

Fibonacci Numbers: A Deep Dive

Stanislav Ostapenko

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Abstract

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Listings

$$\mathcal{O}(1) = \mathcal{O}(\text{yeah})$$

$$\mathcal{O}(\log n) = \mathcal{O}(\text{nice})$$

$$\mathcal{O}(n) = \mathcal{O}(\text{k})$$

$$\mathcal{O}(n^2) = \mathcal{O}(\text{my})$$

$$\mathcal{O}(2^n) = \mathcal{O}(\text{no})$$

$$\mathcal{O}(n!) = \mathcal{O}(\text{mg})$$

$$\mathcal{O}(n^n) = \mathcal{O}(\text{sh}^*\text{t!})$$

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2.8 Recursion Tree

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3 Optimizing Recursion

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3.5 Binet formula in real life

3.6 Conclusion

4 Limitations of Primitive Types for Large Fibonacci Numbers

4.1 Showcase : from int to double

4.2 When double goes wild

4.2.1 $0.1 + 0.2 \neq 0.3$

4.2.2 IEEE 754 Representation