

Fibonacci Numbers: A Deep Dive

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Abstract

TODO

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Listings

$$\mathcal{O}(1) = \mathcal{O}(\text{yeah})$$

$$\mathcal{O}(\log n) = \mathcal{O}(\text{nice})$$

$$\mathcal{O}(n) = \mathcal{O}(\text{k})$$

$$\mathcal{O}(n^2) = \mathcal{O}(\text{my})$$

$$\mathcal{O}(2^n) = \mathcal{O}(\text{no})$$

$$\mathcal{O}(n!) = \mathcal{O}(\text{mg})$$

$$\mathcal{O}(n^n) = \mathcal{O}(\text{sh}^*\text{t!})$$

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