Balanced Cortical Microcircuitry Based on Negative Derivative Feedback Control and STDP

Yuancheng, Xu

Outline

- Persistent activity in working memory
  - Parametric working memory:
    - Firing rate model
    - **Spiking network model**
  - Spatial working memory:
    - Firing rate model
    - **Spiking network model**
- Spike-time dependent plasticity (STDP)
  - Mechanism
  - Simulation in parametric and spatial working memory
- Conclusion and discussion

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Working memory

- Definition: A cognitive system responsible for temporarily holding information available for processing
- **Persistent neural activity** in the absence of a stimulus
  - Leakage of currents out of membranes.
  - $\tau \frac{dr}{dt} = -r$; $\tau: milisecond$
  - Timescale of working memory: $seconds$

Big picture

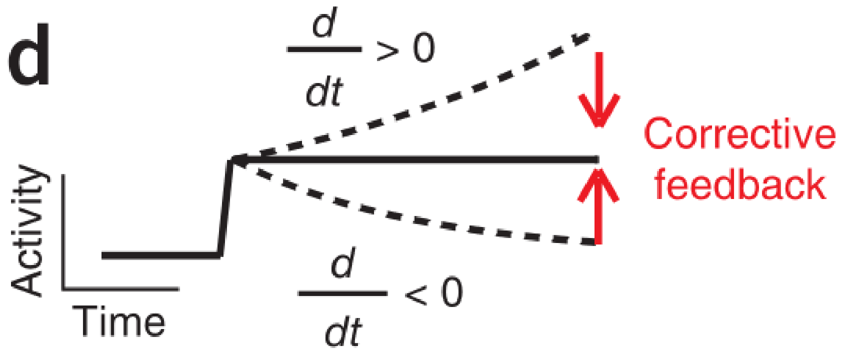
Previous work

- Positive feedback models
- $\tau \frac{dr}{dt} = -r + wr + I(t)$
- Drawback: requires a fine tuning of the level of feedback
  - W>1 : grow exponentially
  - W<1 : decay exponentially

Our work (Sukbin, 2013)

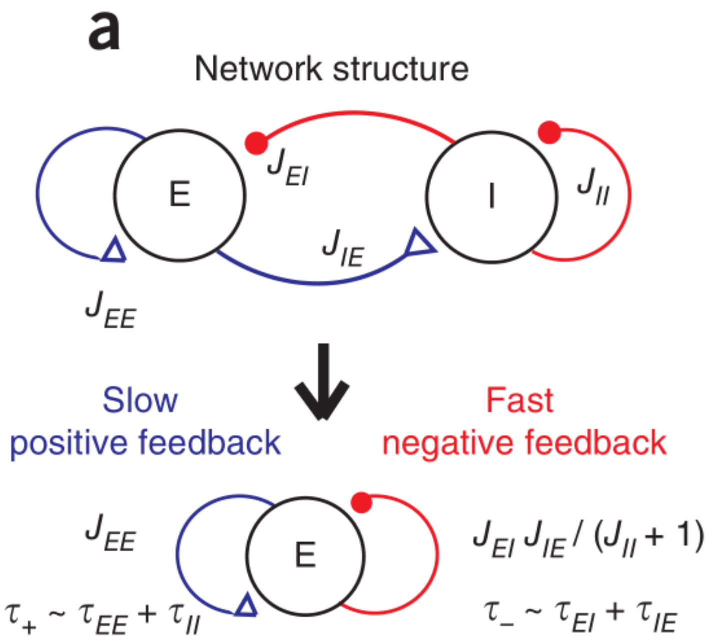
- Negative derivative feedback model

$$\tau \frac{dr}{dt} = -r + W_{\text{pos}} r - W_{\text{der}} \frac{dr}{dt} + I(t)$$



Q: How can this negative derivative feedback arise from interactions between E and I neurons in neocortical circuits?

- Firing rate model- the main setting



$$
\begin{aligned}
\tau_E \dot{r}_E &= -r_E + f_E(J_{EE}s_{EE} - J_{EI}s_{EI} + J_{EO}i(t)) \\
\tau_I \dot{r}_I &= -r_I + f_I(J_{IE}s_{IE} - J_{II}s_{II} + J_{IO}i(t)) \\
\tau_{ij}\dot{s}_{ij} &= -s_{ij} + r_j \qquad \text{for } i, j = E, \text{ or } I
\end{aligned}
$$

Observations in experiments

- Cortical neurons receive massive amounts of both excitation and inhibition that are closely balanced
- EE connections have relatively slow kinetics resulting from an abundance of slow NMDA conductances. EI are faster.

Computational Results

- Conditions:

$$\begin{cases} \dfrac{J_{EE}J_{II}}{J_{EI}J_{IE}} \sim 1 \quad \text{for large } J \text{ values} \\[2ex] \tau_+ = (\tau_{EE} + \tau_{II}) > (\tau_{EI} + \tau_{IE}) = \tau_- \end{cases}$$

$$\begin{aligned} \tau_E \dot{r}_E &= -r_E + f_E(J_{EE}s_{EE} - J_{EI}s_{EI} + J_{EO}i(t)) \\ \tau_I \dot{r}_I &= -r_I + f_I(J_{IE}s_{IE} - J_{II}s_{II} + J_{IO}i(t)) \\ \tau_{ij}\dot{s}_{ij} &= -s_{ij} + r_j \qquad \text{for } i, j = E, \text{ or } I \end{aligned}$$

$$\tau \frac{dr}{dt} = -r + W_{\text{pos}}r - W_{\text{der}}\frac{dr}{dt} + I(t)$$

Derivation of the balanced conditions

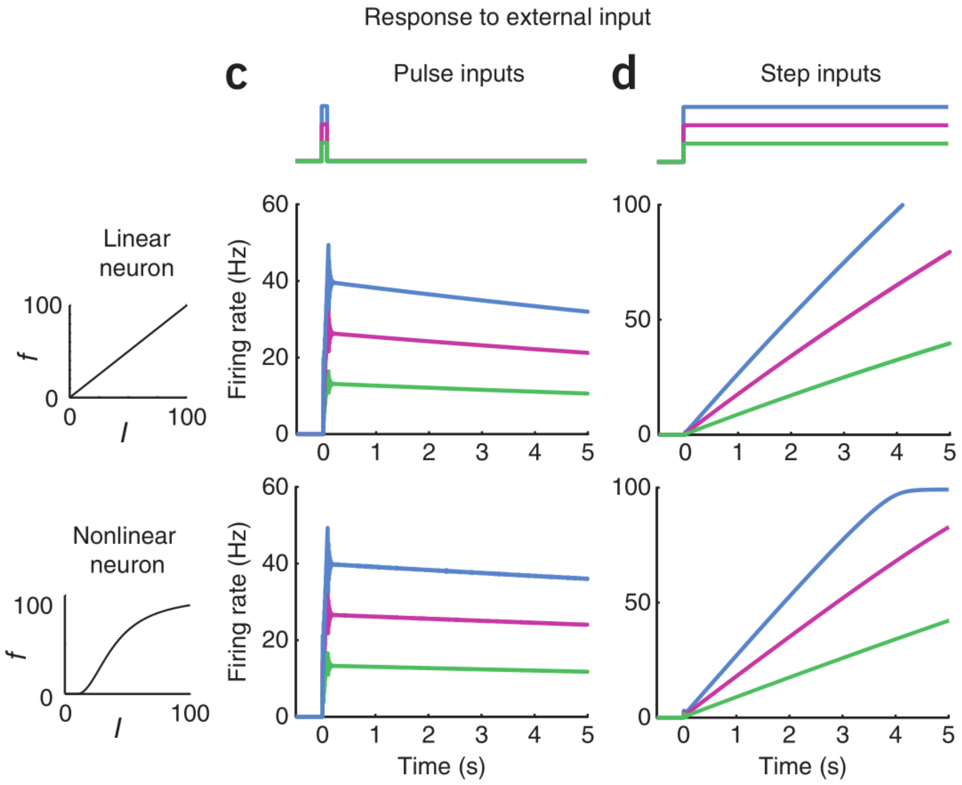
- Laplace transformation: negative derivative approximation
- At least one of the eigenvalues are close to zero

$$a_1 / a_0 = -\sum 1/\lambda_i$$

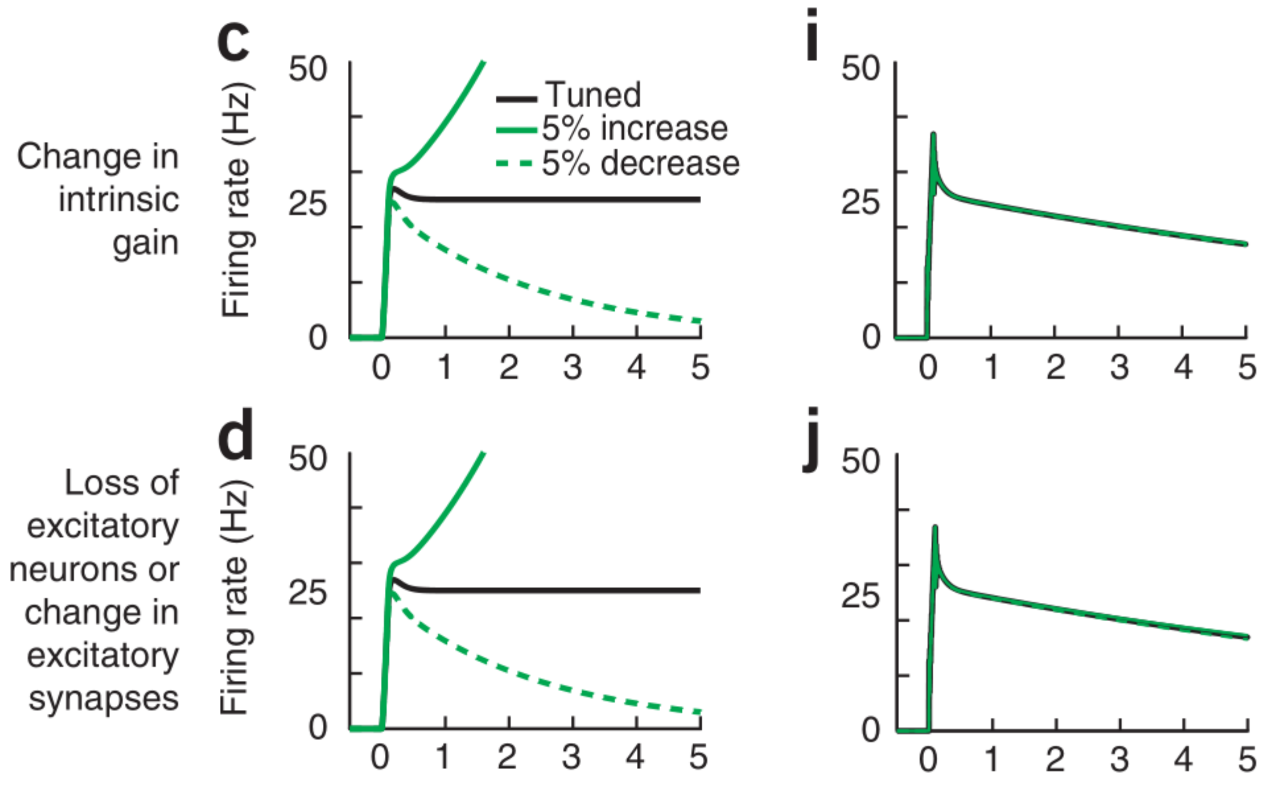
$$= \frac{J_{EI}J_{IE}/(J_{II}+1)\left(\tau_{EE}+\tau_{II}\right)-(J_{EE}-1)\left(\tau_{IE}+\tau_{EI}\right)+(\tau_E+\tau_{EE})-\left(J_{EE}-1\right)/(J_{II}+1)(\tau_I+\tau_{II})}{J_{EI}J_{IE}/(J_{II}+1)-(J_{EE}-1)}$$

  - Positive feedback (previous work)
  - Negative derivative feedback: $J_{EI}J_{IE}/\left(J_{EE}J_{II}\right)\sim 1$
- The state should be stable (attractor) $\tau_{EE}+\tau_{II} > \tau_{IE}+\tau_{EI}$

Simulation results



Robustness to common perturbations



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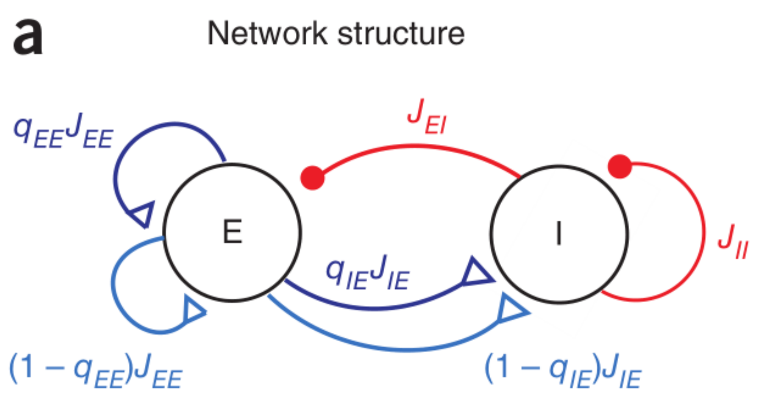
Spiking network

Difference between spiking model and firing rate model

- Discontinuity:
  - Spike timing is discrete
  - Threshold and reset
- Maybe more biologically feasible
- Firing rate is the average effect of spiking.

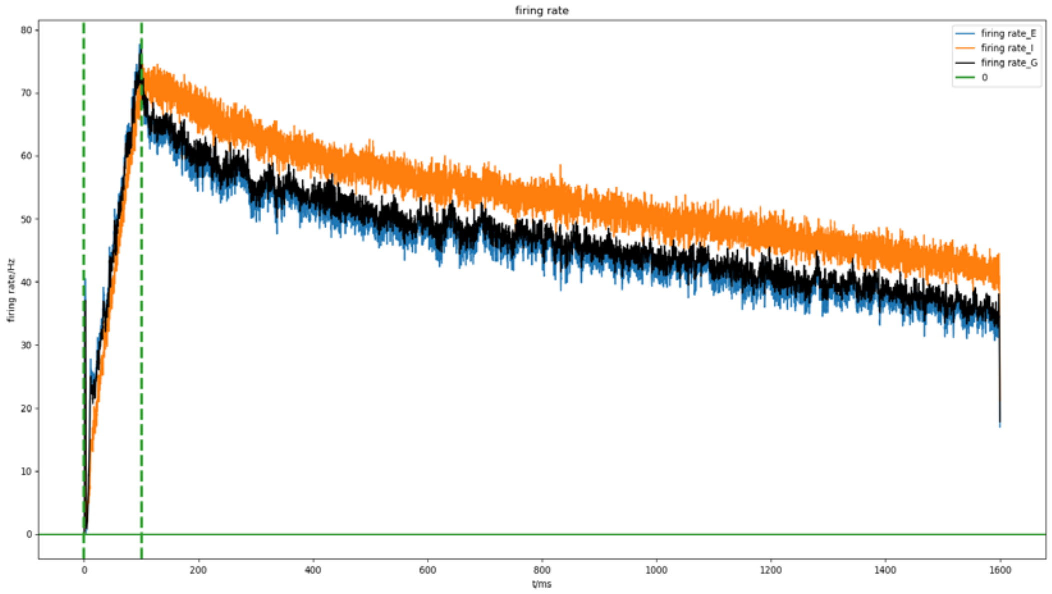
$$\tau_i \frac{dV_i^l}{dt} = -(V_i^l - V_L) + \sum_m \tilde{J}_{iE}^{lm} p_{iE}^{lm} \left(q_{iE}^N s_{iE}^{lm,N}(t) + q_{iE}^A s_{iE}^{lm,A}(t)\right) - \sum_m \tilde{J}_{iI}^{lm} p_{iI}^{lm} s_{iI}^{lm}(t) + \sum_m \tilde{J}_{iO}^{lm} p_{iO}^{lm} s_{iO}^{lm}(t) \quad (3)$$

$$\tau_{ij}^k \frac{ds_{ij}^{lm,k}}{dt} = -s_{ij}^{lm,k} + \sum_{t_j^m} \delta(t - t_j^m), \text{ for } j = E, I, \text{ or } O, \text{ and } k = N \text{ or } A. \quad (4)$$

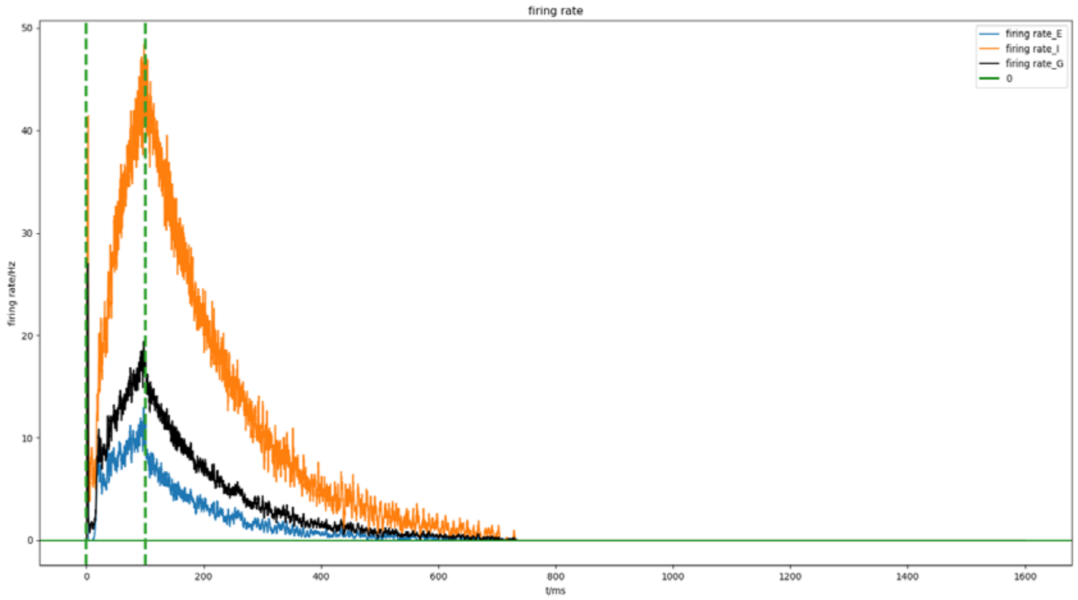


Simulation Results

Balanced



Unbalanced



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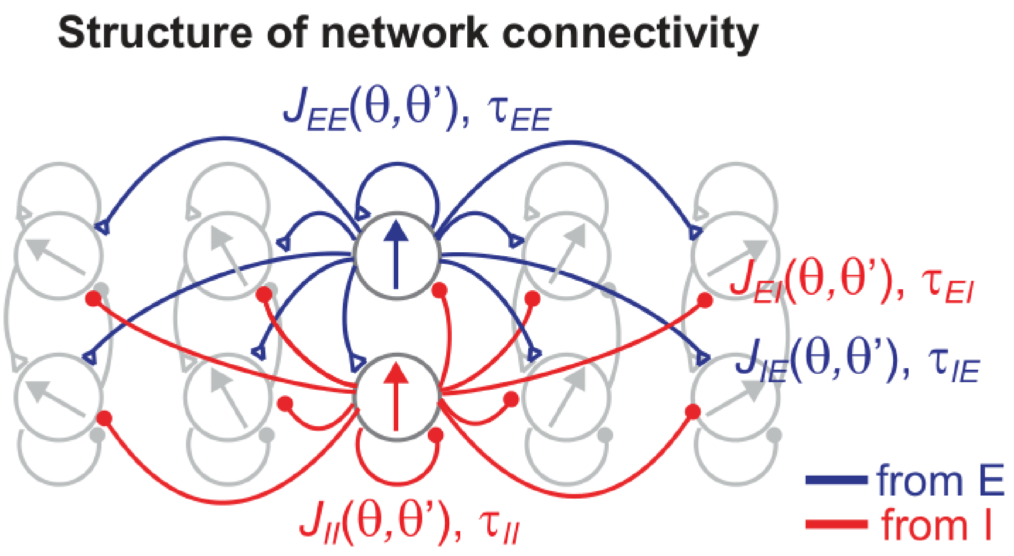
Spatial working memory

- neurons that encode the remembered location of a cue through spatially tuned patterns of **persistent neural firing**



Firing rate model

$$\tau_E \frac{dr_E(\theta,t)}{dt} = -r_E(\theta,t) + f_E\left(\sum_{j=E,I}\int_{-\pi}^{\pi} J_{Ej}(\theta,\theta')\, s_{Ej}(\theta',t)d\theta' + i_E(\theta,t)\right)$$

$$\tau_I \frac{dr_I(\theta,t)}{dt} = -r_I(\theta,t) + f_I\left(\sum_{j=E,I}\int_{-\pi}^{\pi} J_{Ij}(\theta,\theta')\, s_{Ij}(\theta',t)d\theta' + i_I(\theta,t)\right)$$

$$\tau_{ij} \frac{ds_{ij}(\theta',t)}{dt} = -s_{ij}(\theta',t) + r_j(\theta',t) \qquad \text{for } i,j = E \text{ or } I$$

Spiking network model

$$\tau_i \frac{dV_i^l}{dt} = -(V_i^l - V_L) + \sum_m \tilde{J}_{iE}^{lm} p_{iE}^{lm} (q_{iE}^N s_{iE}^{lm,N}(t) + q_{iE}^A s_{iE}^{lm,A}(t)) - \sum_m \tilde{J}_{iI}^{lm} p_{iI}^{lm} s_{iI}^{lm}(t) + \sum_m \tilde{J}_{iO}^{lm} p_{iO}^{lm} s_{iO}^{lm}(t) \quad (3)$$

$$\tau_{ij}^k \frac{ds_{ij}^{lm,k}}{dt} = -s_{ij}^{lm,k} + \sum_{t_j^m} \delta(t - t_j^m), \text{ for } j = E,I, \text{ or } O, \text{ and } k = N \text{ or } A. \quad (4)$$

Spatial structure: Gaussian-shaped profiles of synaptic connectivity (**strength based**)

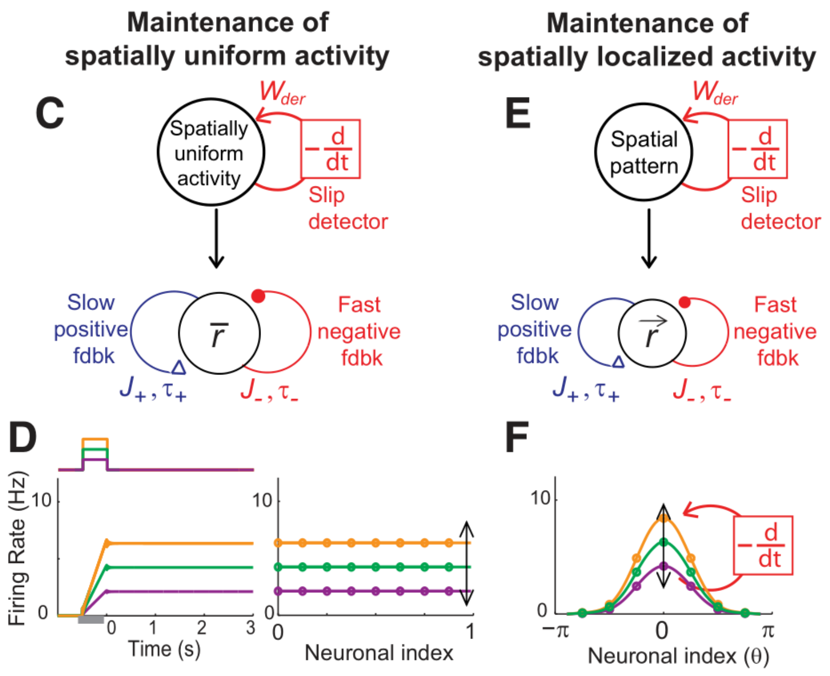
$$\tilde{J}_{iE}^{lm} = \tilde{J}_{iE} \exp[-(\theta_i^l - \theta_E^m)^2/\sigma_{iE}{}^2] \text{ where } \theta_i^l = 2\pi l/N_i - \pi,$$

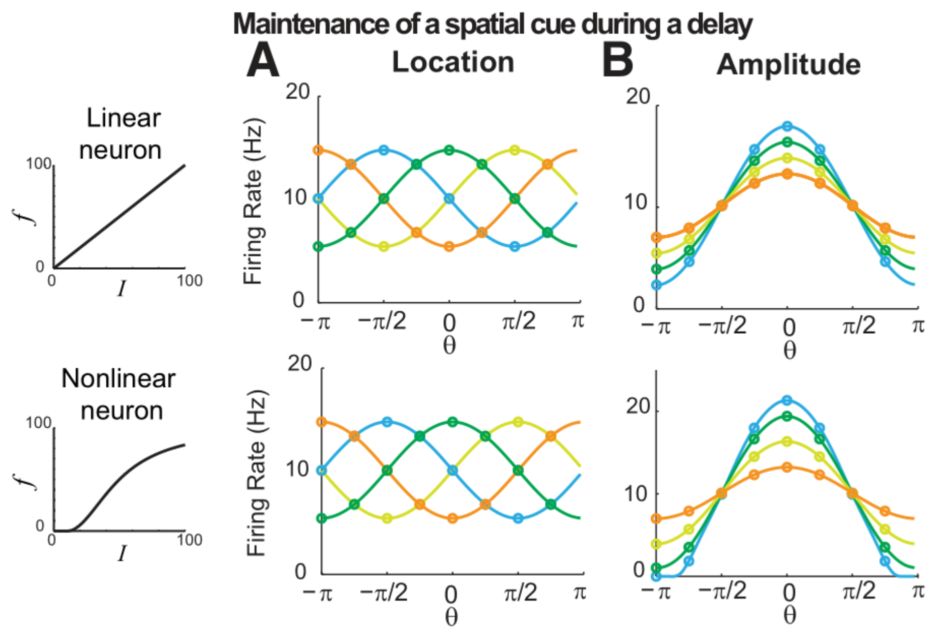
$$\theta_E^m = 2\pi m/N_E - \pi. \quad (5)$$

Probability Based: Gaussian-shaped profiles of synaptic connection probability

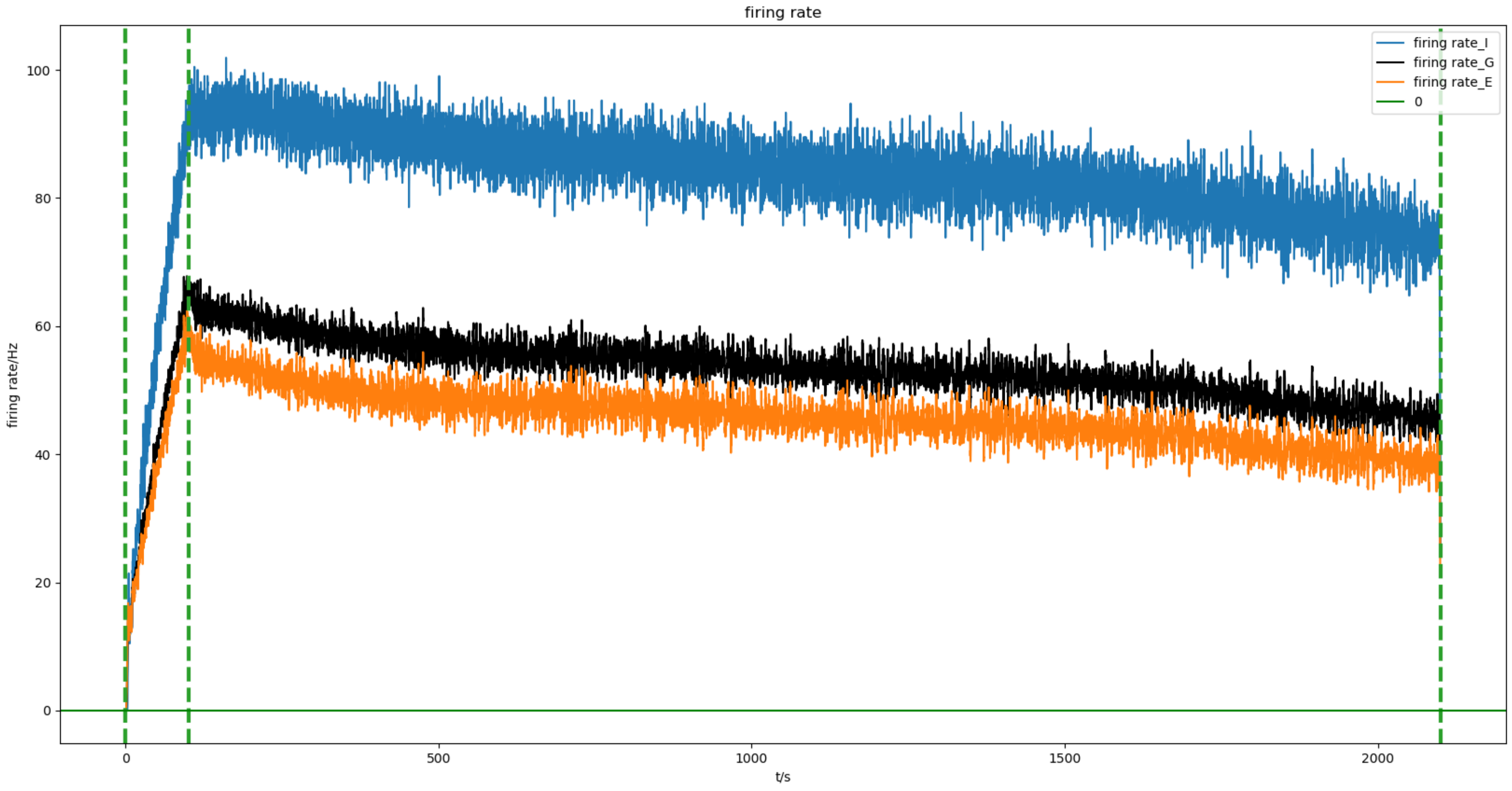
Derivation and results.

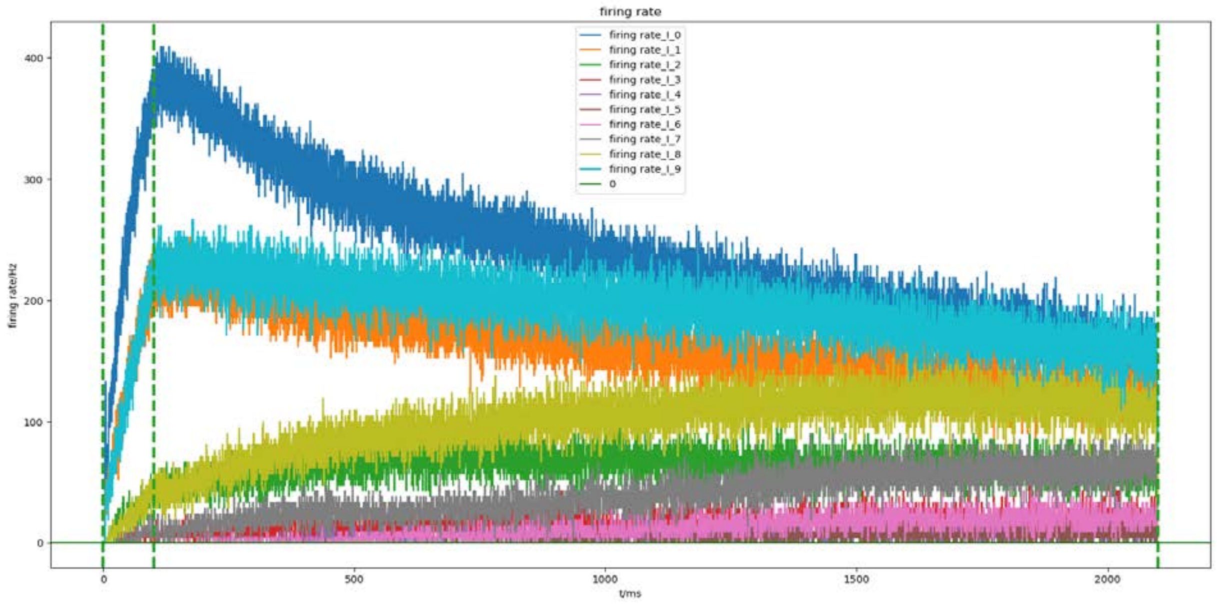
- Theoretical computation (Sukbin, 2014)
- Simulations of firing rate model

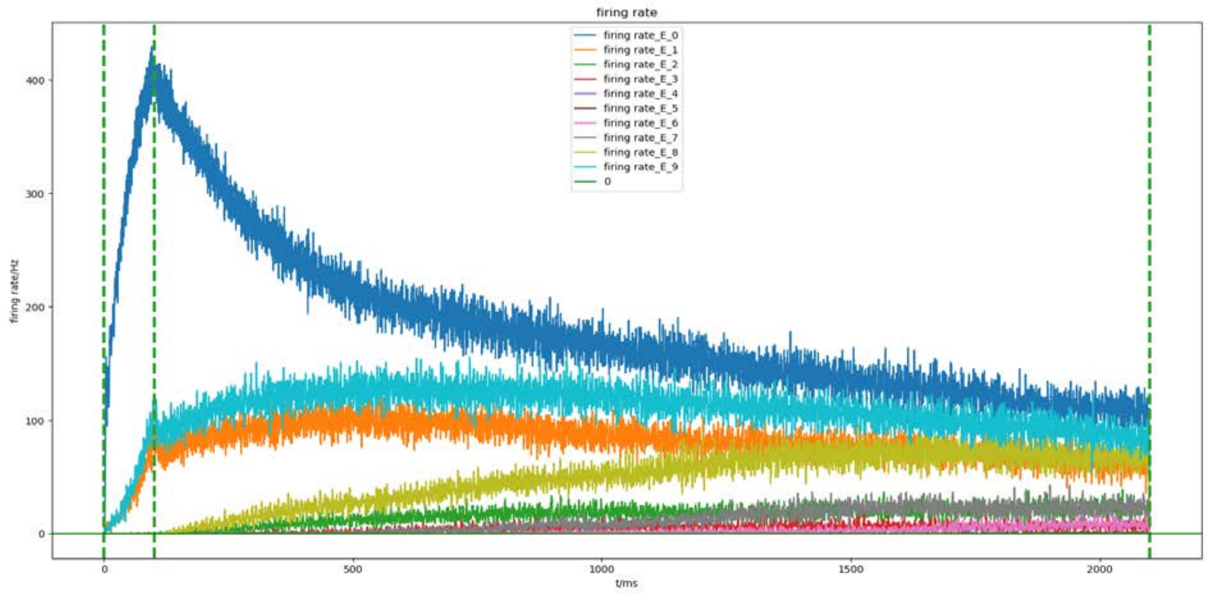




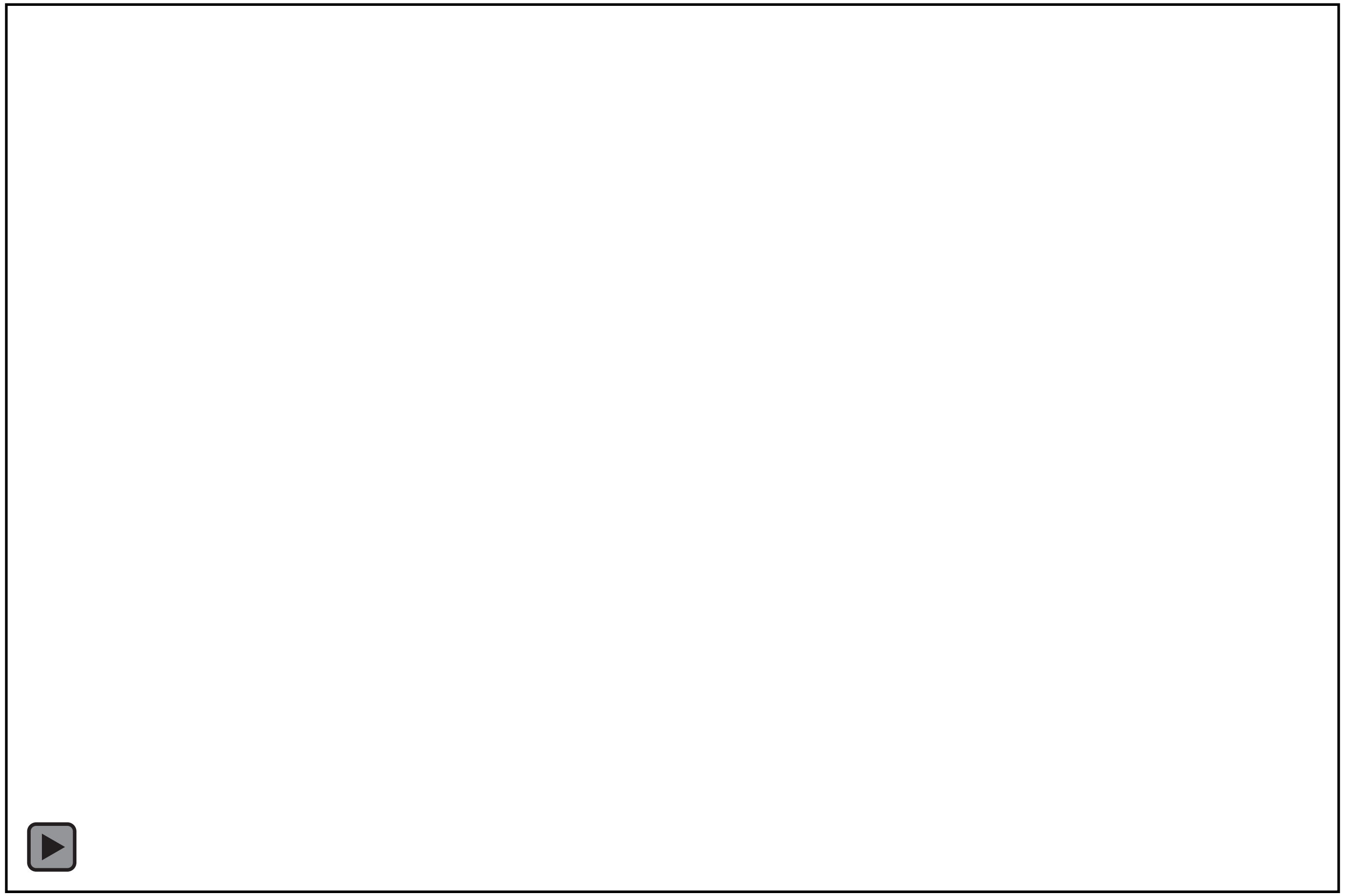
Simulations of spiking network model (probability based)



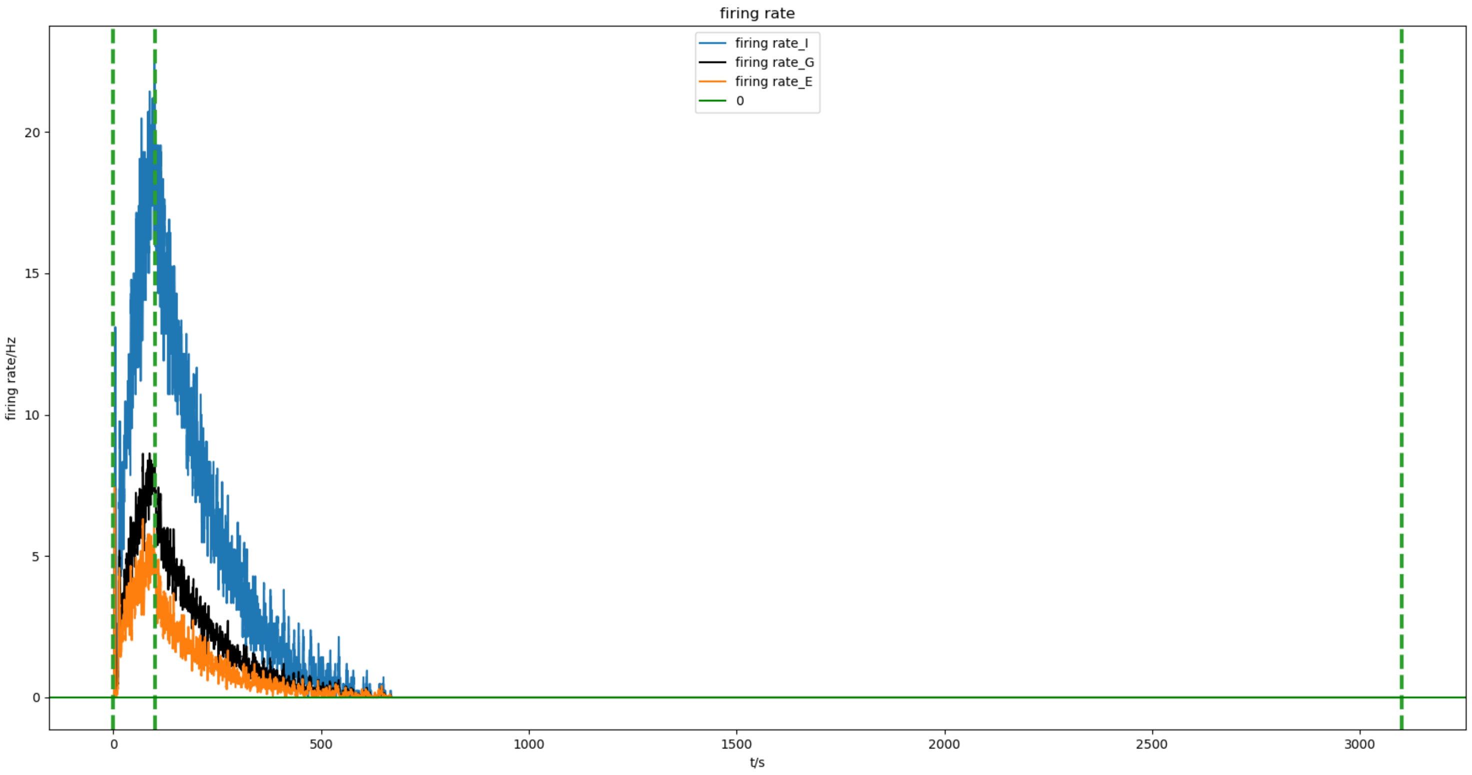

firing rate
firing rate_I_0
firing rate_I_1
firing rate_I_2
firing rate_I_3
firing rate_I_4
firing rate_I_5
firing rate_I_6
firing rate_I_7
firing rate_I_8
firing rate_I_9
0
firing rate/Hz
t/ms


firing rate
firing rate_E_0
firing rate_E_1
firing rate_E_2
firing rate_E_3
firing rate_E_4
firing rate_E_5
firing rate_E_6
firing rate_E_7
firing rate_E_8
firing rate_E_9
0
firing rate/Hz
t/ms

Evolution of firing rate of each population



When the balanced conditions do not hold



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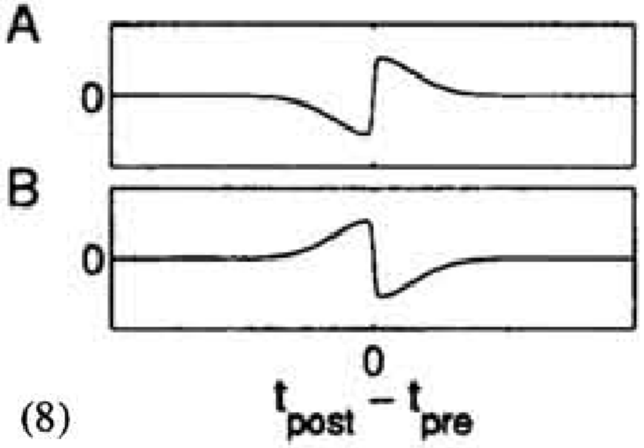
Spike-time dependent plasticity (STDP)

- Experimental observation:
  - Pre then post: increase in strength
  - Post then pre: decrease in strength
  - Depending on the time difference
  - May be opposite (differential anti-Hebbian)

Differential Hebbian

- Modeling (pair based)

$$W_{ij}(T+\lambda) - W_{ij}(\lambda) = \int_0^T dt_j \int_{-\infty}^{\infty} dt_i\, f(t_i - t_j) s_i(t_i)\, s_j(t_j)$$



Results (Sebastian 2000)

$$\dot{W}_{ij} \propto \dot{\nu}_i \nu_j \quad \text{(diff. Hebbian)} \qquad \dot{W}_{ij} \propto -\dot{\nu}_i \nu_j \quad \text{(diff. anti-Hebbian)} \qquad (8)$$

Modeling (Triplet rule)

- Motivation: Some results cannot be produced by pair rules
- Model:

$$\frac{dr_1(t)}{dt} = -\frac{r_1(t)}{\tau_+}$$

if $t = t^{\mathrm{pre}}$, then $r_1 \rightarrow r_1 + 1$.

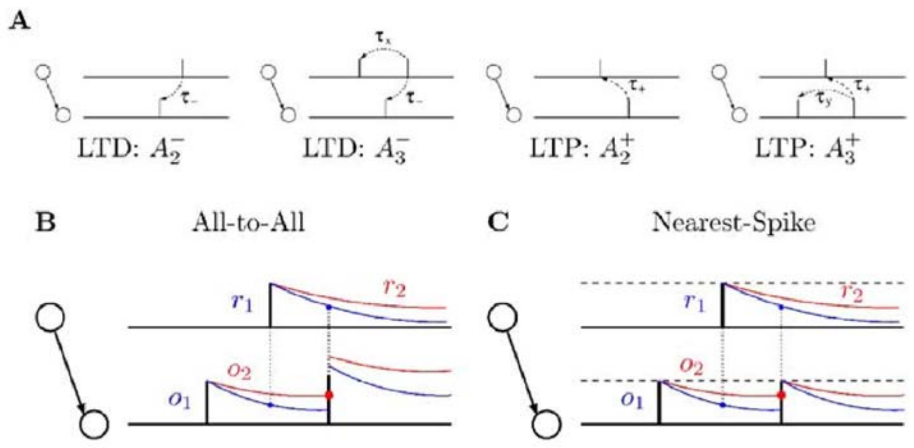
$$\frac{dr_2(t)}{dt} = -\frac{r_2(t)}{\tau_x} \quad \text{if } t = t^{\mathrm{pre}} \text{ then } r_2 \rightarrow r_2 + 1$$

$$\frac{do_1(t)}{dt} = -\frac{o_1(t)}{\tau_-} \quad \text{if } t = t^{\mathrm{post}} \text{ then } o_1 \rightarrow o_1 + 1$$

$$\frac{do_2(t)}{dt} = -\frac{o_2(t)}{\tau_y} \quad \text{if } t = t^{\mathrm{post}} \text{ then } o_2 \rightarrow o_2 + 1.$$

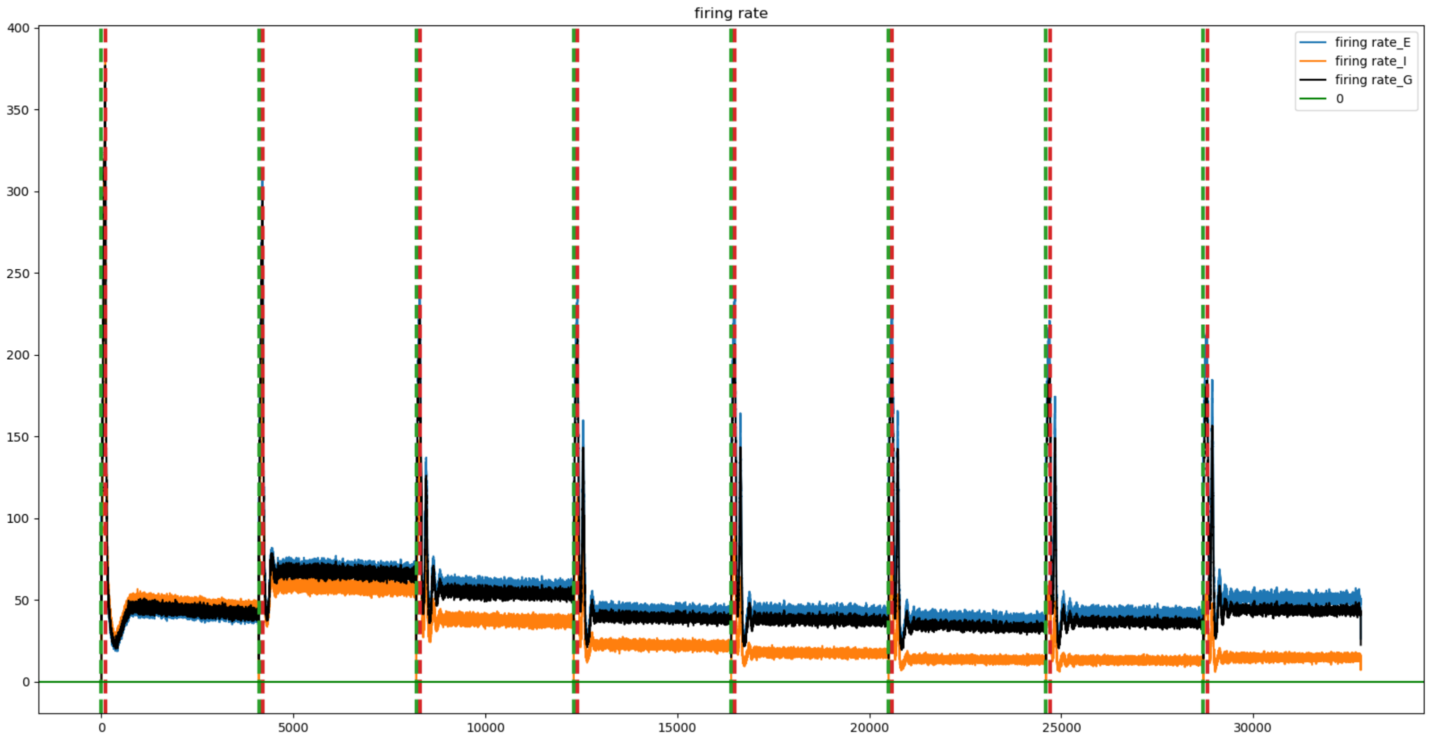
$$w(t) \rightarrow w(t) - o_1(t)[A_2^- + A_3^- r_2(t-\epsilon)] \text{ if } t = t^{\mathrm{pre}}.$$

$$w(t) \rightarrow w(t) + r_1(t)[A_2^+ + A_3^+ o_2(t-\epsilon)] \text{ if } t = t^{\mathrm{post}}$$

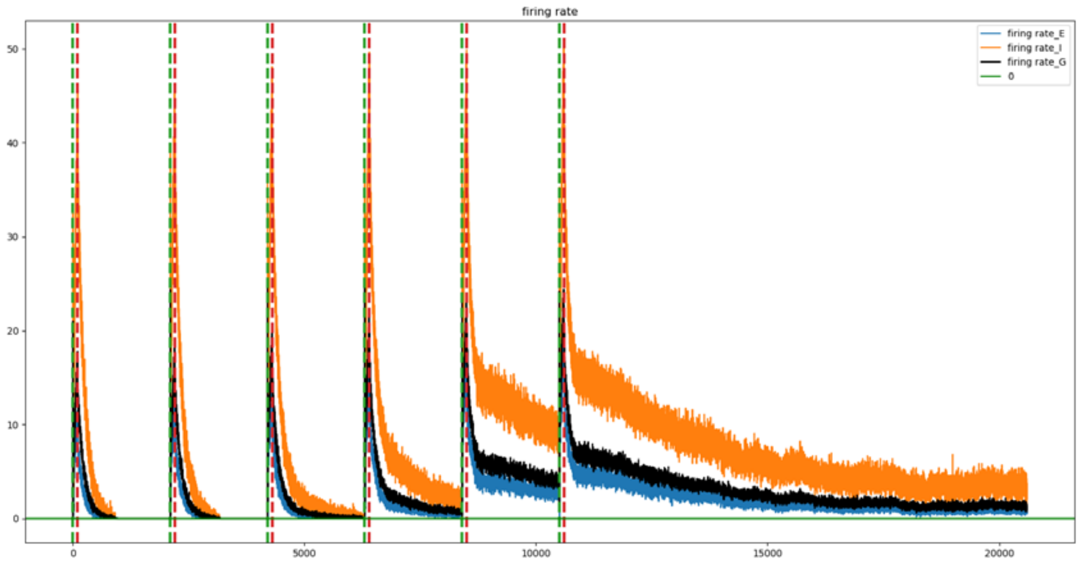


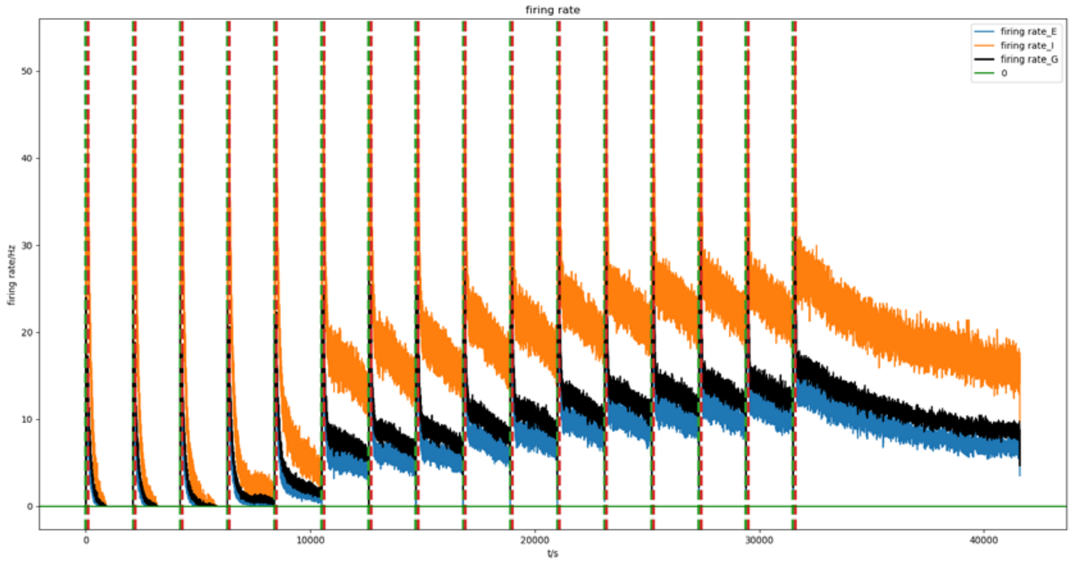
Simulations

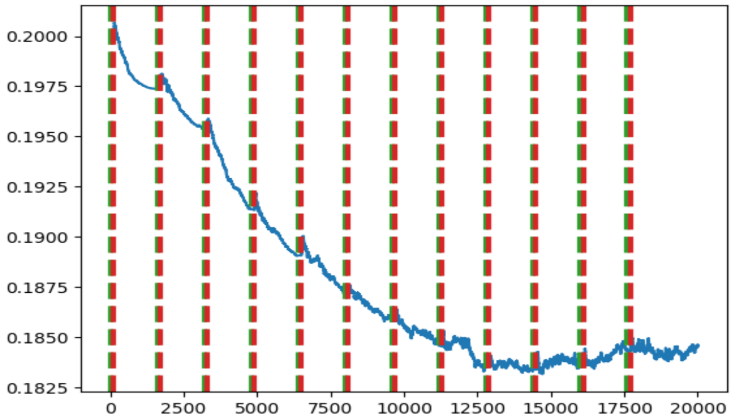
- Parametric network, balanced condition



• STDP:Parametric network, unbalanced condition

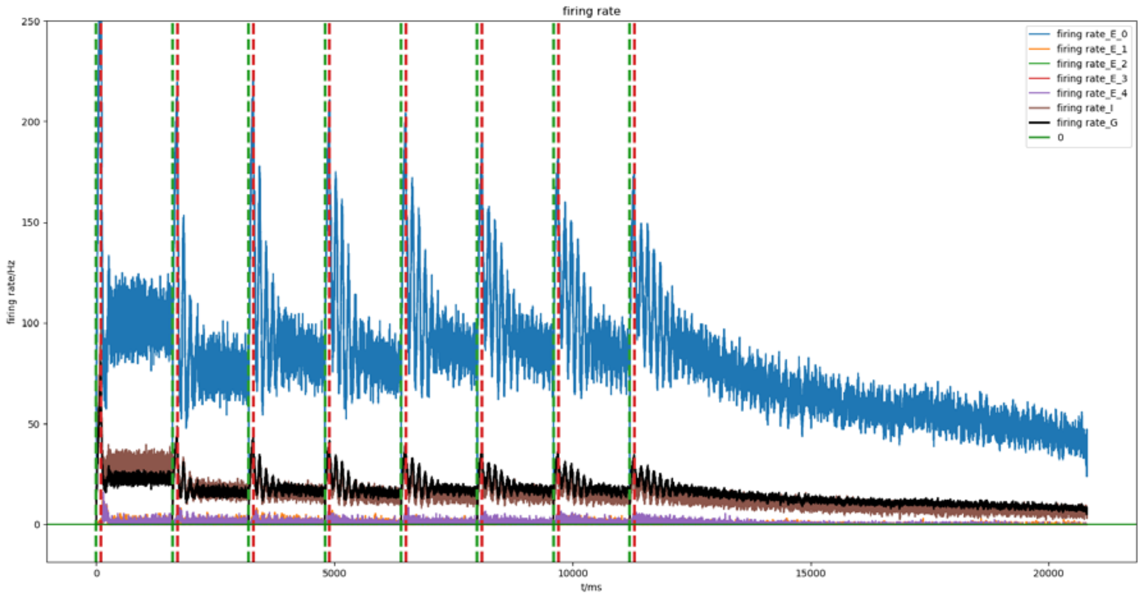




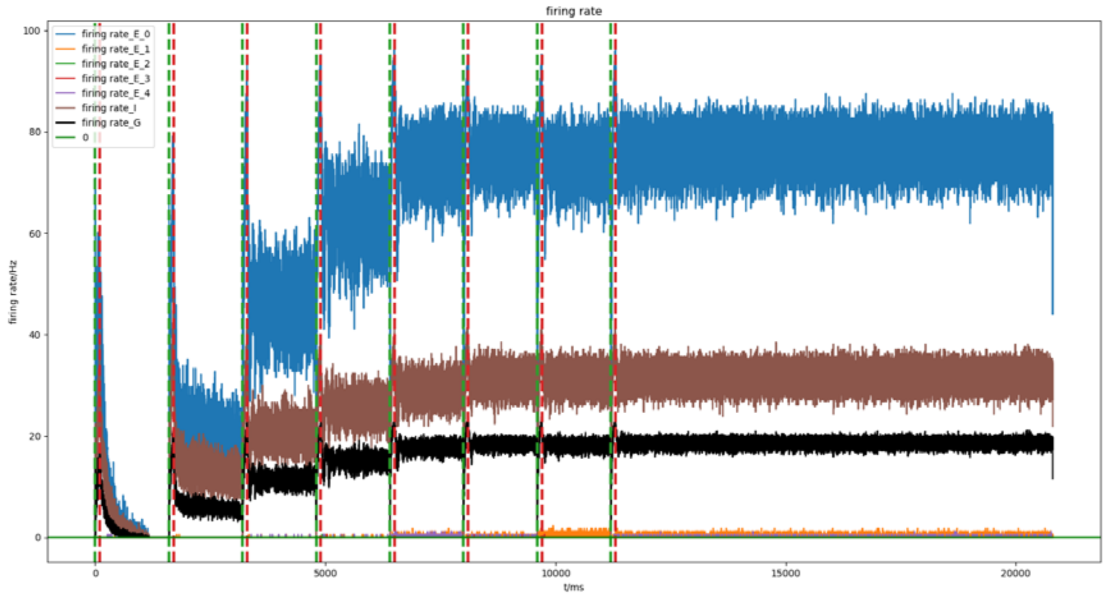


Evolution of synaptic strength

- Spatial working memory



Balanced



Unbalanced

Conclusion

Persistent activity in working memory can be obtained by

- Balanced conditions (both firing rate model and spiking network model)

$$\frac{J_{EE}J_{II}}{J_{EI}J_{IE}} \sim 1 \quad \text{for large } J \text{ values}$$

$$\tau_{+} = (\tau_{EE} + \tau_{II}) > (\tau_{EI} + \tau_{IE}) = \tau_{-}$$

- STDP in spiking network

Discussion

- STDP in spatial working memory: random stimulus
- Does STDP recover the balanced conditions $JIE*JEI = JEE*JII$?
- The effect of duration of delayed period

Acknowledgement