



## Stochastics and Statistics

## A game theory approach in seller–buyer supply chain

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## ABSTRACT

In this paper, several seller–buyer supply chain models are proposed which incorporate both cost factors as well as elements of competition and cooperation between seller and buyer. We assume that unit marketing expenditure and unit price charged by the buyer influence the demand of the product being sold. The relationships between seller and buyer will be modeled by non-cooperative and cooperative games, respectively. The non-cooperative game is based on the *Stackelberg strategy solution concept*, where we consider separately the case when the seller is the leader (*Seller-Stackelberg*) and also when the buyer is the leader (*Buyer-Stackelberg*). *Pareto efficient* solutions will be provided for the cooperative game model. Numerical examples presented in this paper, including sensitivity analysis of some key parameters, will compare the results between different models considered.

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## 1. Introduction

A seller–buyer supply chain represents a manufacturer which wholesales a product to a retailer, who, in turn retails it to a consumer (Yang and Zhou, 2006; Chen et al., 2006; Dai et al., 2005). In the literature, the terms vendor, supplier, and manufacturer have been used interchangeably to represent the seller. Likewise, the word retailer has been used to represent the buyer. In this paper, for the sake of simplicity, we will use the nomenclature buyer and seller. The related literature on finding optimal seller and buyer's policy of production and ordering can be broadly categorized into three groups based on the following assumptions: seller–buyer interaction is seen in light of constant demand, seller and buyer are independently studied where demand varies, and seller and buyer interaction is considered without logistic cost including setup/ordering and holding/carrying costs. We briefly summarize these models in order to compare with our proposed approach.

There are many possible interactive coordination mechanisms that can occur between the two members of a seller–buyer supply chain. Various types of mechanisms have been discussed in the literature on supply chain coordination such as quantity discount, credit option, buy back or return policies, quantity flexibility and commitment of purchase quantity (Sarmah et al., 2006). Quantity discount, a popular tool of coordination mechanism, is considered

in Chiang et al. (1994), Corbett and de Groote (2000) and Viswanathan and Wang (2003). Abad (1994) proposed a model of seller–buyer relationship, where demand is price sensitive and provided procedure of finding the optimal policy for both seller and buyer under a cooperative scenario. A similar model was presented in Abad and Jaggi (2003) where the main assumption is that the seller offers trade credit to the buyer. Several works (Sucky, 2005, 2006; Chan and Kingsman, 2007; Heuvel et al., 2007; Dai and Qi, 2007) have addressed the problem of determining the optimal order quantity (lot size) or order (production) cycles in a cooperative structure in order to achieve maximum savings or enhance profit for the whole supply chain where demand rate is considered fixed.

In contrast, fixed demand is avoided in some research where joint lot sizing and pricing decisions are used to determine the optimal price and order quantity for maximization of the firm's profit. In such cases, price would depend on demand over a planning horizon (Abad, 1994; Lee, 1993; Lee et al., 1996; Kim and Lee, 1998; Jung and Klein, 2001, 2005). Similar approaches have also been used in cases where both marketing expenditure and price influence demand (Freeland, 1982; Lee and Kim, 1993, 1998; Sajadi et al., 2005). A significant shortcoming of all these models is that they only regard seller or buyer supply chain management problem without considering any interaction between buyer and seller.

The effort expended in marketing product is also an important coordination mechanism tool that can occur between the two members of a seller–buyer supply chain. For example, Kotler (1997) noted that effort expended in marketing is an expenditure which can be spent in several ways such as advertising, sales

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promotion, sales force and marketing research expenditure. Huang and Li (2001) and Li et al. (2002) investigated manufacturer–retailer coordination as a cooperative advertising in supply chain problem. They highlighted the impact of investment in brand name, local advertising and sharing policy in three models under a cooperative regime in which the seller agrees to share a fraction of the total local advertising expenditure with the buyer. Yue et al. (2006) proposed a similar model under the assumption that seller offers a price reduction to customers. The profit function in a typical supply chain model contains a logistic cost component. However, in order to avoid the confounding effect of logistic cost, these papers assume that lot size is equal to demand.

In this paper, we propose several models of seller–buyer relationship subject to the impact of marketing effort. In our models, the seller produces a product and wholesales it to the buyer, who then retails the product to the consumer. The production rate of the seller is assumed to be linearly related to the market demand rate, while demand is sensitive to selling price and marketing expenditure. When demand is sensitive to selling price, the unit price imposed by the seller on the buyer does influence the end demand for the product. In such case, if the buyer expends on marketing, then the seller needs to coordinate her pricing and lot sizing to cope with new demand for the product. In this paper, we consider such interaction among the seller and buyer using non-cooperative as well as cooperative game theory. The non-cooperative game aspect will be considered from two perspectives: *Seller-Stackelberg* and *Buyer-Stackelberg* scenarios. In the first scenario, the seller dominates the buyer in a conventional way, whereas in the second, power has shifted from the seller to the buyer. Similarly, we consider the cooperative game model and Pareto efficient solutions will be provided for this model. We will demonstrate that a buyer or seller could benefit more from a cooperative structure than a non-cooperative one.

Another novel aspect of our models is to allow the sellers to impose lot sizes on the buyers. Conventionally, buyer determines lot size but in many large industries such as heavy equipment industry, aerospace industry, automotive industry where highly specialized equipments are supplied to customers, the cost of production is high and in order to reduce this and optimize production, it is common for the seller to fix lot sizes. For example, Kelle et al. (2003) investigate two scenarios for shipment quantity, fixed by the supplier or buyer in a Just in Time (JIT) supply chain system. A buyer in a JIT system would like small, frequent shipments whereas a seller prefer large production lot sizes. The paper compares and contrasts the costs when either the supplier or the seller holds a dominant position and is able to impose lot sizes on the others.

The remainder of this paper is organized as follows. We give the notation and assumptions underlying our models in Section 2. This section also formulate the problem, including a discussion of the model from the buyer's and seller's perspective. In Section 3, the non-cooperative Seller-Stackelberg and Buyer-Stackelberg models are discussed. The cooperative game aspect is presented in Section 4. Section 5 presents some computational results including a number of numerical examples and their sensitivity analysis. Finally, the paper concludes in Section 6 with some suggestions for future work in this area.

## 2. Notation and problem formulation

This section introduces the notation and formulation used in our supply chain problem. Specifically, all decision variables, input parameters and assumptions underlying our models will be stated.

### 2.1. Decision variables

$\mathcal{P}$	the price charged by the seller to the buyer (\$/unit)
$Q$	lot size (units) determined by the seller
$P$	selling price charged by the buyer (\$/unit)
$M$	marketing expenditure incurred by the buyer (\$/unit)

### 2.2. Input parameters

$k$	scaling constant for demand function ( $k > 0$ )
$u$	scaling constant for production function ( $u \geq 1$ )
$i$	percent inventory holding cost per unit per year
$\alpha$	price elasticity of demand function ( $\alpha > 1$ )
$\beta$	marketing expenditure elasticity of demand ( $0 < \beta < 1, \beta + 1 < \alpha$ )
$A_b$	buyer's ordering cost (\$/order)
$A_s$	seller's setup (ordering cost) (\$/setup)
$C_s$	seller's production cost including purchasing cost (\$/unit)
$r$	seller's production rate (units/day)
$D(P, M)$	annual demand; for notational simplicity we let $D \equiv D(P, M)$
$d$	market demand rate (units/day)

Since we have assumed that the seller's setup cost is high relative to the buyer's, we let  $A_s > A_b$ . Also, our demand is assumed to be a function of  $P$  and  $M$  and is based on Lee and Kim (1993):

$$D(P, M) = kP^{-\alpha}M^{\beta}. \quad (1)$$

The fact that our demand function is not constant, as is assumed in standard deterministic inventory models, is fairly common in many current models, see, e.g. Abad (1994), Lee (1993), Lee et al. (1996), Kim and Lee (1998), Jung and Klein (2001, 2005).

### 2.3. Assumptions

The proposed models in this paper are based on the following assumptions:

1. Planning horizon is infinite.
2. Parameters are deterministic and known in advance.
3. Even though the buyer usually determines lot size in conventional supply chain models, here we assume in the contract between seller–buyer that the seller determines lot size. This is appropriate in circumstances where setup, inventory and internal storage costs are high for the seller relative to the buyer. Therefore, our model is restricted to cases where there is a low number of providers and few competitors.
4. The annual demand depends on the selling price and marketing expenditure according to (1).
5. Shortages are not permitted, hence the production rate  $r$  is greater than or equal to demand rate  $d$  and, without loss of generality, we will assume them to be linearly related by the following equation:

$$r = ud, \quad u \geq 1. \quad (2)$$

### 2.4. The Buyer's model formulation

The buyer's objective is to determine the selling price and marketing expenditure such that her net profit is maximized. The selling price and marketing expenditure influence the demand and consequently, the seller's lot size. Here, we consider the buyer's model proposed by Abad (1988) but with the addition of the marketing expenditure,  $M$  as an additional decision variable. Therefore, the buyer's annual profit function is

Buyer's profit = Sales revenue – Purchase cost – Market cost – Ordering cost – Holding cost or expressed mathematically as a function of her decision variables

$$\begin{aligned}\Pi_b(P, M) &= PD - \gamma D - MD - A_b \frac{D}{Q} - 0.5i\gamma Q, \\ &= kP^{-\alpha+1}M^\beta - k\gamma P^{-\alpha}M^\beta - kP^{-\alpha}M^{\beta+1} - A_b kP^{-\alpha}M^\beta Q^{-1} \\ &\quad - 0.5i\gamma Q.\end{aligned}\quad (3)$$

The holding cost is expressed as a percentage of production cost, that is  $i\gamma Q$ . This is further multiplied by  $\frac{1}{2}$  to obtain an average value since the inventory is changing with time.

It can be shown that  $\Pi_b(P, M)$  is a strictly *pseudoconcave* function (refer to Appendix A(i)) with respect to  $P$  for fixed  $M$ . Hence, the first-order condition of  $\Pi_b(P, M)$  with respect to  $P$  determines the unique  $P$  that maximizes the profit function given a fixed  $M$ , i.e.

$$\frac{\partial \Pi_b}{\partial P} = 0.$$

This first-order condition yields

$$P^*(M) = \frac{\alpha(\gamma + M + A_b Q^{-1})}{\alpha - 1}.\quad (4)$$

Substituting (4) into (3) gives

$$\Pi_b(P^*(M), M) = k \left( \frac{\alpha\gamma + \alpha M + \alpha \frac{A_b}{Q}}{\alpha - 1} \right)^{-\alpha} M^\beta \left( \frac{\gamma + M + \frac{A_b}{Q}}{\alpha - 1} \right) - 0.5i\gamma Q,\quad (5)$$

and, using first- and second-order conditions in (5) gives

$$M^* = \frac{\beta(\gamma + A_b Q^{-1})}{(\alpha - \beta - 1)}, \quad \alpha > \beta + 1\quad (6)$$

as the unique maximizer of  $\Pi_b(P^*(M), M)$ . (Refer to Appendix A (ii) for a proof that  $\Pi_b(P^*(M), M)$  has a local maximum at  $M^*$ .) By substituting (6) into (4), we obtain  $P^*$

$$P^* = \frac{\alpha(\gamma + A_b Q^{-1})}{\alpha - \beta - 1}.\quad (7)$$

Alternatively, we can substitute (6) into (3) to eliminate  $M$  and then optimize the resulting expression with respect to  $P$  to obtain  $P^*$ . With the above approach, the buyer obtains the optimal selling price  $P^*$  and marketing expenditure  $M^*$ .

## 2.5. The seller's model formulation

The seller's objective is to determine the optimal lot size and price such that her net profit is maximized. Unlike the models proposed in Weng (1995) and Abad (1988),  $Q$  also appears in the seller's model instead of just in the buyer's model and due to one of our assumptions,  $Q$  is one of the seller's decision variables. Thus, the seller's annual profit is Seller's profit = Sales revenue – Production cost – Setup cost – Holding cost or expressed mathematically as a function of her decision variables,

$$\begin{aligned}\Pi_s(\gamma, Q) &= \gamma D - C_s D - A_s \frac{D}{Q} - 0.5iC_s Q \frac{d}{r} \\ &= k\gamma P^{-\alpha}M^\beta - kC_s P^{-\alpha}M^\beta - A_s kP^{-\alpha}M^\beta Q^{-1} - 0.5iC_s Q u^{-1}.\end{aligned}\quad (8)$$

The function defined by (8) is concave in  $Q$  (refer to Appendix A(iii)). We note that the holding cost term in (8) is a function of the demand and production rate, which represents average inventory multiplied by a percentage of the holding cost. When  $d = r$ , the average inventory equals  $\frac{Q}{2}$  but it is smaller when  $d < r$ , i.e. when  $u > 1$ , due to the seller not having to hold as much inventory in stock. Using the first-order condition, we obtain a unique  $Q^*$  that maximizes  $\Pi_s(\gamma, Q)$  for a fixed  $\gamma$

$$Q^* = \sqrt{\frac{2A_s D}{iu^{-1}C_s}}.\quad (9)$$

Substituting (9) into (8) gives

$$\begin{aligned}\Pi_s(\gamma) &= k\gamma P^{-\alpha}M^\beta - kC_s P^{-\alpha}M^\beta - A_s kP^{-\alpha}M^\beta \sqrt{\frac{iu^{-1}C_s}{2A_s D}} \\ &\quad - 0.5iu^{-1}C_s \sqrt{\frac{2A_s D}{iu^{-1}C_s}}.\end{aligned}\quad (10)$$

Solving  $\Pi_s(\gamma) = 0$  for zero profit gives

$$\gamma_0 = C_s + A_s Q^{-1} + 0.5iC_s Q(uD)^{-1}.\quad (11)$$

Since (10) is an increasing linear function of  $\gamma$ , the optimal  $\gamma^*$  occurs at the highest price it is possible for the seller to charge the buyer. Therefore,

$$\gamma^* = R\gamma_0 = R(C_s + A_s Q^{-1} + 0.5iC_s Q(uD)^{-1})\quad (12)$$

for some  $R > 1$ . For any  $P$  and  $M$  (the unit selling price and the marketing expenditure charged by the buyer), the optimal lot and price for the seller is  $Q^*$  and  $\gamma^*$ , respectively.

## 3. The non-cooperative Stackelberg games

This section considers the seller and buyer relationship by using the non-cooperative structure. Specifically, we will regard the interaction between buyer and seller as a Stackelberg game, where one of the participants, the leader, has the initiative and can enforce her strategy on the other participant, the follower. The leader makes the first move and the follower then reacts by playing the best move consistent with available information. The objective of the leader is to design her move in such a way as to maximize her revenue after considering all rational moves the follower can devise (refer to (Basar and Olsder, 1999) for a comprehensive discussion of a Stackelberg game and its solution).

### 3.1. The Seller-Stackelberg model

The Seller-Stackelberg model has the seller as leader and buyer as follower, and is widely reported in the literature (Huang and Li, 2001; Li et al., 2002; Abad and Jaggi, 2003; Chen et al., 2006). For a given  $\gamma$  and  $Q$  of the seller, the buyer obtains the best marketing expenditure  $M^*$  and selling price  $P^*$  according to the buyer's model, which is given by (6) and (7), respectively. The seller then maximizes her profit  $\Pi_s(\gamma, Q)$  based on the pair  $P^*$  and  $M^*$ . Thus, the problem reduces to

$$\begin{aligned}\text{Max} \quad & \Pi_s(\gamma, Q) = \gamma D - C_s D - A_s \frac{D}{Q} - 0.5iC_s Q u^{-1}, \\ \text{subject to} \quad & P^* = \frac{\alpha(\gamma + A_b Q^{-1})}{(\alpha - \beta - 1)} \\ \text{and} \quad & M^* = \frac{\beta(\gamma + A_b Q^{-1})}{(\alpha - \beta - 1)}, \quad \beta + 1 < \alpha.\end{aligned}\quad (13)$$

Substituting (6) and (7) into (13), the problem transforms into an unconstrained nonlinear function of two variables  $\gamma$  and  $Q$  where the optimal solution can be found using a grid search.

### 3.2. Buyer-Stackelberg model

Most papers in the supply chain literature have focused on the more conventional Seller-Stackelberg Model. Nevertheless, it is not inconceivable that power could shift from the seller to the buyer, see for example Yue et al. (2006). In this section, the Buyer-Stackelberg model is investigated where the buyer act as leader and the seller act as follower. For a given  $P$  and  $M$  of the buyer, the seller

obtains the best price  $\mathcal{V}^*$  and lot size  $Q^*$  of the seller, which is given by (9) and (12), respectively. The buyer then maximizes her profit  $\Pi_b(P, M)$  based on the pair  $\mathcal{V}^*$  and  $Q^*$ . Thus, the problem reduces to

$$\begin{aligned} \text{Max} \quad & \Pi_b(P, M) = PD - \mathcal{V}D - MD - A_b \frac{D}{Q} - 0.5i\mathcal{V}Q, \\ \text{subject to} \quad & Q^* = \sqrt{\frac{2A_b D}{iu^{-1}C_s}} \end{aligned} \quad (14)$$

$$\text{and} \quad \mathcal{V}^* = R(C_s + A_s Q^{-1} + 0.5iC_s Q(uD)^{-1}).$$

Substituting (9) and (12) into (14), the above Buyer-Stackelberg problem reduces to optimizing an unconstrained nonlinear objective function. The optimal solution can again be found using a grid search.

#### 4. The cooperative game

In this section, we apply a cooperative game approach to the seller–buyer supply chain problem with a view to determining whether both parties can increase their profit if they work cooperatively. Using this approach, the seller and the buyer work together to determine  $\mathcal{V}$ ,  $Q$ ,  $P$  and  $M$ . We will obtain the Pareto-efficient solution which is defined as the outcome in which there is no other outcome that would make both participants better off. Such cooperation is carried out through the joint optimization of the weighted sum of the seller's and buyer's objective functions, i.e., the set of Pareto efficient solutions can be characterized by maximizing (Abad and Jaggi, 2003)

$$Z = \lambda \Pi_s + (1 - \lambda) \Pi_b, \quad 0 < \lambda < 1,$$

that is,

$$\begin{aligned} Z = & \mathcal{V}D(2\lambda - 1) + \lambda D(M - P - C_s + (A_b - A_s)Q^{-1}) \\ & + 0.5iQD^{-1}(\mathcal{V} - uC_s) + D(P - M - A_b Q^{-1} - 0.5i\mathcal{V}QD^{-1}). \end{aligned} \quad (15)$$

The first-order condition for maximizing  $Z$  with respect to  $\mathcal{V}$  yields

$$\frac{\partial Z}{\partial \mathcal{V}} = 0 \Rightarrow \lambda = \frac{D + 0.5iQ}{2D + 0.5iQ}, \quad (16)$$

which gives  $\lambda \in (0, 1)$  as is desired. First-order conditions with respect to  $Q$ ,  $P$  and  $M$  further yield

$$Q = \sqrt{\frac{2D(\lambda A_s + A_b(1 - \lambda))}{i(\lambda u^{-1}C_s + (1 - \lambda)\mathcal{V})}}, \quad (17)$$

$$P = \frac{\alpha((\lambda - 1)(\mathcal{V} + A_b Q^{-1}) + \lambda(\mathcal{V} - C_s - A_s Q^{-1}))}{(\alpha - \beta - 1)(\lambda - 1)}, \quad (18)$$

$$M = \frac{\beta((\lambda - 1)(\mathcal{V} + A_b Q^{-1}) + \lambda(\mathcal{V} - C_s - A_s Q^{-1}))}{(\alpha - \beta - 1)(\lambda - 1)}. \quad (19)$$

Pareto efficient solutions can be obtained through a negotiation between the seller and the buyer over a fixed  $\mathcal{V}$ , i.e. where equations (16)–(19) are solved simultaneously to obtain  $\lambda^*$ ,  $Q^*$ ,  $P^*$ , and  $M^*$  for a fixed  $\mathcal{V}$ . Another approach is to assume a value for  $\lambda$  and solve equations (16)–(19) for the other variables.

Using the negotiated  $\mathcal{V}$  approach, it is reasonable to expect that  $\mathcal{V} > C_s + A_s Q^{-1}$  since the seller would never operate under a loss. Under this condition, by comparing (17)–(19) with (6), (7) and (12) we obtain the following results:

- Selling price in a cooperative game is less than in a non-cooperative game. Let  $P_C^*$  and  $P_N^*$  be the optimal selling price in a cooperative and non-cooperative game, respectively, i.e.  $P_C^*$  is given by (18) and  $P_N^*$  by (7). We obtain

$$P_C^* = P_N^* - \frac{\alpha\lambda(\mathcal{V} - C_s - A_s Q^{-1})}{(\alpha - \beta - 1)(1 - \lambda)} \quad (20)$$

where the second term on the right side of the above equation is positive, therefore  $P_N^* > P_C^*$ .

- Marketing expenditure in a cooperative game is less than in a non-cooperative game. Let  $M_C^*$  and  $M_N^*$  be the selling price in a cooperative and non-cooperative game respectively, i.e.  $M_N^*$  is given by (6) and  $M_C^*$  by (19). We obtain

$$M_C^* = M_N^* - \frac{\beta\lambda(\mathcal{V} - C_s - A_s Q^{-1})}{(\alpha - \beta - 1)(1 - \lambda)} \quad (21)$$

which again shows that  $M_N^* > M_C^*$ . We also note that since demand is a function of selling price and marketing price, and price elasticity  $\alpha$  is greater than marketing expenditure elasticity  $\beta$ , we would expect that demand in a cooperative game would be greater than in a non-cooperative game.

- Lot size in cooperative game is less than in a non-cooperative game. This observation follows by considering (9) and (17) and the fact that given our assumptions,  $A_b < A_s$  and  $C_s < \mathcal{V}$ . Note that in a non-cooperative structure the optimal policy for the seller is to have a large lot size, which is directly opposite to the buyer's policy; however, in a cooperative structure the lot size of the seller is smaller even though the setup cost is higher.

#### 5. Computational results

In this section, we present numerical examples which are aimed at illustrating some significant features of the models established in previous sections. We will also perform sensitivity analysis of two main parameters of these models. We note that Examples (1–3) below illustrate the Seller-Stackelberg, Buyer-Stackelberg and cooperative models, respectively. In all these examples, we set  $k = 3500$ ,  $\beta = 0.15$ ,  $\alpha = 1.7$ ,  $i = 10\%$ ,  $A_b = 40$ ,  $A_s = 140$ ,  $u = 1.1$  and  $C_s = 1.5$ .

**Table 1**  
Sensitivity analysis of the Seller-Stackelberg model with respect to  $\alpha$

$\alpha$	1.7	2.0	2.2	2.3	2.4
$P^*$	19.2	11.1	8.9	8.2	7.7
$\mathcal{V}^*$	6.0	4.6	4.1	4.0	3.8
$Q^*$	256.9	270.3	263.5	257.1	249.1
$M^*$	1.7	0.8	0.6	0.5	0.5
$D^*$	25.0	27.7	26.3	25.0	23.5
$\Pi_s^*$	82.5	52.1	36.7	30.4	24.8

**Table 2**  
Sensitivity analysis of the Seller-Stackelberg model with respect to  $\beta$

$\beta$	0.07	0.09	0.15	0.19	0.21
$P^*$	14.6	15.6	19.2	22.3	24.2
$\mathcal{V}^*$	5.3	5.4	6	6.5	6.8
$Q^*$	306.2	292.3	256.9	236.8	227.5
$M^*$	0.6	0.8	1.7	2.5	3.0
$D^*$	35.5	32.3	25.0	21.2	19.6
$\Pi_s^*$	96.8	92.3	82.5	78.0	76.1

**Table 3**  
Sensitivity analysis of the Buyer-Stackelberg model with respect to  $\alpha$

$\alpha$	1.7	2.0	2.2	2.3	2.4
$P^*$	8.1	6.3	5.9	5.7	5.6
$\mathcal{V}^*$	2.7	2.8	2.9	3.0	3.0
$Q^*$	602.2	544.2	488.3	457.4	430.9
$M^*$	0.7	0.5	0.4	0.4	0.3
$D^*$	95.0	77.6	62.5	54.8	48.6
$\Pi_b^*$	356.8	157.4	83.8	57.3	39.5



**Table 4**  
Sensitivity analysis of the Buyer-Stackelberg model with respect to  $\beta$

$\beta$	0.07	0.09	0.15	0.19	0.21
$P^*$	6.8	7.1	8.1	8.9	9.3
$\gamma^*$	2.6	2.6	2.7	2.7	2.8
$Q^*$	682.5	659.3	620.2	571.2	557.2
$M^*$	0.3	0.4	0.7	1.0	1.1
$D^*$	122.0	113.8	95.0	85.5	81.3
$\Pi_b^*$	385.3	374.5	356.8	353.8	354.4

**Table 5**  
Sensitivity analysis of the co-operative game with respect to  $\alpha$

$\alpha$	1.7	2.0	2.2	2.3	2.4
$P^*$	5.1	4.5	4.4	4.3	4.2
$\gamma^*$	3.0	3.0	3.0	3.0	3.0
$Q^*$	215.6	192.5	165.0	157.5	155.1
$M^*$	0.4	0.3	0.3	0.3	0.3
$D^*$	191.9	147.5	111.4	98.9	90.0
$Z^*$	203.1	105.2	66.5	52.4	41.3

**Table 6**  
Sensitivity analysis of the co-operative game with respect to  $\beta$

$\beta$	0.07	0.09	0.15	0.19	0.21
$P^*$	4.2	4.3	5.1	5.6	6.6
$\gamma^*$	3.0	3.0	3.0	3.0	3.0
$Q^*$	281.6	243.4	215.6	137.8	132.1
$M^*$	0.2	0.2	0.4	0.6	0.8
$D^*$	273.1	260.0	191.9	172.3	136.2
$Z^*$	220.3	205.8	203.1	177.0	192.2

### 5.1. Numerical examples

**Example 1.** The Seller-Stackelberg model produces the following optimal values for our decision variables:  $D^* = 25.0$ ,  $P^* = 19.2$ ,  $Q^* = 256.9$ ,  $M^* = 1.7$  and  $\gamma^* = 6.0$ . The corresponding seller's and buyer's profits are  $\Pi_s^* = 82.5$  and  $\Pi_b^* = 204.2$ , respectively.

**Example 2.** The Buyer-Stackelberg model produces the following optimal values for the decision variables:  $D^* = 95.0$ ,  $P^* = 8.1$ ,  $Q^* = 602.2$ ,  $M^* = 0.7$  and  $\gamma^* = 2.7$ . The seller's and buyer's profits are  $\Pi_s^* = 51.4$  and  $\Pi_b^* = 356.8$ , respectively. The second model, in contrast to the first one, utilizes less marketing expenditure, and

has smaller seller's price and selling price charged by the buyer. The higher profit  $\Pi_b^*$  for the buyer and low selling price  $P^*$  charged by the buyer to customer indicate that she is better off under the second model. One of the reasons could be that under this model, the buyer is better informed about the end demand and would therefore gain more if she is leader. Note that demand  $D^*$  is greater in this model. Also, a larger lot size  $Q^*$  is preferred by seller when she is the follower.

**Example 3.** We will obtain Pareto efficient solutions by assuming that the seller and buyer has negotiated an agreement on seller price at  $\gamma = 3$ . Eqs. (17)–(19) are used to obtain  $Q^*$ ,  $P^*$ ,  $M^*$  and finally  $\lambda$  is obtained using (16). The solutions are as follow:  $\lambda^* = 0.4709$ ,  $D^* = 224.2$ ,  $P^* = 4.6$ ,  $Q^* = 238.3$ ,  $M^* = 0.4$ , and  $Z^* = 206.4$ . As expected from the theory in Section 4, selling price here is less than in non-cooperative games and the profit obtained is more than the profits of the seller for both the Seller-Stackelberg and Buyer-Stackelberg models. Therefore, the seller would prefer the cooperative model to the non-cooperative ones because she gains more profit from the former model.

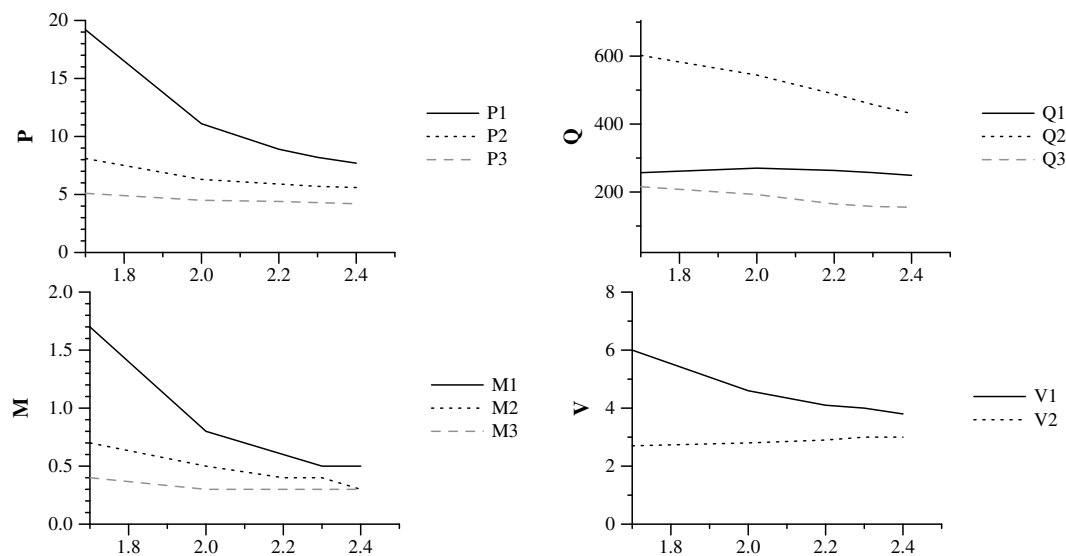
### 5.2. Sensitivity analysis

We investigate the effects of parameters  $\alpha$  and  $\beta$  on  $P^*$ ,  $\gamma^*$ ,  $Q^*$ ,  $M^*$ ,  $D^*$ ,  $\Pi_b^*$  and  $\Pi_s^*$  in the Seller-Stackelberg and Buyer-Stackelberg models through a sensitivity analysis. Similarly, in the cooperative game, the effect of  $\alpha$  and  $\beta$  on  $P^*$ ,  $Q^*$ ,  $M^*$ ,  $D^*$ , and  $Z^*$  will also be investigated. We will fix  $k = 3500$ ,  $i = 10\%$ ,  $A_b = 40$ ,  $A_s = 140$ ,  $u = 1.1$ , and  $C_s = 1.5$  as in the previous examples but allow  $\alpha$ ,  $\beta$  to vary. Results of these sensitivity analysis are summarized in Tables 1–6.

The results in Tables 1, 3 and 5 are also graphically displayed in Fig. 1. In each figure, the number 1, 2 and 3 refers to the Seller-Stackelberg, the Buyer-Stackelberg and the cooperative game, respectively.

As is seen in the figure, the buyer's decision variables are independent of her position as leader or follower, i.e. by increasing  $\alpha$ , parameters  $P^*$  and  $M^*$  decrease, regardless of whether the buyer is a follower or a leader. However, the seller's decision variables  $Q^*$  and  $\gamma^*$  to depend on her position as follower or leader. For example, by increasing  $\alpha$ ,  $\gamma^*$  increases in the Buyer-Stackelberg model, but decreases in the Seller-Stackelberg model.

Finally, the effect of parameter  $\beta$  on  $P_i^*$ ,  $M_i^*$ ,  $Q_i^*$ ,  $\gamma_i^*$ ;  $i = 1, 2, 3$ , i.e. for each of the three models, are graphically displayed in Fig. 2. As



**Fig. 1.** The effect of  $\alpha$  parameter on  $P^*$ ,  $Q^*$ ,  $M^*$  and  $\gamma^*$ .

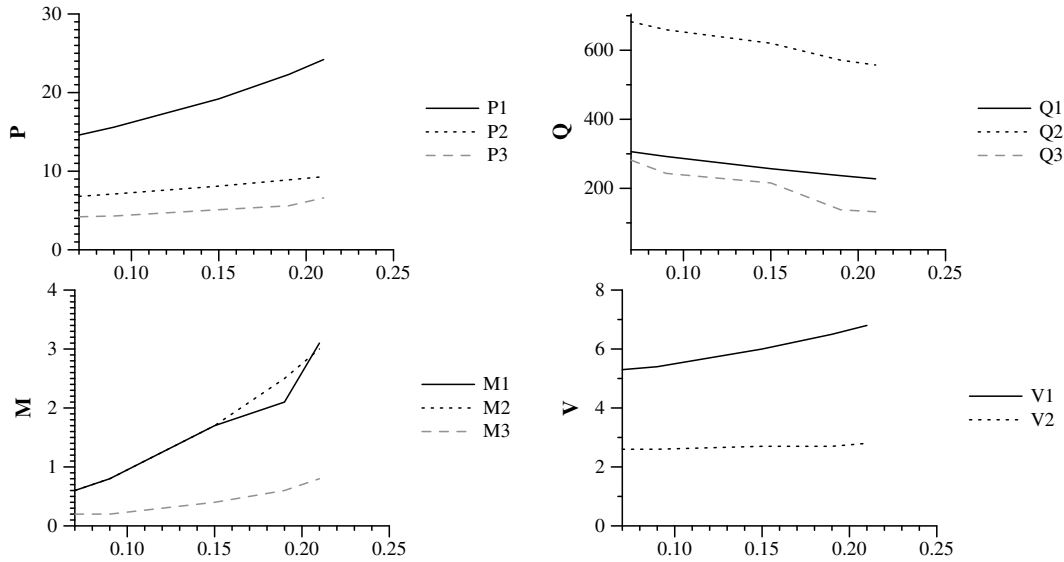


Fig. 2. The effect of  $\beta$  parameter on  $P^*$ ,  $Q^*$ ,  $M^*$  and  $\gamma^*$ .

$\beta$  increases,  $P^*$ ,  $M^*$  and  $\gamma^*$  increase and  $Q^*$  decreases. Therefore with increasing  $\beta$ , the seller's and the buyer's decisions are independent of their leadership position.

According to (20) and (21), we would expect selling price, marketing expenditure and lot size in the cooperative game to be less than in non-cooperative games.

## 6. Conclusion

In this paper, the seller–buyer supply chain management problem is considered, both separately and as interactive games. The seller produces a product and wholesales it to the buyer, with the production rate linearly related to the market demand rate. In addition to selling price, the models presented also incorporate the effect of marketing expenditure on demand, i.e. we assume the demand is sensitive to both the selling price and marketing expenditure. We consider the seller–buyer Supply chain models both as a non-cooperative and as a cooperative game. In the non-cooperative game scenario, two types of games are considered: Seller-Stackelberg, where Seller is leader, and Buyer-Stackelberg, where Buyer is leader. Optimal solutions for both of these cases are obtained. For the cooperative game, Pareto efficient solutions are obtained by optimizing the weighted sum of the seller's and buyer's objective functions. It is shown that both selling price and marketing expenditure are smaller in cooperative than in non-cooperative games, consequently demand is expected to be larger under a cooperative structure. Numerical examples are presented which aim at illustrating the theory. Through a sensitivity analysis, the effect of two main parameters of the model on the buyer and the seller's decisions are also investigated.

There is much scope in extending the present work. For example, marketing expenditure could incorporate advertising expenditure and the seller may agree to share fraction of the advertising expenditure with the buyer by covering part of it. In this case, we could investigate the impact of non-cooperative and cooperative relationships on the shared fraction. Also, we have assumed in this paper that the production rate is greater than or equal to the demand rate, in order to avoid having to consider shortage cost. By not making this assumption, the extra cost could be incorporated into future models. Finally, even though the buyer or seller presumably knows her own costs and price charged to the consumers, it is unlikely that her opponent would be privy to such

information. This would lead to incomplete knowledge on the part of the two participants and result in bargaining models with incomplete information along the line of models such as those discussed in Sucky (2006).

## Appendix A

(i) This appendix contains the proof of the strict pseudoconcavity of  $\Pi_b(P, M)$  with respect to  $P$  for a fixed  $M$ . Let  $S$  be a nonempty open set in  $E_n$ , and  $f : S \rightarrow E_n$  be differentiable on  $S$ . The function  $f$  is said to be *strictly pseudoconvex* if, for each distinct  $x_1, x_2 \in S$  with  $\nabla f(x_1)(x_2 - x_1) \geq 0$ , the inequality  $f(x_2) > f(x_1)$  holds. Equivalently, if  $f(x_2) \leq f(x_1)$  then  $\nabla f(x_1)(x_2 - x_1) < 0$ . In addition,  $f$  is a strictly pseudoconcave function if  $-f$  is a strictly pseudoconvex function (Bazaraa et al., 1993).

Suppose the inequality  $\Pi_b(P_2, M) \leq \Pi_b(P_1, M)$ , which is equivalent to (refer to (3)):

$$D_2(\gamma + M + A_b Q^{-1} - P_2) \leq D_1(\gamma + M + A_b Q^{-1} - P_1). \quad (\text{A.22})$$

In order to prove strictly pseudoconcavity of  $\Pi_b(P, M)$ , it suffices to show that (A.22) implies

$$\nabla \Pi_b(P_1, M)(P_2 - P_1) < 0 \quad (\text{A.23})$$

i.e.

$$D_1(\alpha - 1)P_1 - D_1(\gamma + M + A_b Q^{-1}) < 0. \quad (\text{A.24})$$

which follows from (A.23) on further assuming that (A.25) holds. We next show that (A.22) implies

$$P_2 > P_1 \quad (\text{A.25})$$

$$\text{and } \gamma + M + A_b Q^{-1} - P_i > 0, \quad i = 1, 2. \quad (\text{A.26})$$

We prove (A.25) by contradiction. Suppose  $P_2 \leq P_1$ , then since  $D(P, M) = kP^{-\alpha}M^\beta$  and letting  $D_i = D(P_i, M)$ ,  $i = 1, 2$ , it follows that  $D_1 \leq D_2$ . Therefore,

$$D_1(\gamma + M + A_b Q^{-1} - P_1) \leq D_2(\gamma + M + A_b Q^{-1} - P_2) \quad (\text{A.27})$$

which contradicts to (A.22), hence (A.25) holds.

Next, suppose  $(\gamma + M + A_b Q^{-1} - P_i) \leq 0$ ,  $i = 1, 2$ , contradicting (A.26). Then,  $P_2 > P_1$  implies

$$(\gamma + M + A_b Q^{-1} - P_1) \leq (\gamma + M + A_b Q^{-1} - P_2)$$

and furthermore,  $D_2 < D_1$  leads to

$$D_1(\gamma + M + A_b Q^{-1} - P_1) \leq D_2(\gamma + M + A_b Q^{-1} - P_2). \quad (\text{A.28})$$

Since (A.28) contradicts (A.22), (A.26) holds. Finally, (A.26) implies

$$P_1 < \gamma + M + A_b Q^{-1}.$$

Hence,

$$D_1(\alpha - 1)P_1 - D_1\alpha(\gamma + M + A_b Q^{-1}) < 0$$

since  $0 < \alpha - 1 < \alpha$  by assumption and the proof is complete.

(ii) Using (5), we obtain

$$\begin{aligned} \frac{\partial^2 \Pi_b(P^*(M), M)}{\partial M^2} &= \frac{-\alpha^2 D}{(\alpha - 1)P^*(M)} \left( \frac{\beta P^*(M)}{\alpha M} - 1 \right) + \left( \frac{\beta D}{M} \right) \left( \frac{\beta P^*(M)}{\alpha M} - 1 \right) \\ &\quad + D \left( \frac{\alpha \beta M - \beta(\alpha - 1)P^*(M)}{\alpha(\alpha - 1)M^2} \right). \end{aligned} \quad (\text{A.29})$$

We next observe that  $M^*$  given by (5) satisfies  $M = \frac{\beta}{\alpha} P^*(M)$  and substituting this into (A.29) gives

$$\frac{\partial^2 \Pi_b(P^*(M^*), M^*)}{\partial M^2} = \frac{\beta + 1 - \alpha}{(\alpha - 1)M^*} < 0$$

by assumption.

(iii) The concavity of  $\Pi_S(\gamma, Q)$  with respect to  $Q$  is easily verified since

$$\frac{\partial^2 \Pi_S(\gamma, Q)}{\partial Q^2} = -2A_5 D Q^{-3} < 0.$$

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