

ANOVA FINAL REVIEW

t-test

One-Way ANOVA

Factorial ANOVA

Repeated Measures ANOVA

Mixed Factorial ANOVA

BRIEF REVIEW OF T-TESTS

WHEN TO USE WHAT?

- One IV (2 levels) and one DV
- **Independent Samples t test**
 - Trying to compare the difference between two independent samples (e.g., moose and deer).
- **Dependent Samples t test**
 - Trying to compare the difference between two related samples (e.g., drivers before and after a race).

ASSUMPTIONS

- DV is continuous and normally distributed
- IV is categorical
 - Independent - 2 independent groups (ex: teams, treatment/control, etc.)
 - Dependent - 2 related groups (ex: Time 1 and Time 2)
- Independence of observations
- Homogeneity of variance (indep. t test)

HOMOGENEITY OF VARIANCE

- What is it?
 - Equal variance of dependent variable in each group.
- How do we test for it?
 - Levene's Test.
- What do we do if it's violated?
 - Welch's procedure.
 - Adjusts the p-value and df
 - Could also use non-parametric tests (...to be continued in 4 months...)

EFFECT SIZE

- What is it?
 - Cohen's D
 - .3 (sm) - .5 (mod) - .8(lg)
 - Cohen's D
 - Dependent t-test: $d = M_d / S_{Dd}$
 - Independent t-test: $d = (M_1 - M_2) / S_{Dpooled}$
 - $S_{Dpooled} = (SD_1 - SD_2)/2$
 - What does it mean?
 - The mean difference between two groups in standard deviation units.

INDEPENDENT SAMPLES EXAMPLE

- Two famous bakers like to bake baguettes. Bon Appetit Magazine wanted to know if one bakery baked more baguettes per day than other. Data was collected over the course of one week between Dominique Ansel Bakery and Ken's Artisan.
- IV: Bakery (A = Dominique Ansel Bakery, B = Ken's Artisan)
- DV: Mean daily loaves baked per day

Descriptive statistics by group

group: A

	vars	n	mean	sd	median	trimmed	mad	min	max
Baker	1	40	20.50	11.69	20.50	20.50	14.83	1.00	40.00
Group*	2	40	1.00	0.00	1.00	1.00	0.00	1.00	1.00
Loaves	3	40	28.41	5.28	28.12	28.17	5.00	17.21	40.61
	range	skew	kurtosis	se					
Baker	39.0	0.0	-1.29	1.85					
Group*	0.0	NaN	NaN	0.00					
Loaves	23.4	0.3	-0.31	0.83					

group: B

	vars	n	mean	sd	median	trimmed	mad	min	max
Baker	1	40	60.50	11.69	60.50	60.50	14.83	41.00	80.00
Group*	2	40	2.00	0.00	2.00	2.00	0.00	2.00	2.00
Loaves	3	40	38.12	8.42	37.79	38.14	6.66	17.92	67.33
	range	skew	kurtosis	se					
Baker	39.00	0.00	-1.29	1.85					
Group*	0.00	NaN	NaN	0.00					
Loaves	49.41	0.53	2.28	1.33					

INDEPENDENT SAMPLES T-TEST

Independent Samples T-Test

			statistic	df	p	Cohen's d	Lower	Upper
	Loaves	Student's t	-6.18	78.0	< .001	-1.38	-12.8	-6.58

GIVE A TWO-SENTENCE INTERPRETATION

DEPENDENT SAMPLES EXAMPLE

- A new volunteer organization wants to know if their Daily Encouraging Messages program had an effect on their volunteers desire to help others. The organization sent two texts per day, one in the morning and one in evening. Desire to help others was measured before they started a new project and after the project was over.
- IV: Time (Pre, Post)
- DV: Desire to help others (10-point Likert scale; 1 = Not at all, 10 = Great desire)

	vars	n	mean	sd	median	trimmed	mad	min
Volunteer	1	50	25.50	14.58	25.50	25.50	18.53	1.00
Desire_Pre	2	50	5.00	2.32	4.99	4.98	2.25	0.38
Desire_Post	3	50	7.85	0.89	7.75	7.84	1.10	5.95
diff	4	50	2.85	2.59	2.95	2.91	3.02	-2.19

	max	range	skew	kurtosis	se	
Volunteer	50.00	49.00	0.00	-1.27	2.06	PAIRED SAMPLES T-TEST
Desire_Pre	10.00	9.62	0.03	-0.53	0.33	
Desire_Post	10.00	4.05	0.19	-0.61	0.13	Paired Samples T-Test
diff	7.62	9.81	-0.15	-0.88	0.37	

				statistic	df	p	Cohen's d	Lower	Upper
	Desire_Pre	Desire_Post	Student's t	-7.79	49.0	< .001	-1.10	-3.59	-2.12

GIVE A TWO-SENTENCE INTERPRETATION

ANOVA

What is an ANOVA and why do we use it?

- An inferential statistic method to reach conclusions about a population, based on a sample set of data.
- A method used to determine whether there are any statistically significant differences between the means of three or more groups.
- A type of analysis that analyzes VARIANCE in order to make inferences about means.
- A type of analysis that uses the F distribution
- A null hypothesis significance test

ANOVA: Under the Hood

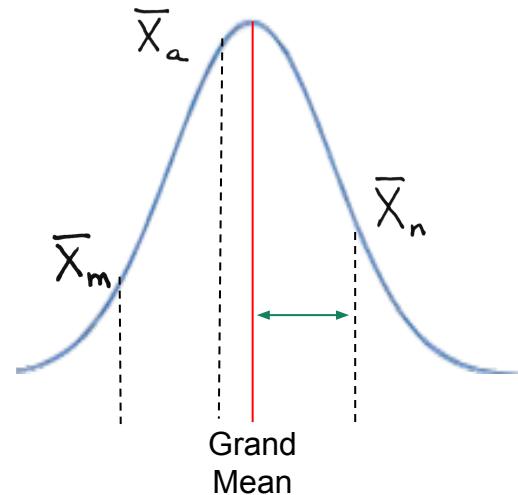
- But what does it actually do?
 - Tests the null hypothesis that the groups are drawn from populations with the same mean values.
- How does it do that?
 - It estimates two different kinds of variance and compares them

One-way ANOVA

What is F: Sums of Squares

F =

Treatment
(Between Group Variance)



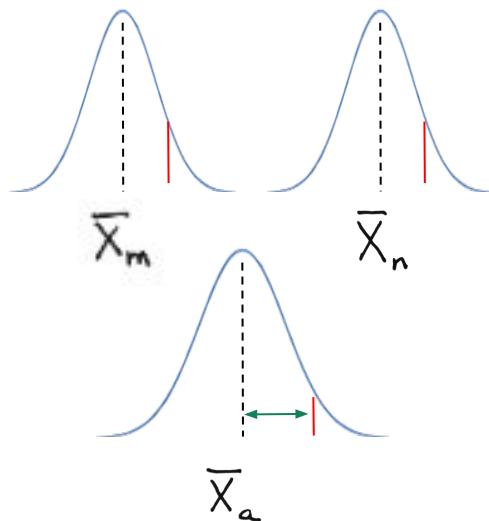
Sums of Squares Between

How much do the group (j) means vary around the grand (T) mean?

÷
Degrees of Freedom ($k-1$)

= **Mean Squares Between**
Estimate of the population variance based on how the group means vary.

Error
(Within Group Variance)



Sums of Squares Error (Within)

How much does each observation (i) vary around its group (j) mean?

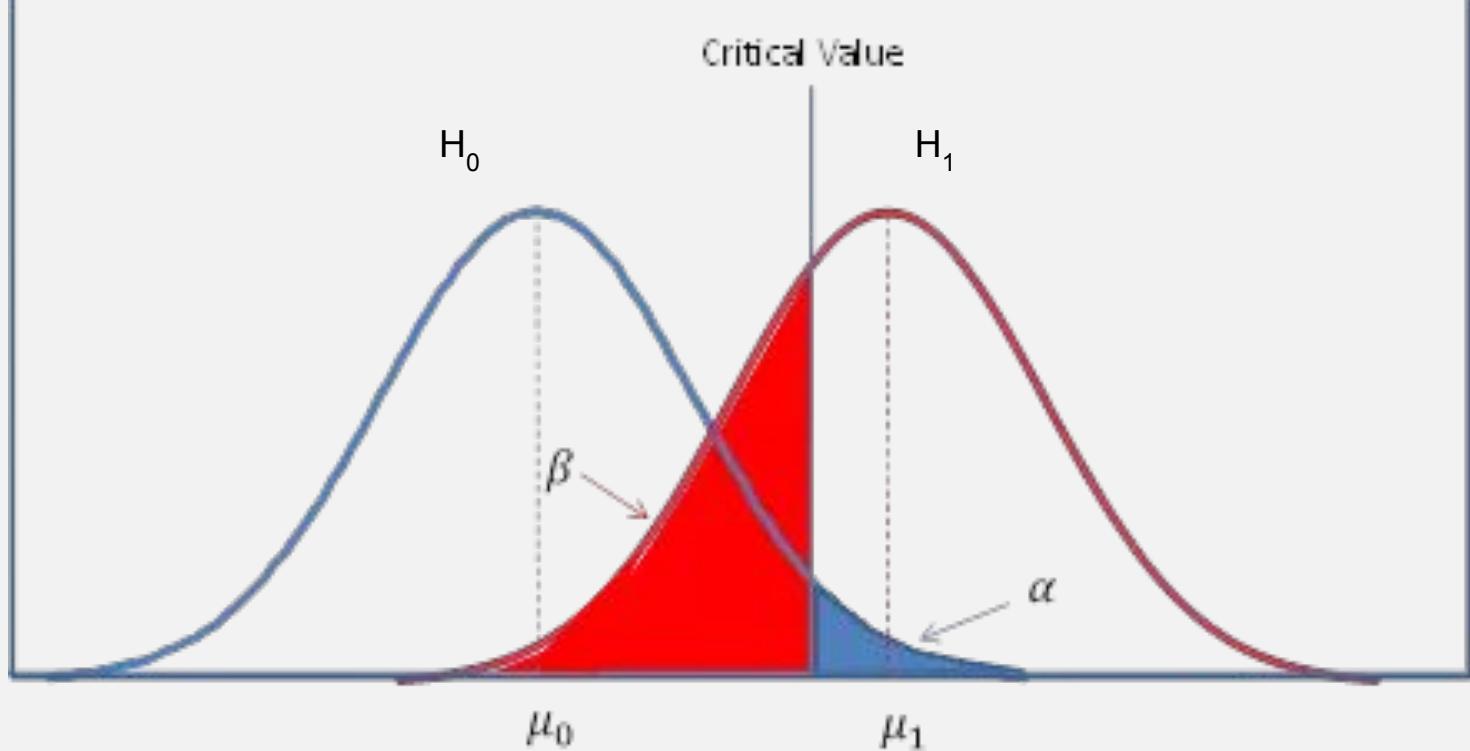
÷
Degrees of Freedom $k(n-1)$ or $N-k$

= **Mean Squares Error (Within)**
Estimate of population variance based on variances of each group

NHST: A Brief Review

		DECISION	
		Reject H_0	Fail to Reject H_0
ACTUAL	H_0 True	Type I Error <i>Producer Risk</i> α -Risk False Positive	Correct Decision Confidence Interval = $1 - \alpha$
	H_a True	Correct Decision Power = $1 - \beta$	Type II Error <i>Consumer Risk</i> β -Risk False Negative

H_0 : Null Hypothesis H_a : Alternative Hypothesis



~~-ANOVA Assumptions-~~

- Continuous dependent variable
- Categorical independent variables
- Independence of Observations

ANOVA Methodological Prerequisites

- Continuous dependent variable
- Categorical independent variables
- Independence of Observations

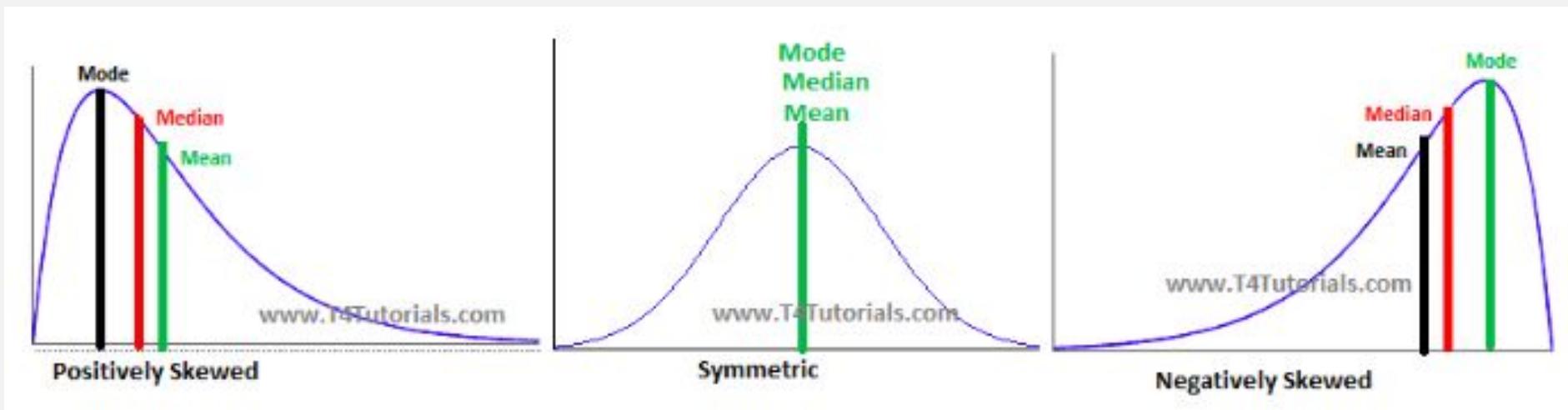
ANOVA Assumptions

NORMALITY

Data is drawn from populations that are normally distributed



WHO CARES: ANOVA is a test that compares sample MEANS, assuming they are the best representation of that population (i.e. the best measure of central tendency). If the data is not normal, this is not the case.



ANOVA Assumptions

HOMOGENEITY OF VARIANCE

Data is drawn from populations that have equivalent variances



WHO CARES: Unequal variance can distort the shape of the F-distribution such that the critical F-value no longer corresponds to your chosen cutoff point (.05). For EX, you may report $p < .05$ when really the significance level is .10 -- Type I error

ANOVA Assumptions

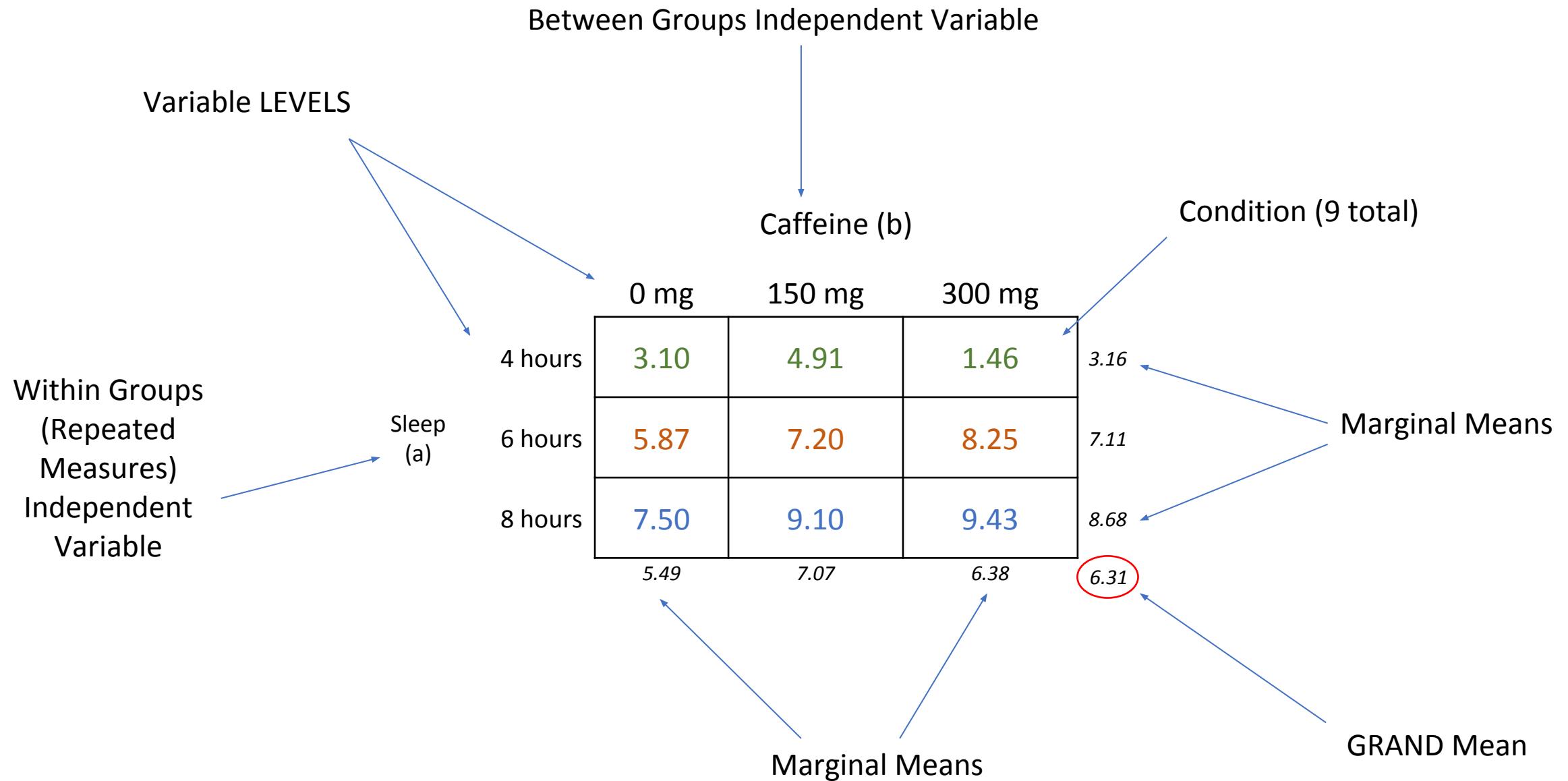
Assumptions	One Way Between Groups ANOVA	Repeated Measures ANOVA
Normality	<ul style="list-style-type: none">How to Test: Histograms and skew/kurtosis of DVIf Violated: Transformations and non-parametric	<ul style="list-style-type: none">How to Test: Histograms and skew/kurtosis values of difference scoresIf Violated: Transformations or non-parametric
Homogeneity of Variance	<ul style="list-style-type: none">How to Test: Levene's TestIf Violated: Cry (Welch's Correction for ANOVAor nonparametric)	Sphericity: Homogeneity of variance of the difference scores <ul style="list-style-type: none">How to Test: Mauchly's Test of SphericityIf Violated: Sphericity Correction (e.g. G-G)
Assumptions	Factorial ANOVA	Mixed Factorial ANOVA
Normality	<ul style="list-style-type: none">How to Test: Histograms and skew/kurtosis of DVIf Violated: Transformations and non-parametric	<ul style="list-style-type: none">How to Test: Histograms and skew/kurtosis values of DV and difference scoresIf Violated: Transformations and non-parametric
Homogeneity of Variance	<ul style="list-style-type: none">How to Test: Levene's TestIf Violated: Cry (Welch's Correction for ANOVAor nonparametric)	<ul style="list-style-type: none">How to Test: Levene's Test (on between); Mauchly's Test of Sphericity (on within)If Violated: Welch's correction and/or sphericity correction

Research Question

One of the Western medical school students who always steals my spot in the library is interested in studying the effect of caffeine and sleep on task performance.

- DV: Task Performance (1-10)
- IV: Caffeine (0 mg, 150 mg, 300 mg)
- IV: Sleep (4 hours, 6 hours, 8 hours)

ANOVA TABLE VOCABULARY

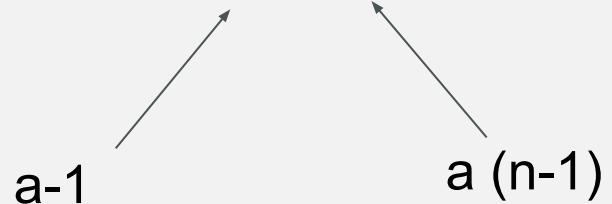


ONE-WAY ANOVA

One Way ANOVA

Caffeine (a)		
0 mg	150 mg	300 mg
5.49	7.07	6.38
		6.31

$$F (\underline{\quad}, \underline{\quad}) = 10.53, p = .025$$



Variables:

- DV: Task Performance (1-10)
- IV: Caffeine (0 mg, 150 mg, 300 mg)

Methods:

- Between groups design
- n = 20

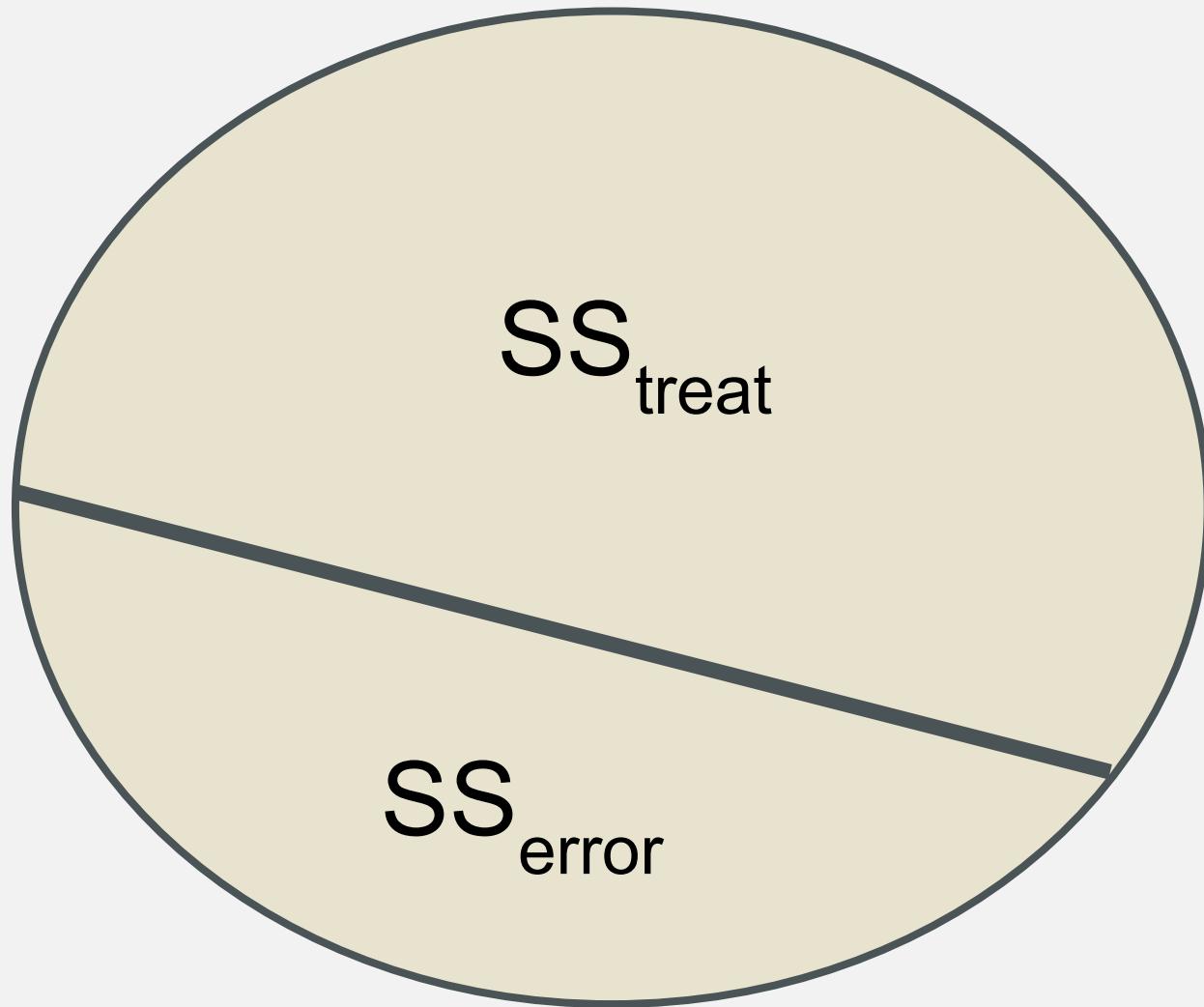
Research Question: Does amount of caffeine have an effect on task performance?

Source	SS	df	MS	F
treat	$n \sum(x_j - \bar{x}_T)^2$	$k - 1$	$\frac{SS_{treat}}{df_{treat}}$	$\frac{MS_{treat}}{MS_{error}}$
error	$\sum(x_{ij} - \bar{x}_j)^2$	$k(n - 1)$	$\frac{SS_{error}}{df_{error}}$	
Total	$\sum(x_{ij} - \bar{x}_T)^2$	$N - 1$		

One Way ANOVA: Effect Size

Effect Size of treatment (A):

$$\eta_p^2 = \frac{SS_{\text{treatment}}}{SS_{\text{treatment}} + SS_{\text{Error}}}$$



Example

You are investigating the effects of playing sports in high school on self-esteem.

DV: Self-Esteem (0 - 50)

IV: Number of Sports Played (Zero, One, Several)

Descriptives

Descriptives		
	Sports	self_esteem
N	Many	30
	None	30
	One	30
Missing	Many	0
	None	0
	One	0
Mean	Many	36.3
	None	32.1
	One	35.0

ANOVA

ANOVA

	Sum of Squares	df	Mean Square	F	p	$\eta^2 p$
Sports	267	2	133.42	40.2	< .001	0.480
Residuals	289	87	3.32			

Post Hocs

Post Hoc Comparisons - Sports

Sports	Sports	Mean Difference	SE	df	t	p-tukey
Many	- None	4.13	0.470	87.0	8.78	< .001
	- One	1.31	0.470	87.0	2.78	0.018
None	- One	-2.82	0.470	87.0	-5.99	< .001

What can we conclude?

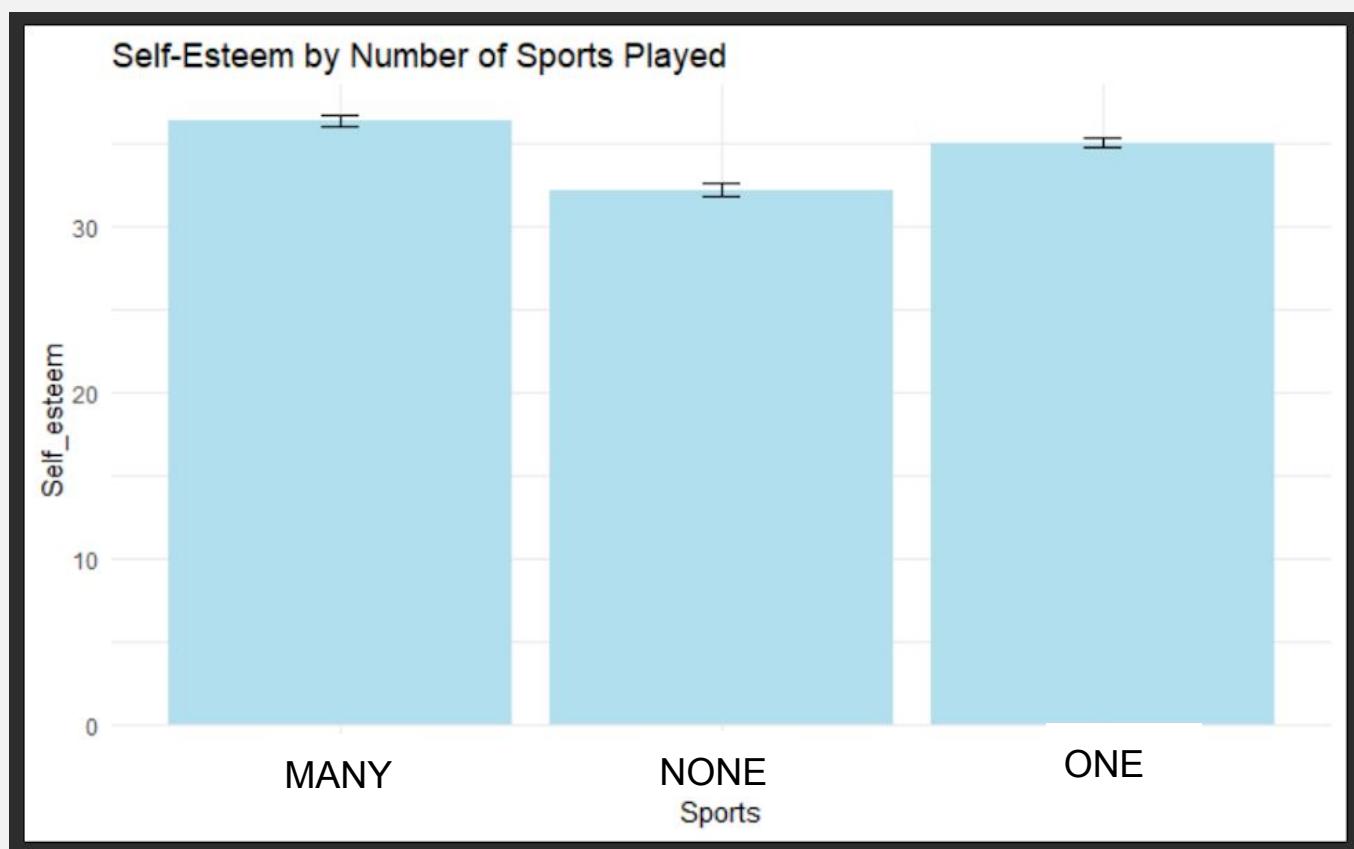
ANOVA: $F(2, 87) = 40.20, p < .001, \eta^2p = .48$

Tukey's HSD:

$$M_{\text{many}} - M_{\text{none}} = 4.13, p < .001$$

$$M_{\text{many}} - M_{\text{one}} = 1.31, p = .018$$

$$M_{\text{none}} - M_{\text{one}} = -2.82, p < .001$$



FACTORIAL ANOVA

Factorial ANOVA

		Caffeine (b)		
		0 mg	150 mg	300 mg
Sleep (a)	4 hours	3.10	4.91	1.46
	6 hours	5.87	7.20	8.25
	8 hours	7.50	9.10	9.43
		5.49	7.07	6.38

Main Effect of Sleep

$$F (\underline{2}, \underline{171}) = 10.53, p = .025$$

a-1 ab (n-1)

Main Effect of Caffeine

$$F (\underline{2}, \underline{171}) = 10.53, p = .025$$

b-1 ab (n-1)

Variables:

- DV: Task Performance (1-10)
- IV: Caffeine (0 mg, 150 mg, 300 mg)
- IV: Sleep (4 hours, 6 hours, 8 hours)

Methods:

- Between groups design
- n = 20

Research Question: Does amount of caffeine and/or amount of sleep have an effect on task performance?

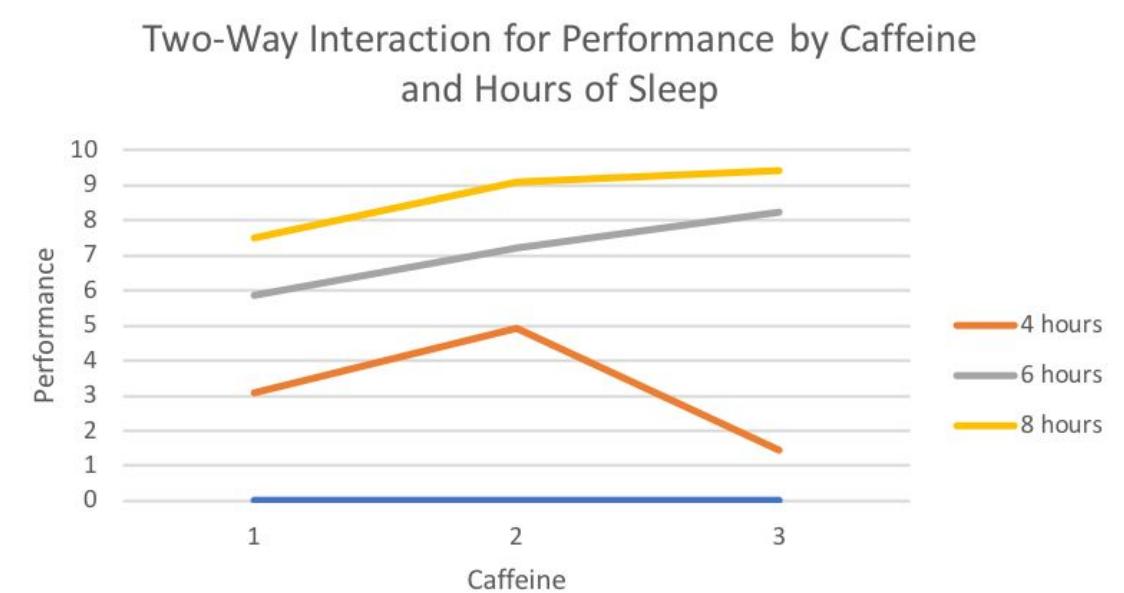
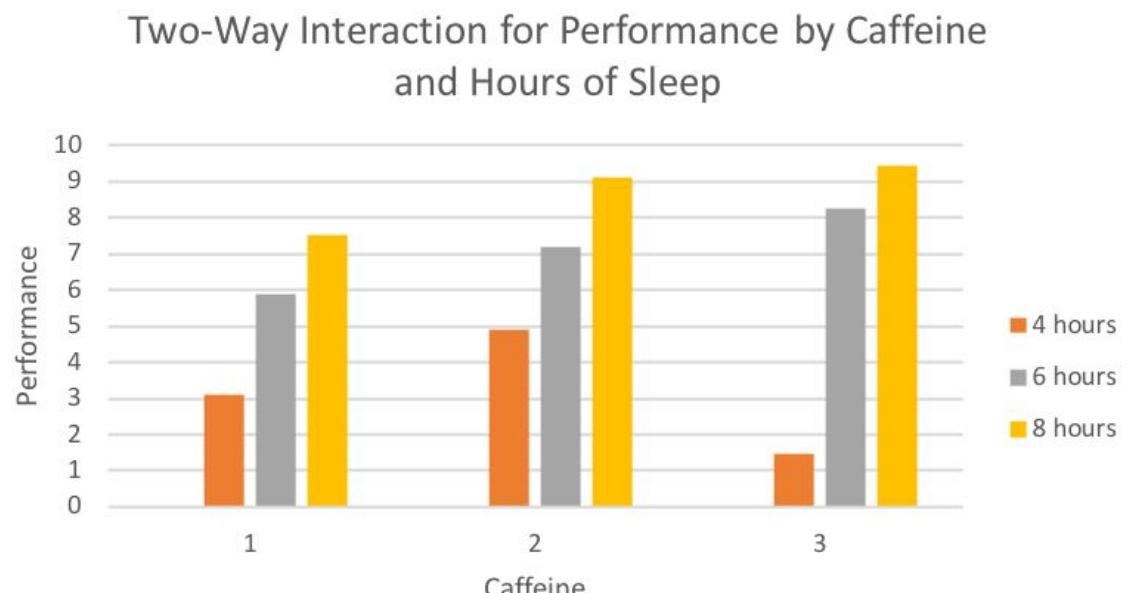
Interaction Between Sleep & Caffeine

$$F (\underline{4}, \underline{171}) = 10.53, p = .025$$

(a-1)(b-1) ab (n-1)

Factorial ANOVA

		Caffeine (b)		
		0 mg	150 mg	300 mg
Sleep (a)	4 hours	3.10	4.91	1.46
	6 hours	5.87	7.20	8.25
	8 hours	7.50	9.10	9.43
		5.49	7.07	6.38
		3.16	7.11	8.68
		6.31		

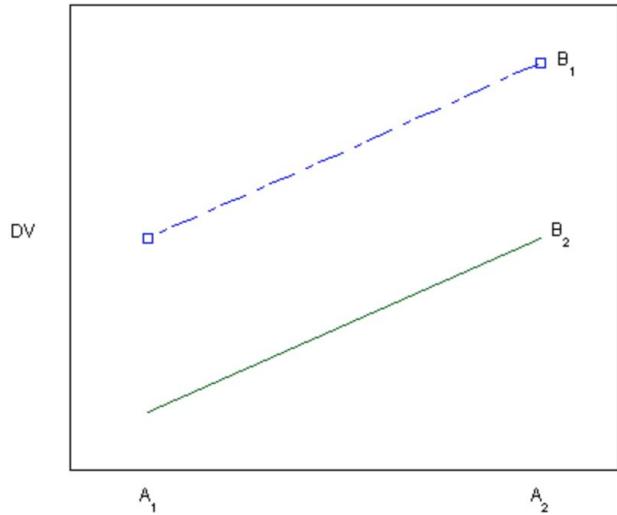


Types of Interactions

Plot 3

Main Effect of A -- Yes
Main Effect of B -- Yes
Interaction -- No

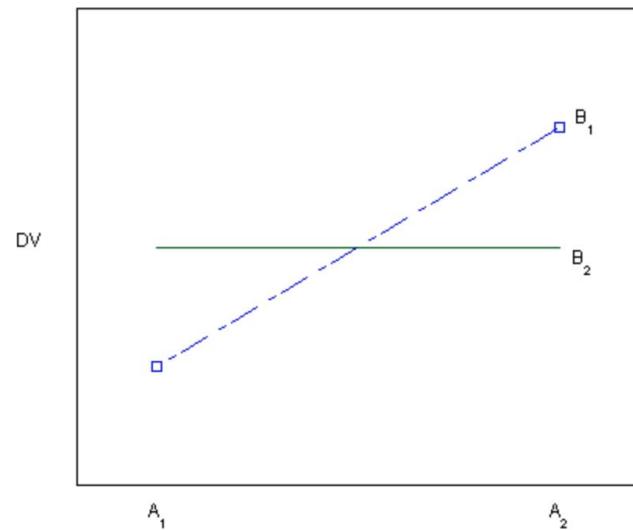
[View as Bar Graph](#)



Plot 5

Main Effect of A -- Yes
Main Effect of B -- No
Interaction -- Yes

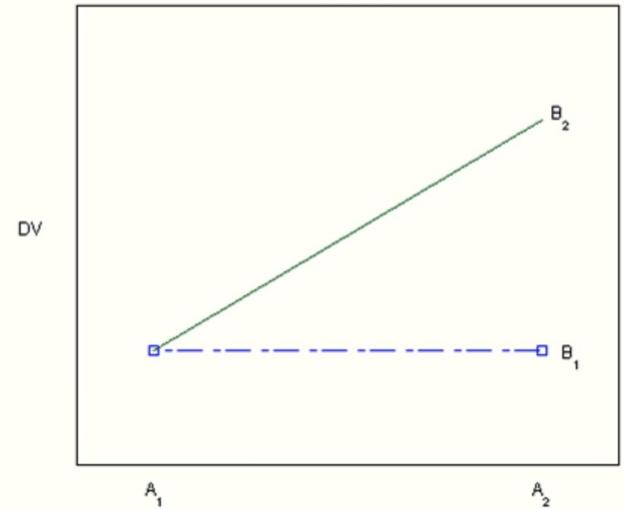
[View as Bar Graph](#)



Plot 7

Main Effect of A -- Yes
Main Effect of B -- Yes
Interaction -- Yes

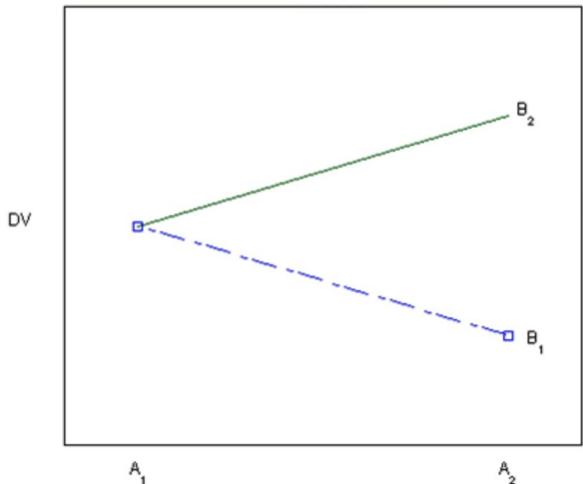
[View as Bar Graph](#)



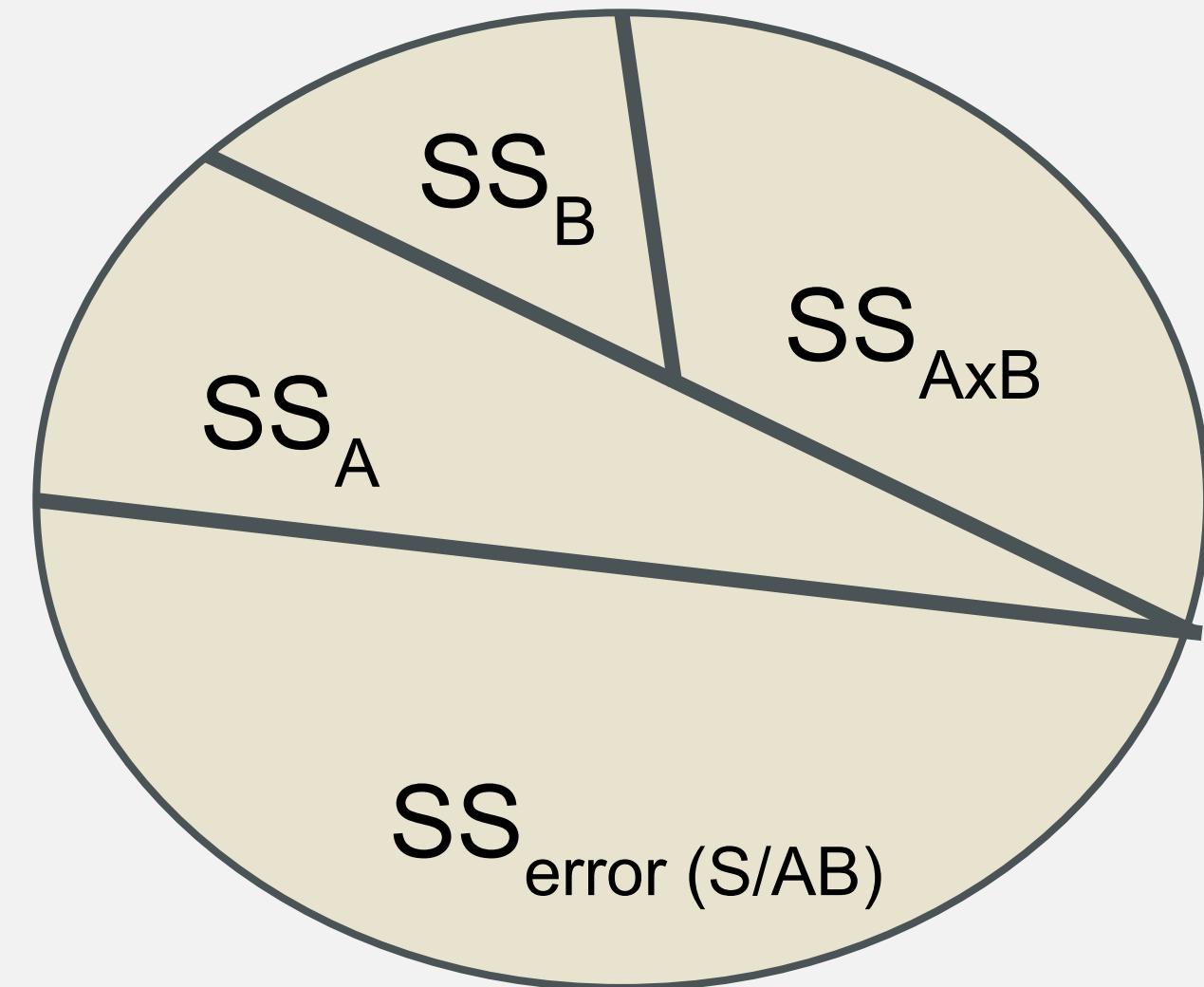
Plot 6

Main Effect of A -- No
Main Effect of B -- Yes
Interaction -- Yes

[View as Bar Graph](#)



Source	SS	df	MS	F
Factor A	$bn \sum (x_{A_j} - x_T)^2$	a - 1	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{error}}$
Factor B	$an \sum (x_{B_k} - x_T)^2$	b - 1	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_{error}}$
Interaction	$SS_{total} - SS_{error} - SS_A - SS_B$	(a - 1)(b - 1)	$\frac{SS_{AXB}}{df_{AXB}}$	$\frac{MS_{AXB}}{MS_{error}}$
Error (S/AB)	$\sum (x_{ijk} - x_{AB_{jk}})^2$	ab(n - 1)	$\frac{SS_{error}}{df_{error}}$	
Total	$\sum (x_{ijk} - x_T)^2$	N - 1		



Effect Size of A (main effect):

$$\eta_p^2 = (SS_A)/(SS_A + SS_{\text{Error}})$$

Effect Size of B (main effect):

$$\eta_p^2 = (SS_B)/(SS_B + SS_{\text{Error}})$$

Effect Size of AxB (interaction):

$$\eta_p^2 = (SS_{AxB})/(SS_{AxB} + SS_{\text{Error}})$$

WHEN DO WE USE THIS?

Conditions:

- At least two independent variables.
 - Categorical.
 - Two or more levels for each variable.
- One dependent variable.
 - Continuous.

Reason:

- Wanting to examine differences between two given conditions on a measured variable. Differences include both main and interactive effects.

ASSUMPTIONS

Statistical:

- Normal distribution of dependent variable by group.
 - Assessed via skew (< abs. value of 3) and kurtosis (< abs. value of 10).
 - Histogram (visual check).
- Homogeneity of variance.
 - Comparison of the variances for each group on the dependent variable yields a non-significant difference.
 - Similar procedure to one-way ANOVA - Levene's, etc.

EFFECT SIZE

Previously, we would use eta-squared in one-way ANOVA - however, since we are working with both main effects and interactions...

...We will use partial eta-squared!

Considering that we want to find the isolated effects of the given variable (or the interaction between two), we change our formula and approach:

For the main effect of A: $SS_a / SS_a + SS_{error}$

For the main effect of B: $SS_b / SS_b + SS_{error}$

For the interaction effect (A*B): $SS_{ab} / SS_{ab} + SS_{error}$

EXAMPLE

A researcher is interested in the variables may affect an individual's evaluation of the quality of a potential job applicant for an entry-level job.

For an upcoming study, the researcher manipulates both the applicant's **educational history** (*associates, bachelors, graduate degree*) **and sex** (*female, male*).

Participants were told that the job was at a local telemarketing firm and the position would require fielding phone calls about products and working with client accounts. All participants viewed the same application and were randomly assigned to one of six conditions.

Following review of the application, participants rated the applicant on **perceived ability** to excel at the position on a 1-10 scale (1 = *No ability*, 10 = *Superior ability*), among other measures.

RESULTS

ANOVA

ANOVA

	Sum of Squares	df	Mean Square	F	p	η^2p
Sex	1.41	1	1.408	1.43	0.234	0.012
Edu	76.05	2	38.025	38.65	<.001	0.404
Sex * Edu	103.72	2	51.858	52.71	<.001	0.480
Residuals	112.15	114	0.984			

Assumption Checks

Test for Homogeneity of Variances (Levene's)

F	df1	df2	p
1.90	5	114	0.100

ANOVA

	Sum of Squares	df	Mean Square	F	p	η^2
Edu	97.2	2	48.617	49.8	< .001	0.636
Residuals	55.7	57	0.977			

POST HOC TESTS

Post Hoc Comparisons - Edu

Edu	Edu	Mean Difference	SE	df	t	p-tukey
Assoc	- Bachelor	2.350	0.313	57.0	7.52	< .001
	- Grad	2.950	0.313	57.0	9.44	< .001
Bachelor	- Grad	0.600	0.313	57.0	1.92	0.143

FEMALE



ANOVA

	Sum of Squares	df	Mean Square	F	p	η^2
Edu	82.5	2	41.267	41.7	< .001	0.594
Residuals	56.5	57	0.990			

MALE



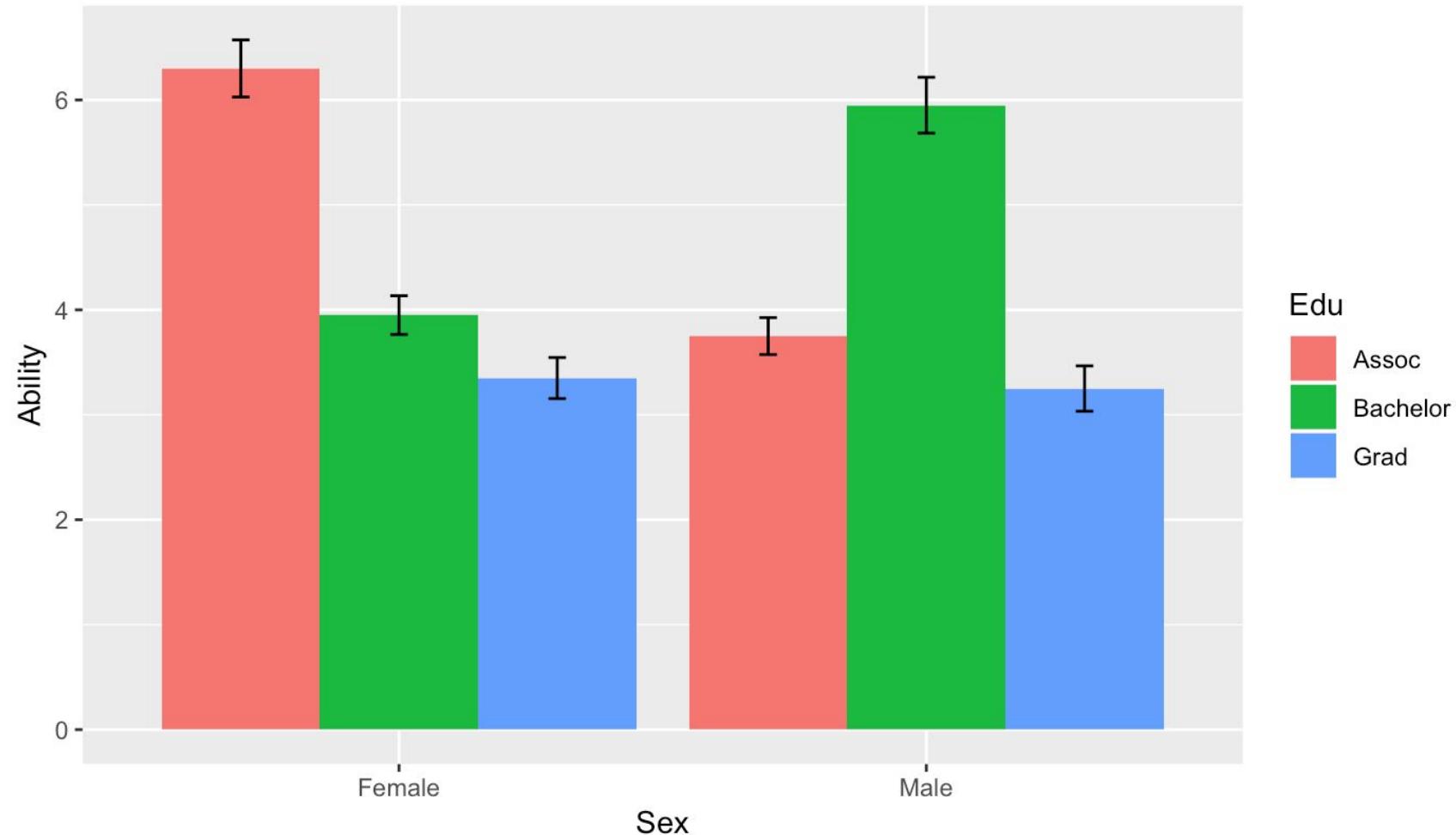
POST HOC TESTS

Post Hoc Comparisons - Edu

Edu	Edu	Mean Difference	SE	df	t	p-tukey
Assoc	- Bachelor	-2.200	0.315	57.0	-6.99	< .001
	- Grad	0.500	0.315	57.0	1.59	0.259
Bachelor	- Grad	2.700	0.315	57.0	8.58	< .001

RESULTS - CONT.: VISUALIZATION

Two-Way Interaction Graph of Perceived Ability for Sex and Education



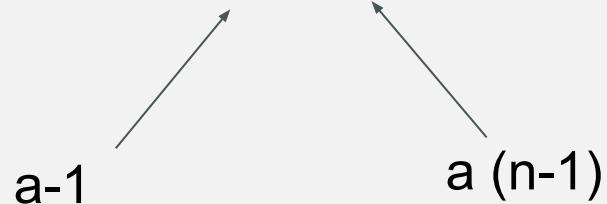
REPEATED MEASURES ANOVA

Repeated Measures ANOVA

Caffeine (a)

0 mg	150 mg	300 mg	
5.49	7.07	6.38	6.31

$$F (\underline{2}, \underline{57}) = 10.53, p = .025$$



Variables:

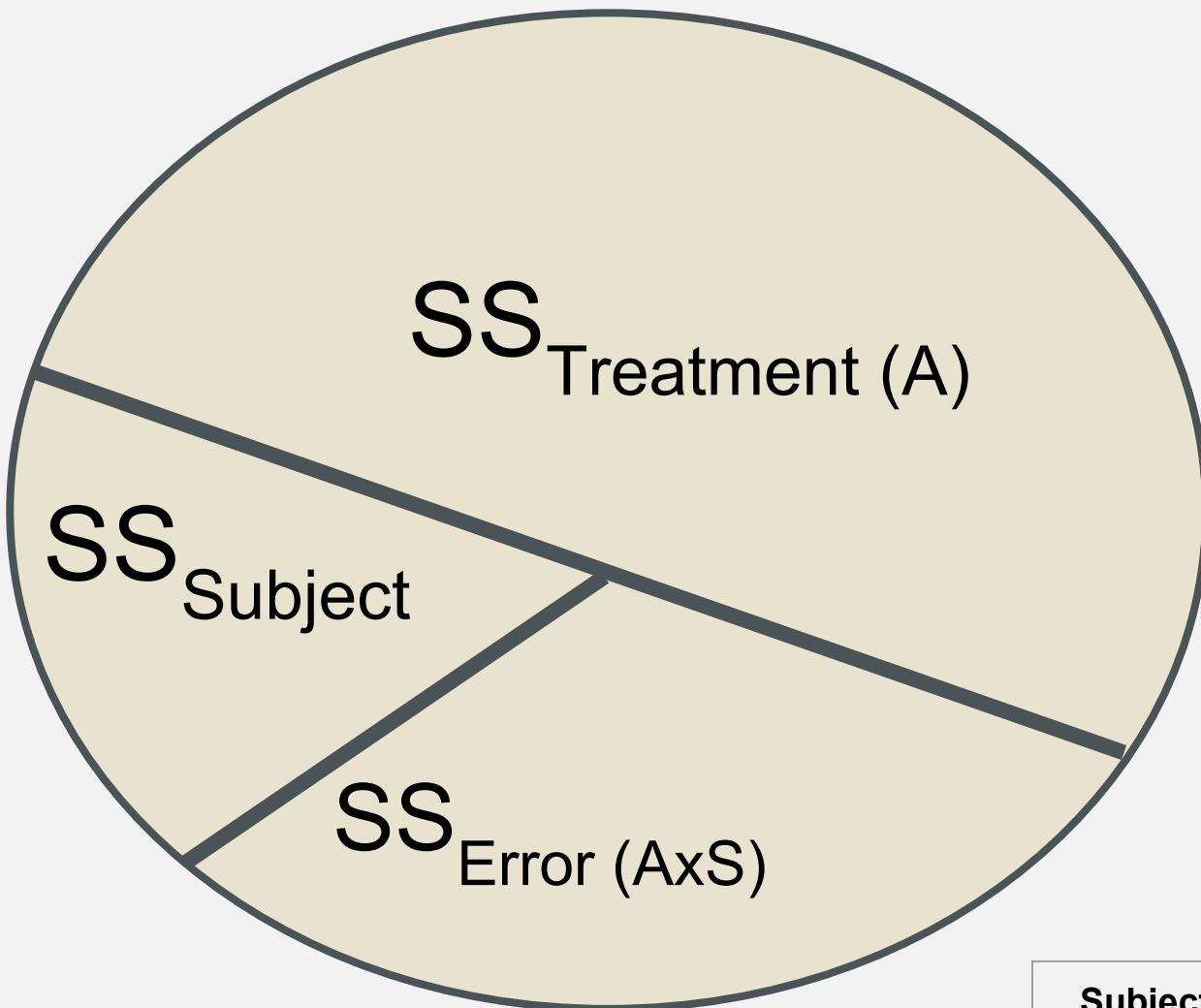
- DV: Task Performance (1-10)
- IV: Caffeine (0 mg, 150 mg, 300 mg)

Methods:

- Within Groups (Repeated Measures) design
- n = 20

Research Question: Does amount of caffeine have an effect on task performance?

Source	SS	df	MS	F
treat	$n \sum(x_j - \bar{x}_T)^2$	$k - 1$	$\frac{SS_{treat}}{df_{treat}}$	$\frac{MS_{treat}}{MS_{error}}$
within	$\sum(x_{ij} - \bar{x}_j)^2$	$k(n - 1)$		
subject	$k \sum(M_i - \bar{x}_T)^2$	$n - 1$		
error	$SS_{within} - SS_{subject}$	$(k - 1)(n - 1)$	$\frac{SS_{error}}{df_{error}}$	
Total	$\sum(x_{ij} - \bar{x}_T)^2$	$N - 1$		



Effect Size of treatment (A):

$$\eta_p^2 = \frac{SS_{\text{treat}}}{SS_{\text{treatment}} + SS_{\text{Error}}}$$

Subject	0 mg	150 mg	300 mg
Sara	7	9	4
Jess	3	7	9

When to use it?

- One IV (3 levels or more) and one DV
- Within subjects: same people in each level
 - Ex IV levels: Pre, During, Post

ASSUMPTIONS

- DV is continuous
- DV is normally distributed
 - Normal distribution of the **DIFFERENCE** scores
- IV is categorical
- Sphericity Assumption

SPHERICITY

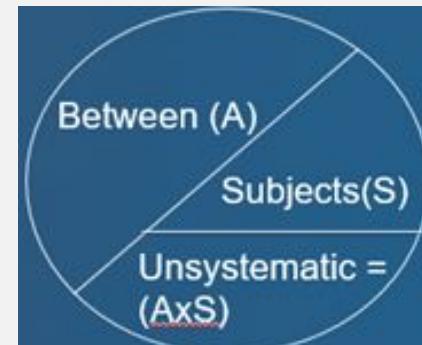
- What is it?
 - Equal variance of the difference between levels of the dependent variable.
- How do we test for it?
 - Mauchly's Test
- What do we do if it's violated?
 - Greenhouse-Geisser or Huynh-Feldt
 - Adjusts the p-value and df

EFFECT SIZE

Partial Eta-Squared: η_p^2

$$\eta_p^2 = SS_{\text{Between}} / (SS_{\text{Between}} + SS_{\text{Unsystematic}})$$

We use η_p^2 instead of η^2 because we do not want to include the subjects variance in the calculation (it would make the effect seem smaller).



RM ANOVA Example

An athletic trainer has developed a new exercise-training intervention and would like to test the effectiveness. Over the 6-months of the intervention, subjects had their **fitness level** measured on three occasions: **pre-**, **3 months**, and **post-intervention**.

REPEATED MEASURES ANOVA

Within Subjects Effects

	Sphericity Correction	Sum of Squares	df	Mean Square	F	p	η^2	partial η^2
RM Factor 1	None	1.51	2	0.756	2.64	0.042	0.060	0.083
	Greenhouse-Geisser	1.51	1.84	0.823	2.64	0.085	0.060	0.083
Residual	None	16.63	58	0.287				
	Greenhouse-Geisser	16.63	53.27	0.312				

Note. Type 3 Sums of Squares

Between Subjects Effects

	Sum of Squares	df	Mean Square	F	p	η^2	partial η^2
Residual	7.10	29	0.245				

Note. Type 3 Sums of Squares

ASSUMPTIONS

Tests of Sphericity

	Mauchly's W	p	Greenhouse-Geisser ϵ	Huynh-Feldt ϵ
RM Factor 1	0.911	0.272	0.918	0.978

POST HOC TESTS

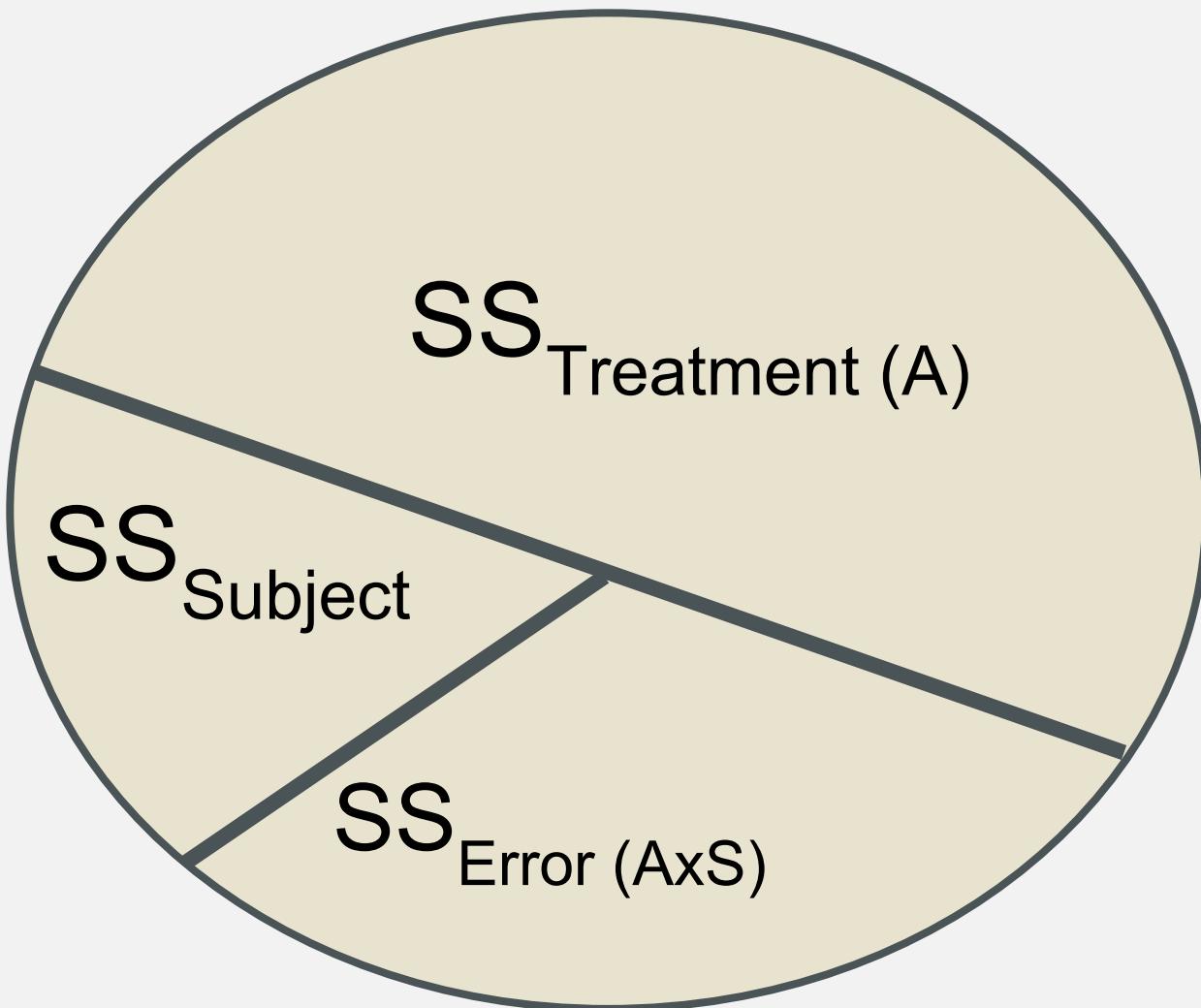
Post Hoc Comparisons - RM Factor 1

RM Factor 1	RM Factor 1	Mean Difference	SE	df	t	p-holm
3 Months	- Pre	-0.2333	0.138	58.0	-1.688	0.049
	- Post	0.0697	0.138	58.0	0.504	0.616
	- Pre	0.3030	0.138	58.0	2.192	0.031

Stats: Using a Repeated Measures ANOVA, there was significant difference between time points, $F(2, 58) = 2.64, p = .042, \eta_p^2 = .08$. This is a medium effect size. A post-hoc analyses, using a Holm-correction, demonstrated that fitness level significantly increased from Pre to 3 Months into the intervention, $M_{diff} = -0.23, p = .049$. There was also a significant increase from Pre to Post intervention, $M_{diff} = -0.30, p = .031$. There was no difference between the 3-Month measurement and Post measurement.

Lay: The intervention showed an increase in fitness scores 3 months into the intervention and after the intervention was complete. Congratulations, the intervention seemed like it worked. I recommend you continue to use it with your clients to see an increase in fitness scores!

Source	SS	df	MS	F
treat	$n \sum (x_j - \bar{x}_T)^2$	$k - 1$	$\frac{SS_{treat}}{df_{treat}}$	$\frac{MS_{treat}}{MS_{error}}$
within	$\sum (x_{ij} - \bar{x}_j)^2$	$k(n - 1)$		
subject	$k \sum (M_i - \bar{x}_T)^2$	$n - 1$		
error	$SS_{within} - SS_{subject}$	$(k - 1)(n - 1)$	$\frac{SS_{error}}{df_{error}}$	
Total	$\sum (x_{ij} - \bar{x}_T)^2$	$N - 1$		



Effect Size of treatment (A):

$$\eta_p^2 = (SS_{\text{treatment}})/(SS_{\text{treatment}} + SS_{\text{Error}})$$

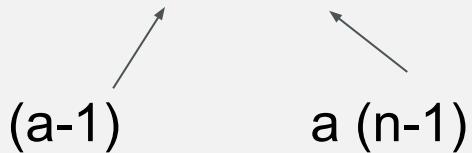
MIXED FACTORIAL ANOVA

Mixed Factorial ANOVA

		Caffeine (b)			
		0 mg	150 mg	300 mg	
		4 hours	6 hours	8 hours	
Sleep (a)	0 mg	3.10	4.91	1.46	3.16
	150 mg	5.87	7.20	8.25	7.11
	300 mg	7.50	9.10	9.43	8.68
		5.49	7.07	6.38	6.31

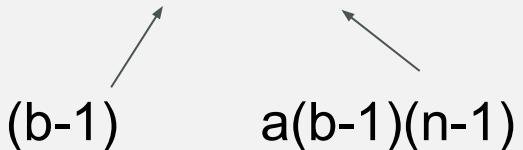
Main Effect of Sleep

$$F (\underline{2}, \underline{57}) = 10.53, p = .025$$



Main Effect of Caffeine

$$F (\underline{2}, \underline{114}) = 19.53, p < .001$$



Variables:

- DV: Task Performance (1-10)
- IV: Caffeine (0 mg, 150 mg, 300 mg)
- IV: Sleep (4 hours, 6 hours, 8 hours)

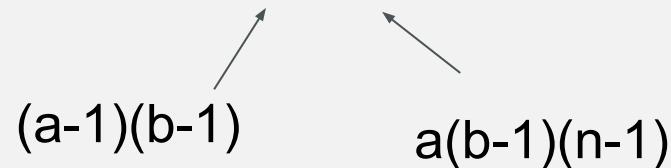
Methods:

- Between groups design
- n = 20

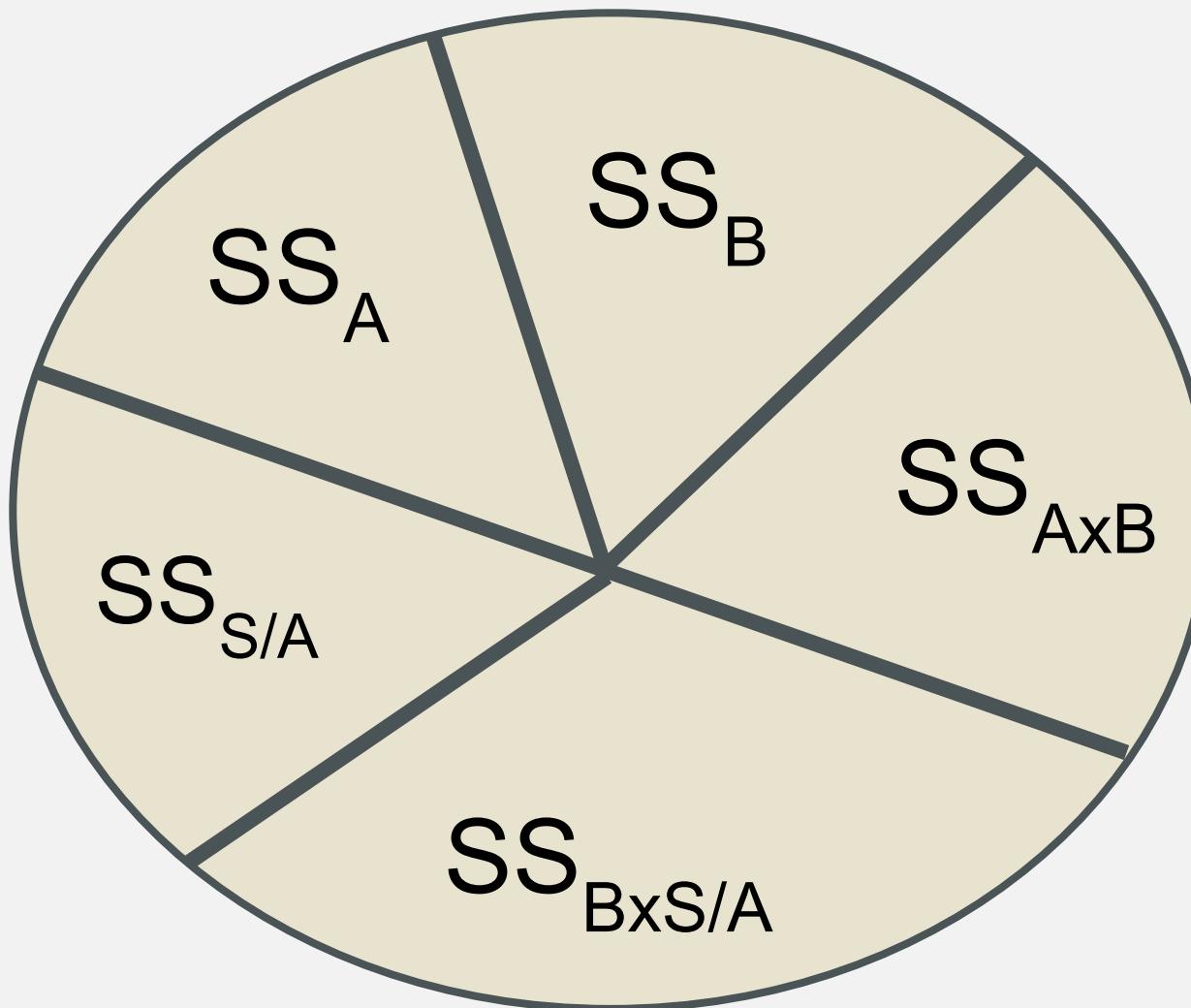
Research Question: Does amount of caffeine and/or amount of sleep have an effect on task performance?

Interaction Between Sleep & Caffeine

$$F (\underline{4}, \underline{114}) = 30.53, p = .001$$



Source	SS	df	MS	F
Between-group				
A	$n \sum (x_j - \bar{x}_T)^2$	a - 1	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{S/A}}$
S/A	$\sum (x_{ij} - \bar{x}_j)^2$	a(n - 1)	$\frac{SS_{S/A}}{df_{S/A}}$	
Within-group				
B	$k \sum (M_i - \bar{x}_T)^2$	b - 1	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_{BxS/A}}$
AxB	$\sum n(Y_{jk} - \bar{Y}_{...})^2$	(a - 1)(b - 1)	$\frac{SS_{AxB}}{df_{AxB}}$	$\frac{MS_{AxB}}{MS_{BxS/A}}$
BxS/A	$\sum (Y_{ijk} - \bar{Y}_{jk})^2$	a(b - 1)(n - 1)	$\frac{SS_{BxS?A}}{df_{BxS?A}}$	
Total	$\sum (x_{ij} - \bar{x}_T)^2$	N - 1 or (a)(b)(n) - 1	$\frac{SS_{Total}}{df_{Total}}$	



Effect Size of A (between groups):

$$\eta_p^2 = (SS_A)/(SS_A + SS_{S/A})$$

Effect Size of B (within groups):

$$\eta_p^2 = (SS_B)/(SS_B + SS_{BxS/A})$$

Effect Size of AxB (interaction):

$$\eta_p^2 = (SS_{AxB})/(SS_{AxB} + SS_{BxS/A})$$

Example

You are an Organizational Psychologist hired by Patagonia to investigate productivity among their employees. You hypothesize that both length of break taken and tenure at the company impacts productivity.

DV: Productivity (0-100)

IV_{Within Subjects}: Length of Break (5 minute, 10 minute, & 20 minute)

IV_{Between Subjects}: Tenure (New & Old employees)

Main Effects?

Within Subjects Effects

	Sphericity Correction	Sum of Squares	df	Mean Square	F	p	partial η ²
Break	None	3475	2	1737.4	158.4	< .001	0.850
	Greenhouse-Geisser	3475	1.52	2285.5	158.4	< .001	0.850
Break:Tenure	None	232	2	115.9	10.6	< .001	0.274
	Greenhouse-Geisser	232	1.52	152.5	10.6	< .001	0.274
Residual	None	614	56	11.0			
	Greenhouse-Geisser	614	42.57	14.4			

Note. Type 3 Sums of Squares

Between Subjects Effects

	Sum of Squares	df	Mean Square	F	p	partial η ²
Tenure	2.34	1	2.34	0.0500	0.825	0.002
Residual	1306.95	28	46.68			

Note. Type 3 Sums of Squares

Interaction?

within Subjects Effects

	Sphericity Correction	Sum of Squares	df	Mean Square	F	p	partial η ²
Break	None	3475	2	1737.4	158.4	< .001	0.850
	Greenhouse-Geisser	3475	1.52	2285.5	158.4	< .001	0.850
Break:Tenure	None	232	2	115.9	10.6	< .001	0.274
	Greenhouse-Geisser	232	1.52	152.5	10.6	< .001	0.274
Residual	None	614	56	11.0			
	Greenhouse-Geisser	614	42.57	14.4			

Note. Type 3 Sums of Squares

Between Subjects Effects

	Sum of Squares	df	Mean Square	F	p	partial η ²
Tenure	2.34	1	2.34	0.0500	0.825	0.002
Residual	1306.95	28	46.68			

Note. Type 3 Sums of Squares

Simple Effects: Short Break

INDEPENDENT SAMPLES T-TEST

Independent Samples T-Test

		statistic	df	p	Cohen's d
Product	Student's t	2.21	28.0	0.035	0.808

Simple Effects: Medium Break

INDEPENDENT SAMPLES T-TEST

Independent Samples T-Test

		statistic	df	p	Cohen's d
Product	Student's t	0.556	28.0	0.583	0.203

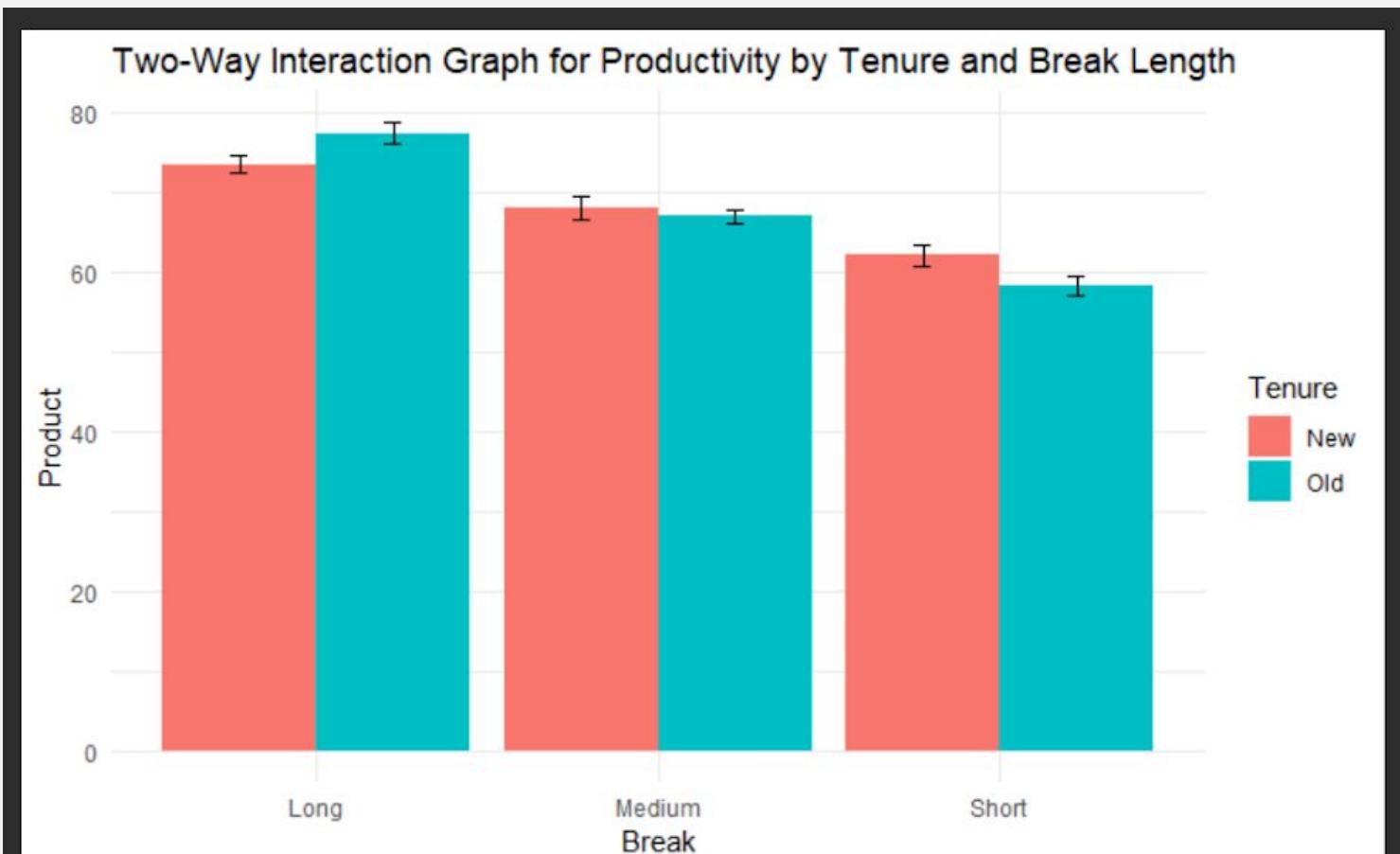
Simple Effects: Long Break

INDEPENDENT SAMPLES T-TEST

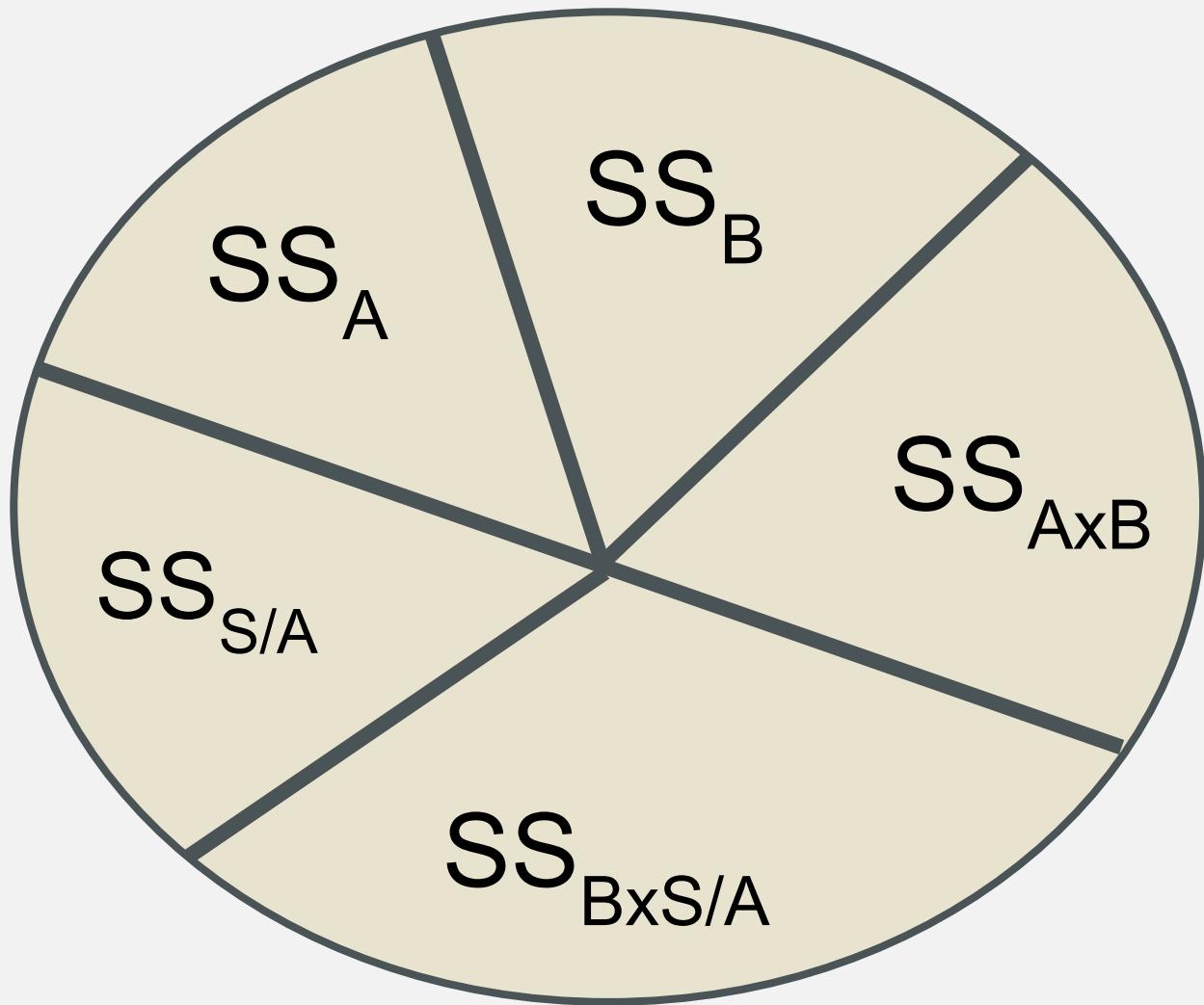
Independent Samples T-Test

		statistic	df	p	Cohen's d
Product	Student's t	-2.24	28.0	0.034	-0.816

What Can We Conclude?



Source	SS	df	MS	F
Between-group				
A	$n \sum (x_j - \bar{x}_T)^2$	a - 1	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{S/A}}$
S/A	$\sum (x_{ij} - \bar{x}_j)^2$	a(n - 1)	$\frac{SS_{S/A}}{df_{S/A}}$	
Within-group				
B	$k \sum (M_i - \bar{x}_T)^2$	b - 1	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_{BxS/A}}$
AxB	$\sum n(Y_{jk} - \bar{Y}_{...})^2$	(a - 1)(b - 1)	$\frac{SS_{AxB}}{df_{AxB}}$	$\frac{MS_{AxB}}{MS_{BxS/A}}$
BxS/A	$\sum (Y_{ijk} - \bar{Y}_{jk})^2$	a(b - 1)(n - 1)	$\frac{SS_{BxS?A}}{df_{BxS?A}}$	
Total	$\sum (x_{ij} - \bar{x}_T)^2$	N - 1 or (a)(b)(n) - 1	$\frac{SS_{Total}}{df_{Total}}$	



Happy Studying
&
Happy Holidays