Problem Chosen	2020 MCM/ICM Summary Sheet	Team Control Number
A		**7420697**

Moving North

Summary

The increasing ocean temperatures are resulting in a significant shift in the distribution of herring and mackerel. Consequently, this impacts small fishing companies in Scotland, as there may not be a sufficient number of fish in their current locations for them to catch. Our job is to identify the migration patterns of herring and mackerel and their most likely distributions in the next 50 years. Based on the mackerel's and herring's distributions from our results, we are to provide practical strategies to these small fishing companies.

In the first model, we first collected the monthly sea surface temperature (SST) data near Scotland from 1870 to 2019. Based on the analysis of its stochastic behavior, we decided to use the Autoregressive Integrated Moving Average (ARIMA) model to forecast the SST over the next fifty years. After the verification of stationarity, we found several potential models by observing the Autoregressive Function (ACF) plot and the Partial Autoregressive Function (PACF) plot. Next, historical data were used to estimate the model parameters. We then used the Out-of-Sample Evaluation to find the best prediction model. Based on model selection criteria RMSE and MAPE, we chose ARIMA(3,0,0). SST over the next fifty years were forecasted using the model, which lays a great foundation for Model II.

Using the forecasted SST data from the first model and the respective species's temperature tolerance range, the second model predicts the migration patterns of the herring and the mackerel. The second model is an agent based model in which the herring and mackerel migrate if the current temperature of their habitat is beyond their best-suited temperature range. Each species migrates to the nearest area of the sea that has the most favorable surface sea temperature. The years in which the mackerel and herring each leave the designated fishing port area is recorded. The results show that schools of herring tend to move southeast while mackerel tend to move northeast. These modeled results are then used to create potential business strategies for Scotland fisheries in the next part.

For potential business strategies, Model III models three potential options: continue operating as before, purchase refrigeration for boats, or completely relocate to a new location. Boat costs, refrigeration costs, and relocation costs were taken into account for each strategy. Furthermore, annual revenue was determined by the the density of the herring's and mackerel's distribution each year, the location's and boat's access to this distribution, and the price of each fish. To determine the most profitable and practical strategy, these three different strategies were compared based on their normalized annual profit trends. We found that the most cost-efficient strategy was to purchase refrigeration for our original boats, as purchasing refrigeration doubles the distance the boats can travel without the fish reducing in quality. After taking into account international waters and limitations, operating with boats with refrigeration still remained the most profitable and practical strategy.

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1 Introduction

Scotland is one of the largest sea fishing nations in Europe, contributing largely to its economy. With the gradual increase in sea temperatures, ocean-dwelling species, such as mackerel and herring, are migrating to areas with better suited sea temperatures. Because the livelihood of these fishing companies are dependent upon catching a sufficient amount of these ocean-dwelling species, the shifting distributions of these species presents a potential threat to these companies.

This paper aims to predict to the potential distributions of both herring and mackerel and to develop a strategy based on these results to aid small Scottish fishing companies.

2 General Assumptions

- **Assumption 1**: We assume that the issue of global warming will not noticeably improve or be resolved in the next 50 years. Given the fact that it is a large-scale issue and significant improvement has not been made despite people's efforts, we ignore the possibility that large-scale change will occur during this relatively short time period.

- **Assumption 2**: We assume that the water's surface temperature is the temperature of the mackerel's and herring's habitats. Majority of herring and mackerel live in depths below 7 meters and 200 meters respectively. Thus we will ignore the change in water temperature at greater depths.

- **Assumption 3**: We assume that all fish migrate together and not individually as mackerel and herring have the habit of gathering and remaining together.

- **Assumption 4**: It is assumed that the suitable living temperature of the fish is always constant, and the fish will not be mutated due to changes in the environment (temperature and geographical location).

- **Assumption 5**: We will assume that the fish can only migrate a certain distance each year (before the temperature changes).

3 Model Overview

The problem requires us to predict the migration of herring and mackerel in the next fifty years and assess fish migration impact on fishery companies. Our work includes the following parts:

- Model I: forecast sea surface temperatures (SST) over the next 50 years.

- Model II: model fish migration and estimate ending location.

- Based on the migration pattern of both fish, discuss possible business strategies that fishery companies can adopt.

4 Model I: Forecasting Change in Ocean Temperatures

4.1 Motivation

The behavior of the sea surface temperature (SST) near UK plays an important role for understanding the migration pattern of Scottish herring and mackerel. We will forecast the SST over the next fifty years in this model, and the forecasted results will be used to predict the migration of these fish in model II.

4.2 Data

The data we used in the model are retrieved from the Met Office Hadley Center observations dataset [1]. We collected the monthly Sea Surface Temperature data (SST) from 1870 to 2019. In this dataset, a temperature is assigned to a 1-degree latitude-longitude grid. The intersection of a longitude line and a latitude line is taken as the center of the grid, and each grid covers 0.5 degree up and down, 0.5 degree left and right.

In this model, we analyze the SST of 300 grids near Scotland, grids centers ranging from -16.5 to 2.5 longitude and 48.5 to 62.5 latitude. We number these 300 grids from left to right (west to east), top to bottom (north to south). Figure 1 shows the SST of these 300 grids in 2020. For example, the first grid is centered at (-16.5, 62.5), and the 300th grid is centered at (2.5, 48.5).

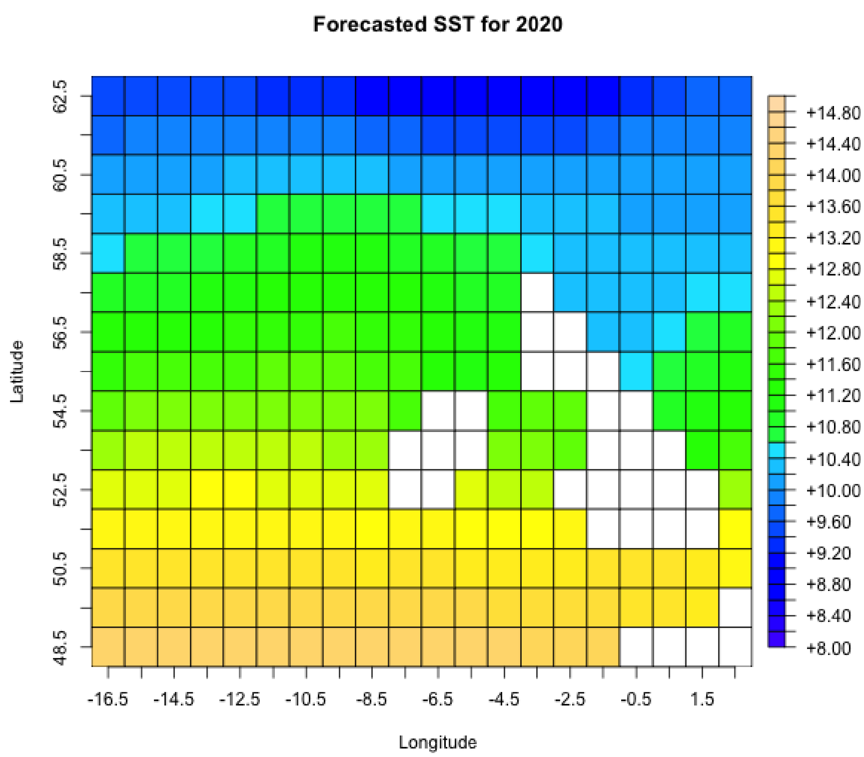

Figure 1: Discretize seas into grids/blocks

4.3 Use Arima Model to Forecast SST over the Next 50 Years

We can view our SST data on the 300 grids as 300 time series. The temperature data in the i-th block is denoted by $\{S_{i,t}\}_{t=1}^{\infty}$. The time series plot shows that each SST series has some mean reverting behavior and is not random white noise. Thus, we will use an Auto Regressive Integrated Moving Average (ARIMA) model to fit the data and predict future values of each series.

Even though many methods can be used to forecast SST, for example, ETS (Exponential Smoothing), Holt Winters Smoothing, AR (Auto-regressive), and MA (Moving Average), We choose ARIMA for its interpretability and parsimony. ARIMA has an intrinsic advantage that we can interpret every parameter of it. It also combines the ideas of AR and MA models into a compact form so that the number of parameters used is kept small, achieving parsimony in parametrization.

ARIMA models have a general form of (p, d, q) where p is the order of the standard autoregressive (AR) term, q is the order of the standard moving average (MA) term, and d is the order the differencing required to make the time series stationary. AR describes how a variable Y_t depends on some of its own past values Y_{t-1} (AR-1), Y_{t-2} (AR-2), etc. MA captures the current and lagged shock effects observed in the white noise terms. These shock effects could be thought of as unexpected events affecting the observation process. The I (for "integrated") indicates that the data values have been replaced with the difference between their values and the previous values, and this differencing process is performed d times to make the time series stationary.

A standard ARIMA can only be used to fit non-seasonal data. So we will seasonally adjust our data by computing the annually average. To further simplify the this forecasting process, we assume that the behavior of SST's in 300 blocks are similar to one another, and hence we can use ARIMA models of the same order (p, d, q) to fit the data. However, the model parameters are different from block to block. The figure 2 shows the time series plot of all 300 blocks over the past 150 years. These 300 time series indeed have similar behaviors, so our additional assumption can by justified.

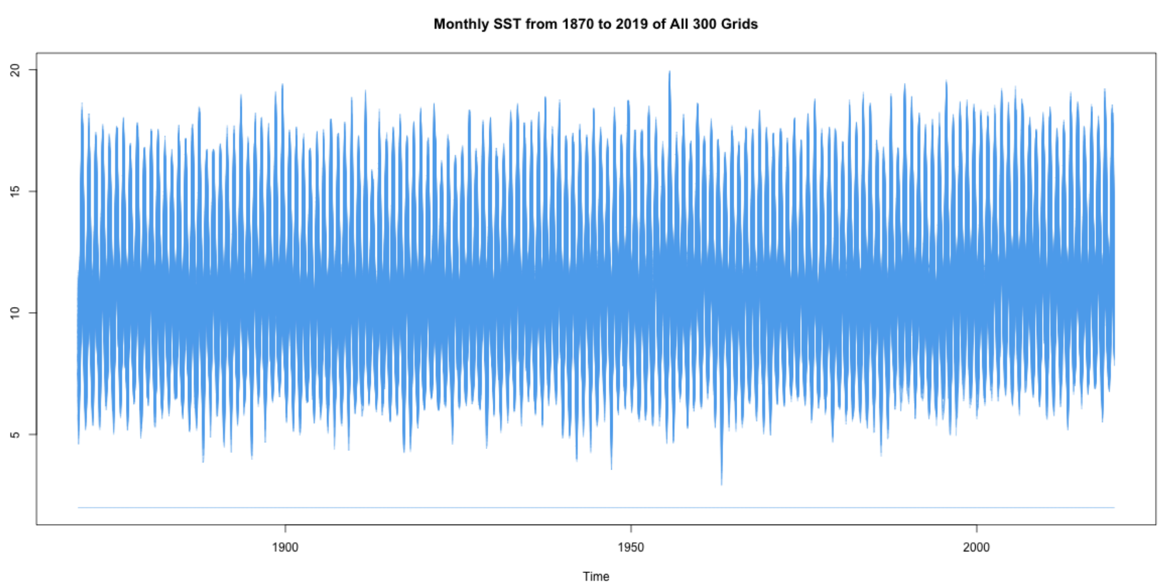

Figure 2: monthly SST in each block over the past 150 years

In this model, each stochastic process $S_{i,t}$ is an ARIMA(p, d, q) and satisfies the following equation:

$$\Phi(L)\Delta^d S_{i,t} = c + \Theta(L)\varepsilon_t \tag{1}$$

where ε_t is a weak white noise $WN(0, \sigma_\varepsilon^2)$ and

$$\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$$
$$\Theta(L) = 1 + \theta_1 L + \cdots + \theta_q L^q.$$

The series $S_{i,t}$ is weakly stationary after d-differencing transformation.

Rearranging the terms, we get

$$\Delta^d S_{i,t} = c_i + \sum_{j=1}^{p} \phi_{i,j}\Delta^d S_{i,t-j} + \sum_{k=1}^{q} \theta_{i,k}\varepsilon_{i,t-k} + \varepsilon_{i,t}. \tag{2}$$

Notice that the orders p, d, and q are the same for each i, whereas parameters c_i, ϕ_i's, and θ_i's will be estimated using each series $\{S_{i,t}\}_{t=1}^{\infty}$ and is different from block to block.

Therefore, the first-step-ahead optimal point forecast of the SST in i-th block is given by

$$f_{i,t,1} = \mu_{i,t+1|t} = \mathbb{E}[\Delta^d S_{i,t+1}|I_{i,t}] \tag{3}$$
$$= \mathbb{E}[\hat{c}_i + \sum_{j=1}^{p} \hat{\phi}_{i,j}\Delta^d S_{i,t-j+1} + \sum_{k=1}^{q} \hat{\theta}_{i,k}\varepsilon_{i,t-k+1} + \varepsilon_{i,t+1}] \tag{4}$$
$$= \hat{c}_i + \sum_{j=1}^{p} \hat{\phi}_{i,j}\Delta^d S_{i,t-j+1} + \sum_{k=1}^{q} \hat{\theta}_{i,k}\varepsilon_{i,t-k+1} \tag{5}$$

4.4 Model Selection

To find the best fit ARIMA model, we first need to determine the order p, d, and q. Once the model order has been identified, we need to estimate the parameters $c, \phi_{i,1}, \phi_{i,2} \ldots, \theta_{i,1}, \theta_{i,2} \ldots$ We use maximum likelihood estimation (MLE) to estimate the model coefficients. This technique finds the values of the parameters which maximize the probability of obtaining the data that we have observed. For ARIMA models, MLE is similar to the LS estimates that would be obtained by minimizing

$$\sum_{i=1}^{T} \hat{e}^2.$$

Since there is no obvious trend in the time series, it's very likely that our series are already stationary. To make sure our observation match the truth, we first use the Augmented Dickey Fuller (ADF) test to examine the stationarity. We run the ADF test on the first time series (SST at the first grid). The test statistic $\tau = -3.98$, and $p = 0.01214$. We therefore reject the null hypothesis and conclude that the these SST series are stationary. Because there is no need to difference our data, $d = 1$.

Next, we use the ACF (autocorrelation function) and PACF (partial autocorrelation function) to determine the possible order p, and q. Based on ACF and PACF in figure 3, there are 4 possible models: ARIMA(1,0,1), ARIMA(3,0,0), ARIMA(2,0,1), ARIMA(1,0,2).

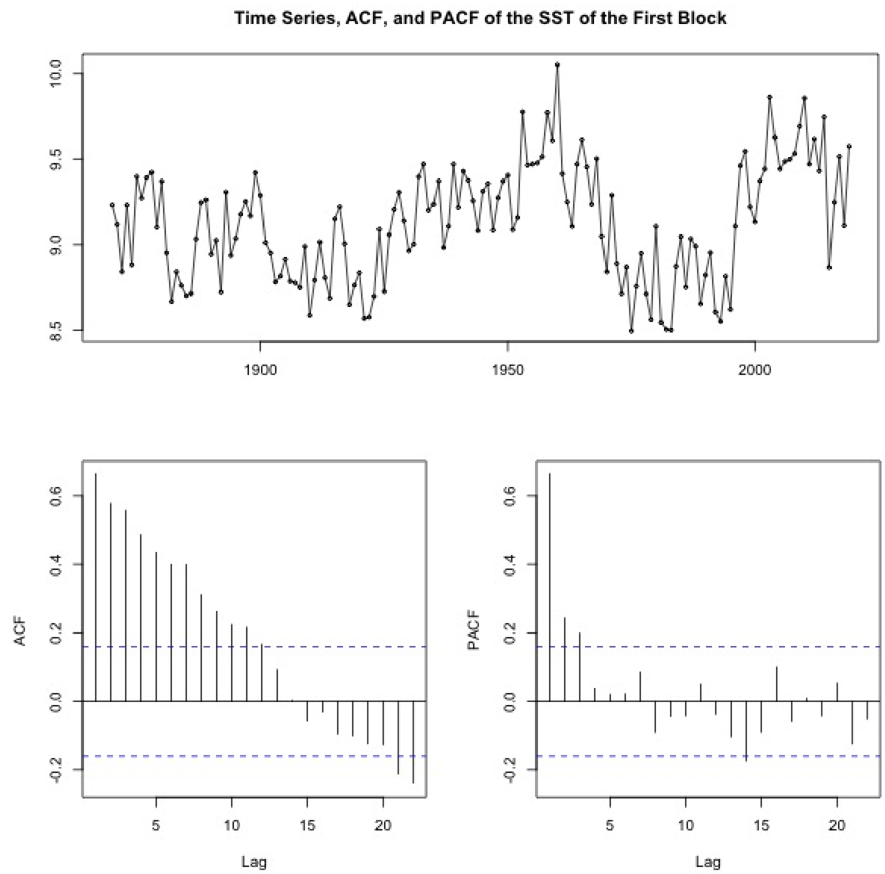

Figure 3: ACF and PACF of SST in the first block over the past 150 years

Now we have four potential models. The next step is to use an out-of-sample evaluation to find the best model. We divide our data into two parts: the training set and the testing set. The training set contains annually SST data from 1870-2009 (140 years), and the testing set contains observations from 2010-2019 (10 years). We use the training set to estimate the model parameters, and then use the model to forecast SST from 2010-2019. We then compare the forecasting results given by each model to real observations, so that we can assess the accuracy of each model.

For example, the first model, ARIMA(1,0,1), can be written as

$$S_{i,t} = c_i + \phi_{i,1} S_{i,t-1} + \theta_{i,1} \varepsilon_{i,t-1} + \varepsilon_{i,t}.$$

In this case, we will use MLE to estimate three parameters, c, $\phi_{i,1}$, $\theta_{i,1}$.

Figure 4 shows the forecasting results given by ARIMA(1,0,1). The purple regions corresponds to 80% and 95% prediction interval; the orange curve represents the observed, true values.

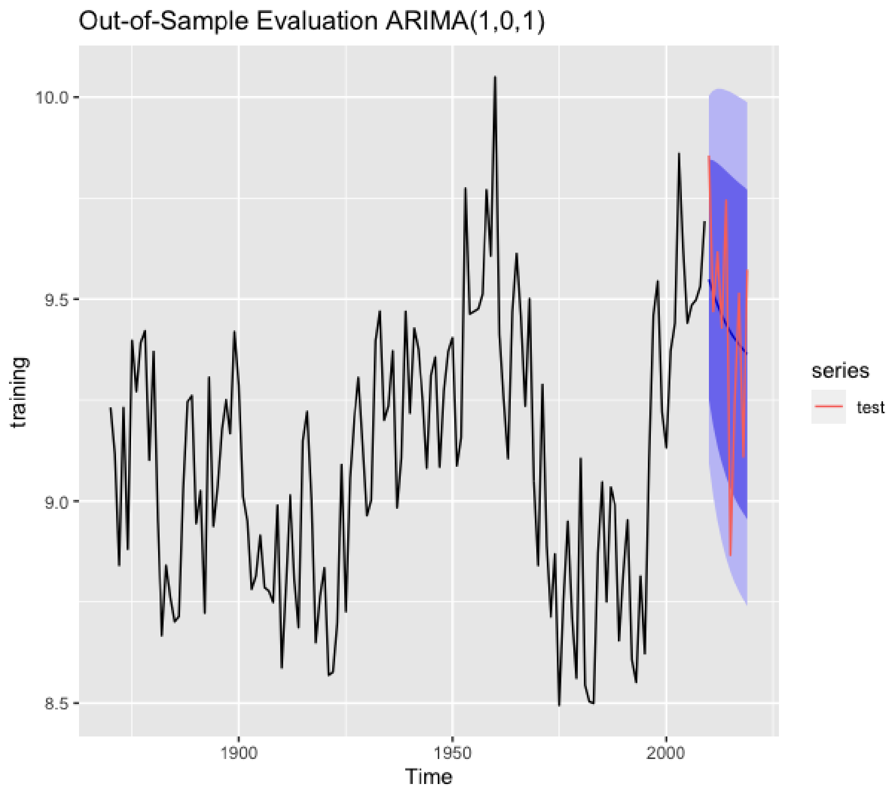

Figure 4: Out-of-sample evaluation using ARIMA(1,0,0)

We will repeat this process three more times for the other three model and use two model selection criteria, RMSE (root mean square error) and MAPE (mean absolute percentage error), to choose the best model. As table 1 shows, both model selection criteria choose ARIMA(3,0,0), therefore, we will use this model to fit and forecast our SST series at each grid.

Model	RMSE (Testing Set)	MAPE (Testing Set)
ARIMA(1,0,1)	0.2585164	2.277296
ARIMA(3,0,0)	0.2566009	2.245809
ARIMA(2,0,1)	0.2603027	2.27744
ARIMA(1,0,2)	0.2605616	2.272831

Table 1: RMSE and MAPE

As an example, we estimate the parameters for the ARIMA model fitted on the first block. The results are shown below in table 2.

Components	Lag	Estimate	Std Error	t Ratio
AR-1	1	0.4543	0.0802	5.66
AR-2	2	0.1212	0.0888	1.36
AR-3	3	0.2033	0.0807	2.53
Intercept	0	8.9911	0.1585	56.7
Drift	0	0.0019	0.0018	1.06

Table 2: MLE estimation results

4.5 SST Forecasting Results

After estimating and fitting 300 ARIMA(3,0,0) models, we use them to forecast the annually SST at each block over the next fifty years. The forecasting result is shown below in figure 5. (Here we only show the forecasted SST in 2069, but our forecasts also contain SST in each year from 2020-2069.)

For most of the blocks, SST increases over the next fifty years. Compared with figure 1 (SST in 2020), the upper, blue region shrinks, and the lower, yellow region expands.

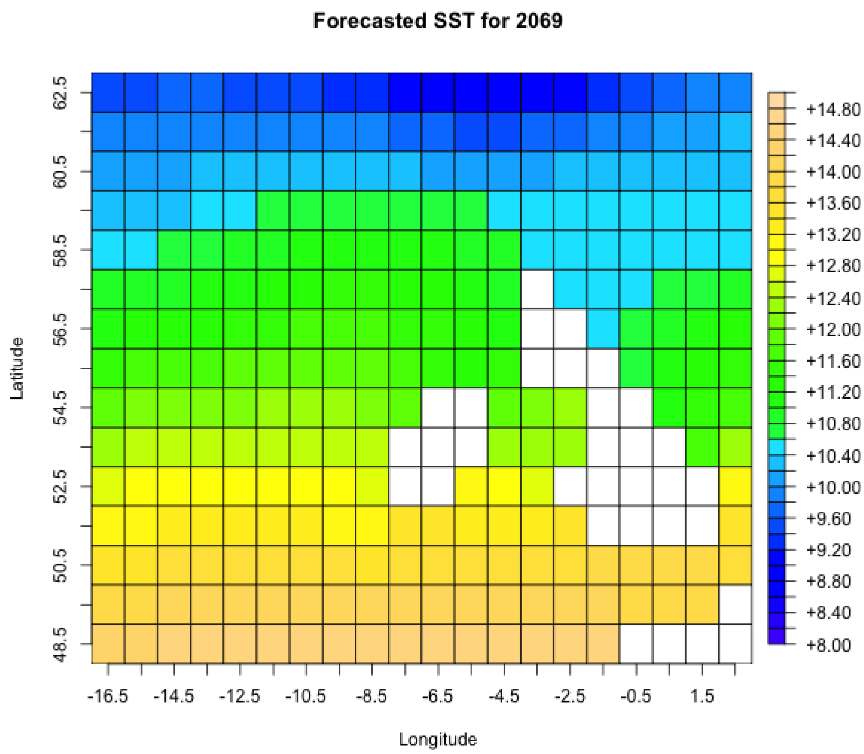

Figure 5: SST forecasting results

5 Model II: Modeling Fish Migration

5.1 Model Assumptions

Using the forecasted temperature data derived from Model I (the grid-map), we predict possible movements/migration of fish schools.

First assumption is that schools of fish are contained in each block and do not appear in two blocks at once. The justification for this assumption is that fish schools rarely take up an area of a 1 by 1 longitude, latitude block. To simplify the model, even If they happen to span two blocks at once, we assume that they are only contained in one of those blocks.

Fish move all together; no individual fish stray from the school. We assume all fish move as a school because tracking individual fish is not practical if we want to provide information for fisheries.

If they do migrate, fish can only migrate one block a year. Atlantic herring and mackerel schools swim at approximately 5.5 meters per second. If their current habitat has waters within their ideal temperature ranges, it is unlikely that they would stray extremely far from that point; thus we assume that they only move the distance of one block per year.

Next is the stochasticity assumption. Because fish do not always have a set path in the wild, we assume that there is always a chance of deviating from the rules that will later be stated. They have probabilities to choose different blocks and paths because they do not have the same deductive reasoning that humans do.

5.2 Fish Movement Parameters

According to official Scottish sea fisheries statistics [2], most herring and mackerel are caught in the North Sea. Thus, our model's fish starting point for the simulations is (57.5, -2.5) latitude, longitude. Herring, in the best case scenario, can live in a SST range parameter of 10.5-11.5 Celsius; in the worst case, they can only survive within 11.05-11.25 Celsius, thus requiring the fish to migrate more often to keep within this range [3]. Mackerel need slightly lower temperatures to survive. In the best case scenario, they are able to live in the range 9.8 to 10.25 Celsius; in the worst case, the SST range parameter is 9.98 to 10.2 degrees Celsius.

We will now estimate the maximum distance that a boat can travel. According to fishery statistics, the average speed of a fishing boat is 4 knots per hour [4], which translates to 7.41 kilometers per hour. Assuming that a boat travels at a constant speed of 7.41 km/hr for 18 hours each day, a boat travels 133 kilometers per day. We then use data [6] that predicts the shelf-life of different types of fish based on different temperatures and find the best fit linear line, which is $y = 10.4 - 0.651x$, where y is the number of days that the fish can last at x temperature in Celsius. The average sea temperature near our starting point is 10 °C [5]. This translates to the fish maintaining a high quality on a non-refrigerated boat for 3.89 days. Thus a boat can travel 517 kilometers. Accounting for the fact that the boat has to return to its starting location, we take half of this distance to obtain a maximum distance from the port location of 260 km. 260 km translates to 2 blocks on our grid. If the boat has refrigeration, we will assume that temperature (our x-value) would be 4 °C, as a refrigerator's temperature on average is 4 °C. Plugging in this value for X and repeating the same calculations, this translates to 4 blocks on our grid. Both of these parameters will show up in Model III. For the purpose of Model II, we will only use the fact that the boat can travel 2 blocks distance away from its current port.

5.3 Fish Model Stochasticity

This model is an agent based model, thus follows a set of rules. Fish will stay in the current block if temperatures are within their respective SST ranges. They will only begin migrating once temperatures leave the desired range. If the current block that fish schools inhabit is out of the range parameter, the model checks all adjacent block (at most 8) temperatures. If more than one block is in the livable temperature range, each block has a probability of 1/n, where n is the number of blocks within temperature range, for the fish to migrate. Each block has an equal probability of being chosen. This model also keeps track of the absolute value of the difference between each adjacent block's temperature against the current block's temperature. If all blocks are too warm, the fish will migrate to the block that has the least temperature difference from the mean of the livable range. However, if the current block is closest to the threshold, the fish have an equal probability of moving to one of the northern blocks (north, northwest, or northeast) because it is a reasonable assumption that as we get further away from the equator, water temperatures are lower. The fish migrate only once a year. Once the fish migrates to a new location, the fish will check the nearby environment again to see if they need to move to the next block. This means that they will stop at some point, if they find a block that maintains livable water temperatures. Once the waters warm, they will begin to move again. However, because fish do not have a set path and cannot predict water temperatures, they have a 0.5 probability of moving into a random adjacent block rather than the one decided upon by previous rules.

5.4 Results

The following histograms show the end longitude and latitudes of fish schools from 100 simulations of the model.

From the graphics, schools of mackerel most likely end up north and east of the starting point. These results seem reasonable because naturally, water temperatures get cooler as one moves further away from the equator, in this case, north. On the other hand, herring are more likely to move south and east because their SST range is higher than mackerel's sst range. The amount of variation depends on how quickly water temperatures rise according to the forecasted model from part I and how wide the sst ranges are for mackerel and herring. Worst cases for both mackerel and herring result in more movement of fish schools since water temperatures more quickly move out of the sst range. Another way to look at the data is seeing what location fish schools end up after each simulation. Because the sst range for mackerel is lower, they tend to move north more quickly and end up at a higher latitude than Atlantic herring. Despite the stochasticity, the end latitude and longitudes are either approximately normal or bimodal. There are some instances where fish stay in warm waters but this model shows that a majority of the simulations result in fish moving towards cooler temperatures especially for the cases of mackerel.

As a result of the model, the most likely elapsed times (the year in which these populations will be too far away for small fishing companies to harvest if the small fishing companies continue to operate out of their current locations) are shown in figure 10 and figure 11.

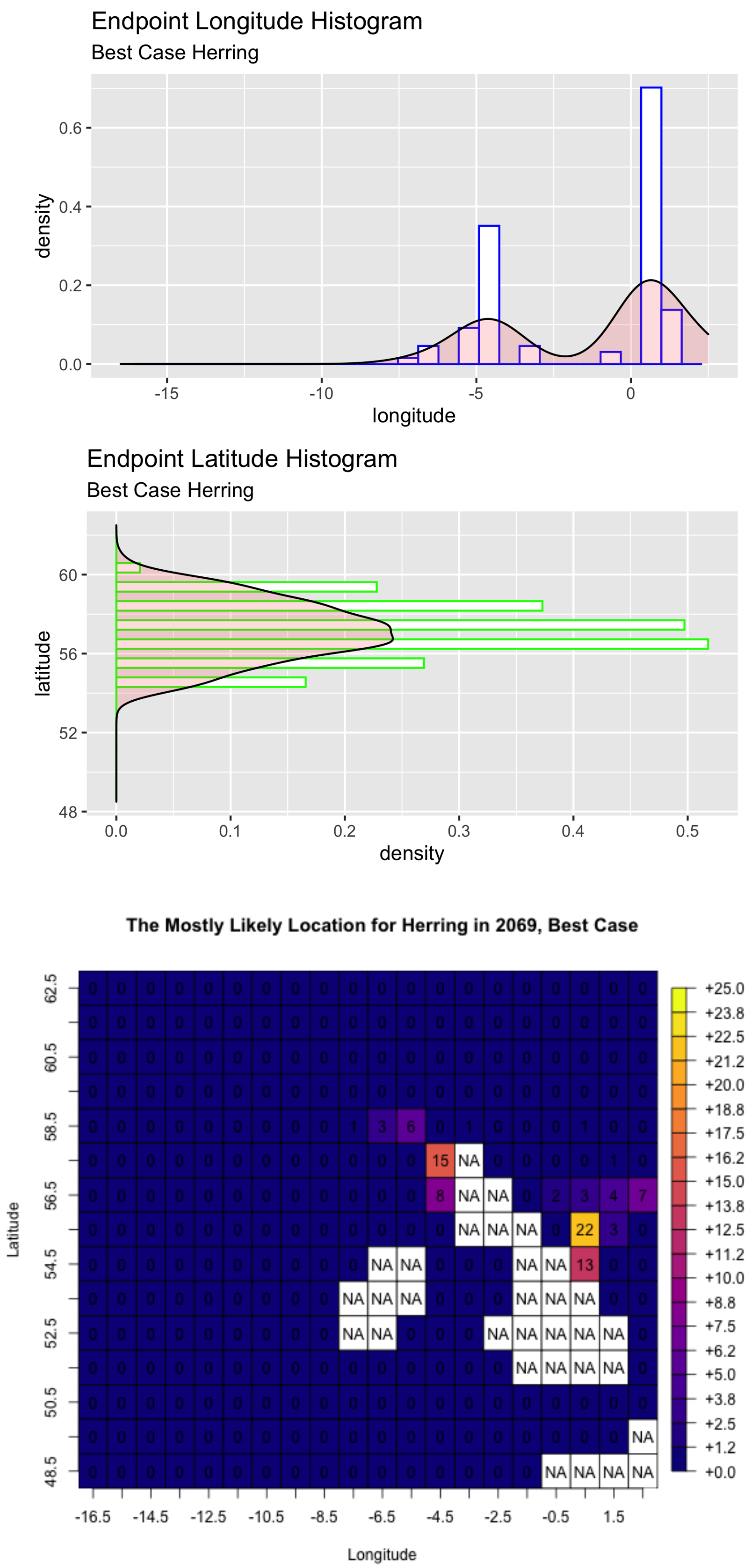

Figure 6: Most likely location for herring under the best case

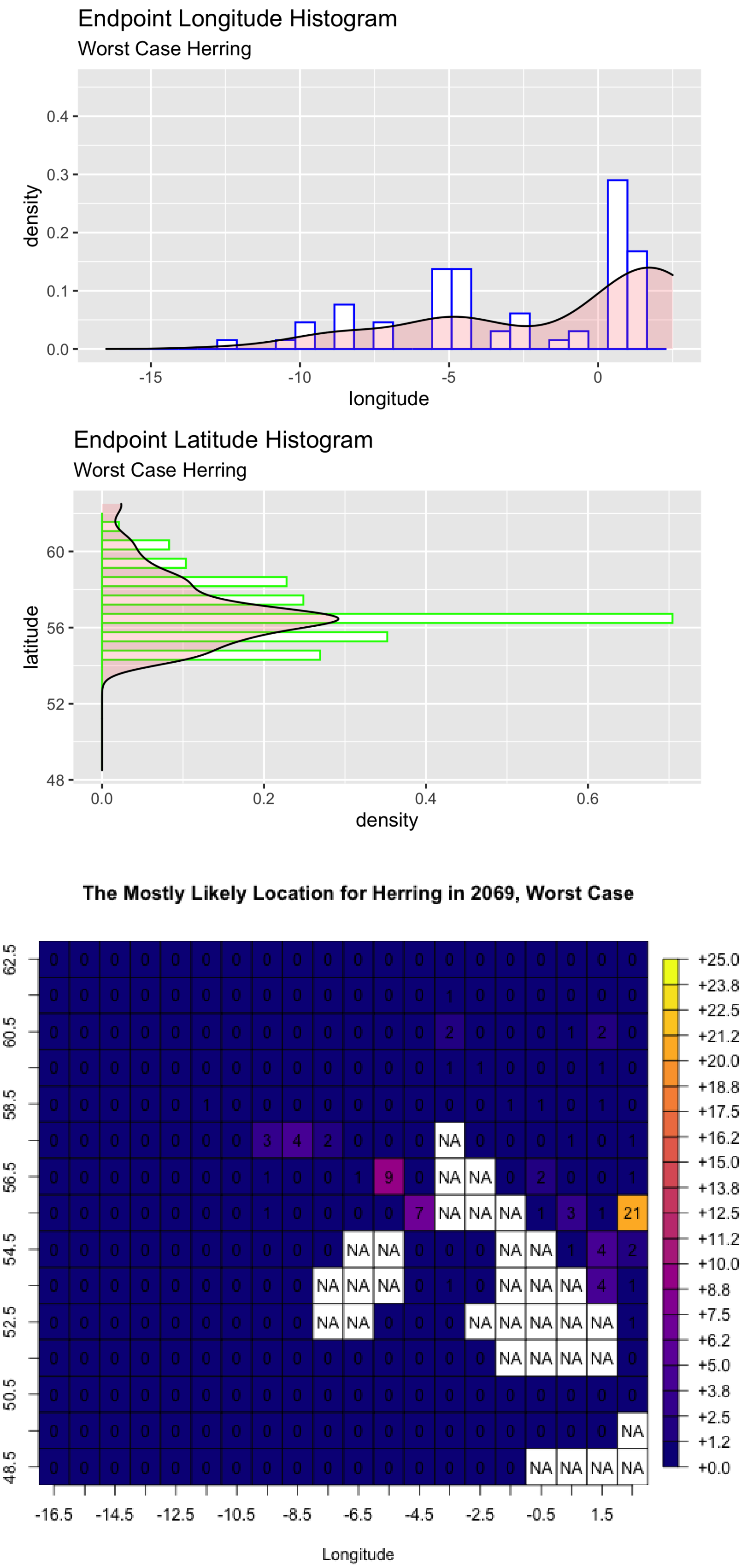

Figure 7: Most likely location for herring under the worst case

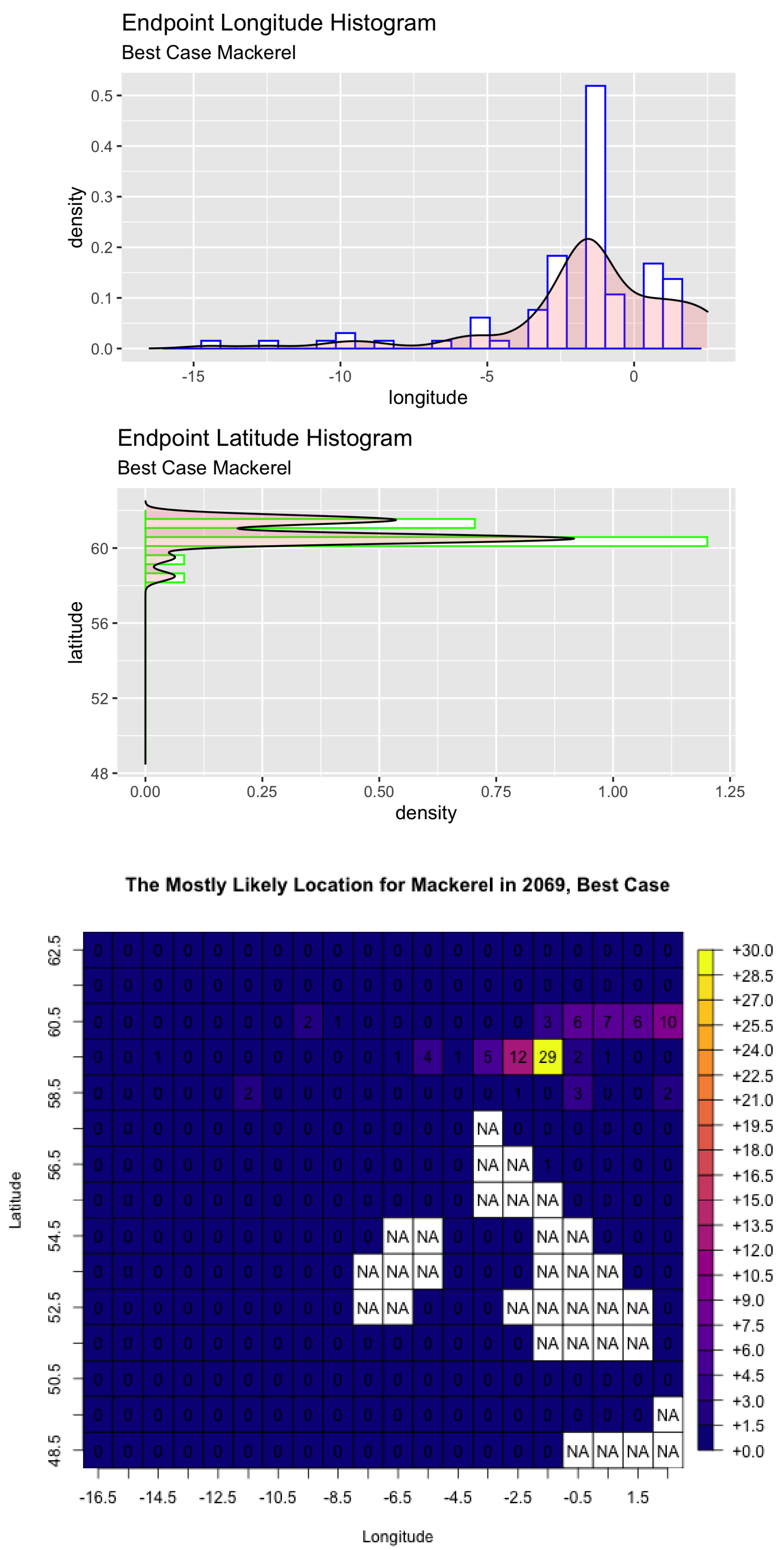

Figure 8: Most likely location for mackerel under the best case

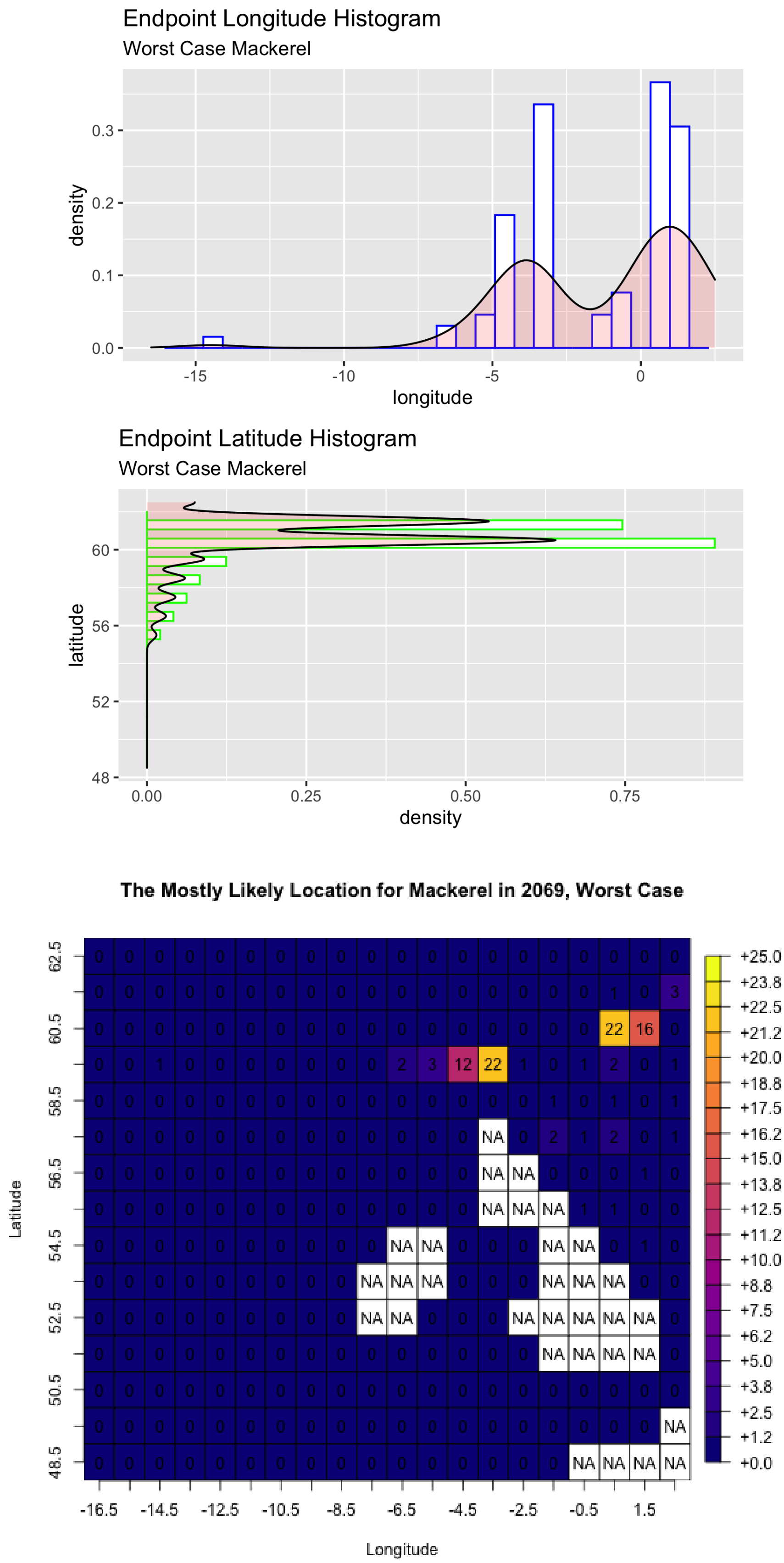

Figure 9: Most likely location for mackerel under the worst case

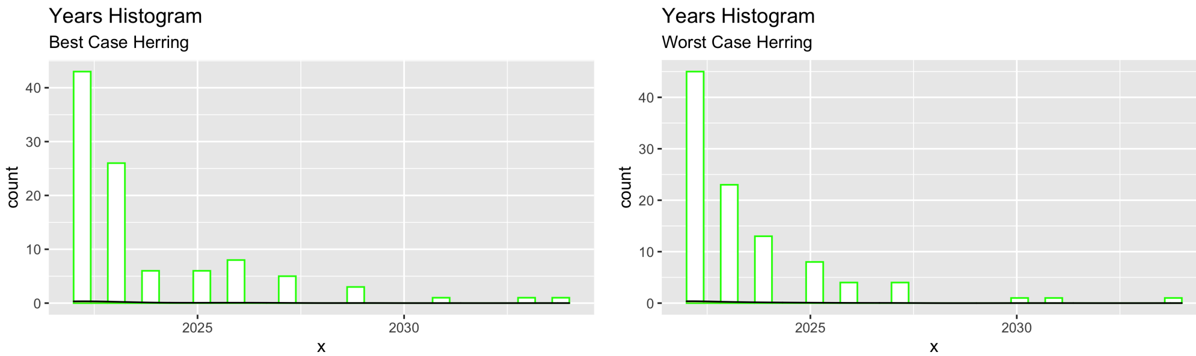

Figure 10: Most likely elapsed time, Herring (Mode: 2022, Median: 2023)

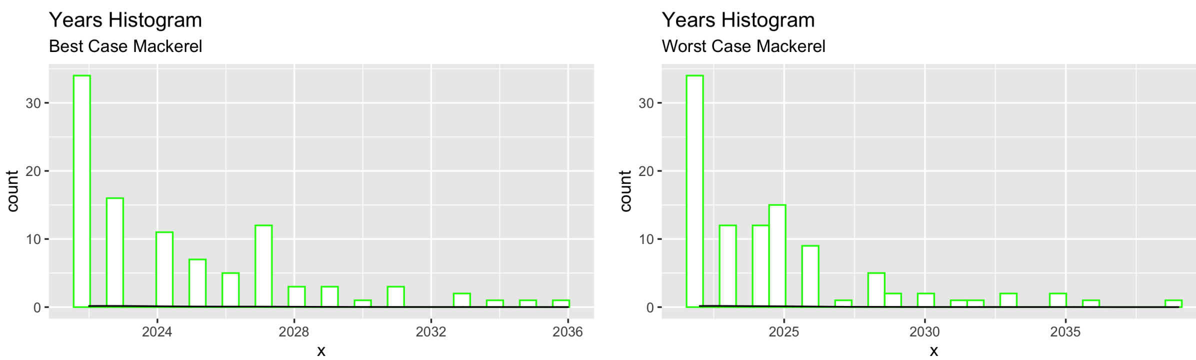

Figure 11: Most likely elapsed time, Mackerel (Mode: 2022, Median: 2024)

6 Potential Strategies

We will consider the following strategies:

- operate with original boat at the same location
- operate with a refrigerated boat at the same location
- operate with original boat and relocate entire fishing port to a new location

We will assume that the company only owns one fishing boat. Model A will be to purchase refrigeration to install in the current boat, and Model B will be to relocate to a new location. While the modeling of Model A and Model B will be described separately, they will be actually using the same model, which we will summarize in section 6.3.

6.1 Model A: Buy On-Board Refrigeration

Assume the company only owns one fishing boat. Currently, the boat has no refrigeration. When needed, the company can choose to purchase refrigeration for the current fishing boat. This model will evaluate the profit trend over the next 50 years in each case to help the company decide whether to buy refrigeration or not.

The firm has two strategies:

$$\begin{cases} i = 1 & \text{operate with original boat} \\ i = 2 & \text{replace the original boat with a refrigerated boat} \end{cases}$$

Here, we only focus on the cost of the fishing boat. Although there are other costs (i.e. fuel consumed), they are relatively small in comparison to the fishing boat and are also very close in value for both strategies, so we will ignore the other costs. Thus the cost of fishing is expressed as

$$C_{i,t} = c_{0,i} \tag{6}$$

where $c_{0,i}$ is the depreciation for each type of boat, and we use a straight line depreciation.

We further assume that a fishing boat can last for 15 years, and it will not age as time goes. We also assume that our current boat is new.

The revenue comes from fishing, which depends on the price (p) and quantity (q) of fish catch. Herring and mackerel each has its own market price, and the price may fluctuate from year to year. The quantity of fish in each block is determined from our simulation in Model II. The number of blocks away from our starting point that our non-refrigerated boat can travel is 2, which is a parameter value calculated in Model II. So the revenue of fishing for choosing the i-th strategy is expressed as

$$R_{i,t} = k_i \sum_{j=1}^{2} p_{j,t} q_{i,j,t} \tag{7}$$

Where i is defined as before; k is the number of times the company fishes per year; $j = 1, 2$ represents the type of fish – we have $j = 1$ for herring and $j = 2$ for mackerel; $t = \{2020, 2021, \ldots, 2069\}$ represents the time.

For $i = 1$,

$$q_{1,j,t} = \frac{1}{100} \times \sum_{b=1}^{2} (\text{number of type j fish in b}^{\text{th}} \text{ nearest block in year t}) \tag{8}$$

Recall that we ran 100 simulations in Model II, we can think of it this way: we have 100 fish, each simulation models the movement of one fish. So we know the location of all 100 type j fish in t-th year. We can just use the best case. The number of blocks away from our starting point that our refrigerated boat can travel is 4, which is a parameter value calculated in Model II.

For $i = 2$,

$$q_{2,j,t} = \frac{1}{100} \times \sum_{b=1}^{4} (\text{number of type j fish in b}^{\text{th}} \text{ nearest block in year t}) \tag{9}$$

We further assume that these company catch both types of fish simultaneously, and the capacity of the both fishing boat is large enough for storing an arbitrary amount of fish.

Our profit for strategy i for year t can thus be expressed as

$$\pi_{i,t} = R_{i,t} - C_{i,t}. \tag{10}$$

6.2 Model B: Relocation

Like in Model A, we will assume that the company only owns one fishing boat. When needed, the company can choose to relocate to a new location. This model will evaluate the profit trend over the next 50 years in each case to help the company whether to decide to relocate to a new location or not. We will also assume that the boat we are operating with before and after relocation is the original boat, which has no refrigeration. The following model will be building off the model from Model A, but we are assuming that the strategy is always i=1, meaning that we are operating with a boat that has no refrigeration.

X is the year of relocation. If $X = 2069$, no relocation occurs. We assume that when the company chooses to relocate, the relocation is instantaneous. We also assume that that everything from the old fishing port is transferred to the new fishing port, so no new purchases are made.

In Model A, we took into account the cost of the boat and the cost of the fuel. We use Model A's fishing cost and introduce another fixed cost. Here, the additional fixed cost is the cost of relocation, which is the cost of the new fishing port. Although there are other costs (i.e. transportation costs), they are relatively small in comparison to the fishing port cost, so we will not account for these costs. So the annual cost of relocation is

$$C_t = a + c_0 \tag{11}$$

where a is the depreciation of the relocation costs; $a = 0$ if there is no relocation. The second part of the equation is from our fishing cost in Model A, assuming $i = 1$.

We assume that the relocation is instantaneous and at the beginning of year X. So, the annual revenue of fishing for relocation at X year is

$$R_t = k \sum_{j=1}^{2} p_{j,t} q_{y,j,t} \tag{12}$$

where

$$\begin{cases} y = 1 & 2019 \leq t < X \\ y = 2 & X \leq t \leq 2069 \end{cases}$$

i.e. $y = 1$ represents the original location and $y = 2$ represents the new location.

Thus our profit for year t can be expressed as

$$\pi_t = R_t - C_t. \tag{13}$$

6.3 Model III: Model A and Model B

Because Model B is essentially an extension of Model A, we can combine Model A and B and express the overall model (Model III) in the following equations. We keep the same intended meaning of the variables and parameters from Models A and B. These can be summarized in the following table.

i	Type of Boat (no fridge, fridge)
t	Year
a	Fishing Port Depreciation
c_0	Boat Depreciation
k	Number of times company fishes per year
p	Price
q	Quantity of Fish Catch
j	Type of Fish (herring, mackerel)
y	Fishing Port Location (original location, new location)
X	Relocation Year

Fishing Cost:

$$C_{i,t} = a + c_{0,i} \tag{14}$$

Fishing Revenue:

$$R_{i,t} = k_i \sum_{j=1}^{2} p_{j,t} q_{i,y,j,t} \tag{15}$$

$$\begin{cases} y = 1 & 2019 \leq t < X \\ y = 2 & X \leq t \leq 2069 \end{cases}$$

Fishing Profit:

$$\pi_{i,t} = R_{i,t} - C_{i,t} \tag{16}$$

The potential business strategies are summarized in the following table:

Option	Description	i	X
1	Operate with the original boat, remain at the same location	1	2069
2	Operate with boat with refrigeration, remain at the same location	2	2069
3	Operate with the original boat, move to a new location	1	$\neq 2069$

Because we are using one model to model our three different strategies, we can compare the three different strategies.

6.4 Parameter Values

We will refer to the different business strategies as Option 1, Option 2, and Option 3 as summarized in the table at the end of section 6.3. We will also use the best-case data results from Model II for both fish when testing this model.

In order to compare the models, we will normalize all parameter values.

First, we determine our fixed costs. The maximum cost of a small fishing boat and minimum cost of a medium fishing boat First, we will select values for our fixed costs parameters. We find that the cost of a small to medium sized boat is approximately $50,000 [7], the cost of adding a refrigerator to a boat is $2,000 [9], and the cost of building a dock for a boat is $14,000[8]. We will depreciate the boat and refrigerator over 15 years and the cost of building a dock over 50 years. So, our annual cost for Option 1 is $3,570, our annual cost for Option 2 is $3,710, and our annual cost for Option 3 is $3,850. Using Option 3's annual cost as our maximum, we normalize the annual cost values:

Option	Annual Cost (Normalized)
1	.93
2	.96
3	1

When testing the model, we assume that the price of the fish remain the same each year. We also found that the prices of herring and mackerel are relatively close, so we choose our normalized p_j values to be

$$p_{1,t} = 0.5 \text{ and } p_{2,t} = 0.5$$

Next, we know that a boat with a refrigerator can travel 6 days without the fish reducing in quality while a boat without a refrigerator can travel 3 days without the fish reducing in quality. Thus, a boat without refrigeration makes a shorter but a greater number of trips. So, our original boat can make twice as many trips as one with refrigeration. Thus, our normalized k_i values are

$$k_1 = 1 \text{ and } k_2 = 0.5$$

We also select our new location for Option 3. From our visuals in Model II, we notice that the mackerel are moving north while the herring are moving south. Because the herring are moving into the UK waters, we will prioritize the mackerel migration when choosing our relocation spot. Thus we will choose the northernmost location in Scotland, with coordinates

$$57.5\ °\text{N},\ 3.5\ °\text{W}$$

To determine the relocation year, we examine the density (q-value) of herring and mackerel each year. Because we take into account both the density of the mackerel and herring, we will graph the sum of the q-values of herring and mackerel of each location each year.

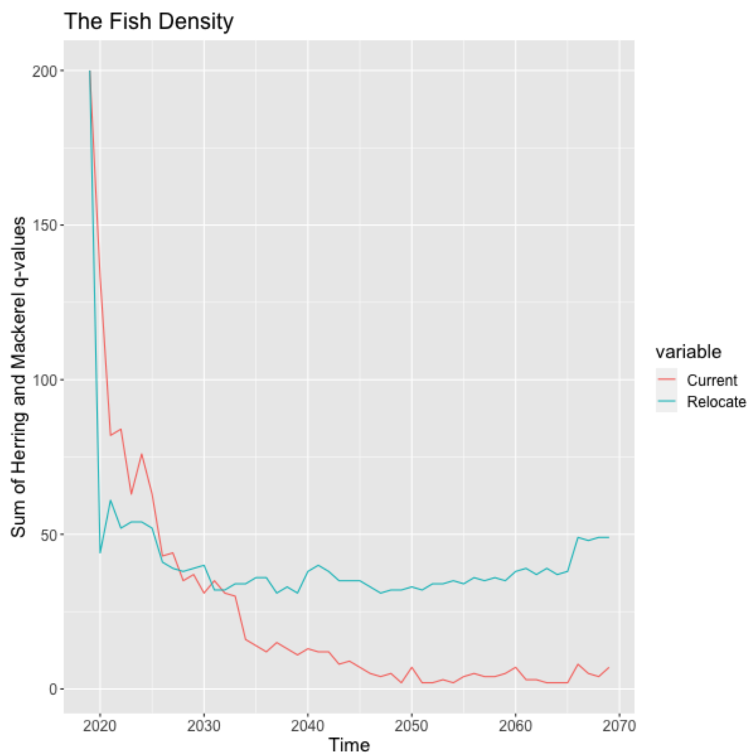

Figure 12: Sum of Herring and Mackerel Density

We see that the mackerel and herring total density of the current port is initially greater than the mackerel and herring density of the relocated port up until the year 2025. Following the year 2025, the fish density is greater in the area of the relocated fishing port. Thus, we select our relocation year to be 2025.

$$X = 2025$$

Our $q_{i,j,t}$ values are obtained from Model II and are already normalized.

6.5 Results

After inputting our parameter values in and running the model for each strategy and obtaining the profits for each year, we get the following graphs:

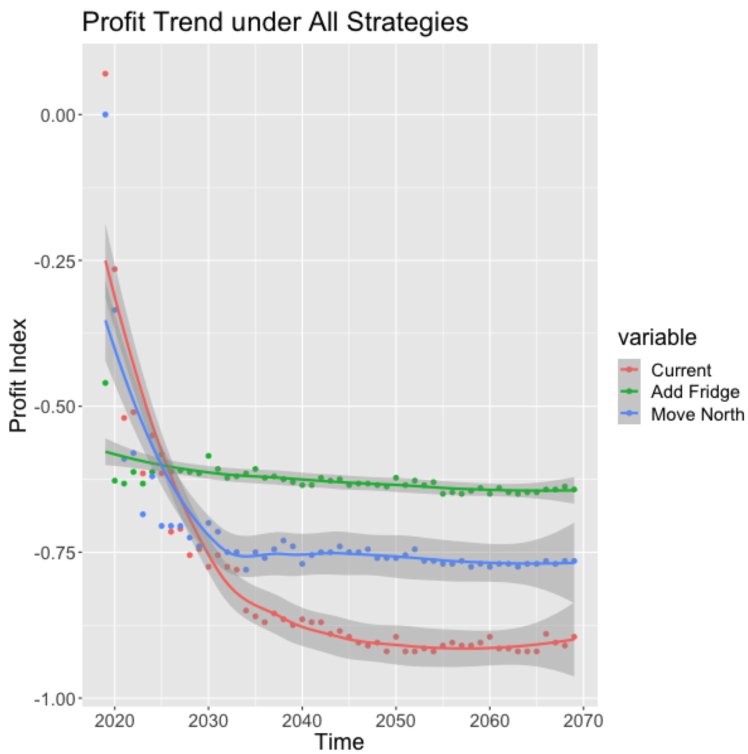

Figure 13: Profit Trend

Looking at the graphs, we see that the most profitable strategy in the 50 year time period is Option 2, operating with a boat with refrigeration. This is a reasonable result because a boat with refrigeration does not cost significantly more than a boat without, yet is able to travel significantly further without the fish reducing in quality. The results for option 3's profit are also reasonable as relocation costs more. However, because the fish density in that new location is increasing, the profits eventually are greater than continuing to operate at the current location.

6.6 Exclusive Economic Zone (EEZ)

After taking international waters into account, we see that the option that is impacted is the boat with refrigeration strategy. As the sea area covered by a boat with refrigeration includes blocks that are considered international waters, we remove those blocks. This will impact our q values, or densities of our herring and mackerel at each year. We then run Model III again for the boat with refrigeration and compare all the strategies.

Looking at the graphs in figure 15, we see that removing the blocks deemed international waters in figure 14 makes a relatively insignificant difference. Thus the boat with refrigeration remains the most profitable and practical strategy.

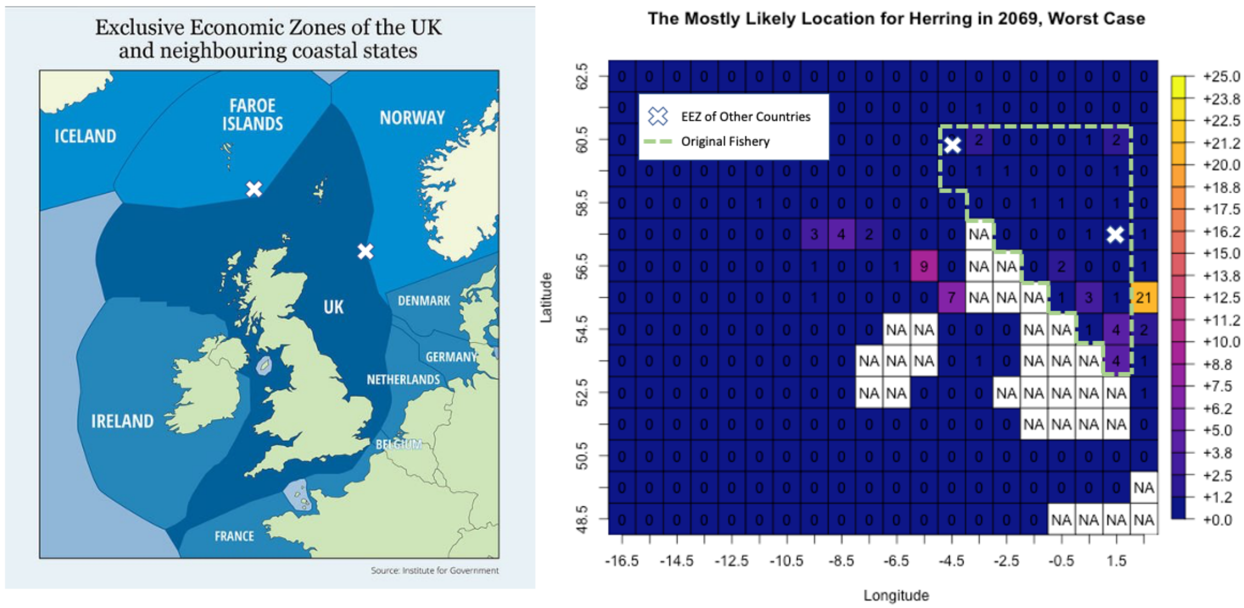

Figure 14: EEZ of other countries

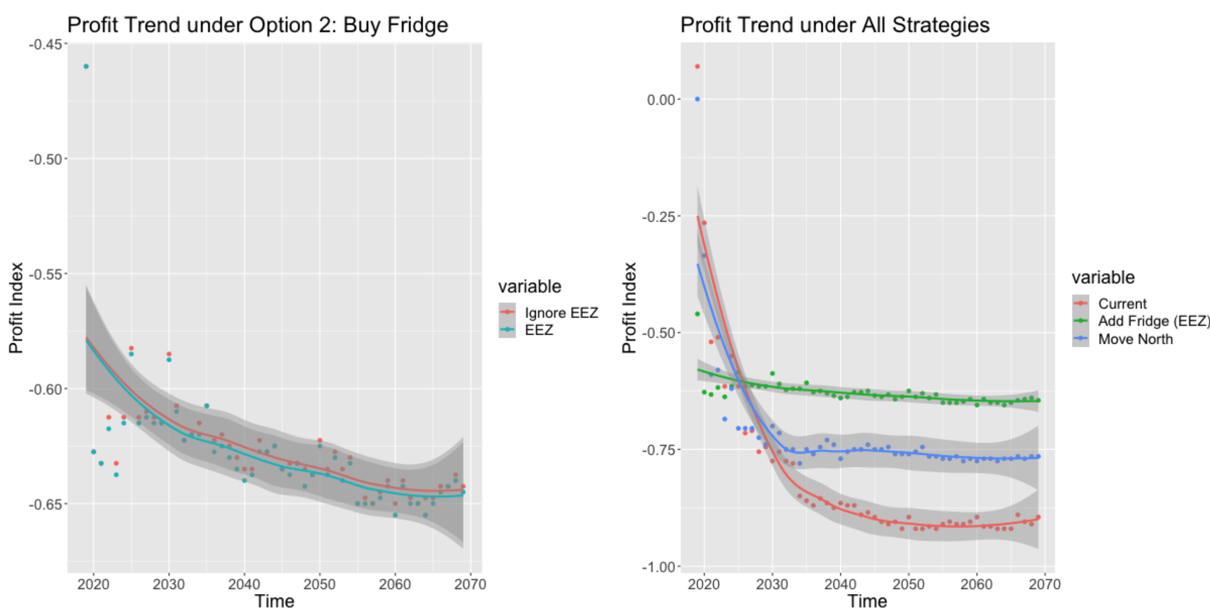

Figure 15: The impact of international waters

7 Model Evaluation

7.1 Sensitivity Test

This model consists of the three separate models, each built on the results of the previous. Thus, changing aspects of each model may affect another model.

Model II is particularly sensitive because it depends on the forecasted results of Model I. In addition, because it is an agent based model, modifying rules and parameters have an impact on the results. The most sensitive parameter of this model is the SST ranges of herring and mackerel. Changing the temperature ranges that these fish prefer impacts how quickly they migrate and in which direction. While we used a herring SST range of 11 degrees Celsius, we also tested a range of 8 degrees Celsius. By changing this parameter, we found that herring migrated north within 4-5 years, a much quicker rate than mackerel whose SST range is 10 degrees Celsius. Another sensitive parameter of this model is the starting location of the fish. If the schools of fish start at a block that is warmers, they tend to migrate more quickly away as opposed to starting on a block within their respective SST ranges. However, this is a less sensitive parameter because most starting points still result in the schools of fish migrating north/east.

After doing several sensitivity tests on the parameters for Model III, Model III was proven to not be sensitive. Model III depends on the results of Model II to determine the mackerel's and herring's fish density, the location of the new fishing port, and the relocation year. However, minor changes to the results of Model II does not significantly impact the results of Model III. We tested the parameters obtained from Model II for sensitivity and the strategy in which we purchase refrigeration still remained the most profitable strategy. Changing the ratios of the normalized parameters also does not significantly impact the results of Model III. We did a sensitivity test for the cost parameters by changing the ratios by +0.01 and -0.01. We also did a sensitivity test for the k-values by changing the ratio by increasing and decreasing k-value for the refrigeration strategy by 0.1. In addition, we tested sensitivity by changing the price ratio for herring and mackerel by -0.1 and +0.1 respectively. After doing a sensitivity test for the each of the cost parameters, the k parameters and the prices of the mackerel and herring, the most profitable option was still the strategy in which we purchase refrigeration.

7.2 Strengths

- Because we account for 300 different ocean grids and over 150 years of data for sea surface temperature, we reasonably forecast the ocean surface temperature changes.
- By normalizing parameters of Model III, we greatly simplify the model so that it only retains pertinent information.

7.3 Possible Improvement

- The analysis of fish migration can be more accurate if know the seasonal migration pattern of these fish.
- We take into account changing population of these fish using birth and death rates.

- We explore more potential business strategies, such as relocation with refrigeration.
- We take into account multiple fishing locations rather than just one.

8 Conclusion

It is important to predict the migration of the mackerel and herring so that Scottish fishing companies can respond and adjust accordingly to this change. Based on our models, we see that the mackerel and herring do migrate out of the fishing port ocean area early on during the fifty-year period, resulting in a significant decrease in profit from the year 2022 on. We also found that out of the three business strategies' proposals, purchasing and installing on-board refrigeration was the most profitable and economical one.

Model I had few parameters and was based largely off of data. While Model III was proven to be resistant to minor parameter changes, Model II was not. Model II's temperature ranges are particularly sensitive parameters. Due to the interdependence of these models, sensitivity is a weakness. After testing sensitivity of this model, the consistently most profitable option during the 50 year period was purchasing and installing on-board refrigeration.

References

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[2] Official Scottish Sea Fisheries Statistics. `https://www.gov.scot/publications/scottish-sea-fisheries-statistics-2018/`

[3] Atlantic Mackerel and Herring Habitable Temperature Ranges `https://www.fisheries.noaa.gov/species/atlantic-mackerel`

[4] Fishing Vessel Types and Speeds. `https://nmssanctuaries.blob.core.windows.net/sanctuaries-prod/media/archive/education/voicesofthebay/pdfs/trawlers.pdf`

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[6] Quality Changes and Shelf Life of Chilled Fish `http://www.fao.org/3/v7180e/v7180e07.htm`

[7] How Much Does A Fishing Boat Cost? `https://www.yachts360.com/how-much-does-a-fishing-boat-cost/`

[8] Home Advisor, How Much Does a Dock Cost? `https://www.homeadvisor.com/cost/outdoor-living/build-a-dock/#:~:text=The%20average%20pier%20runs%20%24100,run%20between%20%2450%2C000%20and%20%2475%2C000`

[9] Fisheries Supply, Marine Refrigeration and Freezer Prices. `https://www.fisheriessupply.com/boat-galley/refrigerators-and-freezers`

Appendices

Appendix A Article for Hook Line and Sinker

With the conditions of global warming worsening, changes in climate are continually occurring and affecting the habitats of wildlife. In particular, sea surface temperatures have increased drastically over the past 50 years and will continue to do so in the future. How does this climate change affect Scotland's fishing industry?

Unfortunately, climate change poses a significant threat to Scotland small fishing companies. As sea temperatures approach numbers beyond mackerel's and herring's tolerance level - two species crucial to the fishing industry - these fish have no choice to but to seek waters with lower temperatures. As climate change continues to raise the temperatures of these waters, these species are continually forced to migrate. In approximately three years, the distribution of these fish will have been reduced by 50%, and by 2050, the herring and mackerel populations in the fishing port sea area will be reduced to nearly zero. This means that revenue created by hauling in these fish will also eventually reduce to zero. Needless to say, the situation is dire. Herring and mackerel are two of the most highly fished species, and Scotland has one of the largest (if not the largest) fishing industries in Europe and plays a major role in Europe's economy. The issue of fish migration cannot be understated.

What can fishing ports do? While profits may be good now, we now know that in just a short time period, the fish will disappear along with these profits. Fortunately, there is an effective and economically practical solution. The issue here is that both mackerel and herring are migrating to locations beyond our fishing boats' reach. How can we increase the distance that our fishing boats can travel without risking the quality of the fish? The answer is simple and economical: refrigeration.

What can refrigeration do? Refrigeration allows fishing boats to travel twice the distance while still maintaining the high quality of the fish. Rather than spending a large amount of money to relocate, fishermen can simply install refrigeration and access the sea area of a potential relocated fishing port while also accessing the area of the current fishing port. It is a win-win situation!

It is critical for Scottish fishing companies to invest in on-boat refrigeration by 2022. If no changes are made, there will not be a sufficient number of fish for fishermen to catch, which will eventually result in fishing companies losing money. Thus the time to purchase on-board refrigeration is now. There is no time to waste!

Appendix B Code

Here is the program we used to forecast SST in MODEL I.

Input R source:

```
#***************************************************************************
# Using ARIMA Model to Forecast SST over the Next 50 Years
# Dataset:
#        "annual_avg_sst_150.csv"        1870—2019, annually average
# Output:
#        "forecast_sst_50.csv"           2020—2069, forecasted annually SST
# Author: Hao Jin
#***************************************************************************

library(tidyverse)
library(forecast)
library(stats)
library(tis)
library(trend)
library(tseries)
library(fpp2)

# Import data
annual_avg_sst = read.csv("annual_avg_sst_150.csv", row.names = 1)

# ———————————————————————— Time Series Plot ———————————————————————— #

first_block = ts(annual_avg_sst[,1], start = 1870, end = 2019)

plot(first_block, ylab = "Annually Average SST", xlab = "Time", lwd = 2,
col = 'skyblue3', main = "SST in Block 1 over the Past 150 Years")
legend(x = "topleft", legend = "Seasonal Adj. Data",
col = "skyblue3", lwd = 2, inset = 0.05)


# Moving average plot
autoplot(first_block) +
  autolayer(ma(first_block, 11), series = "10 yr MA") +
  autolayer(ma(first_block, 21), series = "20 yr MA") +
  autolayer(ma(first_block, 31), series = "30 yr MA") +
  xlab("Time") +
  ylab("SST")


# ————————————————————————— Model Selection ———————————————————————— #

first_block = ts(annual_avg_sst[,1], start = 1870, end = 2019)

tsdisplay(first_block, main = "ACF and PACF of the SST in Block 1")

adf.test(first_block, k = 1)

# ARIMA(1,0,1)
arima101 = Arima(first_block, order = c(1,0,1), include.drift = TRUE)
```

```r
plot(forecast(arima101, 50))
lines(arima101$fitted, col = "red")

# ARIMA(3,0,0)
arima300 = Arima(first_block, order = c(3,0,0), include.drift = TRUE)
plot(forecast(arima300, 50)))
lines(arima300$fitted, col = "red")

# ARIMA(2,0,1)
arima201 = Arima(first_block, order = c(2,0,1), include.drift = TRUE)
plot(forecast(arima201, 50))
lines(arima201$fitted, col = "red")

# ARIMA(1,0,2)
arima102 = Arima(first_block, order = c(1,0,2), include.drift = TRUE)
plot(forecast(arima102, 50))
lines(arima102$fitted, col = "red")

## ------------------- Out of Sample Evaluation ------------------- ##

training <- subset(first_block, end=length(first_block)-10)
test <- subset(first_block, start=length(first_block)-9)

train.101 <- Arima(training, order = c(1,0,1), include.drift = TRUE)
train.101 %>%
  forecast(h = 10) %>%
  autoplot() + autolayer(test) +
  ggtitle("Out-of-Sample Evaluation ARIMA(1,0,1)")

train.300 <- Arima(training, order = c(3,0,0), include.drift = TRUE)
train.300 %>%
  forecast(h = 10) %>%
  autoplot() + autolayer(test) +
  ggtitle("Out-of-Sample Evaluation ARIMA(3,0,0)")

train.201 <- Arima(training, order = c(2,0,1), include.drift = TRUE)
train.201 %>%
  forecast(h = 10) %>%
  autoplot() + autolayer(test) +
  ggtitle("Out-of-Sample Evaluation ARIMA(2,0,1)")

train.102 <- Arima(training, order = c(1,0,2), include.drift = TRUE)
train.102 %>%
  forecast(h = 10) %>%
  autoplot() + autolayer(test) +
  ggtitle("Out-of-Sample Evaluation ARIMA(1,0,2)")

accuracy(forecast(train.101), test)
accuracy(forecast(train.300), test)
accuracy(forecast(train.201), test)
accuracy(forecast(train.102), test)

# ------------------- Forecast for all 300 blocks ------------------- #
```

```
ts.plot(annual_avg_sst)

forecast = matrix(nrow = 50, ncol = 300)

for (b in 1:300) {

  # Get the data at the b-th block: first@(-16.5, 62.5), last@(2.5, 48.5)
  past_150_yr = annual_avg_sst[,b]

  if (is.na(past_150_yr[1])){
    forecast[,b] = rep(NA, 50)

  } else {
    fit = Arima(past_150_yr, order = c(3,0,0), include.drift = TRUE)
    forecast[,b] = forecast(fit, 50)$mean
  }
}

colnames(forecast) = loc[[1]]
rownames(forecast) = seq(2020, 2069)
write.csv(forecast, file = "forecast_sst_50.csv")
```

Here is a program we used to predict herring migration under the best case in MODEL II. The programs for predicting mackerel, and for worst cases, are very similar.

Input R source:

```
#***************************************************************************
# BEST CASE MODEL FOR HERRING
# Dataset:
#     "forecast_sst_50_t.csv"      forecasted data from 2020-2069, transposed
# Author: Daphne Chen
#***************************************************************************
# Best case sst range parameter: 10.5-11.5 C
#***************************************************************************

# library needed packages
library(tidyverse)

set.seed(88)
#test <- function() {
# import non-forecasted data first
sst = read_csv("forecast_sst_50_t.csv", col_names=TRUE)
head(sst)

# data frame to keep track of each run of the model
best_case_herr_df <- data.frame(matrix(NA, nrow = 100, ncol = 51))
# track what year fish leave "within reach"
year_out <- rep(NA, times = 100)

left_edge <- seq(from=1, to=300, by=20)
right_edge <- seq(from=20, to=300, by=20)
#within <- c(53:59, 73:79, 94:99, 115:119, 136:139, 157:159, 178, 179, 199)
within <- c(95, 96, 97, 115, 116, 117, 135, 136, 137, 157)
```

```
for(i in 1:100) {
  best_case_herr_df[i, 1] <- sst[[116, 1]]
  # start block index and year
  current_block <- 116 # 57.5, -1.5
  current_year <- 2020
  prev_block <- 0
  first_yr_out <- TRUE

  j <- 2
  while(current_year < 2070 & current_block > 0) {

    if(first_yr_out) {
      # port coordinate
      #start_loc <- c(-2.5, 56.5)

      #xy <- str_split(sst[[current_block, 1]], " ")
      #latitude <- as.numeric(xy[[1]][1])
      #longitude <- as.numeric(xy[[1]][2])
      #coord <- c(longitude, latitude)

      #d_fr_port <- distm(start_loc, coord, fun = distHaversine)/1000

      #if(d_fr_port > 520) {
      #year_out[i] <- current_year
      #first_yr_out <- FALSE
      #}

      if(!(current_block %in% within) & first_yr_out) {
        year_out[i] <- current_year
        first_yr_out <- FALSE
      }

    }

    # check if current location's temperature is still within habitable range
    current_block_temp <- sst[[current_block, as.character(current_year)]]
    if(10.5 <= current_block_temp & current_block_temp <= 11.5) {
      current_year <- current_year + 1
      best_case_herr_df[i, j] <- sst[[current_block, 1]]
      j <- j+1
      next
    }

    # corresponding indices for surrounding blocks
    north <- current_block-20
    west <- current_block-1
    south <- current_block+20
    east <- current_block+1
    northwest <- current_block-21
    southwest <- current_block+19
    southeast <- current_block+21
    northeast <- current_block-19
```

```r
northern <- c(northwest, north, northeast)


# vector to keep track of which adjacent blocks are
# within temperature range
if(current_block %in% left_edge) {
  adjacent <- c(north, south, east, southeast, northeast)
  northern <- c(north, northeast)
} else if(current_block %in% right_edge) {
  adjacent <- c(north, west, south, northwest, southwest)
  northern <- c(north, northwest)
} else {
  adjacent <- c(north, west, south, east,
                northwest, southwest, southeast, northeast)
}

ncopy <- northern
for(d in northern) {
  if(d <= 0 || is.na(sst[[d,2]])) {
    ncopy <- ncopy[ncopy != d]
  }
}
# average temperature of range in the case that
# none of the block are within range
avg_temp <- 11.0
# best option if no blocks are within temperature range
best <- abs(avg_temp - current_block_temp)
# initialize what the index of the new block to be migrated to
new_block <- current_block
adjacent_copy <- adjacent
stoc_a <- adjacent

# check whether we have data for each direction AND
# if the block is above or below threshold temperature
for(direction in adjacent) {


  # if block is out of range (no data) or not within temperature range,
  # remove the option from the adjacent vector
  if(direction > 300 || direction < 1) {
    adjacent_copy <- adjacent_copy[adjacent_copy != direction]
    stoc_a <- stoc_a[stoc_a != direction]
    next
  }

  temp <- sst[[direction, as.character(current_year)]]

  if(is.na(temp)) {
    adjacent_copy <- adjacent_copy[adjacent_copy != direction]
    stoc_a <- stoc_a[stoc_a != direction]
    next
  }
```

```
      if(!(10.5 <= temp & temp <= 11.5)) {
        if(abs(avg_temp - temp) < best) {
          best <- abs(avg_temp - temp)
          new_block <- direction
        }
        if(new_block == current_block && length(ncopy) > 0) {
          if(length(ncopy) > 1) {
            new_block <- sample(ncopy, 1)
          } else {
            new_block <- ncopy
          }
        }
        adjacent_copy <- adjacent_copy[adjacent_copy != direction]
      }
    }


    # of the blocks that are in temperature range,
    # choose one at random to migrate to
    if(length(adjacent_copy) > 1) {
      new_block <- sample(adjacent_copy, 1)
    }
    if(length(adjacent_copy) == 1) {
      new_block <- adjacent_copy
    }

    # if new block is a previous block and not in temp range, move north
    if(new_block == prev_block) {
      if(!(10.5 <= current_block_temp & current_block_temp <= 11.5)
                                                    && length(ncopy) > 0) {
        if(length(ncopy) > 1) {
          new_block <- sample(ncopy, 1)
        } else {
          new_block <- ncopy
        }
      }
    }

    prob_v <- c(0.5, rep(0.5, times=length(stoc_a)))
    new_block <- sample(c(new_block, stoc_a), size=1, prob=prob_v)


    # increment year
    current_year <- current_year + 1
    prev_block <- current_block
    current_block <- new_block

    # migration_pat <- c(migration_pat, sst[[new_block, 1]])
    best_case_herr_df[i, j] <- sst[[new_block, 1]]
    j <- j + 1
  }

}
```

```
write.csv(best_case_herr_df, file = "bc_herring.csv")
#write.csv(year_out, file = "bch_years.csv")
write.csv(year_out, file = "bch_years.csv")
```

Here is the program we used to compute fish density under each strategy in Part 6.

Input R source:

```
#*****************************************************************************
# Computing Fish Density q USING OLD STARTING POINT
# Dataset:
#       "forecast_sst_50.csv"       2020—2069
#       "bc_herring.csv"            end coordinates of best case herring
#       "bc_mackerel.csv"           end coordinates of best case mackerel
# Author: Hao Jin
#*****************************************************************************

library(tidyverse)
library('plot.matrix')
library(stringr)
library("viridis")
library(RColorBrewer)

forecast = read.csv("forecast_sst_50_t.csv", row.names = 1)
bc_herr = read_csv("bc_herring.csv", col_names = TRUE)
bc_mack = read_csv("bc_mackerel.csv", col_names = TRUE)

# Current fishing port, old fishing boat covers
# 95,  96,  97,
# 115, 116, 117,
#      136, 137
#           157

# Current fishing port, new fishing boat covers
# 53, 54, 55, 56, 57, 58, 59
# 73, 74, 75, 76, 77, 78, 79
#     94, 95, 96, 97, 98, 99
#         115,116,117,118,119
#             136,137,138,139
#                 157,158,159
#                     178,179
#                         199

# Move south, old fishing boat covers
# 137,138,139
# 157,158,159
#     178,179
#         199

# Move south, old fishing boat covers
# 74, 75, 76
# 94, 95, 96
# 114,115,116
#         136
```

```
# ---------------------------------- Herring ---------------------------------- #

years_of_interest = seq(2019, 2069)
years_col = years_of_interest - 2019 + 2

current_herr = numeric(51)
new_boat_herr = numeric(51)
move_south_herr = numeric(51)
move_north_herr = numeric(51)

for (n in years_col){
    # Create empty vectors to store location (lat,lon) info
    latitude <- numeric(100)
    longitude <- numeric(100)

    for (i in seq_len(100)) {
        xy <- str_split(bc_herr[i,n], " ")
        latitude[i] <- as.numeric(xy[[1]][1])
        longitude[i] <- as.numeric(xy[[1]][2])
    }

    # Create empty vectors to store location (row,col) info
    row_num <- numeric(100)
    col_num <- numeric(100)

    for (i in seq_len(100)) {
        row_num[i] = 62.5 - latitude[i] + 1
        col_num[i] = longitude[i] + 16.5 + 1
    }

    # Create empty vectors to store location (block number) info

    # Up to down, left to right
    # First block: 62.5, -16.5
    # 300th block: 48.5, 2.5
    num <- numeric(100)

    for (i in seq_len(100)) {
        num[i] = 20 * row_num[i] + col_num[i]
    }

    table(num)

    # Create empty vectors to count ending block number
    counter = numeric(300)

    # Mark Land
    for (i in seq_len(300)) {
        if(is.na(forecast[i,11]))
        counter[i] = NA
    }
```

```
    for (block in num) {
        counter[block] = counter[block] + 1
    }

    current_herr[n−1] = sum(counter[c(  95, 96, 97,
                                        115,116,117,
                                            136,137,
                                                157)])

    new_boat_herr[n−1] = sum(counter[c( 53, 54, 55, 56, 57, 58, 59,
                                        73, 74, 75, 76, 77, 78, 79,
                                            94, 95, 96, 97, 98, 99,
                                                115,116,117,118,119,
                                                    136,137,138,139,
                                                        157,158,159,
                                                            178,179,
                                                                199)])

    move_south_herr[n−1] = sum(counter[c( 137,138,139,
                                          157,158,159,
                                              178,179,
                                                  199)])


    move_north_herr[n−1] = sum(counter[c( 74, 75, 76,
                                          94, 95, 96,
                                              115,116,
                                                  136)])
}

current_herr
new_boat_herr
move_south_herr
move_north_herr

q_herr = data.frame("current" = current_herr,
                    "new boat" = new_boat_herr,
                    "move north" = move_south_herr,
                    "move_south" = move_north_herr)

rownames(q_herr) = seq(2019,2069)

write.csv(q_herr, "q_herring.csv")
```

Here is the program we used to compute profit trend of each strategy in Part 6.

Input C++ header file:

```
//
//  ModelA.h
//  Math42_Model3
//
//  Created by Christine Mika Hamakawa on 6/12/20.
//  Copyright 2020 Christine Mika Hamakawa. All rights reserved.
```

```
//

#ifndef Model_h
#define Model_h
#include <iostream>
#include <vector>
using namespace std;

class Model {
public:

    // modelA constructor
    Model(double c0, vector<double> herring, vector <double> mackerel,
            double input_k): h(herring),m(mackerel) {
        c_0 = c0;
        k = input_k;
        //no relocation year because no relocation
    }

    // modelB contructor
    Model(double c0,double input_a,vector<double> herring, vector <double> mackerel,
            vector<double> h0, vector<double>m0, double input_k,double input_X):
                h(herring),m(mackerel),h_0(h0),m_0(m0){
        c_0 = c0;
        a = input_a;
        k = input_k;
        X = input_X; // relocation year
    }


    // writes year and profit to file
    void data(string filename) {
        ofstream output;
        output.open(filename);
        for(int i = 0; i<years; i++) {
            double profit = 0;
            if(i<=X) {
                profit = totalRevenue(h[i], m[i]) - totalCost();
                cout<<h[i]<<endl;
            }
            else {
                profit = totalRevenue(h_0[i], m_0[i])-totalCost();
            }
            output<<init_year+i<<","<<profit<<endl;
        }
        output.close();

    }

    // calculates annual total cost
    double totalCost() {
        double cost1 = 0;
        if(X==51) {
            cost1 = 0;
```

```
        }
        else {
            cost1 = a;
        }
        double cost2 = c_0;

        return (cost1+cost2);

    }
    // calculates annual revenue
    double totalRevenue(double q_h, double q_m) {
        return (k*((h_price*q_h) + (m_price*q_m)));
    }


private:
    //fish prices
    double h_price = .5;
    double m_price = .5;

    double k = 0;            // number of times fish per year
    const int years = 51;    // number of years (size of vector, includes 0)
    const int init_year = 2019; //initial year (year 0)
    double X = 51;           // relocation year (in respects to 2019)

    // fixed costs
    double c_0 = 0; // boat
    double a = 0;   // relocation

    // q vectors for herring and mackerel
    vector<double> h;
    vector<double> m;

    // relocation: q vectors for herring and mackerel after relocation
    vector<double> h_0;
    vector<double> m_0;


};

#endif /* ModelA_h */
```

Input C++ source:

```
#include <iostream>
#include <fstream>
#include "Model.h"
#include <sstream>
#include <vector>
using namespace std;


// function updates vectors
void createVectors(vector<double>& a,vector<double>&b,
```

```
    vector<double>&c, vector<double>&d, string filename);

int main(){

    // create herring vectors
    vector<double> h_current;
    vector<double> h_newBoat;
    vector<double> h_moveNorth;
    vector<double> h_moveSouth;
    createVectors(h_current, h_newBoat, h_moveNorth,
        h_moveSouth, "q_herring_int_water.csv");

    // create mackerel vectors
    vector<double> m_current;
    vector<double> m_newBoat;
    vector<double> m_moveNorth;
    vector<double> m_moveSouth;
    createVectors(m_current, m_newBoat, m_moveNorth,
        m_moveSouth, "q_mackerel_int_water.csv");

    // k value:  86
    // boats fish 5 days out of seven each week on average
    // boats can go up to three days w/o refrigeration
    // k = 5*52/3
    double old_k = 1;

    // k value:  43
    // boats fish 5 days out of seven each week on average
    // boats can go up to six days w/o refrigeration
    // k = 5*52/6
    double new_k = .5;

    //fixed costs
    double boat = .93, boat_fridge = .96, port = .07;


    // current model
    Model current(boat,h_current,m_current, old_k);
    current.data("current.csv");

    // fridge model
    Model newBoat(boat_fridge, h_newBoat,m_newBoat, new_k);
        newBoat.data("newBoat_int_water.csv");


    // relocation model
    double relocation_year1 = 2;
    Model moveNorth1(boat,port,h_current,m_current,h_moveNorth, m_moveNorth,
        old_k, relocation_year1); moveNorth1.data("moveNorth2022.csv");
    // before the density becomes too small
    // bar with highest density 2020


    // relocation model
```

```
    double relocation_year2 = 7;
    Model moveNorth2(boat,port,h_current,m_current,h_moveNorth, m_moveNorth,
        old_k, relocation_year2); moveNorth2.data("moveNorth2027.csv");
    // before the density becomes too small
    // bar with highest density 2020


    // relocation model
    double relocation_year3 = 14;
    Model moveNorth3(boat,port,h_current,m_current,h_moveNorth, m_moveNorth,
        old_k, relocation_year3); moveNorth3.data("moveNorth2034.csv");
    // before the density becomes too small
    // bar with highest density 2020

    return 0;
}

// function updates vectors
void createVectors(vector<double>& a,vector<double>&b, vector<double>&c,
                        vector<double>&d, string filename) {
    ifstream input;
    input.open(filename);
    int count = 0;

    while(!input.eof()) {
        string s;
        getline(input, s);
        if(count==0) {
            count++;
            continue;
        }
        else if(count==52) {
            continue;
        }
        else {
            istringstream instr(s);
            double d1, d2, d3, d4, d5;
            char c1, c2, c3, c4, c5,c6;
            instr>>c1>>d1>>c2>>c6>>d2>>c3>>d3>>c4>>d4>>c5>>d5;
            //cout<<d1<<endl;
            a.push_back(d2*1.0/100);
            b.push_back(d3*1.0/100);
            c.push_back(d4*1.0/100);
            d.push_back(d5*1.0/100);
            count++;
        }
    }
    input.close();
}
```