>• User-defined</li> </ul>
BS 7608	<ul style="list-style-type: none"> <li>• None (built-in)</li> </ul>
Uniaxial Stress-Life	<ul style="list-style-type: none"> <li>• Goodman</li> <li>• Gerber</li> <li>• Walker</li> <li>• Soderberg</li> <li>• R-Ratio S-N Curves</li> <li>• User-defined</li> </ul>
Uniaxial Strain-Life	<ul style="list-style-type: none"> <li>• Morrow</li> <li>• Smith-Watson-Topper</li> <li>• Walker</li> </ul>
NASALIFE	<ul style="list-style-type: none"> <li>• Walker (built-in)</li> </ul>

The mean stress correction is specified from the job file. If the default mean stress correction is specified Quick Fatigue Tool will use the correction defined in the material .mat file.

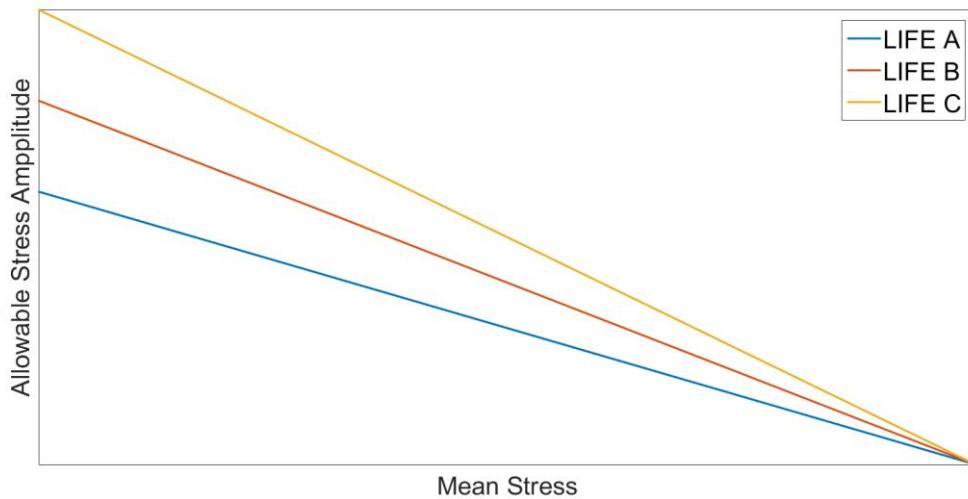
Mean stress correction	Job file option
Default	MS_CORRECTION = [0.0   'default'];
Morrow	MS_CORRECTION = [1.0   'morrow'];
Goodman	MS_CORRECTION = [2.0   'goodman'];
Soderberg	MS_CORRECTION = [3.0   'soderberg'];
Walker	MS_CORRECTION = [4.0   'walker'];
Smith-Watson-Topper	MS_CORRECTION = [5.0   'swt'];
Gerber	MS_CORRECTION = [6.0   'gerber'];
R-ratio S-N curves	MS_CORRECTION = [7.0   'ratio'];
None	MS_CORRECTION = [8.0   'none'];
User-defined	MS_CORRECTION = <filename>.msc;

## 7.2 Goodman

The Goodman mean stress correction assumes that, for mirror polished specimens, the relationship between the allowable stress amplitude and tensile mean stress is linear [42]:

$$\frac{\sigma_a}{\sigma_0} + \frac{\sigma_m}{\sigma_U} = 1 \quad [7.1]$$

Equation 7.1 can be plotted on a Haigh diagram to give Figure 7.2.



**Figure 7.2: Haigh diagram representation of Goodman relationship for different reference lives. The curves meet the material's ultimate tensile strength.**

For each cycle in the loading, Quick Fatigue Tool evaluates the mean stress and finds the equivalent stress amplitude as if the cycle had zero mean stress. This is achieved by re-arranging Equation 7.1 into Equation 7.2:

$$\sigma_0 = \sigma_a \left(1 - \frac{\sigma_m}{\sigma_U}\right)^{-1} \quad [7.2]$$

It should be noted that, when using the Goodman mean stress correction, Quick Fatigue Tool assumes that the equivalent stress amplitude has zero mean stress. Therefore, if custom S-N data is being used which was measured at an R-ratio other than -1 then the Goodman correction will not produce a reliable solution.

## Modified Goodman envelopes

The standard implementation of the Goodman mean stress correction is non-conservative for large values of mean stress and assumes no change in the allowable stress amplitude for negative mean stress. A modified version of the Goodman correction combines the standard slope with the Buch line, shown in Figure 7.3.

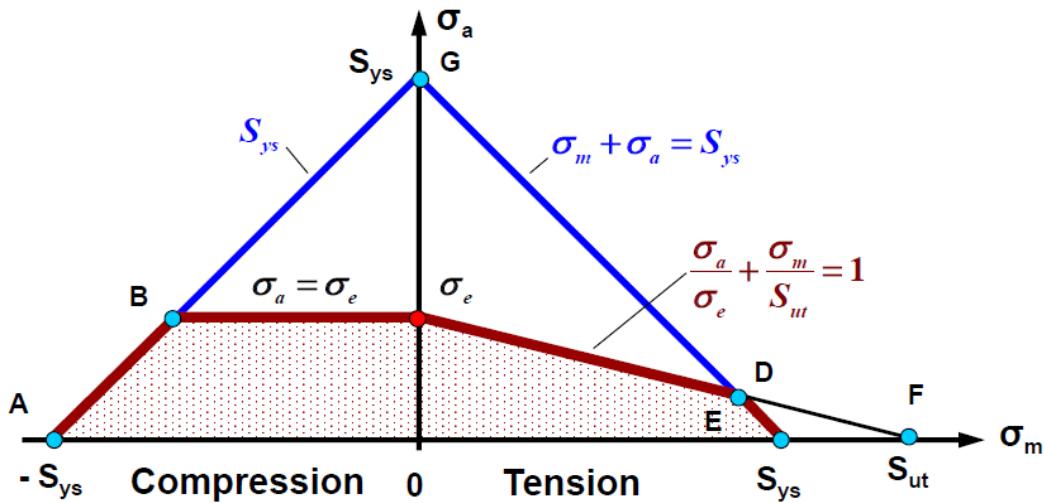


Figure 7.3: Modified Goodman line (red), defined by the intersection of the standard line (black) with the Buch line (blue); copyright © Professor Grzegorz Glinka [75]; reproduced with permission.

The modified Goodman envelope is enabled from the environment file.

*Environment file usage:*

Variable	Value
modifiedGoodman	[0.0   1.0];

The modified Goodman envelope requires a value of the proof stress. If the proof stress is undefined, the standard Goodman envelope is used instead.

Care must be taken when using the Goodman mean stress correction. Since the stress amplitude is zero along the horizontal axis, there is no fatigue in this regime. The assumption that the limiting fatigue strength is equal to the static tensile strength is therefore incorrect, since static and fatigue failure are driven by physically distinct mechanisms. The Goodman correction can often over predict fatigue lives by a factor of three to four. In cases where the Goodman correction fails to produce acceptable results, the Walker correction is recommended as an alternative.

## Setting the Goodman limit stress

By default, the intercept between the Goodman envelope and the horizontal (mean stress) axis is the material ultimate tensile strength. This intercept value can be changed by the user from the environment file.

*Environment file usage:*

Variable	Value
goodmanMeanStressLimit	$G_{lim};$

The value of  $G_{lim}$  can be set as follows:

$G_{lim}$	<b>Limit stress</b>
'UTS'	Material UTS
'PROOF'	Material 0.2% proof stress
'S-N'	S-N intercept (at 1 repeat)
$n$	User-defined value

When the modified Goodman envelope is enabled, the Goodman limit stress is taken as the yield strength and cannot be modified by the user.

### 7.3 Soderberg

The Soderberg mean stress correction is similar to Goodman, but uses the proof stress as the maximum allowable mean stress [43]:

$$\frac{\sigma_a}{\sigma_0} + \frac{\sigma_m}{\sigma_y} = 1 \quad [7.3]$$

It should be noted that, when using the Soderberg mean stress correction, Quick Fatigue Tool assumes that the equivalent stress amplitude has zero mean stress. Therefore, if custom S-N data is being used which was measured at an R-ratio other than -1, then the Soderberg correction will not produce a reliable solution.

## 7.4 Gerber

The Gerber mean stress correction is a non-linear version of Goodman:

$$\frac{\sigma_a}{\sigma_0} + \left( \frac{\sigma_m}{\sigma_U} \right)^2 = 1 \quad [7.4]$$

The Haigh diagram representation is shown in Figure 7.4:

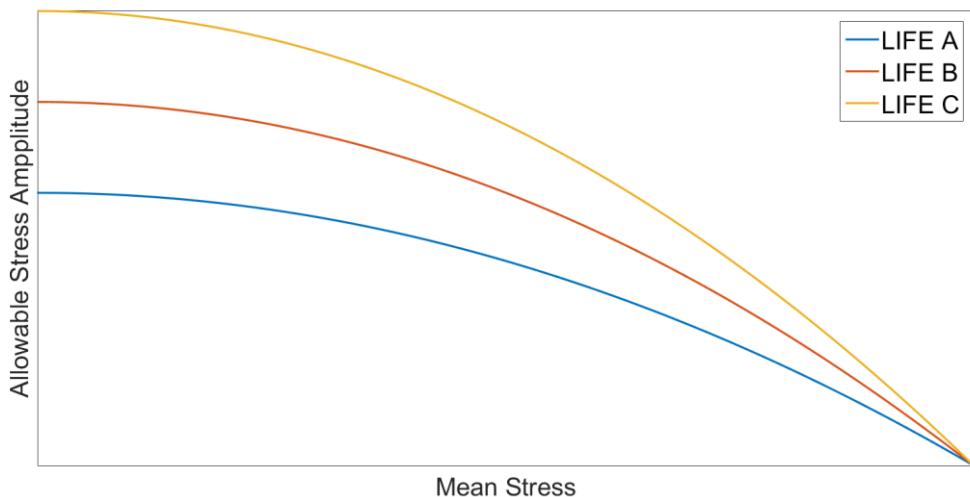


Figure 7.4: Haigh diagram representation of Gerber relationship for different reference lives. The curves meet the material's ultimate tensile strength.

The Gerber correction is more conservative than Goodman; however, for compressive mean stresses, the Gerber correction increases the fatigue life, whereas the Goodman correction has no effect in compression.

## 7.5 Morrow

### 7.5.1 Strain-life

The Morrow mean stress correction modifies the elastic component of the strain-life curve [44]. This reflects the observation that the mean stress has the most noticeable effect in the HCF regime. The correction is expressed in its original form as Equation 7.5:

$$\frac{\Delta\varepsilon}{2} = \frac{(\sigma_f' - \sigma_m)}{E} (2N_f)^b + \varepsilon_f' (2N_f)^c \quad [7.5]$$

Although the Morrow mean stress correction gives reasonable results, it does not reflect the material's true behaviour. The correction postulates that the ratio between the elastic and plastic strain components varies with mean stress. In fact, the hysteresis behaviour of metals is a function of strain only.

### 7.5.2 Stress-life

Stress-life algorithms which use the Morrow mean stress correction only have their fatigue strength term modified:

$$\frac{\Delta\sigma}{2} = \frac{(\sigma_f' - \sigma_m)}{E} (2N_f)^b \quad [7.6]$$

## 7.6 Smith-Watson-Topper

### 7.6.1 Strain-life

The Smith-Watson-Topper (SWT) relationship [41] is obtained by multiplying the damage parameter by the maximum stress in the load cycle, and the right-hand side by the tensile fatigue strength coefficient:

$$\frac{\Delta\varepsilon}{2}\sigma_{max} = \frac{\sigma_f'^2}{E}(2N_f)^b + \sigma_f'\varepsilon_f'(2N_f)^{b+c} \quad [7.7]$$

The SWT correction gives acceptable results for a wide range of materials and load types, and is less conservative than the Morrow correction for compressive mean stress.

### 7.6.2 Stress-life

The stress-life implementation of the SWT mean stress correction is applied to the stress cycle, rather than the stress-life equation. This is achieved using the Walker mean stress correction with  $\gamma = 0.5$ .

## 7.7 Walker

### 7.7.1 Overview

The Walker mean stress correction is similar to the Smith-Watson-Topper correction, with the addition of the material parameter  $\gamma$ . The Walker equation corrects the stress amplitude term in the stress-life equation:

$$\frac{\Delta\sigma_a}{2} = \frac{\Delta\sigma}{2} \left( \frac{2}{1 - R} \right)^{1-\gamma} \quad [7.8]$$

where  $\frac{\Delta\sigma_a}{2}$  is the equivalent stress amplitude at zero mean stress,  $\frac{\Delta\sigma}{2}$  is the stress amplitude of the cycle and  $R$  is the load ratio of the cycle.

### 7.7.2 Walker $\gamma$ -parameter

The Walker  $\gamma$ -parameter can be calculated in three ways:

1. Walker regression fit
2. Standard values for steel and aluminium
3. User-defined

#### **$\gamma$ from Walker regression fit**

The value of  $\gamma$  can be approximated using the regression fit in Figure 7.5.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
walkerGammaSource	1.0;

The value of  $\gamma$  correlates well with steels [45]. However, there exists no such correlation between the Walker parameter and aluminium, as shown by Figure 7.6. It is therefore recommended that the Walker regression fit is not used with non-steels.

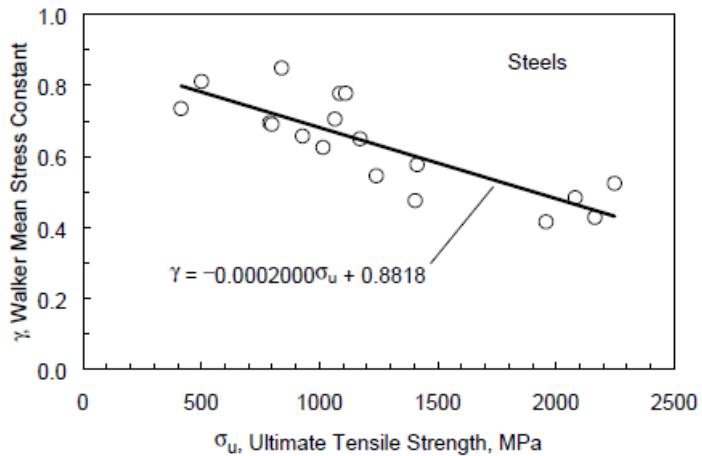


Figure 7.5: Approximation of  $\gamma$  based on the ultimate tensile strength of steel

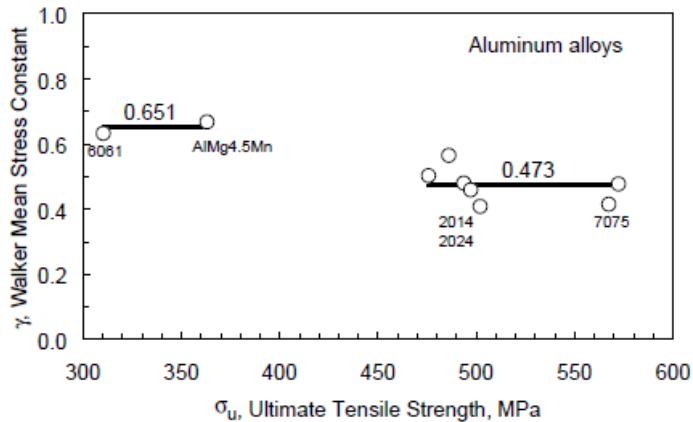


Figure 7.6: Approximation of  $\gamma$  based on the ultimate tensile strength of aluminium

### $\gamma$ from standard values

Work by Dowling [46] has shown that the value of  $\gamma$  can be approximated for most steel and aluminium specimens.

*Environment file usage:*

Variable	Value
walkerGammaSource	2.0;

The following table describes how the value of  $\gamma$  is calculated when this option is selected.

Material type	$\gamma$ -Solution
Steel	0.65
Aluminium	0.45
Other	$1, R < 0$ $0.5, R \geq 0$

Where  $R$  is the load ratio of the cycle. The material type is defined in the general settings of the material editor, under *Derivation*. If the ultimate tensile strength is not defined for the material, Quick Fatigue Tool will automatically find a value of  $\gamma$  based on this method.

### **$\gamma$ from user input**

The value of  $\gamma$  may be defined directly by the user in the environment file.

*Environment file usage:*

Variable	Value
walkerGammaSource	3.0;
userWalkerGamma	$\gamma$ ;

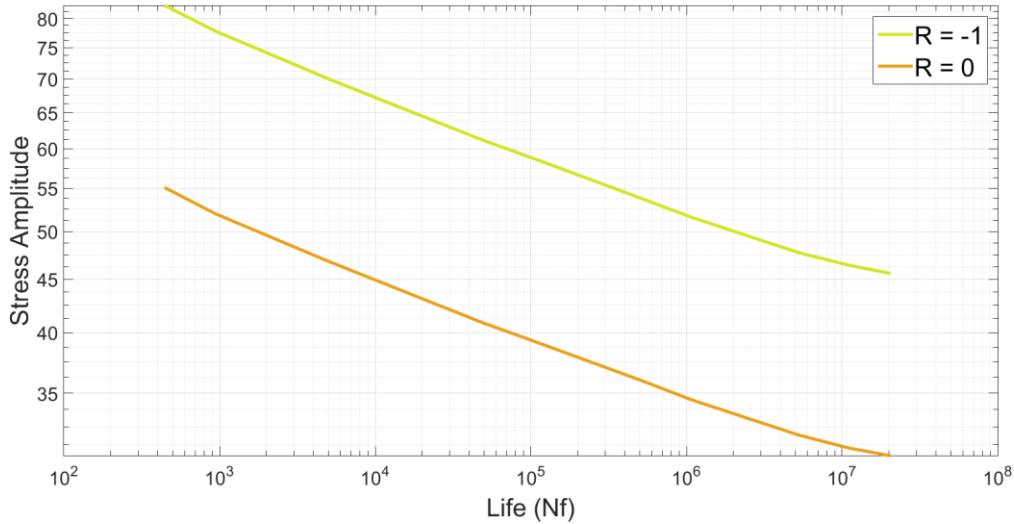
### **7.7.3 Strain-life**

The strain-life implementation of the Walker mean stress correction has been described by Dowling [47] and Ince [48], and is given in the form of Equation 7.9:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E} \left[ 2N_f \left( \frac{1-R}{2} \right)^{(1-\gamma)/b} \right]^b + \varepsilon_f' \left[ 2N_f \left( \frac{1-R}{2} \right)^{(1-\gamma)/b} \right]^c \quad [7.9]$$

## 7.8 R-ratio S-N curves

The loading can be corrected for the effect of mean stress using experimental stress-life data, where the endurance curve for the material has been measured at more than one load ratio. An example of a multi R-ratio S-N curve from the Material Manager is shown in Figure 7.7. Creating S-N data with multiple load ratios is explained in Section 5.4.



**Figure 7.7: Example S-N data from the Material Manager, for a load ratio of -1 and 0**

If R-ratio S-N curves are selected, Quick Fatigue Tool interpolates between the two curves that envelope the load ratio of the current cycle. The standard interpolation formula is used:

$$S_{R_i} = S_2 - \frac{S_2 - S_1}{R_{upper} - R_{lower}} \cdot (R_i - R_{lower}) \quad [7.10]$$

where  $S_{R_i}$  is the stress-life curve at the calculated load ratio  $R_i$ .  $R_{upper}$  and  $R_{lower}$  are the upper and lower load ratios corresponding to the curves  $S_2$  and  $S_1$ , respectively, which envelope the required curve.

If the required S-N curve lies outside the range of S-N data (i.e. the required curve is not bound by a curve to either side), then the stress-life data is extrapolated. In such cases, accuracy is not guaranteed, and a warning will be written to the message file.

R-ratio S-N curves may only be used for  $R < 1$ . Fully-compressive cycles ( $R > 1$ ) will use the S-N curve at  $R = -1$ . S-N curves may not be defined for infinite-valued R-ratios.

### Treatment of the fatigue limit

When the R-ratio S-N Curves mean stress correction is used, the fatigue limit depends on the load ratio of the current cycle. Therefore, the fatigue limit is re-calculated for each cycle based on the interpolated S-N curve; the values of the environment variables `fatigueLimitSource` and `userFatigueLimit` are ignored.

Under certain conditions, Quick Fatigue Tool reduces the fatigue limit such that previously non-damaging cycles may become damaging. Modification of the fatigue limit is enabled with the environment variable `modifyEnduranceLimit`, and is discussed in detail in Appendix A1.5. If modification of the fatigue limit is enabled then the instantaneous fatigue limit,  $\sigma_{f_i}$ , is based on the interpolated S-N curve for the current cycle. Subsequent cycles are compared either to the reduced fatigue limit,  $\sigma_{f_r}$ , or  $\sigma_{f_i}$  (whichever is smaller). If the subsequent cycles are non-damaging,  $\sigma_{f_r}$  will recover towards  $\sigma_{f_i}$ . Hence, the rate of recovery may change depending on the load ratio of the current cycle.

It is not clear how best to treat this “bouncing” endurance limit in combination with multiple R-ratio S-N curves; the above outlined methodology is a suggestion only. In case the user has doubt as to the applicability of the modified endurance limit, it can be disabled from the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>modifyEnduranceLimit</code>	0.0;

## 7.9 User-defined mean stress corrections

User-defined mean stress corrections can be provided for an analysis. The data should describe the allowable stress amplitude at the material's endurance limit as a function of the mean stress. The result is a Haigh envelope which Quick Fatigue Tool uses to calculate a mean stress correction factor.

User-defined mean stress corrections are specified in the job file as a mean stress correction (.msc) file.

*Material Manager usage:* User-defined mean stress corrections are not supported in Material Manager.

*Job file usage:*

<i>Option</i>	<i>Value</i>
MS_CORRECTION	<i>'msc-file-name.msc'</i> ;

where '*msc-file-name.msc*' is a file containing normalized mean stress and allowable stress amplitude data defining the Haigh envelope. The file must be saved in *Data\msc*.

Quick Fatigue Tool includes .msc files for Gray Iron and Inconel 718 Steel. These are shown in Figure 7.8.

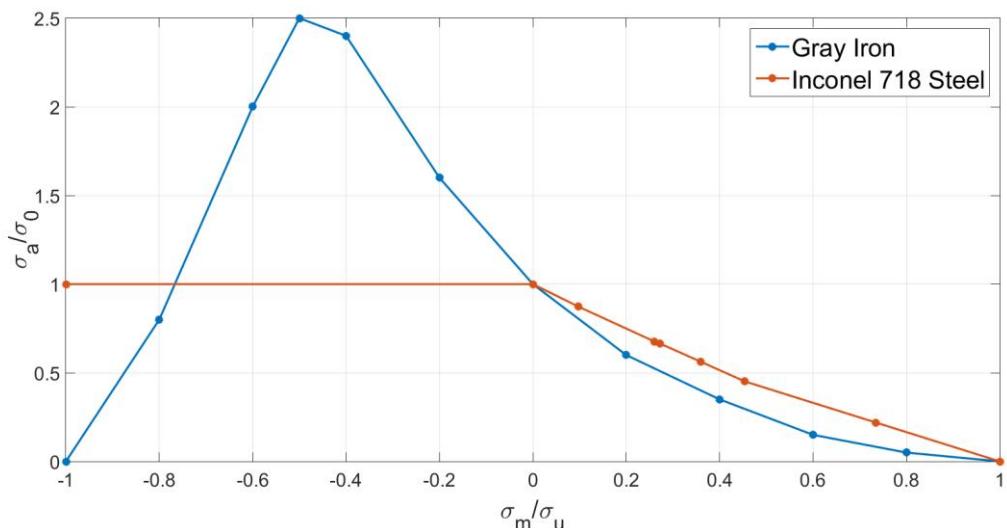


Figure 7.8: Allowable stress amplitude as a function of mean stress for Gray Iron and Inconel 718 Steel

According to the Haigh data, Inconel 718 has an allowable stress amplitude in compression which remains constant, hence compressive cycles are not corrected for the mean stress.

Gray Iron has increased strength in compression at up to 50% of its compressive strength, hence compressive cycles within this region are corrected in a sense that reduces the resulting fatigue damage. Beyond 50% of the compressive strength, the fatigue strength reduces to zero, at which point crushing is expected.

In tension, both materials have decreasing fatigue strength until the mean stress equals the ultimate tensile strength, at which point static failure is expected.

The mean stress correction factor is calculated by comparing the allowable stress amplitude at zero mean stress to the allowable stress amplitude corresponding to the mean stress of the cycle. Consider the following user-defined mean stress data:

$\sigma_m$	$\sigma_a$
0.2	0.8
0.3	0.7

Now consider a stress cycle which has a mean stress of  $88\text{ MPa}$  and stress amplitude of  $100\text{ MPa}$ . If the material's UTS is  $400\text{ MPa}$ , then the normalized mean stress is  $88/400 = 0.22$ . Quick Fatigue Tool determines the corresponding allowable stress amplitude by assuming that intermediate stress amplitude data follows a linear relationship and hence fits the equation of a straight line ( $y = mx + c$ ). The normalized stress amplitude corresponding to the mean stress of the cycle is then  $-(0.22 - 0.3) + 0.7 = 0.78$ . The mean stress correction factor is therefore  $1/0.78 \cong 1.282$ . The equivalent stress amplitude at zero mean stress is  $100 \times 1.282 = 128.2\text{ MPa}$ . In general, the mean stress correction factor is given by Equation 7.11:

$$MSC = \left[ \frac{d\sigma_a}{d\sigma_m} (\sigma_m - \sigma_{mi}) + \sigma_{ai} \right]^{-1} \quad [7.11]$$

where  $\frac{d\sigma_a}{d\sigma_m}$  is the local gradient of the Haigh data,  $\sigma_m$  is the cycle mean stress and  $\sigma_{mi}$  and  $\sigma_{ai}$  are the allowable mean stress and stress amplitude at point  $i$ , respectively. The equivalent stress amplitude at zero mean stress is given by Equation 7.12:

$$\sigma_{a,e} = \sigma_a \times MSC \quad [7.12]$$

User-defined mean stress correction data is usually obtained from existing Haigh diagrams for the given material. The mean stress and stress amplitude are typically normalized by the ultimate tensile strength and the tensile fatigue limit, respectively. If the material's compressive strength is known, this can be specified in the job file. For cycles with negative mean stress, Quick Fatigue Tool will normalize those cycles by the compressive strength. If the compressive strength is not defined, the tensile strength will be used for all cycles.

*Job file usage:*

<i>Option</i>	<i>Value</i>
UCS	$\sigma_{UCS};$

The following file format must be obeyed for user-defined mean stress data:

*First column:* Normalized mean stress values

*Second column:* Normalized stress amplitude values

The values of the normalized mean stress must be decreasing.

If the mean stress of the cycle is less than the minimum, or greater than the maximum value of mean stress defined in the *.msc* file, Quick Fatigue Tool will calculate the mean stress correction factor directly from the first or last stress amplitude value in the data, respectively.

## 8. Safety factor analysis

### 8.1 Background

Some designs require that the fatigue performance of a component is expressed as a safety factor, rather than a number of cycles. The finite life calculation is substituted for a pass/fail criterion, such that a value greater than unity indicates that safety has been achieved by a certain margin; values less than unity indicate that design and/or load modification is required. Quick Fatigue Tool offers two design factor calculations:

1. Fatigue Reserve Factor (FRF)
2. Factor of Strength (FOS)

The two parameters are similar in their objectives. However, their methodologies have some fundamental differences which should be considered before the analysis.

### 8.2 Fatigue Reserve Factor

#### 8.2.1 Overview

The FRF is a linear scale factor which considers the worst stress cycle in the loading and compares the mean stress and stress amplitude to a given target life. There are three types of calculation:

1. Horizontal
2. Vertical
3. Radial

Figure 8.1 shows the Goodman and Gerber life envelopes which can be used for the FRF calculation. The FRF is calculated for the worst cycle in the loading.

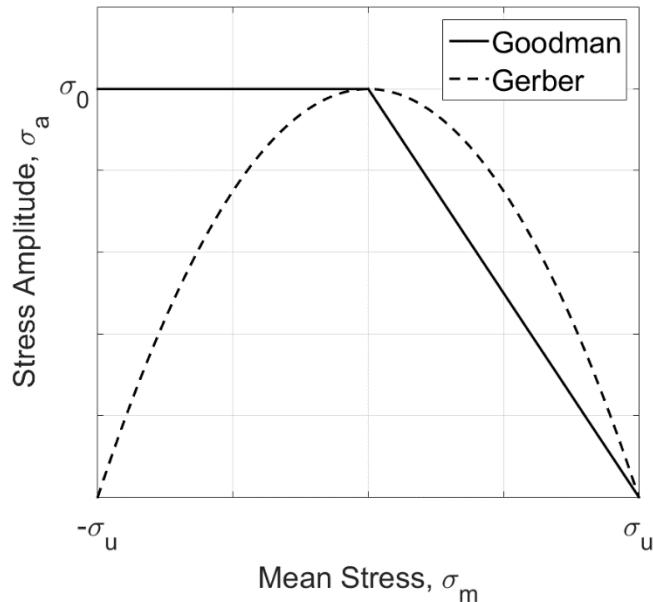


Figure 8.1: Goodman and Gerber envelopes for FRF calculation

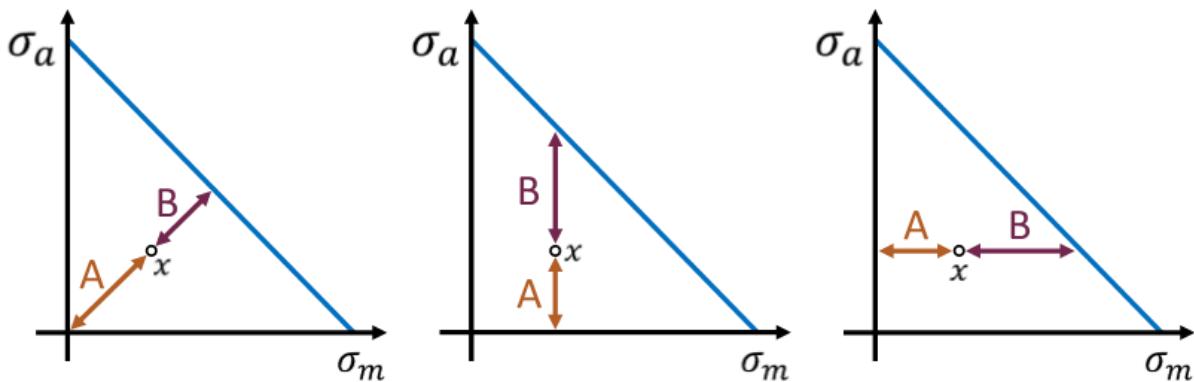


Figure 8.2: Radial, vertical and horizontal fatigue reserve factor relative to an arbitrary cycle,  $x$ , on a Haigh diagram. The blue line represents an arbitrary design envelope

An arbitrary cycle,  $x$ , is plotted on a Haigh diagram in Figure 8.2. The value of the FRF in the radial, vertical and horizontal direction is given by Equation 8.1. The terms  $\sigma_m$  and  $\sigma_a$  correspond to the mean stress and stress amplitude, respectively.

$$FRF = \frac{A + B}{A} \quad [8.1]$$

The pros and cons of the FRF calculation are as follows:

Pros:

- The calculation is fast
- Results are accurate for simple, constant amplitude loading

Cons:

- Results are inaccurate for variable amplitude loading
- Since the FRF is not calculated from the complete loading, it cannot be used to reliably scale the stresses
- The number of loading repeats set by REPEATS in the job file will not affect the FRF calculation

### 8.2.2 Included envelopes

Quick Fatigue Tool includes the Goodman, Goodman B and Gerber design envelopes. The Goodman envelope is used by default and is selected from the job file. Values of 1, 2 and 3 correspond to the Goodman, Goodman B and Gerber envelopes, respectively.

*Job file usage:*

<i>Option</i>	<i>Value</i>
FATIGUE_RESERVE_FACTOR	[1.0   2.0   3.0];

## Goodman

The Goodman envelope is defined by Equation 8.2:

$$\frac{\sigma_a}{\sigma_0} + \frac{\sigma_m}{\sigma_u} = 1 \quad [8.2]$$

The mean stress and stress amplitude is given by  $\sigma_m$  and  $\sigma_a$ , respectively. The envelope is limited by the ultimate tensile strength and the endurance limit,  $\sigma_u$  and  $\sigma_0$ , respectively.

By rearranging Equation 8.2, the values of the Goodman fatigue reserve factors in the horizontal, vertical and radial directions are given by Equations 8.3-8.5, respectively:

$$FRF_H = \frac{\sigma_u}{\sigma_m} \left( 1 - \frac{\sigma_a}{\sigma_0} \right) \quad [8.3]$$

$$FRF_V = \frac{\sigma_0}{\sigma_a} \left( 1 - \frac{\sigma_m}{\sigma_u} \right) \quad [8.4]$$

$$FRF_R = \frac{\alpha}{\beta} \quad [8.5]$$

The terms  $\alpha$  and  $\beta$  are given by Equations 8.6-8.7:

$$\alpha = \sqrt{\gamma^2 + \delta^2} \quad [8.6]$$

$$\beta = \sqrt{{\sigma_m'}^2 + {\sigma_a'}^2} \quad [8.7]$$

where  $\sigma_m'$  and  $\sigma_a'$  is the mean stress and stress amplitude of the cycle, respectively. The terms  $\gamma$  and  $\delta$  are given by Equations 8.8-8.9:

$$\gamma = \frac{\sigma_0}{\frac{{\sigma_a'}'}{{\sigma_m'}'} + \frac{\sigma_0}{\sigma_u}} \quad [8.8]$$

$$\delta = \sigma_a' + \frac{{\sigma_a'}'}{{\sigma_m}'} (\gamma - \sigma_m') \quad [8.9]$$

The above equations apply to the standard Goodman envelope. Quick Fatigue Tool also uses a modified version of this envelope called Goodman B, which is the intersection between the standard Goodman and Buch envelopes. The Goodman B envelope is illustrated in Figure 8.3. If the proof stress is not defined in the material, the standard Goodman envelope is automatically used by default.

## Gerber

The Gerber envelope is the quadratic equivalent of the Goodman envelope, and is defined by Equation 8.10:

$$\left(\frac{\sigma_a}{\sigma_0}\right)^2 + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1 \quad [8.10]$$

By rearranging Equation 8.10, the values of the Gerber fatigue reserve factors in the horizontal, vertical and radial directions are given by Equations 8.11-8.13, respectively:

$$FRF_H = \frac{\sigma_u}{\sigma_m} \sqrt{1 - \left(\frac{\sigma_a}{\sigma_0}\right)^2} \quad [8.11]$$

$$FRF_V = \frac{\sigma_0}{\sigma_a} \sqrt{1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2} \quad [8.12]$$

$$FRF_R = \frac{\alpha}{\beta} \quad [8.13]$$

The terms  $\alpha$  and  $\beta$  are given by Equations 8.14-8.15:

$$\alpha = \sqrt{\gamma^2 + \delta^2} \quad [8.14]$$

$$\beta = \sqrt{{\sigma_m'}^2 + {\sigma_a'}^2} \quad [8.15]$$

The terms  $\gamma$  and  $\delta$  are given by Equations 8.16-8.17:

$$\gamma = \sqrt{\frac{\sigma_0^2}{\left(\frac{\sigma_a'}{\sigma_m'}\right)^2 + \left(\frac{\sigma_0}{\sigma_u}\right)^2}} \quad [8.16]$$

$$\delta = \sigma_a' + \frac{\sigma_a'}{\sigma_m'} (\gamma - \sigma_m') \quad [8.17]$$

Fatigue reserve factors calculated from the Goodman envelope are slightly more conservative than those calculated from the Gerber envelope.

### 8.2.3 Specifying the target life

The FRF target life can either be the infinite life envelope defined by the material's endurance limit, or a user-defined design life. The target life mode is configured in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>frfTarget</code>	$n;$

A value of  $n = 1$  corresponds to the user-defined design life set by DESIGN\_LIFE in the job file, while a value of  $n = 2$  corresponds to the material's endurance limit.

### 8.2.4 Specifying the FRF limits

By default, Quick Fatigue Tool limits the FRF values so that the minimum and maximum reported values are capped at 0.1 and 10.0, respectively. These values can be changed in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>frfMaxValue</code>	$FRF_{max};$
<code>frfMinValue</code>	$FRF_{min};$

### 8.2.5 Enabling FRF output

The FRF algorithm requires field output to be requested from the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
<code>OUTPUT_FIELD</code>	$1.0;$

Although the radial FRF is defined for any combination of stress amplitude and mean stress, the horizontal and vertical FRFs are only defined within certain regimes. For example, with respect to the Goodman B envelope the horizontal FRF is only defined for stress amplitudes below the fatigue limit, whereas the vertical FRF is only defined for mean stresses less than the proof stress. If the FRF cannot be evaluated, a value of  $-1$  is reported to the field output file.

### 8.2.6 User-defined FRF envelopes

The FRF can be calculated with user-defined data. This is a file containing a material life envelope as Haigh diagram ( $\sigma_m - \sigma_a$ ) data. FRF data files are created in the same way as user-defined mean stress correction (.msc) files. Quick Fatigue Tool uses the .msc file for both user mean stress correction and FRF calculations. For guidance on creating custom FRF data, see Section 7.9.

*Job file usage:*

<i>Option</i>	<i>Value</i>
FATIGUE_RESERVE_FACTOR	'msc-file-name.msc';

Quick Fatigue Tool interpolates the user-defined data to find the point on the envelope which forms the intersecting line with the stress cycle coordinate (radial, horizontal and vertical).

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
frfInterpOrder	'<MODE>';

The user can choose between the following techniques:

<b>MODE</b>	<b>DESCRIPTION</b>
'NEAREST'	Nearest neighbour method
'LINEAR'	Linear ( $y = mx + c$ )
'SPLINE'	Cubic, piecewise polynomial
'PCHIP'	Cubic, shape-preserving

The **LINEAR** method is shown in Figure 8.3. A comparison between the **SPLINE** and **PCHIP** methods are shown in Figures 8.4-5.

Linear interpolation is well-suited in cases where few data points are available. Spline interpolation is globally very smooth, while PCHIP interpolation only considers local curvature between data points. Although the PCHIP method results in coarser interpolation, its behaviour is preferable over the spline method in cases where the data contains sudden changes in gradient, as shown by Figure 8.5.

Nearest neighbour interpolation is not recommended as it can produce highly inaccurate results.

### Modelling compressive behaviour for user-defined FRF data

Metals tend to show greater fatigue strength when loaded in compression, compared with tension. This behaviour is often represented by a plateau on the FRF curve as shown by Figure 8.1, in the case of the Goodman envelope. Care should be taken when defining these plateaus with user-defined FRF data, since this can result in undesirable behaviour from the interpolation algorithm in MATLAB.

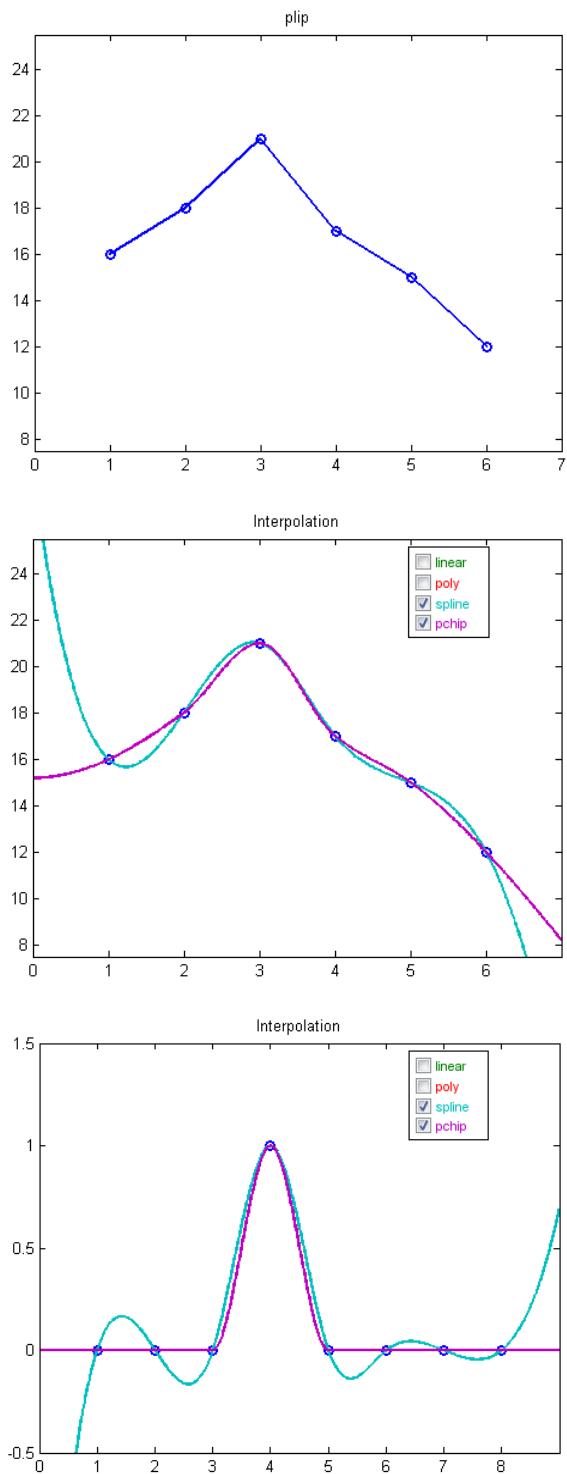
For an endurance envelope with a compressive plateau, the FRF data may be defined as below:

Mean stress	Stress amplitude
1	0
0	1
-1	1
-1	0

Since the mean stress values must be monotonically decreasing, the definition above will result in an error. The problem is resolved by modifying the mean stress values by a very small amount, as shown below.

Mean stress	Stress amplitude
1	0
0	1
-0.9999	1
-1	0

Quick Fatigue Tool automatically adjusts the stress amplitude values to ensure the correct behaviour of the interpolation algorithm.



**Figures 8.3-5:** From top to bottom: Linear; spline vs PCHIP for smooth data; spline vs PCHIP for discontinuous data.

Image credit: Cleve Moler, “Splines and Pchips”, July 16 2012, The Mathworks.

## Specifying normalization parameters for user-defined FRF data

Quick Fatigue Tool normalizes the mean stress and stress amplitude of each fatigue cycle to the limits of the design envelope before performing an FRF calculation with user-defined FRF data. By default, the design envelope limits are those defined by the Goodman FRF: Tensile and compressive mean stresses are normalized by the ultimate tensile and ultimate compressive strength, respectively, while the stress amplitude is normalized by the fatigue limit stress. These default values can be changed with the following environment variables.

*Environment file usage:*

Variable	Value
frfNormParamMeanT	[ <b>'UTS'</b>   <b>'UCS'</b>   <b>'PROOF'</b>   <i>n</i> ];
frfNormParamMeanC	[ <b>'UTS'</b>   <b>'UCS'</b>   <b>'PROOF'</b>   <i>n</i> ];
frfNormParamAmp	[ <b>'LIMIT'</b>   <i>n</i> ];

The parameters **UTS**, **UCS**, **PROOF** and **LIMIT** correspond to the ultimate tensile strength, ultimate compressive strength, 0.2% proof stress and fatigue limit stress, respectively. The normalization parameter can be specified directly with a numerical value, *n*.

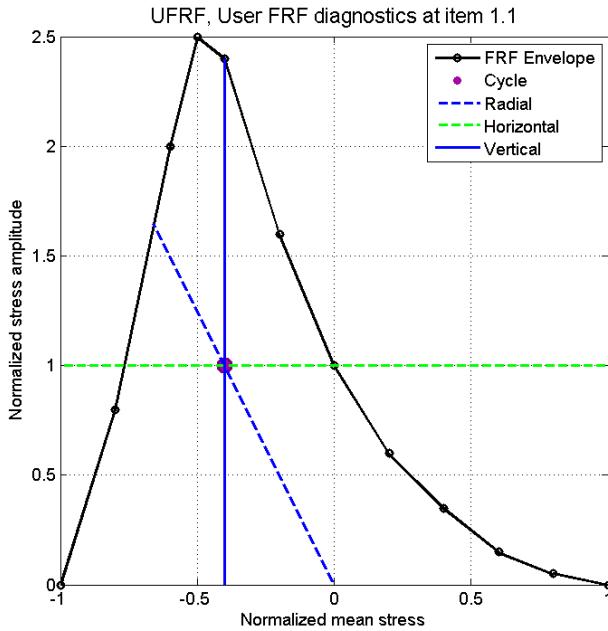
## Visualizing user FRF envelopes

The user-defined FRF envelope can be plotted for a chosen analysis item with its respective radial, horizontal and vertical projections. This MATLAB figure is enabled from the environment file, and requires MATLAB figure output to be enabled for the analysis.

*Environment file usage:*

Variable	Value
frfDiagnostics	<i>n</i> ;

The item number *n* corresponds to the analysis item as defined by its position in the data set file. An example diagnostic output is shown in Figure 8.6 for the uniaxial cycle [28.3, -348.3], with the envelope ‘Gray Iron.msc’.



**Figure 8.6: An example FRF diagnostics with ‘Gray Iron.msc’.** Mean stress and stress amplitude values are normalized by 400 and 188.3, respectively.

### Precautions for user FRF envelopes

The user should ensure that the user-defined FRF data is defined in such a way that valid FRF calculations are possible. If Quick Fatigue Tool is unable to calculate the FRF based on the user data, a value of  $-1$  will be assigned. If the data is considered unusable, the analysis will exit with an error. User FRF data is checked against the following conditions before the start of the analysis:

- there must be exactly two columns of data;
- there must be at least two FRF data pairs;
- mean stress values must be decreasing down the column;
- stress amplitude values must be positive;
- the radial from the origin, through the cycle, must not be able to cross the FRF envelope more than once; and
- duplicate stress amplitude values on a given side of the amplitude axis are not permitted (the same amplitude value can be specified as long as it is on the other side of the amplitude axis).

The following observations apply to user-defined FRF data:

- mean stress values should not be over unity;
- adjacent stress amplitude values are automatically adjusted to prevent zero gradients;
- the FRF envelope should be closed at both ends (zero amplitude at mean stress limits); and
- horizontal FRF values will default to  $-1$  if there is more than one possible solution.

### 8.2.7 Treatment of residual stresses

If a residual stress is defined, the FRF calculation is modified such that the origin of the Haigh diagram is shifted from  $(0, 0)$  to  $(\sigma_R, 0)$ .

## 8.3 Factor of Strength

### 8.3.1 Overview

The FOS is a linear scale factor which, when applied to the fatigue loading, results in the specified design life. Although traditionally the FRF was preferred over the FOS due to it being relatively inexpensive, modern computers are able to perform FOS calculations in a reasonable amount of time and as such the FOS is generally preferred over the FRF.

The FOS takes into account the nonlinear relationship between the damage parameter and the fatigue life by iterating to recalculate the life until a stop condition is met.

Quick Fatigue Tool performs the following procedure when calculating the FOS:

1. Assume an initial FOS of 1.0
2. Compare the calculated fatigue life with the target life:
  - a. If the calculated life is less than the target life, decrease the FOS by a fixed increment
  - b. If the calculated life is greater than the target life, increase the FOS by a fixed increment
3. Apply the current FOS to the original loading and re-calculate the life
4. Repeat steps 2 and 3 until one of the following stop conditions is met:
  - a. The maximum number of iterations has been reached
  - b. The specified tolerance has been achieved
  - c. The target life lies between the last two computed values of fatigue life
  - d. The current FOS is less than or equal to the minimum specified FOS
  - e. The current FOS is greater than or equal to the maximum specified FOS

The FOS is far more computationally expensive than the FRF because it repeats the damage calculation at every analysis item. Furthermore, for the multiaxial algorithms such as the Stress-based Brown-Miller, a new critical plane analysis must be performed each time. This is because changes in the mean stress caused by re-scaling of the load history can affect the orientation of the critical plane.

### 8.3.2 Enabling the FOS calculation

The FOS calculation is enabled by setting the relevant option in the job file:

*Job file usage:*

<i>Option</i>	<i>Value</i>
FACTOR_OF_STRENGTH	[0.0   1.0];

The FOS algorithm requires field output to be requested from the job file:

*Job file usage:*

<i>Option</i>	<i>Value</i>
OUTPUT_FIELD	1.0;

The FOS target life can either be the infinite life envelope defined by the material's endurance limit, or a user-defined design life. The target life mode is configured in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
fosTarget	[1.0   2.0];

A value of 1 corresponds to the user-defined design life set by the DESIGN\_LIFE option in the job file, while a value of 2 corresponds to the material's endurance limit.

### 8.3.3 Setting FOS band definitions

By default, Quick Fatigue Tool limits the FOS values so that the minimum and maximum reported values are capped between 0.5 and 2.0, respectively. These values can be changed in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
fos.MaxValue	$FOS_{max};$
fos.MinValue	$FOS_{min};$

During each iteration, the FOS is increased or decreased by a fixed increment. The value of the increment depends on whether the current FOS lies within the fine band set by the user. The default minimum and maximum fine band sizes are 0.8 and 1.5, respectively.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>fosMaxFine</code>	$FB_{max};$
<code>fosMinFine</code>	$FB_{min};$

The FOS band values must be decreasing:

$$FOS_{max} \geq FB_{max} \geq FB_{min} \geq FOS_{min}$$

Furthermore,

$$FOS_{max} > FOS_{min}$$

FOS values within this band are incremented with a smaller value. This fine value must be smaller than or equal to the coarse value. The increment size for the fine and coarse bands are set by the user in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>fosCoarseIncrement</code>	0.1;
<code>fosFineIncrement</code>	0.01;

The default stop conditions can also be modified from the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>fosMaxCoarselтерations</code>	8.0;
<code>fosMaxFinelтерations</code>	12.0;
<code>fosTolerance</code>	5.0;

The pros and cons of the FOS calculation are as follows:

Pros:

- Results are accurate for both simple and complex, variable amplitude loading
- Since the whole damage calculation is repeated, the FOS takes into account changes in the critical plane orientation, mean stress, surface finish definition, plasticity correction etc.

Cons:

- The calculation is computationally expensive
- Since the calculation accounts for the global load state, the FOS is only valid when used to scale the stress at every point in the model. For example, models containing bolts where the preload level is fixed may yield overly conservative FOS values

#### 8.3.4 Additional guidance on FOS parameters

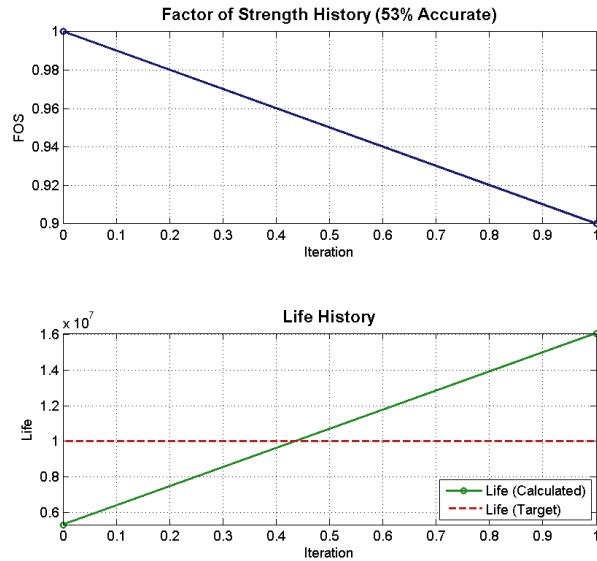
Quick Fatigue Tool estimates the solution to the FOS by linearly scaling the original loading by successive estimates of the FOS. If the target life is very far away from the calculated life, or if the target life is very small, the FOS calculation can become unreliable. This is due to the finite allowable increments in the first case and the severe nonlinearity of the S-N curve at low lives in the second case. In fact, there is no single configuration of the FOS parameters which is guaranteed to achieve a reliable solution for all target lives.

The user is strongly encouraged to review the accuracy of the FOS calculation, which is printed to the log file (*Project\output\<jobName>\<jobName>.log*). Detailed information about the calculation can be obtained by limiting the analysis to the worst item using the ITEMS option in the job file, and requesting FOS diagnostics.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
fosDiagnostics	[0.0   1.0];

This creates a MATLAB figure showing the successive FOS and life values over each iteration, which allows the user to easily check whether the scaled loading results in a fatigue life sufficiently close to the target value. The iteration history is printed to the log file, and the FOS accuracy for each analysis item is written to the file *Project\output\<job>\Data Files\fos\_accuracy.log*. If automatic export is enabled, the FOS accuracy per analysis item is written to the .odb file as an additional results field. MATLAB figures must be enabled from the job file in order to generate the FOS diagnostic figure.



**Figure 8.7: Example FOS diagnostics for *tutorial\_intro* with a target life of 1E7 repeats. The coarse and fine increments are 0.1 and 0.01, respectively.**

An example FOS diagnostic output is shown by Figure 8.7 for the job *tutorial\_intro*. In this case only one iteration was performed because the value of the FOS crossed the target life. The value of the FOS is reported as 1.0 because the original value of life is closer to the design target than the initial FOS calculation would produce. This represents an accuracy of 53%. It is clear that the default FOS configuration for this analysis failed to achieve an acceptable value.

The coarse and fine increments are reduced to allow Quick Fatigue Tool to perform more iterations before the target life is crossed.

*Environment file usage:*

Variable	Value
<code>fosCoarseIncrement</code>	0.01;
<code>fosFineIncrement</code>	0.001;

Furthermore, the maximum number of iterations is increased.

*Environment file usage:*

Variable	Value
<code>fosMaxCoarselIterations</code>	16.0;
<code>fosMaxFinelIterations</code>	24.0;

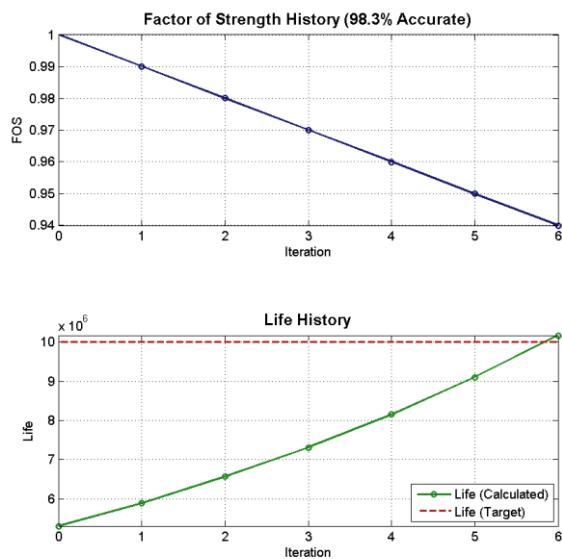
The FOS diagnostic for this run is shown in Figure 8.8. The calculated FOS is 0.94 and scaling the loading by this value results in a life of  $1.017E7$  repeats. This solution is within 1.7% of the target life. Therefore, the value is considered to be acceptable.

If the target life is below one million cycles, linearly scaling the loading by even very small amounts can have a dramatic effect on the fatigue life. This can cause numerical difficulties when Quick Fatigue Tool attempts to find a solution for the FOS. An example of “chattering” is shown in Figure 8.9, where the FOS never converges on the target life. This can be prevented by enabling a bracketing condition which ends the FOS calculation if the current calculated life crosses the target life.

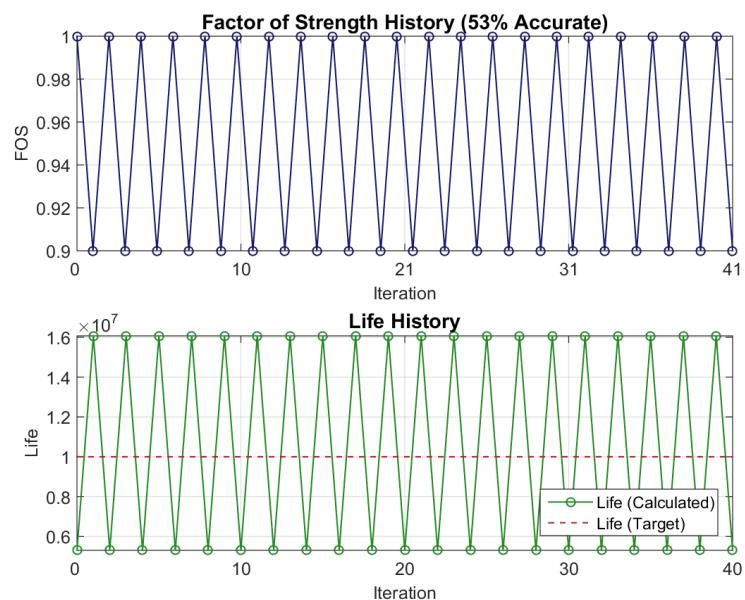
*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>fosBreakAfterBracket</code>	[0.0   1.0];

If the FOS algorithm detects a chattering condition, bracketing is automatically enabled to prevent additional iterations. Quick Fatigue Tool will automatically accept the best solution based on the tolerance.



**Figure 8.8:** Example FOS diagnostics for *tutorial\_intro* with a target life of 1E7 repeats. The coarse and fine increments are 0.01 and 0.001, respectively.



**Figure 8.9:** An example of FOS “chattering” where the algorithm is unable to converge on the target life.

### 8.3.5 FOS augmentation

Selection of the correct incrementation settings is often unobvious and can lead to a time-consuming process of trial and error. Even with well-tuned settings, the default incrementation scheme is linear, and it is easy to unnecessarily spend many iterations. FOS augmentation is a scheme which attempts to accelerate FOS convergence by modifying the user-defined incrementation parameters if the current iteration is judged to cause unsatisfactory convergence behaviour. Thus, FOS augmentation aims to alleviate the manual aspect of the FOS algorithm by applying a certain level of automation to the incrementation scheme.

Since the effect of estimating the FOS depends on a number of factors, such as loading, critical plane analysis, material properties (and so on), even the augmented FOS scheme is required to make guesses as to the appropriate incrementation.

The augmented FOS scheme is enabled in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>fosAugment</code>	[0.0   1.0];

When augmentation is enabled, Quick Fatigue Tool compares  $N_l$ , the difference in fatigue life between the two most recent iterations, with  $N_t$ , the difference between the previously calculated fatigue life and the target life. If the ratio between  $N_l$  and  $N_t$  is less than a threshold value,  $k_{thresh}$ , the next FOS increment is increased by a factor  $k_{factor}$ .

The threshold value and augmentation factor is set in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>fosAugmentThreshold</code>	$k_{thresh}$ ;
<code>fosAugmentFactor</code>	$k_{factor}$ ;

The default values of the augmentation threshold and factor are 0.2 and 5.0, respectively.

Figures 8.10-11 illustrate the improvement in convergence behaviour when FOS augmentation is enabled with the default settings using `tutorial_intro.m` as an example (note that the load factor and algorithm have been modified to improve the illustration).

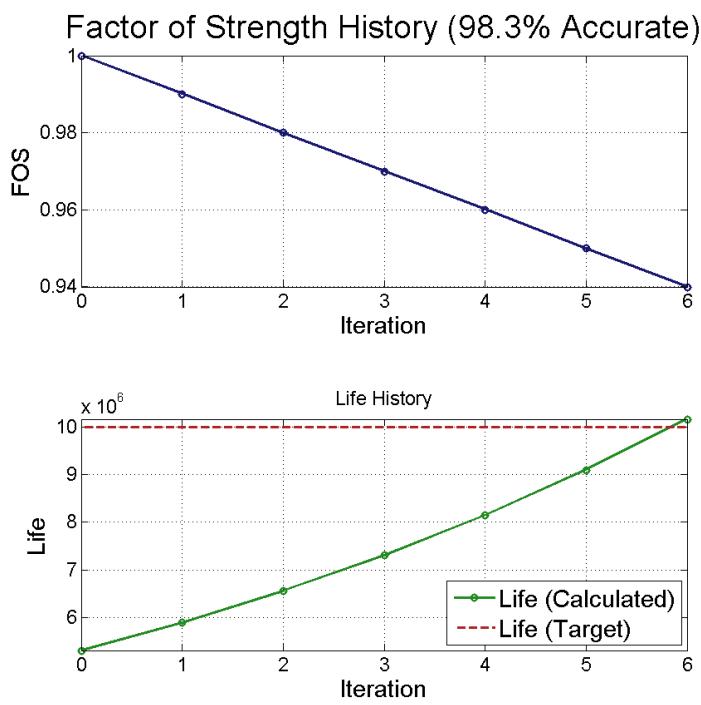


Figure 8.10: FOS diagnostics with augmentation disabled

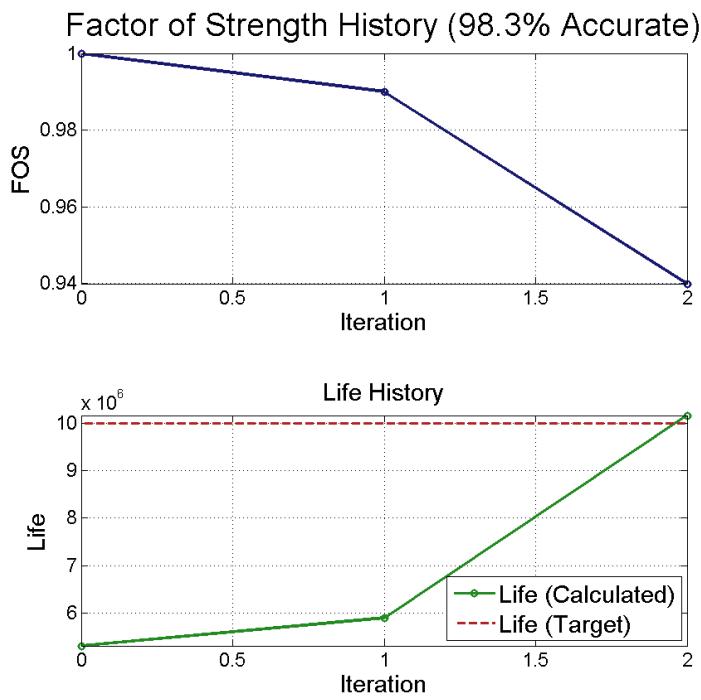


Figure 8.11: FOS diagnostics with augmentation enabled

## **9. Job and environment files**

This section has been released in the document *Quick Fatigue Tool User Settings Reference Guide*.

## 10. Output

### 10.1 Background

Quick Fatigue Tool reports extensive analysis output in addition to the fatigue life and safety factors at the worst analysis item.

There are four categories of output:

1. Worst item summaries
2. Fields
3. Worst item histories
4. Whole model histories

The worst item is defined as the item with the largest fatigue damage. In cases where more than one item has the largest damage, the item with the largest principal stress over the set of worst items is taken.

Field and history data is written to a set of text files. Pre-selected history variables can be plotted automatically and saved in the results directory as MATLAB figures. If FEA stresses were analysed from an Abaqus model, field data can be written to an output database (.odb) file and viewed on the finite element mesh with Abaqus/Viewer. This functionality is discussed in Section 10.4.

Field and history data is requested in the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
OUTPUT_FIELD	1.0;
OUTPUT_HISTORY	1.0;

Pre-selected MATLAB figures are also requested from the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
OUTPUT_PICTURE	1.0;

## 10.2 Output variable types

When an analysis runs to completion, a compact summary of a selection of variables is displayed in the command window and written to the log file, an example of which is shown in Figure 10.1.

If extensive output is requested in the job file, the fields and histories are written to separate output files.

```
FATIGUE RESULTS SUMMARY:  
=====  
Worst Life-Repeats : 5.3e+06  
at item 1.1  
  
Number of cycles in loading : 1  
  
Worst FRF : 0.9415  
at Item 1.1  
  
Maximum stress (MPa) : 200  
at Item 1.1  
  
Maximum stress/yield : 0.6154  
at Item 1.1  
  
Maximum stress/UTS : 0.5  
at Item 1.1  
  
Worst cycle mean stress (MPa) : 0  
at Item 1.1  
  
Worst cycle stress amplitude (MPa) : 200  
at Item 1.1  
  
Worst cycle damage parameter (MPa) : 200  
at Item 1.1  
  
Analysis time : 0:00:0.398  
=====  
Job tutorial_intro completed successfully (14-Jul-2016 15:59:42)
```

Figure 10.1: Example fatigue results summary from the log file

## Field variables

Field output is data representing spatially varying quantities over the model. The following fields are written, with their respective identifiers:

<b>L</b>	Fatigue life (linear scale) at each item in the model. The units depend on the string value set by LOAD_EQ in the job file.
<b>LL</b>	The $\log_{10}$ value of <b>L</b> . Note that values of <b>LL</b> are capped at the material's endurance limit. For example, if the endurance limit is 2E+07 reversals, then the maximum reported value of <b>LL</b> will be 7.0.
<b>D</b>	Fatigue damage ( $L^{-1}$ ) at each item in the model.
<b>DDL</b>	Fatigue damage at design life. Calculated by multiplying the damage, <b>D</b> , by the user-specified design life.
<b>FOS</b>	Factor of strength at design life. A linear scale which, when multiplied by the loading, results in the design life.  This field has a value of -1.0 if the FOS calculation was not requested.  The FOS calculation is enabled by setting FACTOR_OF_STRENGTH = 1.0 in the job file.
<b>SFA</b>	Ratio between the material fatigue limit and the maximum stress amplitude, <b>WCA</b> , at each item in the model. The endurance limit is discussed in Appendix I.
<b>FRFH</b>	Horizontal fatigue reserve factor.  For user-defined FRF data, a value of -1.0 is returned if the calculation was unsuccessful.
<b>FRFV</b>	Vertical fatigue reserve factor.  For user-defined FRF data, a value of -1.0 is returned if the calculation was unsuccessful.

<b>FRFR</b>	Radial fatigue reserve factor.  For user-defined FRF data, a value of -1.0 is returned if the calculation was unsuccessful.
<b>FRFW</b>	Fatigue reserve factor (worst of above three).
<b>SMAX</b>	Largest stress in loading. If the absolute value of the third principal stress is greater than the absolute value of the first principal stress, the maximum stress will be the third principal.  The calculation of the maximum stress does not include the effect of residual stress.
<b>SMXP</b>	<b>SMAX</b> divided by the 0.2% proof stress.
<b>SMXU</b>	<b>SMAX</b> divided by the material's ultimate tensile strength.
<b>TRF</b>	Stress triaxiality factor (ratio between hydrostatic and von Mises stress):  $TRF = -\frac{tr(\sigma)I_1}{3\sqrt{3J_2}}$ <p>where <math>\sigma</math> is the Cauchy stress tensor, <math>I_1</math> is the first stress invariant and <math>J_2</math> is the second deviatoric stress invariant.</p>
<b>WCM</b>	Worst cycle mean stress. Note that the mean stress is taken as the mean value of the worst cycle at each item in the model defined by the damage parameter. Therefore, the value of <b>WCM</b> depends on the selected fatigue analysis algorithm.

**WCA**

Worst cycle stress amplitude. The stress amplitude is the cycle counted quantity according the selected analysis algorithm:

**Uniaxial Stress-Life:** Uniaxial stress  $\sigma_{11}$  (defined directly as the stress history).

**Stress-based Brown-Miller:** Sum of the shear and normal stress on the critical plane.

**Normal Stress:** Normal stress on the critical plane.

**Findley's Method:** Shear stress on the critical plane (note that the normal stress is not included in the definition of the stress amplitude).

**Stress Invariant Parameter:** von Mises equivalent stress.

**BS 7608:** Normal, shear or (normal + shear) stress on the critical plane, depending on the value of FAILURE\_MODE in the job file

**NASALIFE:** Effective stress, defined by the environment variable `nasalifeParameter`. The effective stress variables are described in Section 6.8.3.

**WCATAN**

Worst cycle arctangent between **WCM** and **WCA**.

**WCDP**

Worst cycle damage parameter. The damage parameter is the stress used in the fatigue damage calculation and is usually the stress amplitude.

The damage parameter includes the effect of the mean stress correction (except in the case of the Morrow and R-ratio S-N curves corrections, since these are applied indirectly). Therefore, if mean stress correction is applied to the loading, the values of **WCDP** and **WCA** will differ.

If no mean stress correction is applied, or if the mean stress in the loading is zero, the value of **WCDP** and **WCA** is the same. The exception is with the use of Findley's Method with  $k > 0$ , since the amplitude parameter is different from the damage parameter.

## **YIELD**

Items in the model which are yielding.

This field has a value of 1.0 if the items have yielded according to the selected criterion, 0.0 if the item has not yielded, -2.0 if the yield criterion could not be evaluated or -1.0 if the yield calculation was not requested.

In addition to the yield stress, the yield calculation requires the cyclic stress-strain material properties  $K'$  and  $n'$  and the Young's Modulus  $E$ . The stresses are corrected for plasticity using the Ramberg-Osgood multilinear cyclic hardening model.

The algorithm accounts for the effect of hysteresis and material memory. Therefore, the instantaneous elastic-plastic stress is a function of all previous stress states in the history. Stress-hardening due to small amounts of ratcheting may result in delayed yielding as the yield surface is shifted along the  $\sigma$ -axis.

Yield is defined by one of the following strain energy criteria:

### **Beltrami-Haigh isotropic total strain energy theory**

The stored energy associated with elastic deformation at the point of yield is independent of the specific stress tensor. Thus yield occurs when the strain energy per unit volume is greater than the strain energy at the elastic limit in simple tension. For the three-dimensional stress state this is given by [49]:

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3) \leq \sigma_y^2$$

This yield criterion is enabled by setting '`yieldCriterion`' = 1.0 in the environment file.

A value of the Poisson's ratio,  $\nu$ , is required for this calculation.

### **Shear strain energy theory**

Failure occurs when the shear strain energy in the actual case exceeds the shear strain energy in a simple tension test. For the three-dimensional stress state this is given by:

$$0.5 \times [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2] \leq \sigma_y^2$$

This yield criterion is enabled by setting '`yieldCriterion`' = 2.0 in the environment file.

#### **Writing to an output database file**

The value of **YIELD** can be written to an Abaqus output database (*.odb*) file by specifying the model database in the job file (consult Section 4.6 for instructions on associating a job file with an output database). The **YIELD** variable is specified from the environment file using the variable `autoExport_YIELD`.

Writing the yielding items to an *.odb* file through the Export Tool is not currently supported.

#### **TSE**

If the variable **YIELD** has a value of 1.0 anywhere in the model, the total strain energy per unit volume is written to ‘warn\_yielding\_items.dat’.

#### **PSE**

If the variable **YIELD** has a value of 1.0 anywhere in the model, the plastic strain energy per unit volume is written to ‘warn\_yielding\_items.dat’.

## History variables

History output represents the load-varying quantities for the most damaged analysis item in the model. The following histories are written, with their respective identifiers:

<b>ST</b>	Stress tensor at worst item, on the critical plane, for the loading.
<b>HD</b>	Haigh diagram for the worst cycle on the critical plane.
<b>VM</b>	von Mises stress for the loading.
<b>PS1</b>	Maximum (first) principal stress for the loading.
<b>PS2</b>	Middle (second) principal stress for the loading.
<b>PS3</b>	Minimum (third) principal stress for the loading.
<b>CN</b>	Maximum normal stress history for the loading. For Uniaxial Stress-Life or von Mises analyses, the normal stress is the hydrostatic stress. For multiaxial analyses, it is the maximum normal stress history on the critical plane.
<b>CS</b>	Maximum shear stress history for the loading. For Uniaxial Stress-Life or von Mises analyses, the shear stress is the Tresca stress. For multiaxial analyses, it is the maximum shear stress history on the critical plane, determined either by the maximum chord method or the maximum resultant shear stress. This is specified by the environment variable <a href="#">cpShearStress</a> .
<b>DP</b>	Damage vs. plane angle.
<b>DPP</b>	Damage parameter vs. plane angle.
<b>LP</b>	Life vs. plane angle.
<b>DAC</b>	Damage accumulation at worst analysis item. Only available if more than one cycle in the loading.
<b>RHIST</b>	Rainflow histogram of cycle counted stresses.
<b>RC</b>	Stress range distribution.
<b>SIG</b>	Uniaxial load history (before and after gating if applicable).

## **Whole model variables**

Whole model histories are load histories at every item in the model:

**ANHD**

Worst cycle Haigh diagram for each item in the model.

## MATLAB figures

Certain history data can also be plotted to a series of figures. The default figure type is the MATLAB *.fig* file. This default may be changed in the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>figureFormat</code>	'<format>';

Any valid file format is accepted, e.g. '`png`', '`jpeg`', '`jpg`' etc.

<b>ANHD + HD</b>	Worst cycle Haigh diagram for all items and the critical plane.
<b>KDSN</b>	S-N curves for materials using knock-down factors.
<b>CN + CS</b>	Normal and shear stress history on the critical plane at the worst item.
<b>DP</b>	Damage vs. angle at the worst item.
<b>DPP</b>	Damage parameter vs. angle at the worst item.
<b>LP</b>	Life vs. angle at the worst item.
<b>CPS</b>	Normal stress and resultant shear stress vs. plane angle at the worst item.
<b>PS</b>	Principal stresses at the worst item.
<b>VM</b>	von Mises stress at the worst item.
<b>DAC</b>	Cumulative damage at the worst item.
<b>RHIST</b>	Rainflow cycle histogram at the worst item.
<b>RC</b>	Stress range distribution at the worst item.
<b>SIG</b>	Uniaxial load history (before and after gating, if applicable).
<b>FOS</b>	Factor of strength diagnostics.

## 10.3 Viewing output

### Output location

Field and history output for a particular analysis is stored under *Project\output\<jobName>\Data Files*. The output directory has the file structure shown in Figure 10.2.

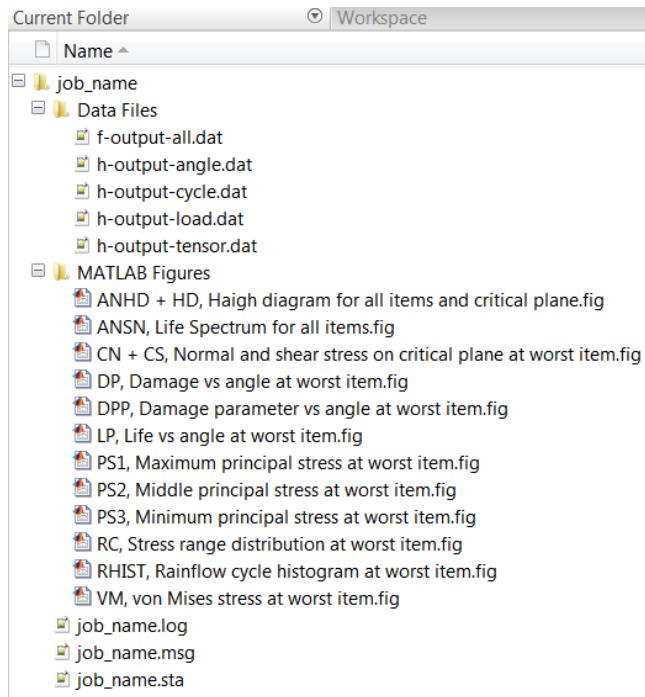


Figure 10.2: Output directory structure

The name of the output directory shares the name of the job with which the output is associated. If a job is submitted with the same name as an existing output directory, Quick Fatigue Tool will overwrite the previous results files. Therefore, care should be taken when choosing the job name in the job file.

Output is stored as human-readable ASCII text, and can be viewed in MATLAB or with any text editor.

### Changing the output format

By default, floating-point values are converted to text using fixed-point notation ('%f'). This can be changed using the following environment variables.

*Environment file usage:*

Variable	Value
fieldFormatString	'<format>';
historyFormatString	'<format>';

For example, '.2f' represents two digits after the decimal mark, '12f' represents twelve characters in the output and '.0f' represents integer output. The user is not required to specify the '%' format identifier. More information on formatting text can be found in the official MATLAB documentation.

## 10.4 The ODB Interface

### 10.4.1 Overview

Quick Fatigue Tool includes an interface which is capable of writing fatigue results to an Abaqus output database (*.odb*) file. This allows the user to visualize selected field output variables in Abaqus/Viewer. There are two methods for accessing the ODB interface:

1. Via the job and environment files (text-based)
2. Via the Export Tool application (UI-based)

The first method allows the user to configure the interface to automatically write fatigue results to the *.odb* file immediately after the analysis, whereas the second method allows the user to write results to an *.odb* file based on a selected field data file, after the analysis has already been performed.

Both methods represent the same functionality; the differences lie in how and when the interface is accessed.

The ODB interface requires that Abaqus is installed on the host machine. If an installation cannot be found, the interface will exit with an error.

### 10.4.2 Accessing the ODB interface via the Export Tool

The Export Tool is a MATLAB GUI application which gives the user interactive access to functions of the ODB interface.

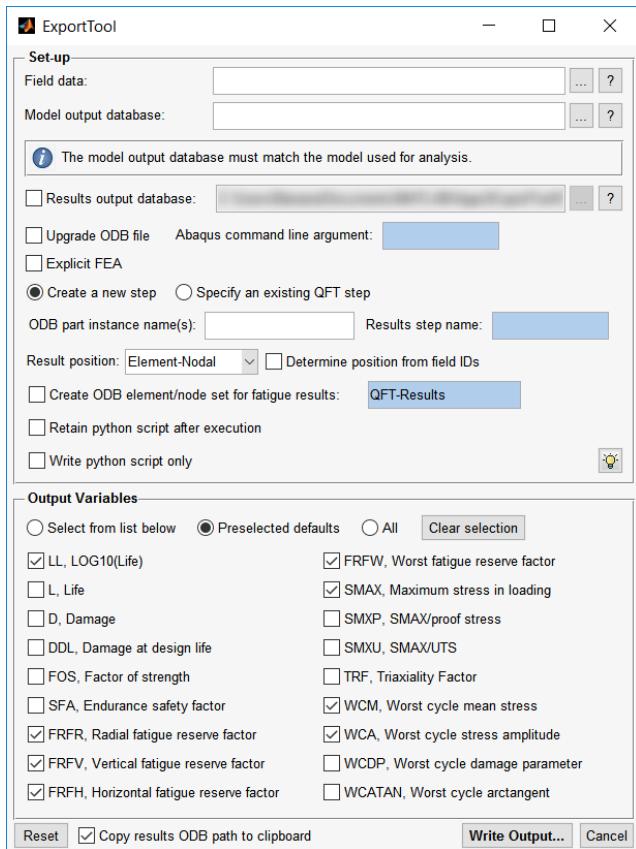
The Export Tool is launched either by running the file *ExportTool.m* in the *Application\_Files\source\odb\_interface* directory, or by installing and running the tool as a MATLAB app. The app installer can be found in *Application\_Files\toolbox*. The Export Tool interface is shown in Figure 10.3.

### 10.4.3 Enabling the ODB interface via the environment file

As long as the ODB interface is enabled, Quick Fatigue Tool will automatically export field results to an output database file if it finds a valid definition in the job file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<i>autoExport_ODB</i>	[0.0   1.0];



**Figure 10.3: Export Tool interface**

#### 10.4.4 Configuring the ODB interface GUI

##### Specifying the field data to export

The ODB interface copies results data from the field output file in the output directory. This is typically located in `\Project\output\<jobName>\Data Files\f-output-all.dat`. The field output file must be specified by the user in advance.

###### *Export Tool usage:*

Set-up region: Select the field data file either by entering the absolute path in the file window, or by selecting the file via the file browser using the button.

###### *Job/environment file usage:*

The field data file is created during the job and automatically located after the analysis. The user is not required to specify the field data file.

## Specifying the model ODB file

The ODB interface must be able to locate the original (model) .odb file so that it can make a copy of the file and append the result step.

*Export Tool usage:* Set-up region: Select the model output database file either by entering the absolute path in the file window, or by selecting the file via the file browser using the  button.

*Job file usage:*

<i>Option</i>	<i>Value</i>
OUTPUT_DATABASE	' <i>model-output-database-file-name.odb</i> ';

The user must always specify the absolute path of the .odb file.

## Specifying the results ODB file

Quick Fatigue Tool automatically chooses the location of the results .odb file. By default, the file is stored in the *Project\output\<jobName>\Data Files* directory. This behaviour may be overridden with the Export Tool.

*Export Tool usage:* Set-up region: Check the option **Results output database**. Select the results output database location either by entering the absolute path in the directory window, or by selecting the location via the file browser using the  button.

*Job/environment file usage:* The results .odb file location cannot be modified by the user. It will always be stored under *Project\output\<jobName>\Data Files*.

## Specifying the FE procedure

Due to rules defined within the Abaqus modelling framework, general static procedures are not permitted directly after an Abaqus/Explicit procedure. By default, the ODB interface will attempt to append a static step to the model .odb file for fatigue results. Therefore, if the previous FEA step originated from an Abaqus/Explicit analysis, the user must indicate this directly.

*Export Tool usage:* Set-up region: Check the option **Explicit FEA**.

*Job file usage:*

<i>Option</i>	<i>Value</i>
EXPLICIT_FEA	[0.0   1.0];

## Specifying the Abaqus API version

The user can specify which version of the Abaqus API will be used to create the results .odb file. This is done by specifying the Abaqus command line argument. The command line argument is the name of the batch file corresponding to the Abaqus version. These files are typically located in <Abaqus\_installation\_directory>\Commands.

*Export Tool usage:* Set-up region: Fill out the **Abaqus command line argument** edit box.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
autoExport_abqCmd	'abaqus-command';

By default, the command 'abaqus' is used. Assuming a standard Abaqus installation exists on the host machine, this argument points to the most recently installed Abaqus version.

If the installed Abaqus version is more recent than the model .odb file, the upgrade utility can be used to upgrade the file to a more recent version. The command line syntax for this is as follows:

```
>> <abaqus_command> upgrade –job <jobName> -odb <oldOdbFileName>
```

This functionality can be accessed via the ODB interface.

*Export Tool usage:* Set-up region: Check the option **Upgrade ODB file**.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
autoExport_upgradeODB	[0.0   1.0];

If the user specified to upgrade the .odb file, the ODB interface chooses the Abaqus version specified by 'abaqus-command'.

## Specifying the part instance

Quick Fatigue Tool can usually only recognise stress datasets originating from a single part instance, since Abaqus may assign duplicate element-node numbers for each instance. Therefore, the part instance name must be specified to resolve potential ambiguity. Part instance names are inherited from the part itself. For example, if the part was named *BOLT*, then the part instance will be named *BOLT-n*. To check the name of the part instance, from Abaqus/Viewer, query an element on a region of the model from where stress data was exported. In the message window, the element type and corresponding part instance name is shown.

*Export Tool usage:* Set-up region: Fill out the **ODB part instance name** edit box.

*Job file usage:*

<i>Option</i>	<i>Value</i>
PART_INSTANCE	'part-instance-name';

If the Abaqus job was run from a flat input file, there is only a single part instance in the output database called *PART-1-1*. If a flat input file was used, this name should be specified.

If the field data spans multiple part instances, these can be specified together.

*Export Tool usage:* Set-up region: Fill out the **ODB part instance name** edit box. Separate part instance names with double quotes ("").

*Job file usage:*

<i>Option</i>	<i>Value</i>
PART_INSTANCE	{'part-instance-1', ..., 'part-instance-n'};

### Specifying the step name

The ODB interface can either append a new step to the results .odb file, or it can write fatigue results to an existing step that was created by the interface on a previous occasion.

*Export Tool usage:* Set-up region: Choose either **Create a new step** or **Specify an existing QFT step** from the radio button selector. If a new step is being created, the name can be specified; otherwise a default name will be used. If an existing step is specified, the name of the step must be given.

The step name is specified by filling out the **Results step name** edit box.

*Job file usage:*

<i>Option</i>	<i>Value</i>
STEP_NAME	'step-name';

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
autoExport_stepType	n;

The value of  $n$  dictates the following:

1. A new step is created
2. Results are exported to an existing QFT step

If a new step is chosen, the step name is optional. If no name is specified, Quick Fatigue Tool will choose a name automatically based on a combination of the job name and the part instance.

If an existing step is chosen, the name of the step must be specified. The following limitations apply:

- the step must have been written by the ODB interface on a previous occasion;
- the same field output variables must be written as when the step was created

### Specifying the element result position

The result position is required because the ODB interface must tell the Abaqus API where on the element to write the fatigue results.

*Export Tool usage:* Set-up region: Select the position from the **Result position** drop-down menu.

*Job file usage:*

<i>Option</i>	<i>Value</i>
RESULT_POSITION	[ 'ELEMENT NODAL'   'UNIQUE NODAL'   'INTEGRATION POINT'   'CENTROID'];

If the result position is unknown, the ODB interface can “guess” the most suitable position based on the format of the field data.

*Export Tool usage:* Set-up region: Check the option **Determine position from field IDs**

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
autoExport_autoPosition	$n;$

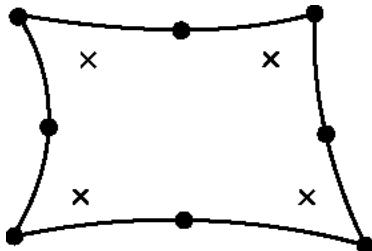
The value of  $n$  dictates the following:

0. The position is not determined automatically
1. The ODB interface will attempt to select the most suitable results position

It is possible for the code to determine the incorrect position, so the user should ensure that the selected result position matches the selection made when exporting the RPT file.

Figure 10.4 is an example of a plane stress element. If the result position is unique or element-nodal, data is written to the outer points of the element. Integration point data is written to the Gauss points in the element subsurface. Centroidal data is written to the geometric centre of each element.

### Creating an element/node set



**Figure 10.4: S8R Shell element with four integration points and nine nodes**

The ODB interface can create an element or node set in the results ODB file to facilitate easier results visualization in Abaqus/Viewer.

*Export Tool usage:* Set-up region: Check the option **Create ODB element/node set for fatigue results** and fill out the edit box.

*Environment file usage:*

Variable	Value
autoExport_createODBSet	[0.0   1.0];
autoExport_ODBSetName	'ODBSetName'

The format of the ODB set is as follows:

Result position	ODB set type
Element-nodal	Node and element
Unique nodal	Node
Integration point	Element
Centroidal	Element

If an ODB set is written to the same ODB file by a subsequent analysis, the API will return the following error:

*OdbError: Duplicate set or surface name QFT\_PART-1-1\_QFT*

ODB sets can only be written to the output database if the selected field data file exactly matches the element-nodes for the selected part instance. ODB sets will not be written if more than one part instance name is specified.

## Specifying how the python script is handled

The Export Tool works by writing a Python script containing all the instructions necessary to create a copy of the model ODB with an additional step containing the fatigue analysis result data. The script is submitted to the Python interpreter within the Abaqus/CAE framework, which then creates the output database. The process schematic is shown in Figure 10.5. The Python script used for the export operation may be retained, or the user can specify that only the python script should be written and it should not be submitted to the Abaqus API. In this case a results .odb file will not be created.

*Export Tool usage:* Set-up region: Check the option **Retain python script after execution** or **Write python script only**. The options are mutually exclusive.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
autoExport_executionMode	<i>n;</i>

The value of *n* dictates the following:

1. Export results and discard the python script
2. Export results and retain the python script
3. Write the python script only. No results are exported

## Specifying the field output variables

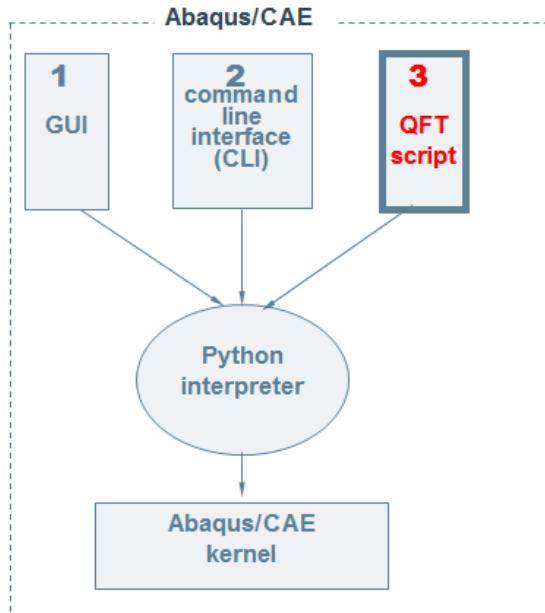
The user can choose which field output variables to export to the output database file.

*Export Tool usage:* Output Variables region: Choose either **Select from list below**, **Preselected defaults** or **All** from the radio button selector.

If **Select from the list below** is selected, check the field output variables you wish to export to the output database file

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
autoExport_selectionMode	<i>n<sub>1</sub>;</i>
autoExport_fieldName	<i>n<sub>2</sub>;</i>



**Figure 10.5: Interface between Quick Fatigue Tool and Abaqus/CAE**

The value of  $n_1$  dictates the following:

1. Select the field output variables manually
2. Use preselected defaults
3. Export all field output variables

The value of  $n_2$  dictates the following:

0. Do not export the field output variables
1. Export the field output variable

#### **Exporting yield criterion variables to the output database**

If the user enabled the yield criterion, the yield flag and associated strain energies can be exported to the results output database file.

*Export Tool usage:* The YIELD variable is not supported in Export Tool.

#### *Environment file usage:*

<i>Variable</i>	<i>Value</i>
autoExport_YIELD	1.0;

#### **10.4.5 Mismatching ODB files**

While it is important that the source model matches the results data, Quick Fatigue Tool will attempt to match field data to corresponding elements and nodes in the model ODB, even if the finite element mesh is different. If the user selects the “Unique Nodal” or “Centroidal” options, Quick Fatigue Tool will write field data directly to the ODB based on the position labels provided in the field output file. If the user selects the “Element-Nodal” or “Integration Point” options, Quick Fatigue Tool will request the nodal connectivity matrix from the Abaqus API. This is required due to the fact that the order of the element-node or element-integration point data in the field output file will not necessarily match the order required in the ODB.

If the ODB interface finds nodes in the ODB file which cannot be found in the field data, that node and all others belonging to the same element will be ignored. If the Export Tool finds elements in the ODB which cannot be found in the field data, that element and all of its nodes or integration points will be ignored. As such, it is not necessary to export stress datasets from every element in the part instance when generating the RPT file(s), since the ODB interface will automatically ignore regions of the part instance which weren’t included in the fatigue analysis.

#### **10.4.6 Large ODB files**

For very large ODB files, it is unnecessary (and often impractical) to copy all of the FEA data to the results ODB. In cases where copying the ODB would consume a significant amount of time, the user is advised to create a *data check* ODB. These files contain the mesh from the FE model, but no FE results data. Hence, *data check* ODBs are more efficient as a container for fatigue analysis results.

*Abaqus/CAE usage:*      Expand the jobs container from the model tree. Right-click on the job and select **Data Check**.

*Command line usage:*      `abaqus job=<jobName> datacheck`

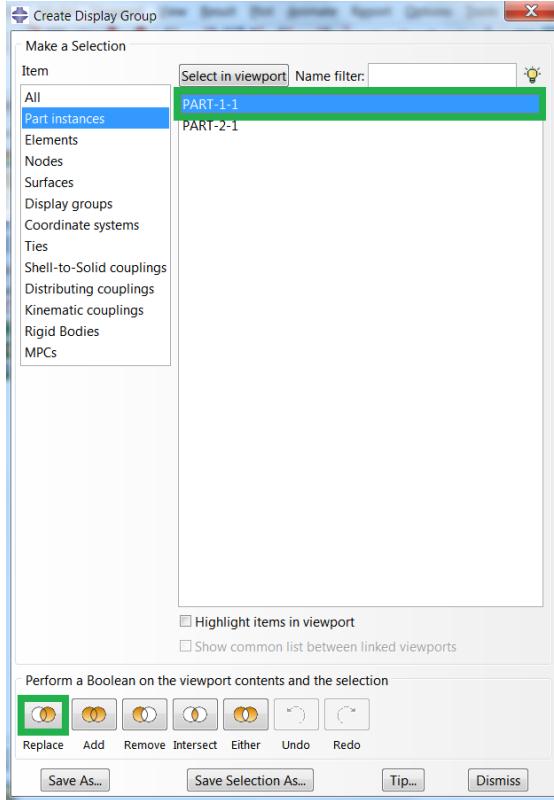
#### **10.4.7 Exporting field data to multiple Abaqus ODB part instances**

Since Quick Fatigue Tool only uses the element-node numbers written to the field report file, it does not distinguish between individual part instances. Although it is possible to write field data to multiple part instances, the user must ensure that element-node numbers are not reused between instances, otherwise results will not be written to the output database correctly. There are three approaches for writing field data to multiple part instances:

1. Write multiple steps to the same *.odb* file, each step containing results at one part instance
2. Append field output from subsequent part instances to a previously created results step
3. Run the FEA from a flat input file

##### **Example: Write multiple steps to the same *.odb* file**

If the user only has access to the *.odb* file and is unable to modify the attributes of the original model, field data may be written to multiple part instances by running a separate fatigue analysis job for each instance. Upon completion of each job, field data is written back to the previous result *.odb* file as a new step.



**Figure 10.6: Create display group tool showing how to replace the viewport contents with the required part instance**

Individual part instances are selected in Abaqus/Viewer using the **Create Display Group** tool. In order to create the necessary stress dataset file, only the required part instance should be displayed in the viewport. This is achieved by selecting the required part instance and selecting **Replace**, as shown in Figure 10.6.

The stress dataset file is then created by following the steps outlined in Section 3.2, corresponding to the first part instance of interest. For the first job, the Abaqus ODB options are configured from the job file.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATASET	'dataset-A.rpt';
OUTPUT_DATABASE	'<absolutePath>\model-ODB.odb';
PART_INSTANCE	'part-instance-name-A';
STEP_NAME	'step-name-A';

A second job is run for the dataset corresponding to the second part instance of interest. The model output database is specified as the results output database from the previous analysis and the name of the second part instance is specified.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATASET	'dataset-B.rpt';
OUTPUT_DATABASE	'<absolutePath>\results-ODB.odb';
PART_INSTANCE	'part-instance-name-B';
STEP_NAME	'step-name-B';

This will append a new step to the existing results output database containing field data for the second part instance. The STEP\_NAME option adds an additional string to the name of the results step and is not compulsory. However, adding a step name for each analysis minimizes the risk of Quick Fatigue Tool attempting to create two steps with the same name in a single .odb file, which would result in an error.

The advantage of writing field output to individual steps is that the field output variable selection can differ between part instances. However, the drawback of this method is that it is not possible to view a field over all part instances simultaneously; rather, the user must switch between steps in order to view results at different regions in the model.

**Example: Append field output from subsequent part instances to a previously created results step**

An alternative to the previous method is to append results from subsequent analyse to a results ODB step created from a previous analysis. Output is written to the first part instance.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATASET	'dataset-A.rpt';
OUTPUT_DATABASE	'<absolutePath>\model-ODB.odb';
PART_INSTANCE	'part-instance-name-A';
STEP_NAME	'step-name-A';

The analysis is then repeated for the second part instance.

*Job file usage:*

<i>Option</i>	<i>Value</i>
DATASET	'dataset-B.rpt';
OUTPUT_DATABASE	'<absolutePath>\results-ODB.odb';
PART_INSTANCE	'<partName_B>';
STEP_NAME	'<stepName>';

In this case, the path to the output database is defined as the path to the results output database from the first analysis. The step names must match between the two models. By default, Quick Fatigue Tool creates a new step for the results data. Therefore, in order to instruct the program to append results to the previous step, the following environment variable must be set from the environment file.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
autoExport_stepType	2.0;

The following caveats must be observed when appending output to an existing step:

- The previous step must have been written by Quick Fatigue Tool
- The fields must exactly match those written in the original step, otherwise the Python API will exit with errors

The advantage of appending results data to a previous step is that fields can be visualized over all part instances simultaneously, without having to switch between steps. The results position of the elements does not have to be the same between part instances, although exporting data at different positions will result in duplicate field variables being written to the ODB frame. Results which are appended may also overwrite results which were written during previous analyses.

#### **Example: Run the FEA from a flat input file**

The alternative solution is to avoid writing individual part instances and assemblies to the input file. This is achieved by selecting **Do not use parts and assemblies in input files** in the **Edit Model Attributes** dialogue in Abaqus/CAE.

After the analysis, Quick Fatigue Tool will write to the message file the number of regions found in the .rpt file.

#### **10.4.8 Minimum requirement for output**

The Abaqus API imposes minimum requirements for the amount of field data which must be written to a given result frame. These requirements are detailed in the table below:

<b>Result position</b>	<b>Requirement</b>
Element-nodal	All the nodes defining a single element
Unique Nodal	At least two nodes
Integration Point	All the integration points defining a single element
Centroid	At least two centroids

Failure to adhere to above requirements will cause the ODB interface to exit with the following error message:

<b>Element-nodal/Integration Point</b>	Error: No matching position labels were found from the model output database
<b>Unique Nodal/Centroid</b>	ODBgetSeqSeqDoubleFromArray() num dims (1) != 2

## 11. Modelling techniques

### 11.1 Background

This section applies to users who wish to use stress datasets from a finite element analysis. Detailed guidance and illustrations focus on SIMULIA Abaqus, but the concepts can be applied to any FEA package.

### 11.2 Preparing an FE model for fatigue analysis

Consider the model in Figure 11.1. We wish to determine the fatigue life of the shaft when subject to a fully-reversed bending load. The first consideration is whether it is viable to analyse the whole model. The shaft is made from 56,720 elements and 60,867 nodes. Analyses with Quick Fatigue Tool can be cumbersome for models containing more than a few thousand nodes, so it would be unadvisable to analyse every element. At this stage, the user should ask the following questions:

- a) Is analysing the whole FE model reasonable considering the size of the mesh?

**If the answer to a) is no, then ask the following question:**

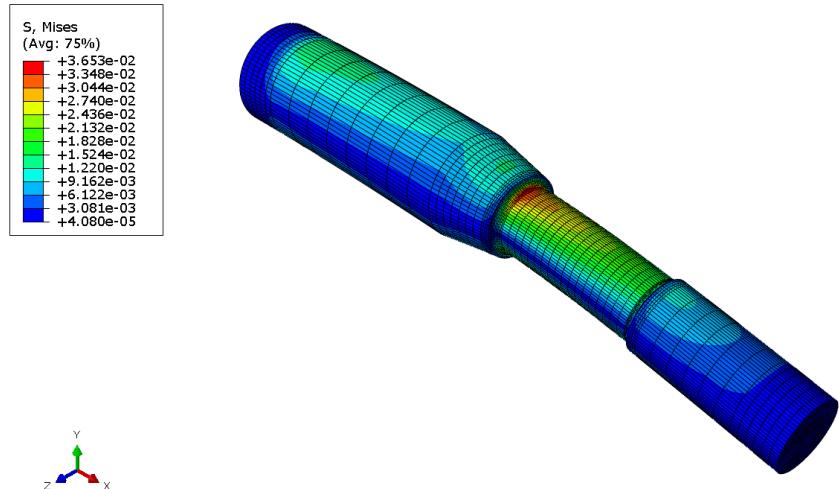
- b) Are the stresses concentrated at a particular location? i.e. are there regions in the model where the stresses are clearly non-damaging?

**If the answer to b) is yes, then reduce the number of elements for analysis**

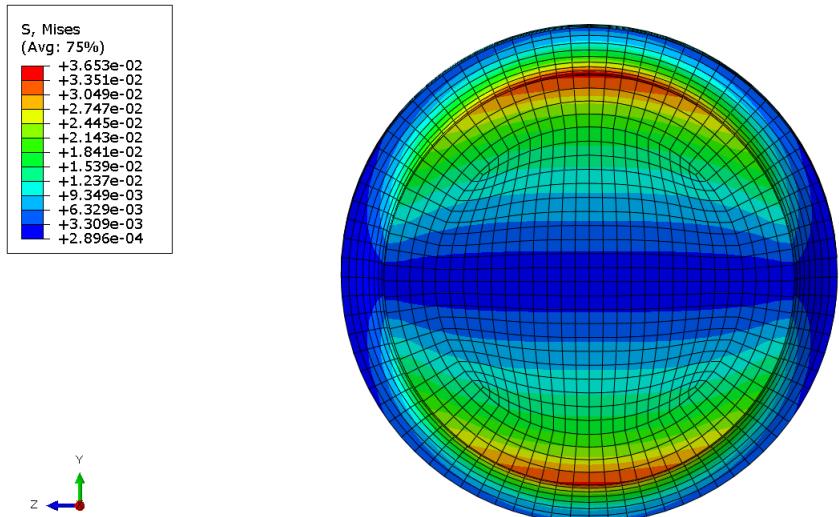
In the example of the shaft model, the bending load has resulted in a stress concentration at one of the fillet radii, so it is only necessary to analyse this region.

Using the Display Group Manager, the elements at the notch containing the stress concentration are isolated, shown by Figure 11.2. The model could be cut once more in the x-z plane to exploit the symmetry of the result. However, for this example the full notch geometry will be considered.

Quick Fatigue Tool defines “failure” as the complete propagation of a crack across a material surface defined by critical plane analysis. In most cases, these cracks initiate on the component surface, therefore subsurface elements may be excluded in cases where the stress gradient is sufficiently large. Subsurface elements are excluded from the analysis by creating a new display group and choosing to select elements “by angle”, shown in Figure 11.3. The final element set for the analysis is shown in Figure 11.4.

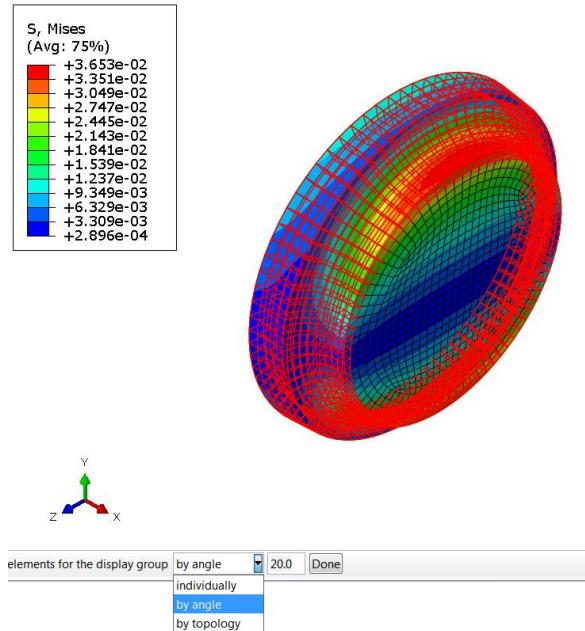


**Figure 11.1: SAE shaft model in bending**



**Figure 11.2: Isolating the stress concentration**

Before exporting the stress tensors for analysis, it is recommended that result averaging is turned off so that the stresses written to the *.rpt* file are a closer representation of the calculated gauss point values. From the main menu bar, go to **Result → Options...** and make sure that “Average element output at nodes” is unchecked (Figure 11.5).



11.3: Picking surface elements by angle

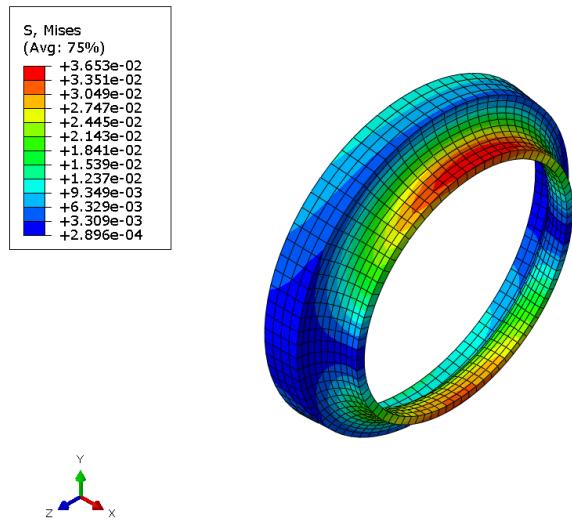


Figure 11.4: Final element analysis group

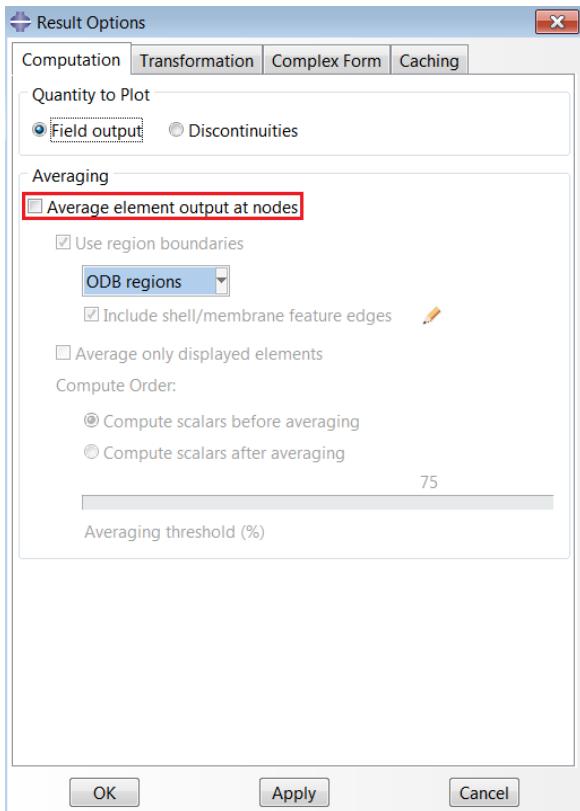


Figure 11.5: Result options dialogue in Abaqus/CAE

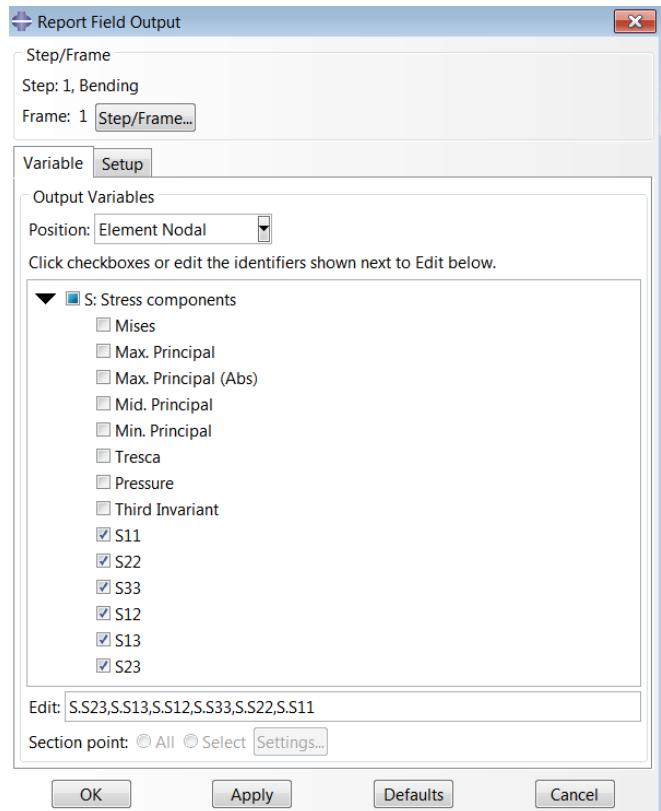


Figure 11.6: Selecting stresses for the .rpt file

The stresses are then exported by going to **Report -> Field Output...** and choosing the stress components, as shown in Figure 11.6. The user has the option to select the position for the field output. The choices of positions compatible with Quick Fatigue Tool are described below.

Position	Description
Integration Point	<ul style="list-style-type: none"> <li>• “True” stress solution</li> <li>• Size of output depends on the integration order</li> <li>• Produces accurate results for brittle materials where crack initiation is in the element sub-surface</li> <li>• Not recommended for ductile metals</li> </ul>
Centroid	<ul style="list-style-type: none"> <li>• Single, averaged value at the geometric centre of each element</li> <li>• Size of output depends only on the size of the mesh. Generates the least amount of output</li> <li>• Good for quick fatigue estimates</li> <li>• Offers the worst solution accuracy</li> </ul>
Element Nodal	<ul style="list-style-type: none"> <li>• Un-averaged result for each node of each element (nodes belonging to N elements have N solutions)</li> <li>• Size of output depends on the element geometric order</li> <li>• Produces accurate results for ductile materials where crack initiation is on the element free surface</li> <li>• Not recommended for brittle metals</li> </ul>
Unique Nodal	<ul style="list-style-type: none"> <li>• Single, averaged value at each node</li> <li>• Size of output depends on the element geometric order</li> <li>• Recommended for ductile metals</li> <li>• Offers slightly shorter analysis time than Element Nodal at the expense of some solution accuracy</li> </ul>

If the stresses are written using the Unique Nodal position, the number of nodes written for analysis for the example shaft model is 2,240. Since this is approximately 27 times less nodes than originally present, the expected analysis should also be approximately 27 times faster.

## 12. Tutorial A: Analysis of a welded plate with Abaqus

### 12.1 Background

This tutorial outlines the procedure for analysing a welded structure using an Abaqus output database file with Quick Fatigue Tool. The analysis is based upon the continuum shell model of a welded T-joint in bending, shown in Figure 12.1.

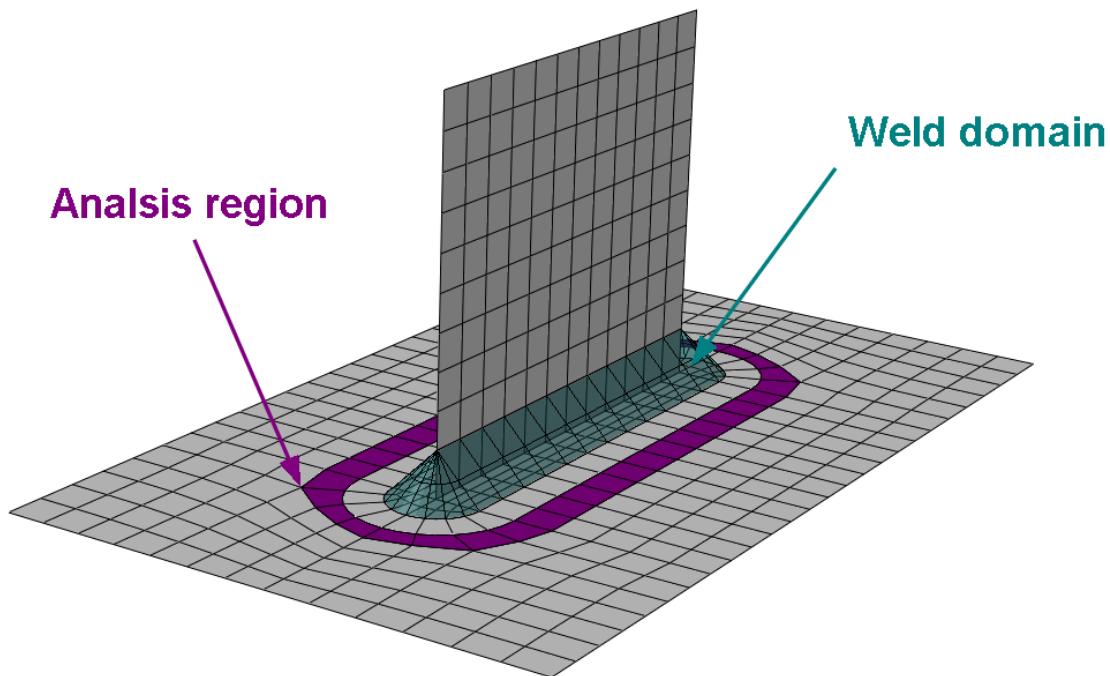


Figure 12.1: Weld plate model

The model will be analysed using the British Standard BS 7608 method for the analysis region indicated by the magenta elements in Figure 12.1. The methodology corrects the Stress-Life curve to account for the presence of a stress concentration at the weld toe; therefore, the stresses used for analysis should be a small distance away from the weld line, in order to avoid excessively conservative results. For more information about the BS 7608 method, refer to Section 6.6.

The stress datasets are first extracted from the output database and written to an RPT file. A fatigue analysis is then performed with Quick Fatigue Tool and the results are written back to the ODB using the Export Tool.

## 12.2 Preparing the RPT file

If you do not have Abaqus installed, you can skip this step; the file *weldPlate.m* has already been added to the *Data\datasets* folder. You must copy this file into the *Project\input* folder before continuing.

To generate an RPT file for the analysis region, open the Create Display Group dialogue box. From the Item region, select Elements. Select the element group PLATE-1\_ANALYSE and select Replace from the list of Boolean operations.

The exported stress tensors should not have any averaging applied to them. From the main menu, select **Result → Options....** From the Averaging region, deselect “Average element output at nodes” and select **OK**. From the main menu, select **Report → Field Output...** From the Variable tab, select Element-Nodal as the result position and select all four stress tensor components S11, S22, S33 and S12. From the Setup tab, specify the absolute path to the Quick Fatigue Tool input folder *Project\input* and name the file *weldPlate.rpt*. Deselect *Column totals* and *Column min/max* from the data region and select **OK**.

## 12.3 Running the analysis

From the *Project\input* folder, open the job file named *tutorial\_A.m* and review the contents. Ensure that the DATASET option points to the correct file. A summary of the other pertinent options are listed below.

Option	Details
PLANE_STRESS = 1.0;	The elements are plane stress
ALGORITHM = 8.0;	The algorithm is set to BS 7608
OUTPUT_FIELD = 1.0;	Field output is required to write results to the ODB
WELD_CLASS = 'F2';	Weld classification. For guidelines on choosing the weld class, consult the document BS 7608
YIELD_STRENGTH = 325; UTS = 400;	Mechanical properties of the weld plate material (MPa)
DEVIATIONS_BELOW_MEAN = 2.0;	Confidence interval (95% probability of failure)
FAILURE_MODE = 'NORMAL';	Failure criterion
CHARACTERISTIC_LENGTH = 1.0;	Plate thickness (mm)
SEA_WATER = 0.0;	Atmospheric condition (fresh air)

Run the analysis by right-clicking the job file and selecting *Run*. The analysis may take a few minutes depending the computer’s hardware specification. The result summary is displayed in the command window (Figure 12.2).

```

FATIGUE RESULTS SUMMARY:
=====
Worst Life-Repeats : 3.73e+04
at item 299.49

Number of cycles in loading : 1

Maximum stress (MPa) : -226.1
at Item 299.49

Worst cycle mean stress (MPa) : 18.08
at Item 351.442

Worst cycle stress amplitude (MPa) : 113.1
at Item 299.49

Worst cycle damage parameter (MPa) : 113.1
at Item 299.49

Analysis time : 0:00:0.457

=====
Job tutorial_A completed with warnings. See message file for details. (15-Jul-2016 18:55:37)

```

**Figure 12.2: Fatigue results summary**

The predicted life is 37,300 repeats at element 299, node 49. Open the message file (located in *Project\output\tutorial\_A*) to view information about the analysis. Take note of which element face Quick Fatigue Tool analysed. By default, the negative shell face is used for analysis.

## 12.4 Post processing the results

This step can only be completed if the user has access to Abaqus/Viewer.

The field data for the entire analysis region can be written back to the Abaqus output database. Start the Export Tool either by clicking on the App icon, or by running the file *ExportTool.m* in *Application\_Files\source\python*. Configure the dialogue so that it appears as shown in Figure 12.3.

For the *Field Data* input, select the file *f-output-all.dat* from *Project\output\Tutorial\_A\Data Files*.

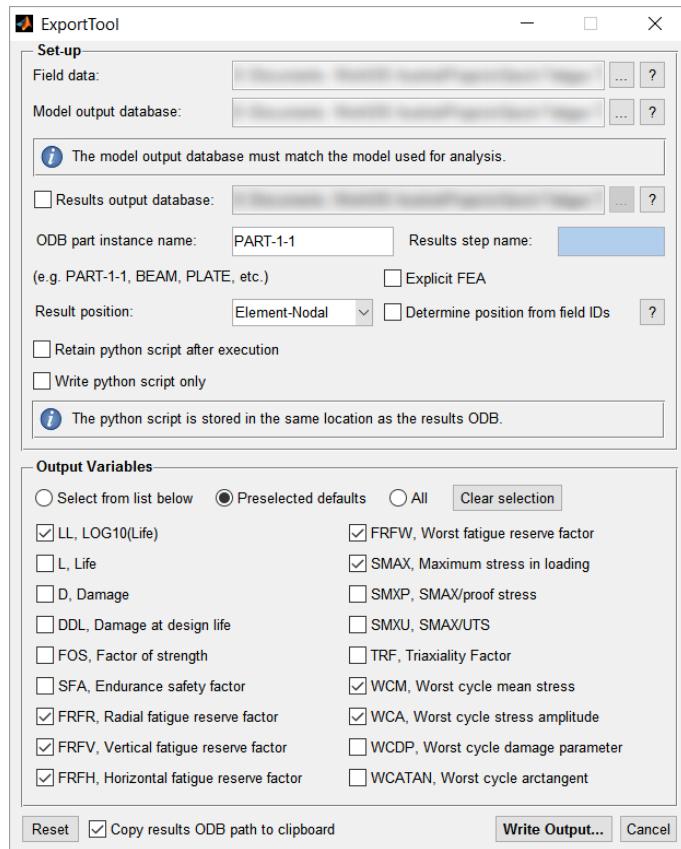
For the *Model output database* input, select *weldPlate\_614.odb* from *Data\abaqus*.

Check **Results output database** and select *Project\output\Tutorial\_A* as the result directory.

Accept the default field output and select **Write Output....** The progress of the export is displayed in the command window.

The contents of the export log file are shown in Figure 12.4. Since the fatigue analysis only considered a subset of elements, 615 of the 655 total elements in the ODB were ignored.

The fatigue results ODB path is copied to the clipboard. Open Abaqus/CAE and press Ctrl+O. In the Open Database dialogue, paste the ODB path into the file box and press Enter. The fatigue results are shown in Figure 12.5.



**Figure 12.3: Export Tool setup**

```

Quick Fatigue Tool 6.8-00 ODB Interface Log

User-selected results position: Element-Nodal
Allow Quick Fatigue Tool to determine results position based on field IDs: NO

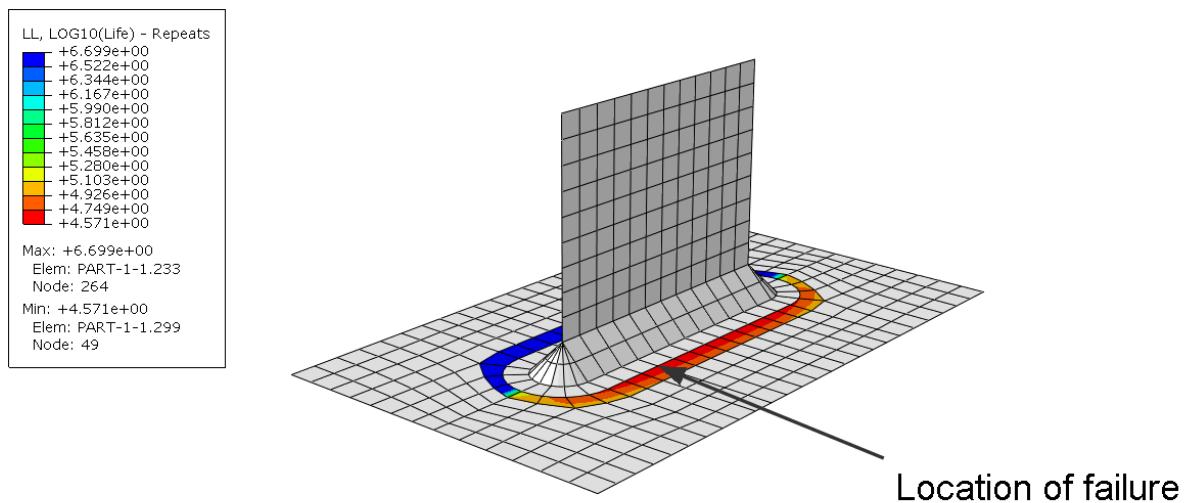
Collecting field data...
    Based on user selection, field data will be read as ELEMENT_NODAL
    Requesting nodal connectivity matrix... Success
        Detected 1983 nodes belonging to 655 elements
        Checking for redundant nodes... 615 connected node groups removed
        Checking for redundant elements... 0 elements removed
    Taking position labels from ODB element listing
    Generating position labels... 1 fields requested

Preparing field data...
    (1 of 1)

Writing field data to ODB... Success

```

**Figure 12.4: ODB export log file**



ODB: weldPlate\_614Results.odb Abaqus/Standard 6.14-2 Sun Oct 04 12:15:04 W. Europe Daylight Time 2015

Step: Quick Fatigue Tool, version 6.4-00; Job: tutorial\_A, Loading: 1 Repeats

Fatigue results field data

Primary Var: LL, LOG10(Life) - Repeats

Deformed Var: not set Deformation Scale Factor: not set

Figure 12.5: Fatigue results ODB

## 13. Tutorial B: Complex loading of an exhaust manifold

### 13.1 Background

This tutorial outlines the procedure for analysing an exhaust manifold using an Abaqus output database file with Quick Fatigue Tool. The manifold model used is that shown in Figure 13.1.

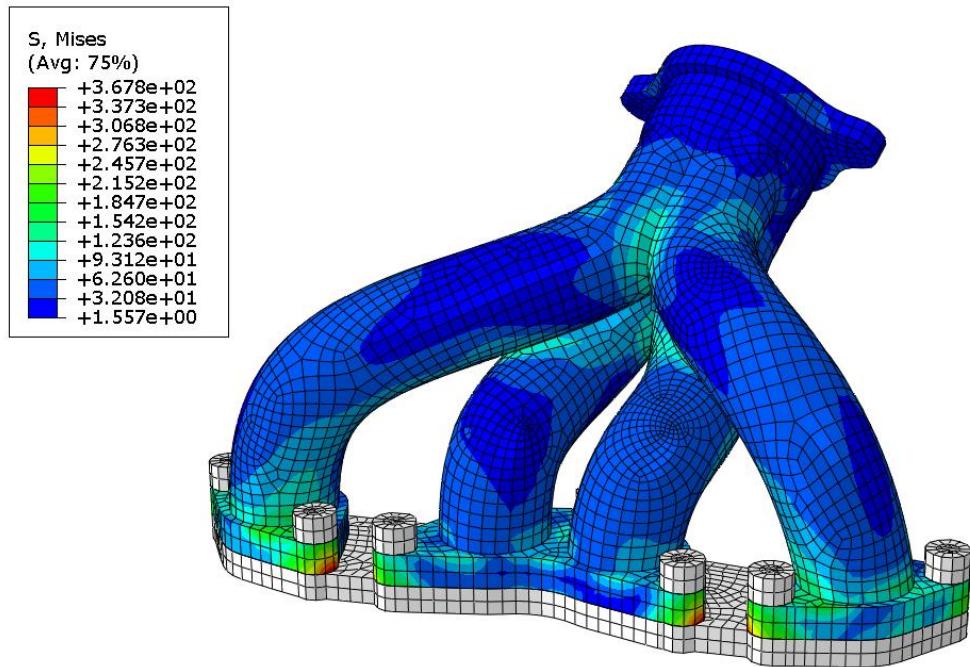
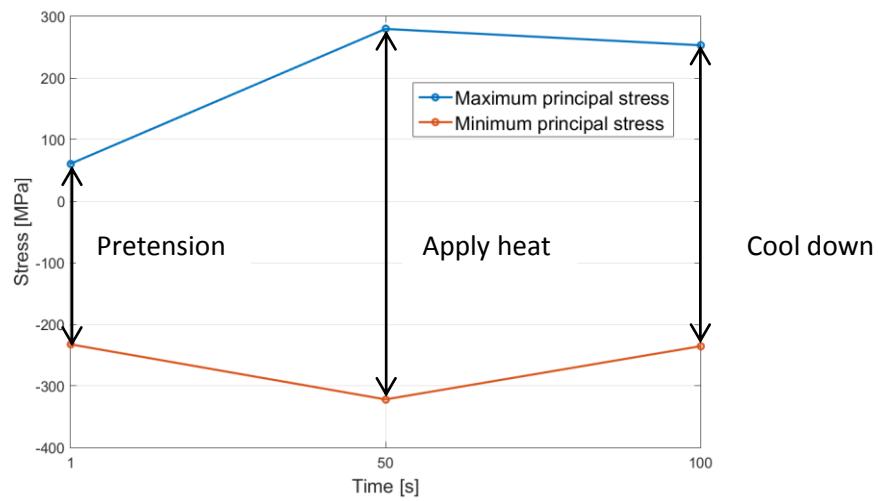


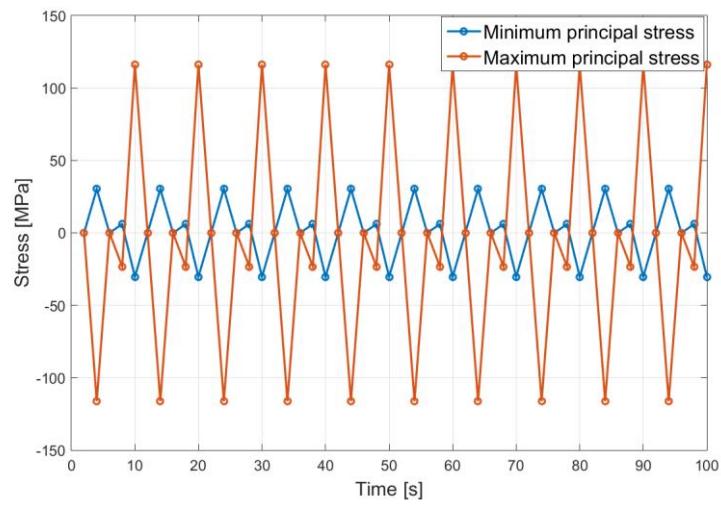
Figure 13.1: Exhaust manifold model

The analysis consists of three loading steps. First, a pre-tension is applied to the bolts. The manifold is then subjected to a transient thermal load. The load is then removed and the model is allowed to return to ambient temperature. The stresses are obtained at each load step and analysed as a stress dataset sequence. The peak stress history is shown in Figure 13.2. In addition to the thermal loading, the mechanical load history shown in Figure 13.3 is superimposed onto the thermal stress as a high frequency stress dataset. The mechanical load is defined in Quick Fatigue Tool as a simple loading of the stress data from the pre-tension step with a user-defined load history.

This tutorial demonstrates the use of several features in Quick Fatigue Tool. However, the model data itself is arbitrary. Sections of the tutorial use Abaqus/Viewer for results post-processing. If the user does not have access to Abaqus/Viewer, these sections may be skipped.



**Figure 13.2: Peak thermal stress**



**Figure 13.3: Peak mechanical load**

## 12.2 Preparation

Analysing finite element models with complex load histories with Quick Fatigue Tool can sometimes be time-consuming; therefore, the analysis will be split into two procedures. First, the whole model will be analysed with a simplified loading configuration to find the location of maximum damage. Afterwards, a second analysis will be performed on the node which experiences maximum damage with the complete loading definition.

Before running the analysis, verify that the following files exist in the *Project\input* directory:

'manifold_1.rpt'	Stress data for step 1 (pretension)
'manifold_2.rpt'	Stress data for step 2 (thermal load)
'manifold_3.rpt'	Stress data for step 3 (cool-down)
'manifold_history_hf.dat'	Normalized history data for the mechanical load

Copy the datasets and history file from *Data\datasets* and *Data\histories*, respectively, into the *Project\input* folder. The input folder should appear as in Figure 13.4.

The datasets may be created in Abaqus/CAE following the procedure outlined in Section 3.2, using unique nodal as the result position.

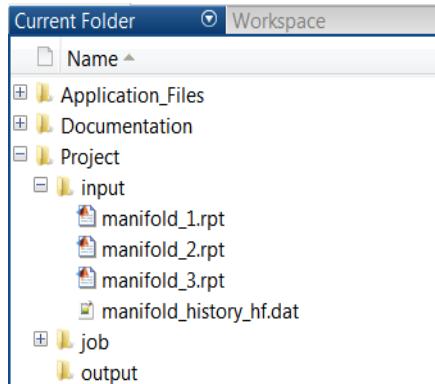


Figure 13.4: Required input files for  
Tutorial B

In order to customise the analysis, setting in the environment file can be changed. However, future analyses should not be affected by this change, so a separate environment file will be stored locally, which corresponds to the analysis in Tutorial B. To create a local environment file, copy the file *environment.m* from *Application\_Files\default* and paste it into *Project\job*. Rename the file to *tutorial\_B\_env.m*. If the file is named differently, it will be ignored during analysis. File names are case-sensitive. The job folder should appear as in Figure 13.5.

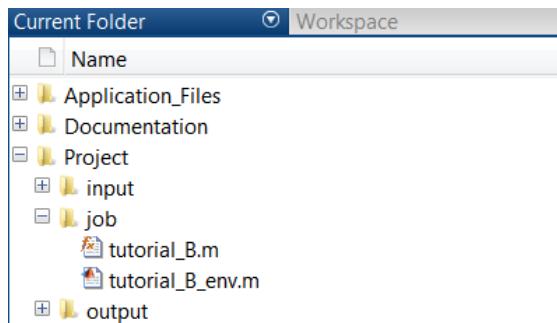


Figure 13.5: Required job files for Tutorial B

### 12.3 Defining the material

The analysis job file *tutorial\_B.m* references a material called *material\_tutorial\_B.mat*. This material needs to be created using the material manager. To launch the material manager, run the file *materialManager.m* from *Application\_Files\source\material\_manager* or run the material manager app from the app menu. The main dialogue is shown in Figure 13.6.

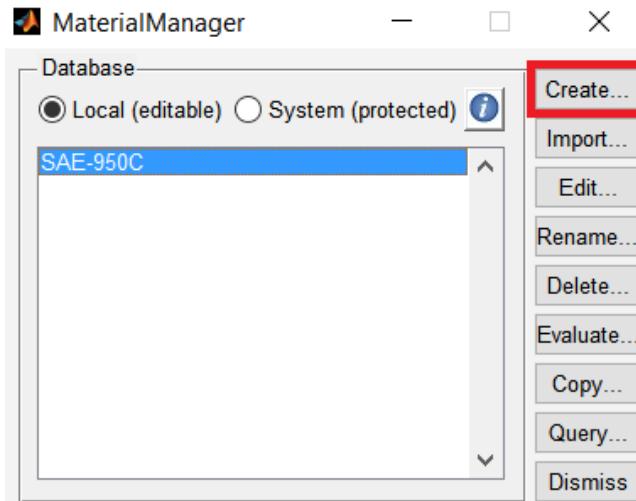
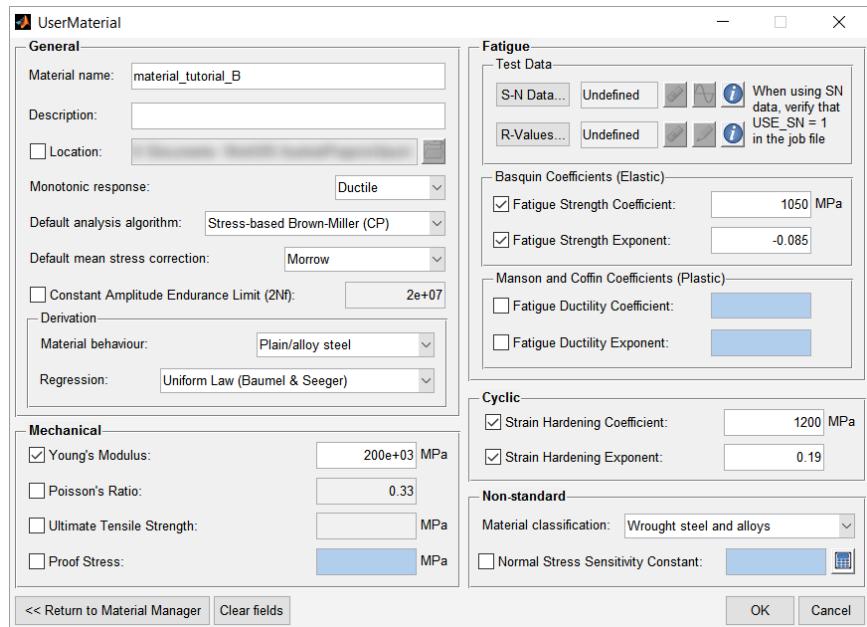


Figure 13.6: Material Manager dialogue

Click **Create...** to create a new material. The material editor is shown in Figure 13.7. Enter the parameters as shown by the green boxes. The material properties are as follows:

Material name	material_tutorial_B
Young's Modulus	200GPa
Fatigue Strength Coefficient	1050MPa
Fatigue Strength Exponent	-0.085
Strain Hardening Coefficient	1200MPa
Strain Hardening Exponent	0.19

Press **OK** to save the material to the workspace. The newly created material should now appear in the list of workspace materials in the Material Manager main dialogue box. Exit the Material Manager by clicking **Dismiss**.



**Figure 13.7: Material editor dialogue**

### 13.4 Running the first analysis

The first analysis is run to identify the location of maximum damage. The loading is restricted to the thermal stress and the critical plane step size is increased to reduce the analysis time.

Open the file *tutorial\_B\_env.m* from the job folder. Ensure that nodal elimination is enabled and that the critical plane step size is set to a value of 10:

```
setappdata(0, 'nodalElimination', 1.0)

setappdata(0, 'stepSize', 10.0)
```

Close the environment file and open *tutorial\_B.m* from the job folder and review the settings:

- The thermal load is defined by DATASET as a sequence of stress datasets representing each loading step from the finite element analysis
- The high frequency mechanical loading defined by HF\_DATASET and HF\_HISTORY is commented out for this analysis
- The default algorithm and mean stress correction are set by ALGORITHM and MS\_CORRECTION, respectively
- ITEMS is used to indicate that all items in the model will be analysed
- A residual stress of 10MPa is specified using RESIDUAL
- Field output is requested using OUTPUT\_FIELD

For a complete description of analysis options, consult the document  
*Fatigue Tool User Settings Reference Guide*.

*Quick*

To run the analysis, right-click on *tutorial\_B.m* and select *Run*, or press F5 while the file is open in the editor. Analysis progress is displayed in the command window. Figure 13.8 shows the result of the analysis.

```
FATIGUE RESULTS SUMMARY:  
=====  
Worst Life-Repeats : 2.93e+06  
at item 21426.1  
  
Number of cycles in loading : 1  
  
Maximum stress (MPa) : 483.3  
at Item 21426.1  
  
Maximum stress/yield : 1.673  
at Item 21426.1  
  
Worst cycle mean stress (MPa) : 274.4  
at Item 21426.1  
  
Worst cycle stress amplitude (MPa) : 330.3  
at Item 21426.1  
  
Worst cycle damage parameter (MPa) : 330.3  
at Item 21426.1  
  
Analysis time : 0:00:0.457  
=====  
Job tutorial_B2 completed successfully (16-Jul-2016 13:35:36)
```

**Figure 13.8: Fatigue analysis results indicating node 21413 as the location of failure**

The stress data from Abaqus was extracted at the nodes and averaged, therefore the worst life is quoted at the unique nodal position. Since the mechanical stresses were not included for analysis the life result is unimportant. However, the analysis result indicates that failure will occur at node 21413 on the finite element model.

## 13.5 Viewing the results with Abaqus/Viewer

This step may be skipped if the user does not have Abaqus 6.14 or later installed on their machine.

To view the life data on the manifold model, launch the Export Tool by running the file *exportTool.m* from *Application\_Files\source\python*, or by running the app from the app bar. Configure the dialogue box so that it appears as in Figure 13.9.

1. Select the field data file *f-output-all.dat* from *Project\output\tutorial\_BData Files*
2. Select the model output database *manifold\_614.odb* from *Data\abaqus*
3. Specify the part instance name *PART-1-1*
4. Uncheck *Determine position from field IDs* and select *Unique Nodal* as the result position
5. Click *Clear selection* to deselect all fields, then select *LL, LOG10(Life)*
6. Click *Write Output...* to write the life data to an Abaqus ODB file

For a detailed explanation of the Export Tool, consult Section 10.4.

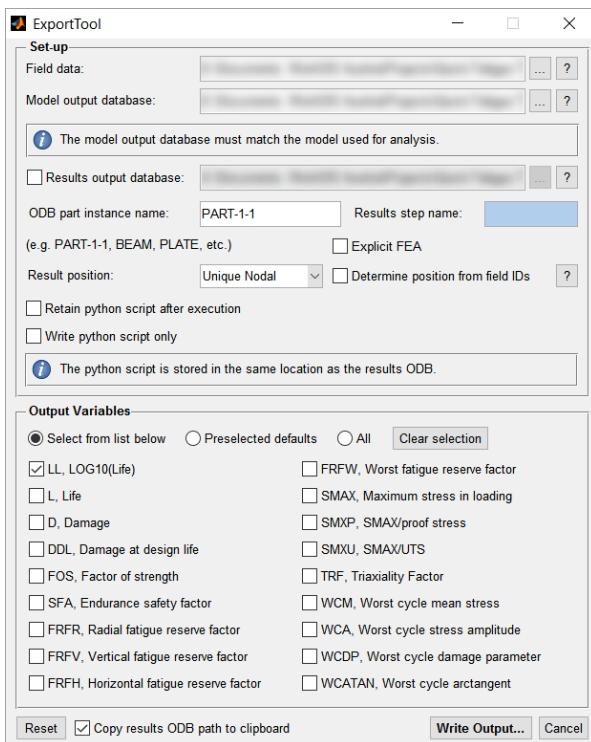
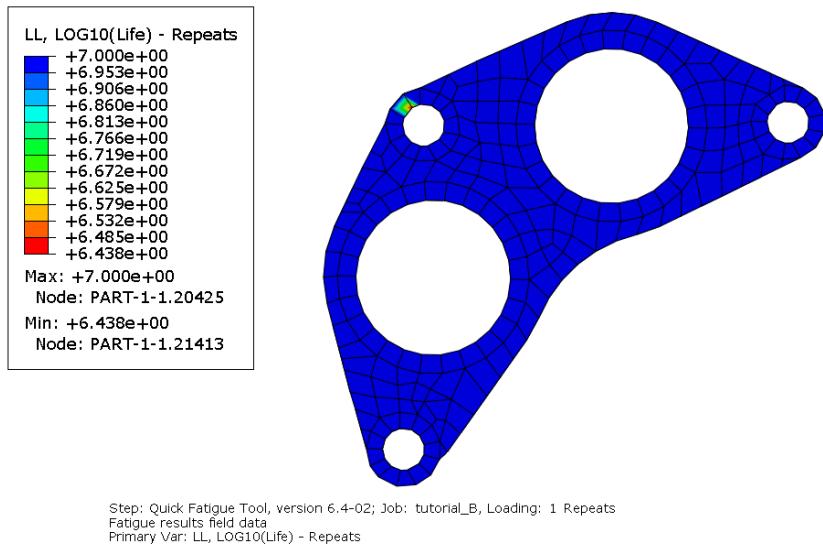


Figure 13.9: Export Tool settings for the results ODB

To view the ODB file in Abaqus/Viewer, start Abaqus and open the ODB in the usual way. The full path to the results ODB file has been copied to the clipboard, so pressing *Ctrl+V* can be used. Isolate the location of failure by selecting *Create Display Group*. Select *Elements* as the item and *Element Sets* as the method of creating the display group. Select *PART-1-1.FL2* and click *Replace*. The location of failure is shown in Figure 13.10.



**Figure 13.10: Location of failure shown in Abaqus/Viewer**

### 13.6 Running the second analysis

The previous analysis determined that the manifold will fail at node 21426. By inspecting the message file, Quick Fatigue Tool tells us that this node corresponds to item number 10518 in the dataset. A second analysis will be performed on this item only. From the job file, specify this item as follows:

```
ITEMS = 10518;
```

The mechanical load must now be considered. Define the high frequency data as follows:

```
HF_DATASET = 'manifold_1.rpt';
HF_HISTORY = 'manifold_history_hf.dat';
```

Commented versions of these entries are already provided below the previous definitions.

Request histories and MATLAB figures in addition to fields:

```
OUTPUT_FIELD = 1.0;
```

```
OUTPUT_HISTORY = 1.0;
```

```
OUTPUT_PICTURE = 1.0;
```

Open `tutorial_B_env.m` and disable stress tensor gating:

```
setappdata(0, 'gateTensors', 0.0)
```

MATLAB should be restarted before running the analysis, to ensure that all data from the previous analysis is cleared. After running the analysis, a summary of the fatigue results are displayed in the command window, as in Figure 13.11.

The analysis indicates a life of 116,000 repeats of the loading until failure.

```
FATIGUE RESULTS SUMMARY:  
=====  
Worst Life-Repeats : 1.16e+05  
at item 21426.1  
  
Number of cycles in loading : 14  
  
Maximum stress (MPa) : 494.9  
at Item 21426.1  
  
Maximum stress/yield : 1.713  
at Item 21426.1  
  
Worst cycle mean stress (MPa) : 98.85  
at Item 21426.1  
  
Worst cycle stress amplitude (MPa) : 539.2  
at Item 21426.1  
  
Worst cycle damage parameter (MPa) : 539.2  
at Item 21426.1  
  
Analysis time : 0:00:1.242  
=====
```

**Figure 13.11: Fatigue analysis results from the second analysis**

## 13.7 Post processing the results

The results of the analysis are stored in *Project\output\tutorial\_B*.

- Field and history data is written to a set of tabulated data files
- Certain results data is automatically plotted to a set of MATLAB figures
- An overview of the analysis is written to *tutorial\_B.log*
- Warnings and messages are written to *tutorial\_B.msg*

Open the message file to view possible issues with the analysis:

MESSAGES:

=====

```
***NOTE: The proof stress for material material_tutorial_B.mat (group 1)
was not specified
-> A derived value of 368.4MPa will be used
```

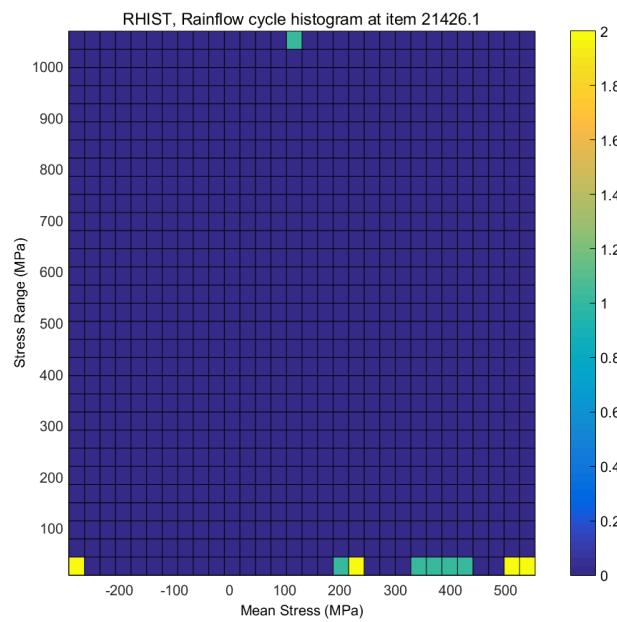
```
***NOTE: In at least one group, the UTS is undefined. The following fields
are unavailable:
-> FRFR, FRFH, FRFV, FRFW, SMXU
```

```
***NOTE: Worst damage at design life (1e+07) is 82.3
```

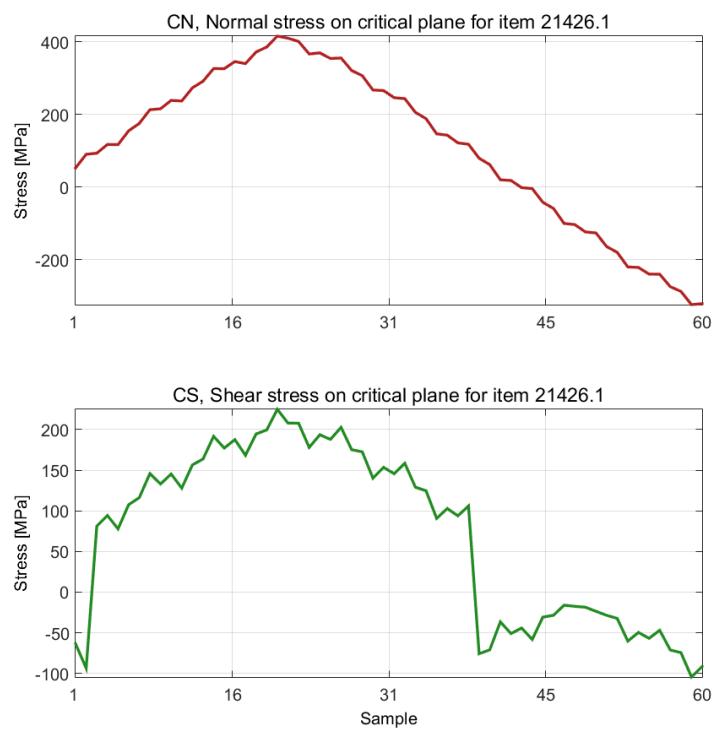
```
***WARNING: 1 items are exhibiting low cycle fatigue (lives less than 1e+06
Repeats)
```

- The proof stress was derived based on the cyclic material properties using the 0.2% strain offset rule
- Certain variables are not available because the ultimate tensile strength was not defined
- The manifold will not survive the default design life of 10 million loading repeats
- The stress-life methodology is not well-suited to low-cycle fatigue problems (lives below 1 million repeats)

A summary of the cycle counting process can be viewed either as a Haigh diagram or a rainflow histogram. The rainflow histogram is shown by Figure 13.12. This indicates that the damage is primarily caused by one cycle in the loading where the stress range is significantly higher than every other location in the history.



**Figure 13.12: Rainflow histogram of cycles**



**Figure 13.13: Normal and shear stress on the critical plane**

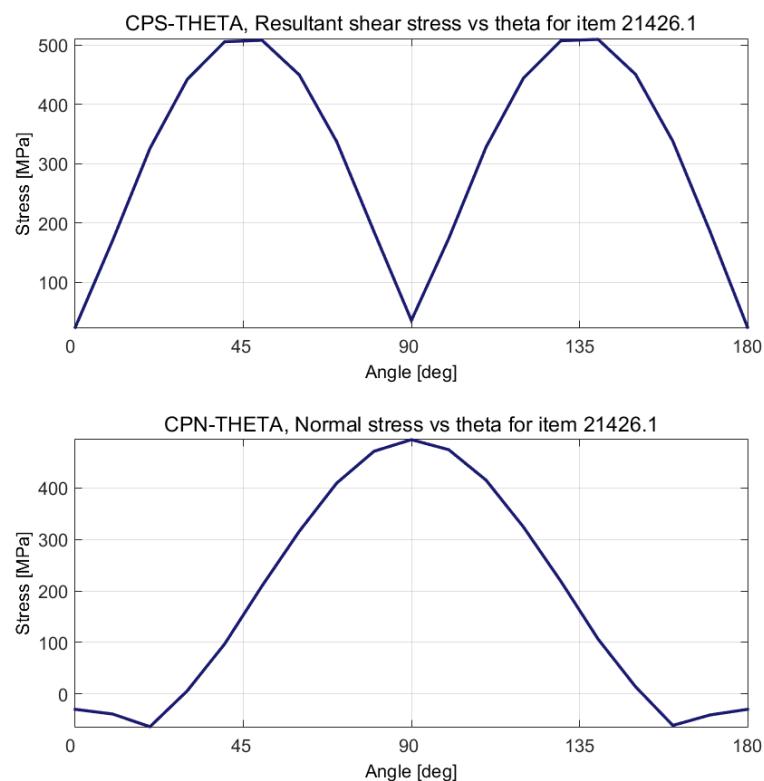
The normal and shear stress on the critical plane is shown by Figure 13.13. It is clear from the data that the mechanical stress was superimposed onto the thermal stress correctly.

The results of the critical plane analysis are shown by Figures 13.15-16. Planes of maximum shear stress occur at 45 degrees, which is expected since ductile metals in pure tension fail along planes at 45 degrees to the free surface. According to the log file, the critical plane occurs where theta is 110 degrees (Figure 13.14). This represents the angle at which the combination of shear and normal stress is maximised.

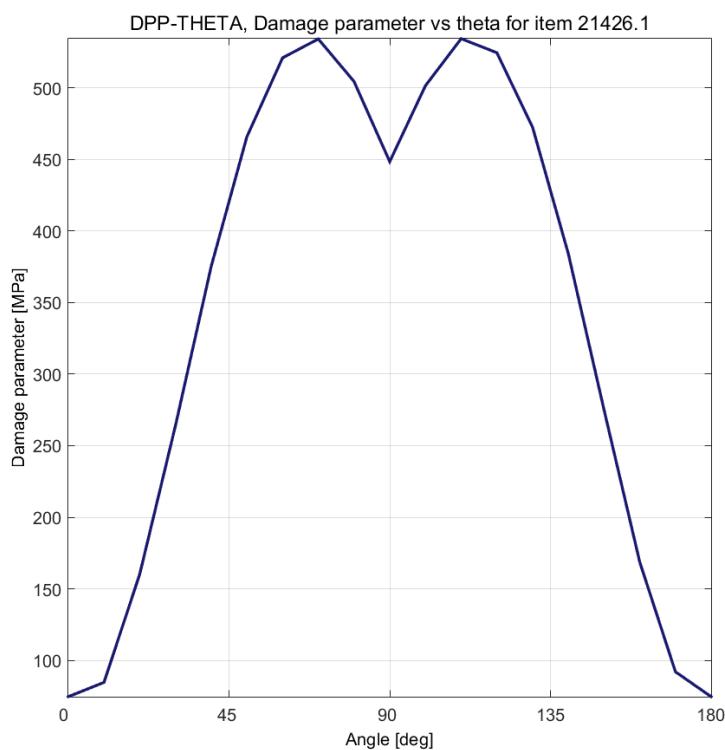
```
CP SUMMARY AT WORST ITEM:  
=====  
    CP Step Size: 10 degrees  
    361 planes searched  
    Coordinates (degrees) :  
        THETA = 110, PHI = 90
```

**Figure 13.14: Sample log file output indicating the orientation of the critical plane**

By inspecting Figure 13.16, it is clear that the damage parameter is maximised when theta equals 110 degrees. In fact, the damage profile is symmetrical about 90 degrees. This is the point at which the shear stress is zero.



**Figure 13.15: Critical plane analysis results**



**Figure 13.16: Damage parameter on the critical plane**

## Appendix I. Fatigue analysis techniques

### A1.1 Background

Quick Fatigue Tool is capable of assessing the fatigue life of entire FEA models. This is achieved by reading the stress tensor at each analysis item (node, integration point or centroid) and performing a damage calculation at each location in the model.

Before the analysis begins, the FEA definitions are combined with the load history to form a scaled time history of stresses at each location in the model. This is known as a *Scale and Combine* analysis.

The stresses at each location must be considered in multiple directions in order to account for multiaxial stress states. The plane which experiences the largest stress is isolated, and the load signal is cycle counted to obtain the stress amplitudes, so that a damage calculation can be performed.

The damage of each cycle in the loading is summed for each analysis item. The item with the largest damage is identified as the *worst* item and the fatigue life is quoted at this location. During the damage calculation process, the endurance limit may be modified to account for the effect of microscopic plasticity at the crack tip.

This section provides a concise description of how the code achieves the above steps.

### A1.2 Combining FEA stresses with a loading

FE models usually comprise of multiple analysis steps, each representing a distinct load configuration for which the stress state is solved. Provided these stresses are elastic, each loading state may be combined with its respective load history to create a single scaled, combined loading, as depicted in Figure A1.1.

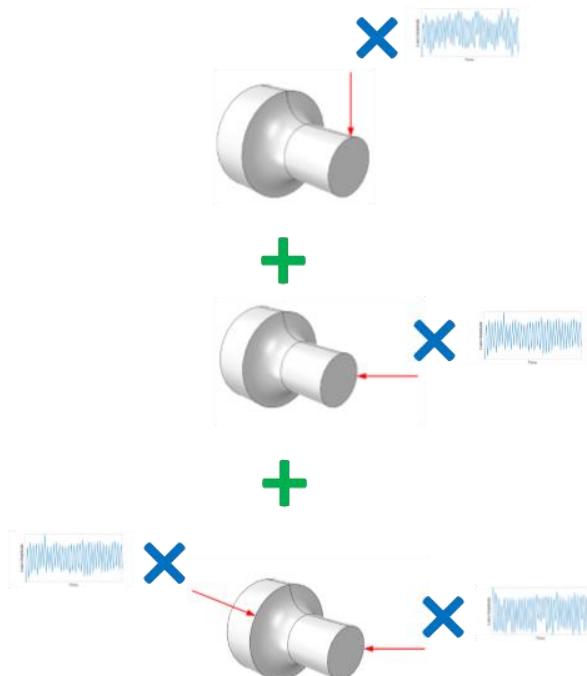


Figure A1.1: Illustration of a scale and combine loading

### A1.3 Critical plane search algorithm

Critical plane analysis is a widely used engineering methodology to determine multiaxial fatigue damage where the most damaging stresses are not readily apparent [50] [51] [52] [53]. It has been argued that fatigue criteria based on critical plane methods are the most effective [28].

Quick Fatigue Tool uses a critical plane approach based on a combination of work by R. Rabb and Luca Susmel.

The stress tensor history at each node is transformed onto a series of planes according to Figure A1.1 [54].

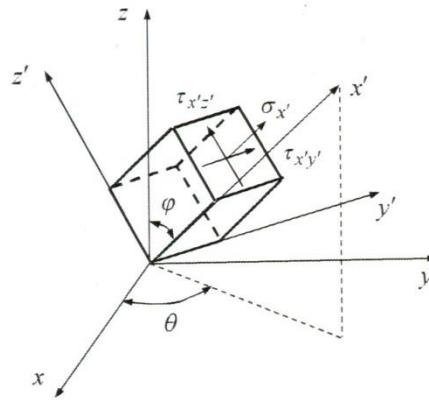


Figure A1.1: Tensor transformation

The rotated tensor,  $\sigma'$ , is calculated from the original tensor by Equation A1.1.

$$\sigma' = Q^T \sigma Q \quad [\text{A1.1}]$$

Where  $Q$  is the transformation matrix given by Equation A1.2.

$$Q = \begin{bmatrix} \sin \varphi \cos \theta & -\sin \theta & -\cos \varphi \cos \theta \\ \sin \varphi \sin \theta & \cos \theta & -\cos \varphi \sin \theta \\ \cos \varphi & 0 & \sin \varphi \end{bmatrix} \quad [\text{A1.2}]$$

The angles  $\theta$  and  $\varphi$  correspond to the cylindrical coordinate system described by Figure A1.1. The rotated tensor is split into one normal and two shear components:

$$\begin{aligned} \sigma_n &= \sigma'(1,1) \\ \tau_{xy} &= \sigma'(1,2) \\ \tau_{xz} &= \sigma'(1,3) \end{aligned} \quad [\text{A1.3}]$$

Depending on the algorithm, the normal and/or shear stresses are cycle counted in order to calculate fatigue damage.

## A1.4 Rainflow cycle counting

The rainflow counting method is used for the analysis of fatigue loadings which reduces a spectrum of variable amplitude stresses into a matrix of simple stress reversals. The advantage of such a method is that it allows for efficient summation of fatigue damage using Miner's rule [55] [56]. The original rainflow method was devised by Tatsuo Endo and M. Matsuishi [57]. However, as of 2008, several counting algorithms are available [58] [59] [60] [61].

Quick Fatigue Tool uses a custom-written algorithm which follows a similar logic to that of the original rainflow methodology. Cycles are counted on the basis of comparing adjacent pairs of stress ranges and removing these points from the loading until no more cycles are closed.

The cycle counting strategy is as follows:

1. Convert the load history into a sequence of peaks and valleys (no intermediate values between inflection points)
2. Re-arrange the history so that the absolute maximum stress is at the start of the signal. Perform the same re-arrangement on the indexes so that the true position of the data points is retained
3. Remove leading and trailing tails (identical values forming flat paths on either side of the load history)
4. Remove plateaus (adjacent points with the same value) which can arise from step 2
  - a. If the plateau lies between two diametrically opposed inflection points (a peak-valley pair), remove all plateaued points
  - b. If the plateau lies between two diametrically similar inflection points (two peaks or two valleys), remove only the current plateau
5. For the current pair of history points, compare the current stress range to the previous stress range
  - a. If the current stress range exceeds the previous stress range, count a cycle and remove the previous stress range from the loading
6. Repeat step 5 until no more cycles can be extracted
7. Move to the next pair of history points and repeat steps 5 and 6 until all cycles have been removed from the loading
8. Check for unmatched half cycles and add these to the cycle matrix until there is no more history data

The algorithm does not consider the nonlinear relationship between stress and strain. Therefore, the effect of cyclic hysteresis and material memory is not included. Since changes in the mean stress due to plasticity are ignored, the performance of the cycle counting algorithm will degrade as fatigue lives decrease below one million repeats.

The method described above is implemented from version 6.6 onwards. The previous version of the cycle counting algorithm can be enabled from the environment file. Values of 1.0 and 2.0 correspond to the old and new versions, respectively.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
rainflowAlgorithm	[1.0   2.0];

## A1.5 Treatment of the endurance limit

### A1.5.1 Overview

The endurance limit is the region on the stress-life curve where fatigue damage does not occur, irrespective of the number of applied stress cycles [62]. Ferrous and titanium alloys exhibit a distinct limit below which the fatigue life is observed to be infinite. However, other nonferrous alloys such as aluminium and copper do not have a distinct limit and will eventually fail regardless of the applied stress amplitude. The concept of the endurance limit is illustrated by the plateau in Figure A1.3.

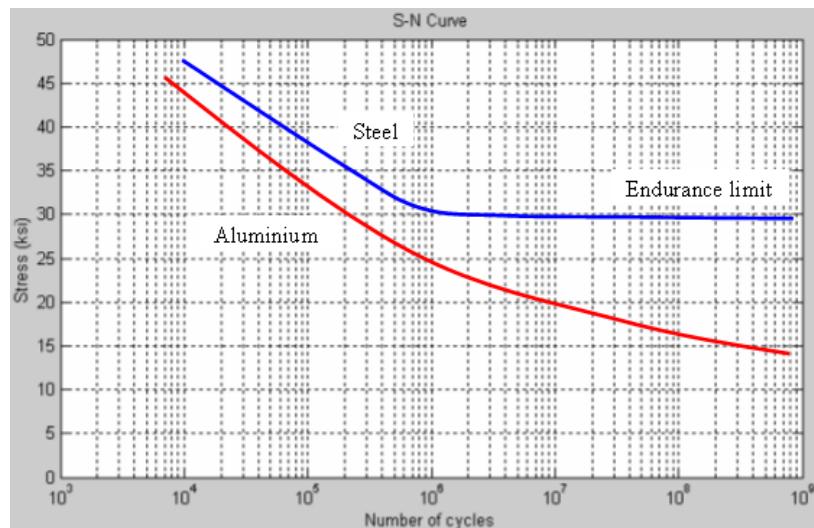


Figure A1.3: Representative Stress-Life curves for steel and aluminium, indicating the presence (and lack thereof) an endurance limit stress below which no fatigue damage occurs.

Image courtesy of Andrew Dressel (Wikimedia Commons: CC-BY-SA-3.0)

### A1.5.2 Enforcement of the endurance limit

Quick Fatigue Tool may ignore fatigue damage for cycles below the endurance limit depending on the setting of the following environment variable:

*Environment file usage:*

Variable	Value
ndEndurance	[0.0   1.0   2.0];

If `ndEndurance` is set to 0.0, Quick Fatigue Tool determines whether to account for the endurance limit based on the material classification defined in the material `.mat` file. Values of 1.0 and 2.0 disable or enable treatment of the endurance limit, respectively.

Material Behaviour	Ignore damage below endurance limit?
Aluminium/copper alloys	NO
All other material classifications	YES

### A1.5.3 Endurance limit of non-ferrous metals

Due to the difficulties associated with testing specimens to very long lives, it is not always feasible to determine the exact endurance limit of the material. Furthermore, for some nonferrous metals as well as very strong steels, the stress-life curve is observed to continuously decrease as far as the test data are available. As a result, materials are often considered to cross into infinite life at a pre-determined limit (usually  $10^7$  cycles), beyond which it is usually safe to assume that no damage will occur. Thus, if the material is classified as *Other* in the material definition, the endurance limit is enforced.

### A1.5.4 The “bouncing endurance limit”

Investigations have shown that cycles above the endurance limit can result in fatigue damage from otherwise non-damaging cycles [63]. This phenomenon can be explained by considering a cycle just above the endurance limit. Since the cycle is large enough to cause fatigue damage, it acts to open the crack tip resulting in a very small plastic zone ahead of the crack. This microscopic plasticity temporarily reduces the fatigue strength of the specimen at the crack tip, until the crack propagates through the plastic zone. The phenomenon of a “bouncing endurance limit” has been investigated in considerable detail [64] [65] [66] [67] [68].

This behaviour is modelled in Quick Fatigue Tool such that the fatigue limit is reduced by a factor,  $k_{scale}$ , if the current cycle is damaging and only applies when enforcement of the endurance limit is enabled.

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>ndEndurance</code>	$[0.0^7 \mid 2.0];$
<code>modifyEnduranceLimit</code>	$[0.0 \mid 1.0];$
<code>enduranceScaleFactor</code>	$k_{scale};$

Over the course of the next  $n_{recover}$  consecutive non-damaging cycles, the fatigue limit is linearly incremented back to its original value. By default,  $k_{scale} = 0.25$  and  $n_{recover} = 50$  [69].

*Environment file usage:*

<i>Variable</i>	<i>Value</i>
<code>cyclesToRecover</code>	$n_{recover};$

If any subsequent cycle exceeds the fatigue limit, the limit will be scaled back to a factor of  $k_{scale}$  of its original value and the recovery process will start from the beginning.

---

<sup>7</sup> If `ndEndurance` = 0.0, the “bouncing endurance limit” is enabled unless the material behaviour is aluminium, in which case damage below the endurance limit is always allowed.

## Appendix II. Materials data generation

### A2.1 Background

Despite extensive efforts over years in characterizing the fatigue behaviour of metals, obtaining fatigue materials data is often time consuming and expensive. Therefore, it is important to be aware of methods which are capable of producing reasonable estimates of material data based on readily available information.

Quick Fatigue Tool offers several approximation algorithms which can estimate the fatigue materials data:

1. Uniform Law (Bäumel-Seeger)
2. Universal Slopes (Manson)
3. Modified Universal Slopes (Muralidharan)
4. 90/50 Rule

### A2.2 Bäumel-Seeger Method

The Bäumel-Seeger method approximates the fatigue materials data as follows [70] [71] [72]:

Fatigue Property	Plain/Alloy Steel	Aluminium
Fatigue strength coefficient $\sigma_f'$	$1.5\sigma_U$	$1.67\sigma_U$
Fatigue strength exponent $b$	-0.087	-0.095
Fatigue ductility coefficient $\varepsilon_f'$	$a = 1, \frac{\sigma_U}{E} < 3^{-3}$ $a = 1.375 - 125 \frac{\sigma_U}{E}, \frac{\sigma_U}{E} > 3^{-3}$	0.35
Fatigue ductility exponent $c$	-0.58	-0.69
Cyclic strain hardening coefficient $K'$	$1.65\sigma_U$	$1.61\sigma_U$
Cyclic strain hardening exponent $n'$	0.15	0.11

### A2.3 Manson

The Manson method approximates the fatigue materials data as follows [73]:

Fatigue Property	Plain/Alloy Steel	Aluminium	Other
Fatigue strength coefficient $\sigma_f'$	$1.9\sigma_U$	$1.9\sigma_U$	$1.9\sigma_U$
Fatigue strength exponent $b$	-0.12	-0.12	-0.12
Fatigue ductility coefficient $\varepsilon_f'$	$0.76 \log \frac{1}{1 - 0.1541}^{0.6}$	$0.76 \log \frac{1}{1 - 0.4394}^{0.6}$	$0.76 \log \frac{1}{1 - 0.1541}^{0.6}$
Fatigue ductility exponent $c$	-0.6	-0.6	-0.6
Cyclic strain hardening coefficient $K'$	$\frac{\sigma_f'}{{\varepsilon_f'}^{0.2}}$	$\frac{\sigma_f'}{{\varepsilon_f'}^{0.2}}$	$\frac{\sigma_f'}{{\varepsilon_f'}^{0.2}}$
Cyclic strain hardening exponent $n'$	-0.2	-0.2	-0.2

## A2.4 Modified Manson

The Modified Manson method approximates the fatigue materials data as follows [74]:

Fatigue Property	Plain/Alloy Steel	Aluminium	Other
Fatigue strength coefficient $\sigma_f'$	$0.623\sigma_U^{0.823}E^{0.168}$	$0.623\sigma_U^{0.823}E^{0.168}$	$0.623\sigma_U^{0.823}E^{0.168}$
Fatigue strength exponent $b$	-0.09	-0.09	-0.09
Fatigue ductility coefficient $\varepsilon_f'$	$0.196 \log \frac{1}{1 - 0.1541} \frac{0.155}{E} \sigma_U^{-0.53}$	$0.196 \log \frac{1}{1 - 0.4394} \frac{0.155}{E} \sigma_U^{-0.53}$	$0.196 \log \frac{1}{1 - 0.155} \frac{0.155}{E} \sigma_U^{-0.53}$
Fatigue ductility exponent $c$	-0.56	-0.56	-0.56
Cyclic strain hardening coefficient $K'$	$\frac{\sigma_f'}{{\varepsilon_f'}^{0.2}}$	$\frac{\sigma_f'}{{\varepsilon_f'}^{0.2}}$	$\frac{\sigma_f'}{{\varepsilon_f'}^{0.2}}$
Cyclic strain hardening exponent $n'$	-0.2	-0.2	-0.2

## A2.5 90/50 Rule

The 90/50 rule is an informal approximation of the stress-life relationship. It assumes the following two points on the endurance curve if only the ultimate tensile strength is known:

Life ( $N_f$ )	Stress (S)
1000	$0.9\sigma_U$
$N_\infty$	$0.5\sigma_U$

Basquin's exponent is then the gradient of the straight line defined by the above two points when plotted on log-log axes:

$$b = \frac{\log 1.8}{\log \frac{1e3}{N_\infty}} \quad [A2.1]$$

Where  $N_\infty$  is the constant amplitude endurance limit. Note that the value of  $b$  depends only on the endurance limit and not on the ultimate tensile strength.

The relationship between the ultimate tensile strength and the fatigue limit becomes increasingly nonlinear with increasing ultimate tensile strength. Therefore, if the UTS exceeds 1000MPa, the endurance limit stress is fixed to 500MPa and Equation A2.2 is used to approximate Basquin's exponent:

$$b_{\sigma_U > 1000MPa} = \frac{\log \frac{0.9\sigma_U}{500}}{\log \frac{1e3}{N_\infty}} \quad [A2.2]$$

If the stress-life relationship is given by Equation A2.3, then the value of the tensile fatigue strength coefficient,  $\sigma_f'$ , may be derived using Equation A2.4 which assumes that  $S(1e3) = 0.9\sigma_U$ .

$$S = \sigma_f' (2N_f)^b \quad [A2.3]$$

$$\sigma_f' = \frac{0.9\sigma_U}{(2 \times 10^3)^b} \quad [A2.4]$$

If  $N_\infty = 1e6$  cycles, then the stress life equation is given by Equation A2.5:

$$S = 1.72\sigma_U N_f^{-0.085} \quad [A2.5]$$

In general, the stress-life relationship is given by Equation A2.6:

$$S = \frac{0.9\sigma_U}{(2 \times 10^3)^b} (2N_f)^b, \quad b = \frac{\log 1.8}{\log \frac{1e3}{N_\infty}} \quad [A2.6]$$

## Appendix III. Gauge fatigue toolbox

### A3.1 Background

#### A3.1.1 Overview

The Gauge Fatigue Toolbox is a set of MATLAB apps which facilitate the analysis of measured strain data. The following apps are included:

- *Multiaxial Gauge Fatigue*
- *Rosette Analysis*
- *Virtual Strain Gauge*

In addition to the above, the toolbox also includes *Mohr Solver* for calculating principal stresses and strains for a given stress tensor.

The toolbox is located in *Application\_Files\toolbox*. To install the app, double click on the *.mlappinstall* file. The app appears in the *Apps* tab of the MATLAB ribbon UI.

#### A3.1.2 Gauge conventions

The strain gauge is defined as a three-armed rosette and is placed relative to the reference  $X - Y$  coordinate system shown in Figure A3.1.

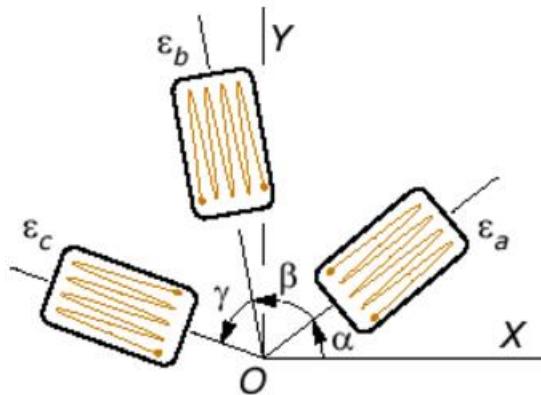


Figure A3.1: Gauge orientation relative to the reference  $X - Y$  axis. Rosette arms are separated by angles  $\alpha$ ,  $\beta$  and  $\gamma$

The user must define the gauge orientation relative to the reference coordinate system. Where appropriate, the GUI contains an icon which displays the active gauge orientation. The orientation can be defined in three ways:

1. Rectangular
2. Delta
3. User-defined

## Rectangular

The rectangular orientation is the special case where the strains  $\varepsilon_A$ ,  $\varepsilon_B$  and  $\varepsilon_C$  from the rosette arms A, B and C, respectively, coincide with the reference axes.

$$\varepsilon_{11} = \varepsilon_A$$

$$\varepsilon_{22} = \varepsilon_C \quad [A3.1]$$

$$\varepsilon_{12} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C$$

The principal strains,  $\varepsilon_1$  and  $\varepsilon_2$ , are readily calculated from the gauge strains:

$$\varepsilon_{1,2} = \frac{\varepsilon_A + \varepsilon_C}{2} \pm \frac{1}{\sqrt{2}} \sqrt{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_C)^2} \quad [A3.2]$$

## Delta

The delta orientation is the special case where the rosette arms are separated by 60 degrees and rosette arm A is rotated 30 degrees counter-clockwise from the positive X reference direction.

The principal strains are readily calculated from the gauge strains:

$$\varepsilon_{1,2} = \frac{\varepsilon_A + \varepsilon_B + \varepsilon_C}{3} \pm \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_C)^2 + (\varepsilon_C - \varepsilon_A)^2} \quad [A3.3]$$

## User-defined

User-defined gauge orientations allow the user to specify the angles  $\alpha$ ,  $\beta$  and  $\gamma$  directly. The reference strains are then solved as a system of linear equations.

$$\begin{aligned} \chi_1 &= \frac{\varepsilon_{11} + \varepsilon_{22}}{2} + \frac{\varepsilon_{11} - \varepsilon_{22}}{2} \cos 2\alpha + \frac{\varepsilon_{12}}{2} \sin 2\alpha = \varepsilon_A \\ \chi_2 &= \frac{\varepsilon_{11} + \varepsilon_{22}}{2} + \frac{\varepsilon_{11} - \varepsilon_{22}}{2} \cos 2(\alpha + \beta) + \frac{\varepsilon_{12}}{2} \sin 2(\alpha + \beta) = \varepsilon_B \\ \chi_3 &= \frac{\varepsilon_{11} + \varepsilon_{22}}{2} + \frac{\varepsilon_{11} - \varepsilon_{22}}{2} \cos 2(\alpha + \beta + \gamma) + \frac{\varepsilon_{12}}{2} \sin 2(\alpha + \beta + \gamma) = \varepsilon_C \end{aligned} \quad [A3.4]$$

The principal strains are calculated from the reference strains:

$$\varepsilon_{1,2} = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} \pm \sqrt{\left(\frac{\varepsilon_{11} - \varepsilon_{22}}{2}\right)^2 + \left(\frac{\varepsilon_{12}}{2}\right)^2} \quad [A3.5]$$

## Stress determination

Assuming an elastic relationship, the principal stresses are determined from the principal strains:

$$\sigma_{1,2} = \frac{E}{(1 - \nu^2)} (\varepsilon_{1,2} + \nu \varepsilon_{2,1}) \quad [\text{A3.6}]$$

where  $E$  is Young's Modulus and  $\nu$  is Poisson's ratio. The reference stresses  $\sigma_{11}$ ,  $\sigma_{22}$  and  $\sigma_{12}$  may be determined from Equation A3.6 by substituting  $\varepsilon_1$  and  $\varepsilon_2$  with  $\varepsilon_{11}$  and  $\varepsilon_{22}$ , respectively.

## Reference orientations

The reference orientation is the angle between the reference axes and the principal/shear axes. The orientations are calculated as follows:

$$\begin{aligned} \phi_D &= \frac{1}{2} \tan^{-1} \frac{\varepsilon_{12}}{\varepsilon_{11} - \varepsilon_{22}} \\ \phi_S &= -\frac{1}{2} \tan^{-1} \frac{\varepsilon_{11} - \varepsilon_{22}}{\varepsilon_{12}} \end{aligned} \quad [\text{A3.7}]$$

where  $\phi_D$  is the angle between the reference axes and the principal axes, and  $\phi_S$  is the angle between the reference axes and the direction of maximum shear stress.

## A3.2 Multiaxial Gauge Fatigue

Multiaxial Gauge Fatigue is a strain-life analysis application which takes measured strain gauge data as the fatigue loading input. The application GUI is shown in Figure A3.2.

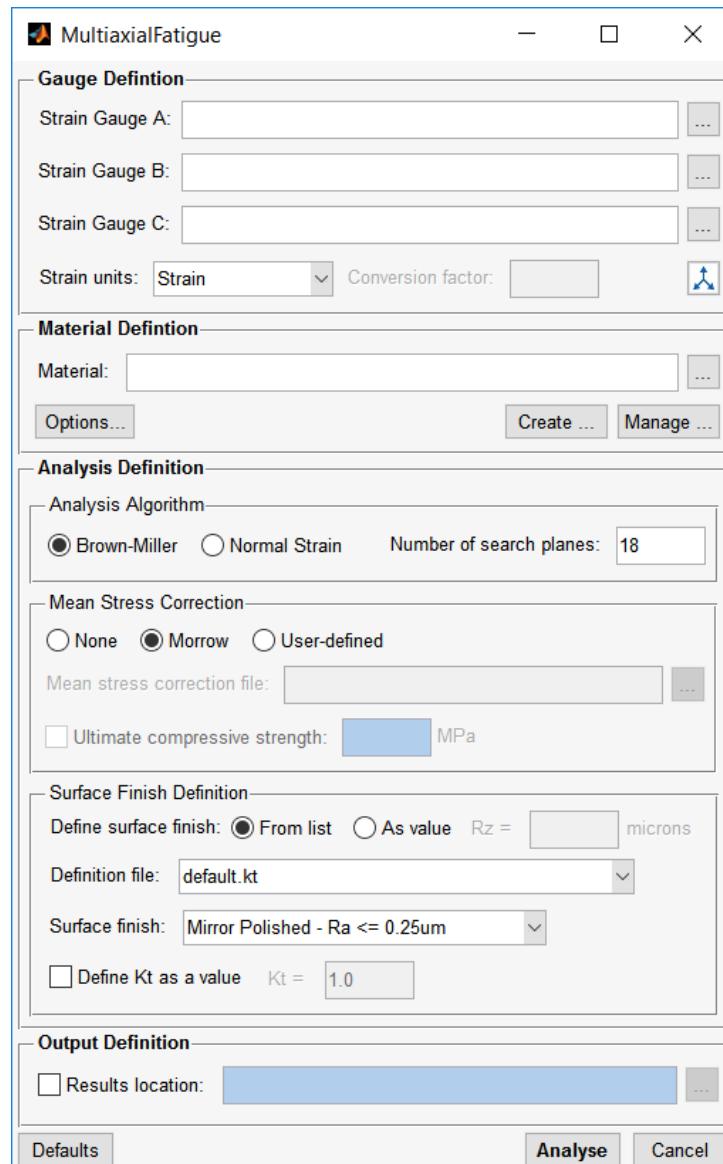


Figure A3.2: Multiaxial Gauge Fatigue GUI

### A3.2.1 Gauge Definition

By default, the gauge is assumed to be orientated in a rectangular fashion, i.e.  $\alpha = 0^\circ$ ,  $\beta = 45^\circ$  and  $\gamma = 45^\circ$ . The gauge orientation can be modified by selecting the  button. This displays the gauge orientation dialogue shown in Figure A3.3.

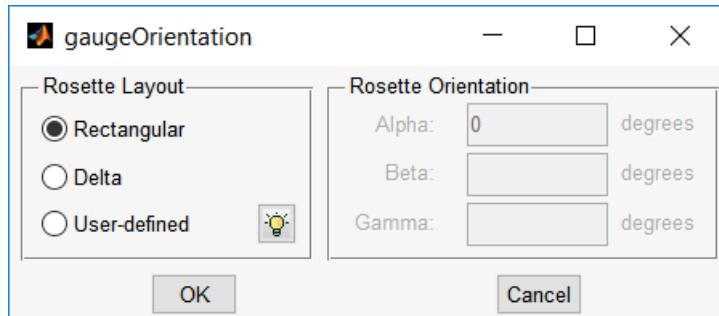


Figure A3.3: Gauge orientation dialogue

The gauge input is defined as a text file containing the measured strain history from its respective rosette arm. Absolute (full) and relative paths may be used. If no path is specified, the current working directory will be searched. The file extension must be included.

If the sample rate is not the same for all three gauges, an optional time history may be included for each gauge. The data file is formatted as follows:

Strain history	Time history (optional)	
$\varepsilon_1$	$t_1$	← First strain point
$\varepsilon_2$	$t_2$	
.	.	
.	.	
$\varepsilon_N$	$t_N$	← Last strain point

The data can be formatted as  $N \times a$  or  $a \times N$ , where  $N$  is the number of strain samples.  $a = 1$  if no time history is specified and  $a = 2$  if a time history is specified.

If time histories are not specified, Quick Fatigue Tool assumes that the lapsed time between samples is equal. If the three signals contain the same number of samples, the phase will be preserved. If the strain histories are not the same length, zeros will be added to the end of the signals where applicable so that they all have the same number of sample points.

If the histories contain a different number of samples, and/or the lapsed time between measurements is not constant, the phase between the signals will be lost if time points are not specified, and inaccurate fatigue results may be obtained. When time histories are defined alongside the strain measurements, Quick Fatigue Tool interpolates the signals so that all three gauges contain measurements at the exact same time points. This ensures that the phase between the signals is preserved.

Time points do not have to be specified for all three gauges. Gauges which do not have time points will be assumed to be increasing monotonically through time:

Strain	$\varepsilon_1$	$\varepsilon_2$	...	$\varepsilon_n$
Time	0	1		$n$

Time points do not have to be specified for every sample point. Missing time points will be interpolated automatically. When using time points, the first point must always be zero, and at least the first and last points must be defined.

The user should exercise caution when using time points. The underlying assumption remains that the strain measurements were obtained at identical points in time. The correction applied by Quick Fatigue Tool is intended for signals with small measurement delay. If the time points between the three gauges differ greatly, the resulting strain history may suffer from aliasing. This happens because of interpolating linearly a strain history which varies nonlinearly.

The default gauge units are strain [*length/length*]. The user may specify units of microstrain [ $\mu\varepsilon$ ], else user-defined units may be specified with a conversion factor such that:

$$[\text{User units}] \times \text{Conversion Factor} = [\text{strain}] \quad [\text{A3.8}]$$

### A3.2.2 Material definition

The material is selected either by browsing for an existing material (\*.mat) file, or by creating a new material with the Material Manager application. The selected material must be located in `<root>\Data\material\local`. Absolute (full) or relative paths may be used. The file extension does not have to be included.

Strain-life analysis requires the fatigue ductility (Coffin-Manson) parameters: The fatigue ductility coefficient ( $\varepsilon_f'$ ) and the fatigue ductility exponent ( $c$ ).

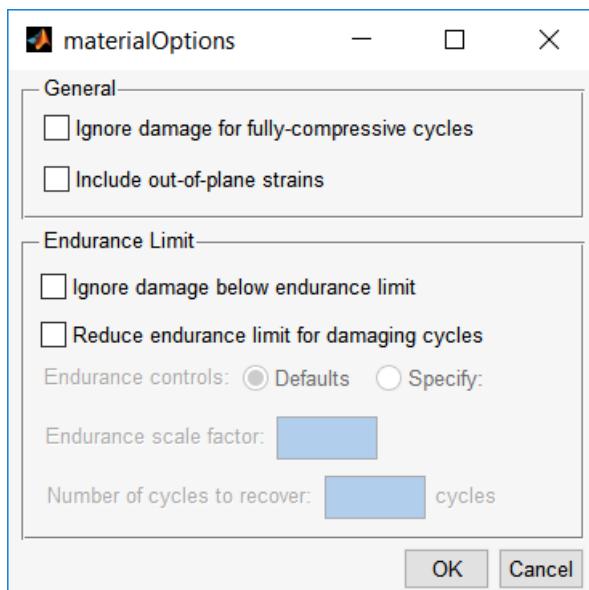


Figure A3.4: Material options

Additional options relating to the material behaviour are accessed by selecting **Options...** from the Material Definition region of the user interface. The options are shown in Figure A3.4.

### **Ignore damage for fully-compressive cycles**

By default, the analysis calculates damage for all cycles above the fatigue limit. Selecting this option will ignore fatigue damage for any cycle whose maximum and minimum points are less than-or-equal-to zero. This behaviour is appropriate for various irons and plastics.

### **Include out-of-plane strains**

Rosette strain gauges can only detect plane strain and the analysis ignores out-of-plane strains by default. The assumption is that the thickness of the specimen is sufficient to constrain out-of-plane displacements, and that through-thickness strains would not cause fatigue damage. Selecting this option will enable calculation of the out-of-plane principal strain according to Equation A3.7.

$$\varepsilon_3 = \frac{-\nu}{1-\nu} (\varepsilon_1 + \varepsilon_2) \quad [\text{A3.9}]$$

This calculation assumes a constant section Poisson's ratio and should only be used in cases where such an estimation is valid.

### **Ignore damage below endurance limit**

Most steels exhibit a clearly defined knee-point, below which fatigue is not observed. In such cases, selecting this option will ignore cycles whose stress amplitude is below the pre-determined fatigue limit. The fatigue limit is defined as the strain amplitude which would result in a fatigue life equal to the material's constant amplitude endurance limit.

Materials such as aluminium do not exhibit a fatigue limit. For these materials, this option should not be selected.

### **Reduce endurance limit for damaging cycles**

If the user selects to ignore damage below the endurance limit, this option can be additionally selected which will automatically reduce the endurance limit in response to a damaging cycle. The resulting behaviour is that previously non-damaging cycles (below the original fatigue limit), will become damaging.

By default, the endurance limit is scaled by a factor of 0.25, and recovers linearly to its original value over the course of 50 non-damaging cycles. If any cycle exceeds its instantaneous fatigue limit during the recovery process, the endurance limit is reset to the scaled value. The user may change these defaults by selecting the **Specify:** radio button and manually entering the values of the endurance limit scale factor and the number of cycles to recover.

### A3.2.3 Analysis definition

The gauge fatigue analysis can be run with either the Brown-Miller or the Principal Strain algorithm. These two algorithms work in a similar fashion to the Stress-based Brown-Miller and Normal Stress algorithms described in Sections 6.2 and 6.3, respectively. Strains are resolved onto a series of planes and fatigue damage is calculated on the plane which experiences the largest combination of normal and shear strain (Brown-Miller) or the largest normal strain (Principal Strain).

The number of search planes can be selected from the GUI. The use of 18 search planes is found to be sufficient in most cases. Increasing the number of search planes drastically increases the analysis time.

### A3.2.4 Mean Stress Correction

The analysis can be run without mean stress correction, or with either the Morrow correction or a user-defined mean stress correction file.

The Morrow mean stress correction modifies the elastic region of the strain-life equation:

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma_f' - \sigma_m}{E} N_f^b + \varepsilon_f' N_f^c \quad [\text{A3.10}]$$

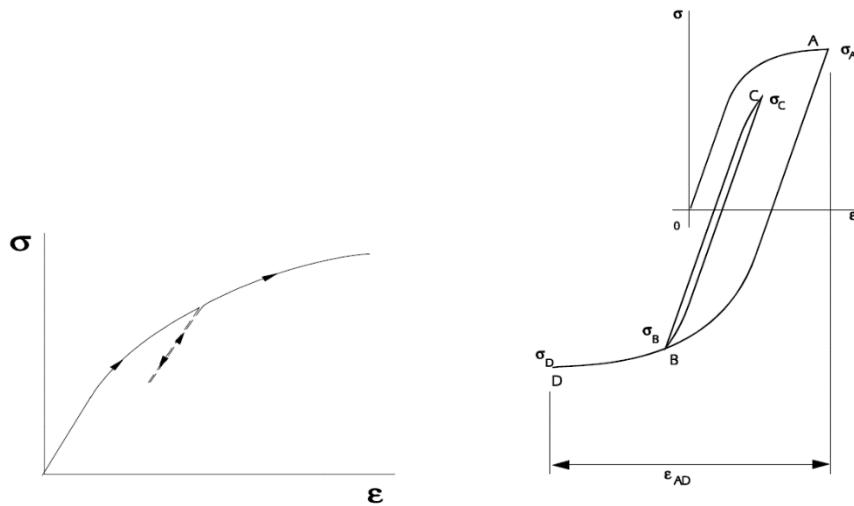
The mean stress has a more significant effect on the material's endurance at higher lives. The Morrow correction subtracts the mean stress from the fatigue strength coefficient.

User-defined mean stress correction can be performed by selecting a mean stress correction (\*.msc) file. Absolute (full) and relative paths may be used. If no path is specified, the current working directory will be searched. The file extension must be included. User-defined mean stress corrections are discussed in Section 7.9. If data are specified in the compressive regime, the ultimate compressive strength may also be defined by selecting the *Ultimate compressive strength* checkbox. If the compressive strength is not specified, the tensile strength is used by default.

To perform mean stress correction, the true stresses must be obtained from the measured strain history. If mean stress correction is specified, the program converts the principal strain history into principal stresses before the analysis, and uses the stress and strain for the critical plane analysis.

The relationship between true stress and strain is assumed to obey a multilinear cyclic strain-hardening law. This is accomplished with the Ramberg-Osgood relationship for cyclic stress and strain:

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{E} + 2 \left( \frac{\Delta \sigma}{2K'} \right)^{1/n'} \quad [\text{A3.11}]$$



**Figure A3.5: The material memory effect. After cycle closure, the hysteresis response resumes from that of the previous excursion**

Where  $\frac{\Delta\epsilon}{2}$  and  $\frac{\Delta\sigma}{2}$  is the true strain and stress range, respectively. Since the true strain is used, elastic-plastic correction such as Neuber's rule is not required. The effect of material memory is considered whereby if a cycle is closed, the material stress-strain response continues as though the reversal had not occurred. This behaviour is illustrated by Figure A3.5. This allows for the effect of ratcheting to be considered.

The current approach for calculating the stresses is limited by the assumption that the strain history is already cyclically stable. Therefore, the analysis does not make additional correction for the effect of kinematic hardening. Depending on the hardening characteristics of the material, fatigue results obtained from the Multiaxial Gauge Fatigue application may be on the non-conservative side. In such cases, additional care and consideration should be taken by the user.

### A3.2.5 Surface Finish Definition

Surface finish is specified either directly or by surface finish type. The effect of surface finish is discussed in Section 4.3.

### A3.2.6 Output Definition

In addition to a fatigue analysis summary, the Multiaxial Gauge Fatigue application writes the following history output:

<b>ST, NT, DPP, DP, LP</b>	Maximum shear strain, maximum normal strain, damage parameter, damage and life as a function of plane angle $\theta$
<b>HD</b>	Mean stress and strain amplitude for each cycle in the load history
<b>CN, CS</b>	Normal and resultant shear strain history on the critical plane
<b>EST</b>	Principal tensor history. Includes the principal stress if mean stress correction is used

The following MATLAB figures are produced:

<b>CN + CS</b>	Normal and shear strain history on the critical plane
<b>CPS</b>	Normal strain and resultant shear stress vs. plane angle
<b>CSSn</b>	Cyclic stress-strain curves for maximum, middle and minimum principal stress-strain histories
<b>DP</b>	Damage vs. angle
<b>DPP</b>	Damage parameter vs. angle
<b>LP</b>	Life vs. angle
<b>PS</b>	Time-correlated principal stresses
<b>PE</b>	Time-correlated principal strains
<b>RHIST</b>	Rainflow cycle histogram

### A3.3 Rosette Analysis

The Rosette Analysis GUI is shown in Figure A3.6.

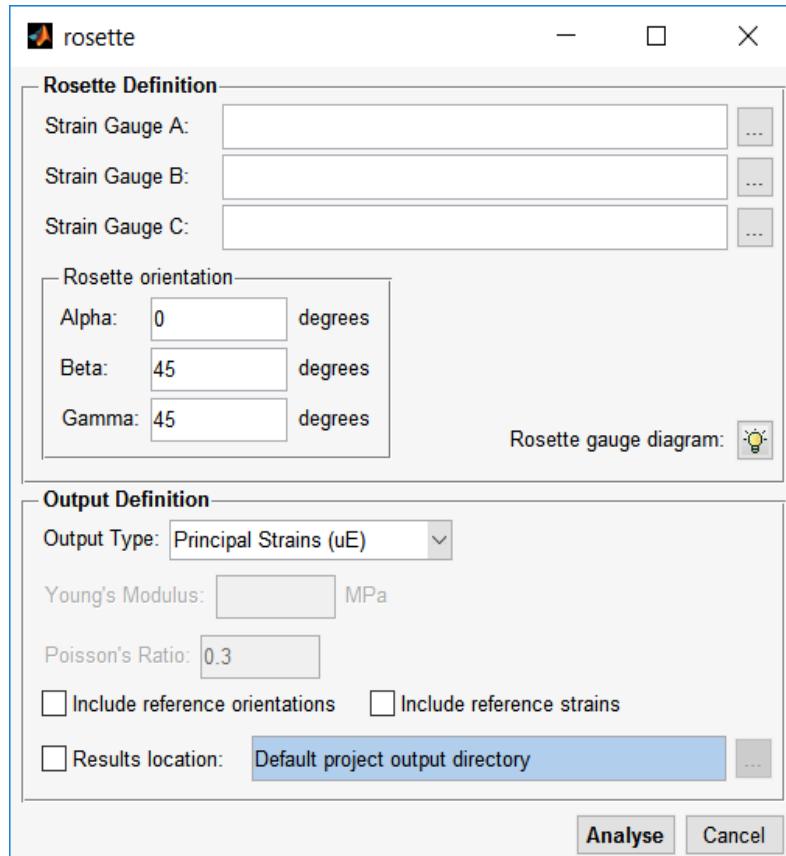


Figure A3.6: Rosette Analysis GUI

The purpose of this application is to process data from a strain gauge to convert it into principal stresses and strains. Strain data is provided in three separate text files representing the three rosette gauge arms. The format for each file must be  $1 \times N$  or  $N \times 1$ . The strain is defined in units of  $[length/length]$ .

The gauge orientation must be defined. A rectangular orientation corresponds to  $\alpha = 0^\circ$ ,  $\beta = 45^\circ$  and  $\gamma = 45^\circ$ , whereas a delta layout would be defined as  $\alpha = 30^\circ$ ,  $\beta = 60^\circ$  and  $\gamma = 60^\circ$ . However, any user gauge orientation may be specified.

The user may request the output in terms of principal strain or principal stress. Additionally, the reference orientations and stress/strain may be requested. The reference orientations and stress/strain are given by Equation A3.7 and Equation A3.4, respectively.

All available output variables and their identifiers are listed below.

<b>PE1</b>	Maximum principal strain
<b>PE2</b>	Minimum principal strain
<b>E12<sup>(MAX)</sup></b>	Maximum shear strain
<b>E11<sup>(R)</sup></b>	Normal strain in reference <i>X</i> -direction
<b>E22<sup>(R)</sup></b>	Normal strain in reference <i>Y</i> -direction
<b>E12<sup>(R)</sup></b>	Shear strain in reference <i>XY</i> -direction
<b>PS1</b>	Maximum principal stress
<b>PS2</b>	Minimum principal stress
<b>S12<sup>(MAX)</sup></b>	Maximum shear stress
<b>S11<sup>(R)</sup></b>	Normal stress in reference <i>X</i> -direction
<b>S22<sup>(R)</sup></b>	Normal stress in reference <i>Y</i> -direction
<b>S12<sup>(R)</sup></b>	Normal stress in reference <i>XY</i> -direction
<b>Φ<sub>D</sub></b>	Angle between principal and reference axes
<b>Φ<sub>S</sub></b>	Angle between maximum shear and reference axes

Output stress, strain and angle units are *MPa*,  $\mu\epsilon$  and *degrees*, respectively.

### A3.4 Virtual Strain Gauge

The Virtual Strain Gauge GUI is shown in Figure A3.7.

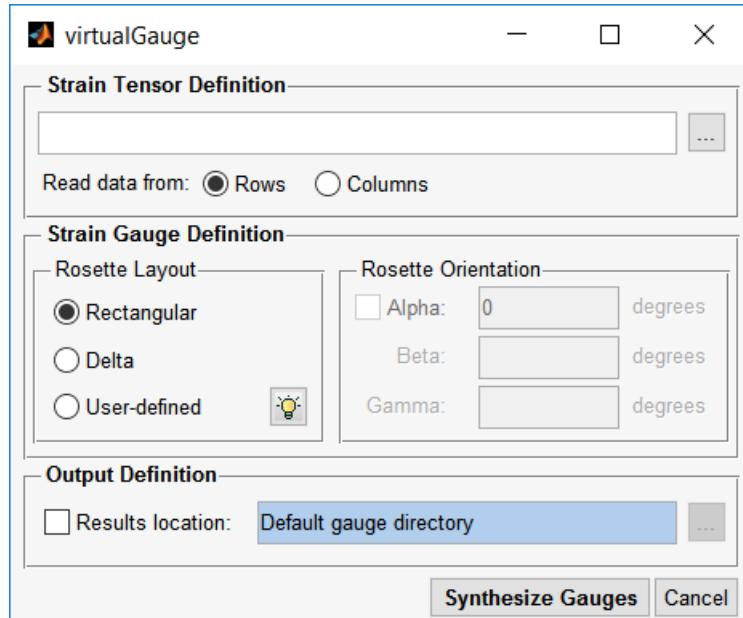


Figure A3.7: Virtual Strain Gauge GUI

The purpose of this application is to mimic the output of a virtual strain gauge in response to a strain tensor. The strain tensor represents the response of the structure onto which the virtual gauge would be adhered. This tool is useful for determining the theoretical response of a rosette strain gauge at a specific orientation.

Strain data is read from a text file and has the following format:

$$\begin{array}{ccc} \boldsymbol{\varepsilon}_{11} & \boldsymbol{\varepsilon}_{22} & \boldsymbol{\varepsilon}_{12} \\ \cdot & \cdot & \cdot & \leftarrow \text{First strain point} \\ \cdot & \cdot & \cdot \\ \boldsymbol{\varepsilon}_{11N} & \boldsymbol{\varepsilon}_{22N} & \boldsymbol{\varepsilon}_{12N} & \leftarrow \text{Last strain point} \end{array}$$

In the above example, the strain components are defined by three separate columns representing the reference  $X$ ,  $Y$  and shear  $XY$  directions, respectively. However, the strain data may be defined by rows or columns, provided that the user specified this with the *Read data from* radio buttons. The strain is defined in units of [ $\text{length}/\text{length}$ ].

The process of converting the input data in strain gauge histories is described in Section 4.9.

### A3.5 Mohr Solver

The Mohr Solver GUI is shown in Figure A3.8.

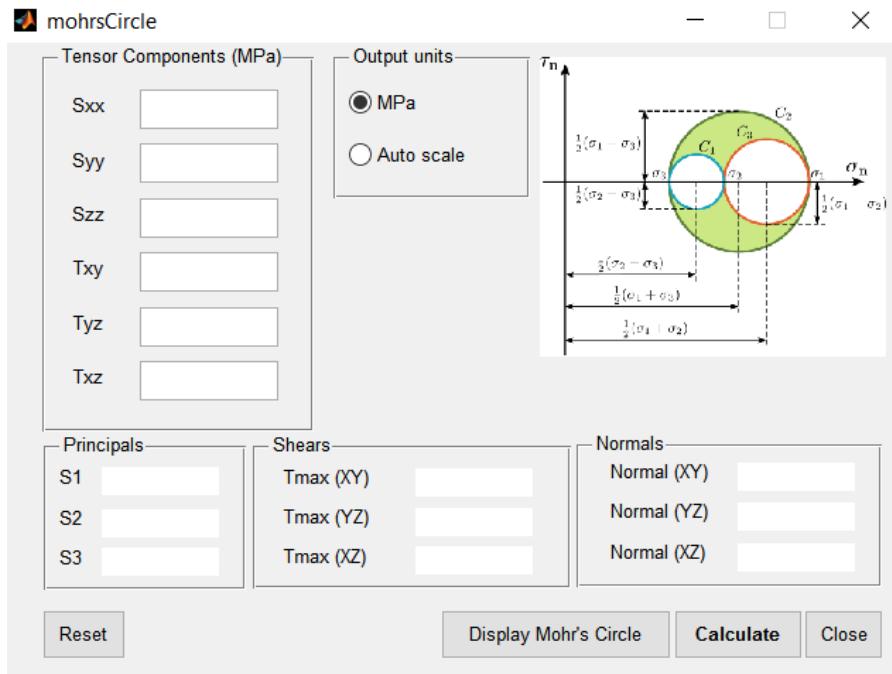


Figure A3.8: Mohr's Circle Solver GUI

The principal stresses are given by Equation A3.12.

$$[\sigma_1 \sigma_2 \sigma_3] = \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \cdot & \sigma_{yy} & \tau_{yz} \\ \cdot & \cdot & \sigma_{zz} \end{vmatrix} \quad [A3.12]$$

The maximum shear stresses are given by Equation A3.13.

$$\tau_{max,xy,yz,xz} = \frac{1}{2} (\sigma_{11,22,11} - \sigma_{22,33,33}) \quad [A3.13]$$

The maximum normal stresses are given by Equation A3.14.

$$\sigma_{N_{max,xy,yz,xz}} = \frac{1}{2} (\sigma_{11,22,11} + \sigma_{22,33,33}) \quad [A3.14]$$

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