

# 修 士 論 文

窒化シリコン  
リング共振器を用いた  
広帯域周波数もつれ光子生成

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令和2年2月1日

# **Broadband frequency entangled photon generation using silicon nitride ring cavities**

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## **Abstract**

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# Chapter 1

## Introduction

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## **1.1 BACKGROUND**

### **1.1.1 Chip-scale quantum information processing**

### **1.1.2 Entangled photon sources for quantum communication and sensing**

## **1.2 CURRENT PROBLEMS**

# Chapter 2

## Principal Theory

In chip scale quantum optical platforms, to confine the light in a sub-micron dimension, the mean how light propagate is different from the TEM mode or Gaussian mode in free space. Based on the conventional guided wave theory, the fundamental mode, which is mainly used in integrated optical applications, can be solved in some simple cases with some approximations. Furthermore, due to the broadband motivation in this study, it is essential to the dispersion into consideration, not only in the part of propagation but also the nonlinear progress where the photon pairs are generated.

In this chapter, the waveguide theory is roughly introduced and sequentially, mode coupling theory is discussed in the case of ring resonator. Second section will focus on nonlinear optics in the view of waveguide.

### 2.1 GUIDED MODES

In an ideal optical waveguide, the core layer and the cladding layer are usually composed of two different materials, where the refractive index is larger in the core. As an analogue of optical fibers, only in the higher index region can the light propagate, and meanwhile dissipate in a wavelength scale in the lower index region.

Usually, we assume the core and the cladding layer are made of nonmagnetic (magnetic permeability  $\mu = \mu_0$ ) and dielectric material (conductivity  $\sigma = 0$ ). Furthermore, we neglect the nonlinear response of the polarization of electric field ( $\mathbf{P} \simeq \varepsilon_0 \chi \mathbf{E}$ ).

Since the waveguide in numerous research objects, is deposited or sputtered using chemical or physical methods, the uneven den-

sity in the waveguide layer can not be negligible. Hence, the propagation equation derived from Maxwell's equation is

$$(\nabla_{\perp}^2 + k^2 n^2 - \beta^2) \mathbf{E} = -(\nabla_{\perp} + i\beta \hat{\mathbf{z}})(\mathbf{E}_{\perp} \cdot \nabla_{\perp} \ln n^2) \quad (2.1)$$

where  $\perp$  denotes the transverse component,  $\nabla_{\perp}^2 = \nabla_x^2 + \nabla_y^2$ . And  $k, n, \beta$  are the wave vector in vacuum, refractive index and propagation constant, respectively. While, with the negligible film anisotropy, ?? can be approximated into

$$(\nabla_{\perp}^2 + k^2 n^2 - \beta^2) \mathbf{E} = 0 \quad (2.2)$$

This is the normal *Helmholtz equation*, indicating the relation between propagation constant  $\beta$  and material refractive index, i.e. *chromatic dispersion*.

Next, the boundary conditions determining the solution to Eqn. ??, arise from the Maxwell's equations as well.

$$\begin{aligned} \hat{\mathbf{n}} \cdot (\mathbf{E}_a - \mathbf{E}_b) &= 0 \\ \hat{\mathbf{n}} \times (\mathbf{H}_a - \mathbf{H}_b) &= 0 \end{aligned}$$

which is the continuity condition of both electric and magnetic field at all dielectric material interfaces. Here,  $\hat{\mathbf{n}}$  is the normal direction at the material boundary and the subscript  $a, b$  denote different regions.

In the case of channel waveguides, the index discontinuity from both vertical and horizontal sides can be decomposed into two sets of independent and complete conditions, i.e. the horizontal boundary condition and vertical boundary condition, with the discontinuity on the waveguide corners neglected. In other words, approximately the equation has two independent particular solutions, which is the mathematical origin of *transverse electric* (TE) modes and *transverse magnetic* (TM) modes.

Hence, we can study the eigenequation by selecting only one set of boundary condition, as used in the *the effective index method*. For example, in a planar waveguide shown in ?? ,  $d^2/dy^2 = 0$ <sup>1</sup>, the TE mode features  $E_x = 0$  and consider only  $y$ -component,

$$\frac{d^2 E_y}{dx^2} + (k^2 n^2 - \beta^2) E_y = 0 \quad (2.3)$$

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<sup>1</sup>Since the planar waveguide is infinite at the  $y$ -direction, thus the solution is identical in arbitrary  $xz$ -plane, which means no gradient along  $x$ -axis.

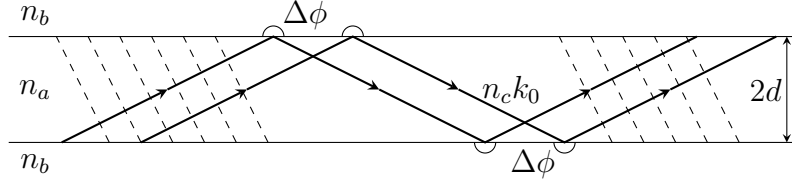


Figure 2.1: **Planar waveguide.** The upper and bottom layer are cladding and the middle is core layer.  $\Delta\phi$  represents the Goos-Hänchen shift at the boundary.

and  $E_y$  is continuous at  $x = \pm d/2$ , where  $d$  is the thickness of core layer.

For the region  $|x| > d/2$ , the light evanesces at  $x$ -direction at rate  $\kappa$  and in contrast, in the region of core layer, the light performs like stationary wave, denoting with  $k_x$ . By substituting these conditions, phase continuity is achieved between two interface

$$2k_x d = m\pi + 2 \arctan(\kappa/k_x) \quad (2.4)$$

where  $m$  is the index of stationary wave. The second term can be treated as the Goos-Hänchen phase shift. Overall, the waveguide modes characterize that the phase shall maintain itself with an  $m\pi$  shift along with the shift at the boundaries.

In the case of TM modes, the eigen equation is

$$2k_x d = m\pi + 2 \arctan(\delta\kappa/k_x) \quad (2.5)$$

where  $\delta = n_a/n_b$  is the index ratio and only differs from ?? with this parameter. conclusively, the less is  $\delta$  parameter, the propagation constant of TE and TM modes are closer.

Based on ?? and ??,  $k_x$  can be solved and then utilized to calculate propagation constant  $\beta$ , since  $n_a^2 k^2 = k_\perp^2 + \beta^2$ . In the case of channel waveguides, the TE and TM solutions are both necessary. Therefore, propagation constants  $\beta$  can be expressed as the product of free space wave vector  $k$  and the *effective index*  $n_{\text{eff}}$

$$\beta = n_{\text{eff}} k = n_{\text{eff}} \frac{2\pi}{\lambda} = n_{\text{eff}} \frac{\omega}{c} \quad (2.6)$$

along with the differential form

$$\frac{d\beta}{dk} = n_{\text{eff}} + k \frac{dn_{\text{eff}}}{dk} = n_{\text{eff}} - \lambda \frac{dn_{\text{eff}}}{d\lambda} \equiv n_g \quad (2.7)$$



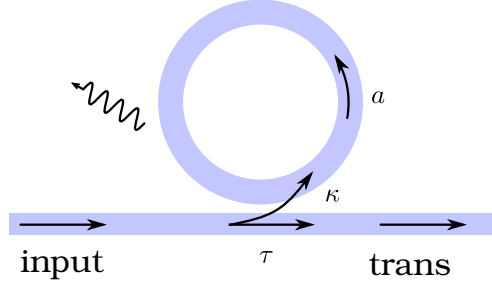


Figure 2.2: **An all-pass type ring resonator.** The transmitted spectrum is filtered periodically by the ring waveguide, in the case satisfying resonance condition.

Figure 2.3: **The transmission spectrum of a ring resonator.**

which defines the group index  $n_g$ .

This formula linking  $\beta-k$  or  $\beta-\omega$  is named as dispersion relation, which gives the physics that light with different color propagates at different *speed*. Furthermore, ?? and ?? also indicate that the dispersion relation intrinsically depends on waveguide geometry.

## 2.2 RING RESONATORS

The ring resonators comprise of a bus waveguide and a ring waveguide, are usually demonstrated as optical filters or modulators at a wide range of platforms. The working principle of ring resonator can be derived completely [?] as an analogue to Fabry-Pérot etalon, based on the coupling mode theory.

In the model illustrated in ??, the self-coupling coefficient  $\tau$  and the cross-coupling coefficient  $\kappa$  can be evaluated analytically or using numerical simulation. Assuming the coupling only occur at the very close area,  $\tau, \kappa$  are the power splitting ratios of the coupler and satisfy  $\tau^2 + \kappa^2 = 1$  if the coupling section is lossless.  $a$  is the single-pass amplitude transmission, including both propagation loss in the ring and loss in the couplers.

The transmission rate of a all-pass type ring cavity takes the form of

$$T = \frac{I_{\text{pass}}}{I_{\text{input}}} = \frac{a^2 - 2a\tau \cos \phi + \tau^2}{1 - 2a\tau \cos \phi + a^2\tau^2} \quad (2.8)$$

where  $\phi = \beta L$  is the phase shift in a single round trip.

### 2.2.1 Coupling condition

By plotting the function in ??, we can see, the extinction ratio of absorption peak is defined by the self-coupling coefficient  $\tau$  and the single-pass amplitude transmission  $a$  due to device geometric differences, like the gap between the bus waveguide and the ring cavity. Namely,  $a$  and  $\tau$  both determine the coupling condition, which can be categorized in three cases

- $a > \tau$  **weak coupling**. The loss inside the ring is larger than the power coupled from bus waveguides.
- $a = \tau$  **critical coupling**. The loss and self-coupling are in balance. The optical power restored in the resonator achieve the minimum.
- $a < \tau$  **over coupling or strong coupling**. The coupling is too strong for the light to dissipate in a single round trip.

Previous work [?] proposed a method to evaluate the coupling condition above using the experimentally measured device transmission. Considering the loss in the coupler, bent segment of ring and higher mode perturbation, usually the critical coupling varies from modes and the cross section of waveguides [?].

### 2.2.2 Spectrum characteristics

Meanwhile, the minimum of transmission rate  $T$  can be achieved periodically as  $\phi = 2\mu\pi$ , which defines the resonance of ring resonators. Therefore, the resonance condition is derived as

$$\beta L = 2\mu\pi \quad (2.9)$$

Specifically, the propagation constant  $\beta$ , shall be an integral times of a quasi wave vector  $2\pi/L$ . With this condition, the free spectrum range (FSR) of wavelength and frequency are obtained

$$\Delta\lambda_{\text{FSR}} \approx \frac{\lambda_{\text{res}}^2}{n_g L} \quad (2.10)$$

$$\Delta\omega_{\text{FSR}} \approx \frac{n_g L}{2\pi c} \quad (2.11)$$

In both wavelength and frequency domain, FSR determines the spacing of neighbouring resonant peak. This is a significant factor when the ring resonators are designed.

Figure 2.4: Caption

Furthermore, from ??, the full width at half maximum (FWHM) of the resonance spectrum is derived as  $\delta\lambda$

$$\delta\phi = \frac{2(1 - a\tau)}{\sqrt{\tau a}} \quad (2.12)$$

Likely, since the phase  $\phi$  is related with the wave vector  $k$  by ??. Substituting  $\delta\phi = Ln_g\delta k$ , the half width of wavelength and frequency are

$$\delta\lambda = \frac{d\lambda}{dk}\delta k = \frac{\lambda_{\text{res}}^2}{2\pi Ln_g} \frac{2(1 - a\tau)}{\sqrt{\tau a}} \quad (2.13)$$

$$\delta\omega = \frac{d\omega}{dk}\delta k = \frac{c}{Ln_g} \frac{2(1 - a\tau)}{\sqrt{\tau a}} \quad (2.14)$$

Note in ?? ?? ?? and ??, the group index  $n_g$  is explicit instead of the effective index  $n_{\text{eff}}$  because both free spectrum range and full width depend on the differential form, ??.

And the finesse  $F$  of the resonator is defined

$$F \equiv \frac{2\pi}{\delta\phi} = \frac{\pi\sqrt{\tau a}}{2(1 - a\tau)} \quad (2.15)$$

Finally, we define the quality factor, a measure of the sharpness of the resonance relative to its central frequency.

$$Q = \frac{\lambda_{\text{res}}}{\delta\lambda} = \frac{\pi Ln_g\sqrt{\tau a}}{\lambda_{\text{res}}(1 - a\tau)} \quad (2.16)$$

Usually, the  $Q$ -factor can be decomposed into two parts by formula  $Q^{-1} = Q_i^{-1} + Q_l^{-1}$ . And  $Q_i, Q_l$  are intrinsic  $Q$ -factor and loaded  $Q$ -factor, referring to the loss inside the ring waveguide and at the coupler, respectively. The physical meaning of the finesse and  $Q$ -factor relates to the number of round-trips before being lost to internal loss and the bus waveguides when the power is depleted to  $1/e$  of its initial value.

## 2.3 THRID-ORDER NONLINEAR OPTICS

Although the nonlinear effect is neglected in the derivation of waveguide modes in ??, for numerous materials, the nonlinear response

of electric field is significant even at mW level, which is easy to occur with assistance of modern lasers. The origin of nonlinear optic phenomena is similar to the movement of the object in a potential field, such as the ball-spring model. In the nonlinear material, the atoms or molecules are driven by the external electric field, due to the around chemical bonds or molecular orientation, the displacement of atoms or molecules perform nonlinear dependence on the strength of field. In real-world materials, interaction coming arising from various frequency leads to the addition or subtraction of these frequency components. This explains the frequency conversion nature in nonlinear optics.

It is worth mentioning that not only in the bulk crystals, but also in the sub-micron scale [?], the nonlinear response is still efficient, even over a single-layer two-dimensional material.

Here, a brief theoretical derivation is elucidated and in the following part, degenerate four wave mixing is emphasized. In an isotropic nonlinear medium, assuming only instantaneous dielectric response, the relation between the polarization and the electric field is expressed by a power series in the electric field

$$\mathbf{P} = \varepsilon_0(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E}^2 + \chi^{(3)}\mathbf{E}^3) = \varepsilon_0\chi^{(1)}\mathbf{E} + \mathbf{P}_{\text{NL}} \quad (2.17)$$

Note in ??, the nonlinear susceptibilities  $\chi^{(2)}$  and  $\chi^{(3)}$  are second-rank and third-rank tensors, corresponding to the tensor product with  $\mathbf{E}^2$  and  $\mathbf{E}^3$ . The higher order response is neglected and sequentially, only  $\chi^{(2)}$  processes and  $\chi^{(3)}$  processes are to be introduced.

$\chi^{(2)}$  **processes** In centrosymmetric crystals such as silicon, the second-order susceptibility term is absent. However, in other materials like lithium niobate ( $\text{LiNbO}_3$ ) and aluminium nitride (AlN), the second-order nonlinearity are essential to realize electro-optic modulation and second harmonic generation.

$\chi^{(3)}$  **processes** Silicon and silicon nitride are both cubic crystal. Due to the third-order dependence, another factor equivalent to the optical intensity is involved, the  $\chi^{(3)}$  process is also named as intensity-dependent effect or Kerr effect.

Consider three frequency components of  $\mathbf{E}^3$ , using the complex expression of electric field

$$\mathbf{E}(\mathbf{r}, t) = \sum_{k=1}^3 \mathbf{E}_{\omega_k}(\mathbf{r}, t) = \frac{1}{2} \sum_{k=1}^3 (\mathbf{E}_{\omega_k}(\mathbf{r})e^{i\omega_k t} + c.c.) \quad (2.18)$$

Substituting into third-order term in ??

$$\mathbf{P}^{(3)} = \frac{3}{4}\varepsilon_0\chi^{(3)} [|\mathbf{E}_{\omega_1}|^2\mathbf{E}_{\omega_1} + \dots] \quad \text{SPM} \quad (2.19)$$

$$+ \frac{6}{4}\varepsilon_0\chi^{(3)} [(|\mathbf{E}_{\omega_2}|^2 + |\mathbf{E}_{\omega_3}|^2)\mathbf{E}_{\omega_1} + \dots] \quad \text{XPM} \quad (2.20)$$

$$+ \frac{1}{4}\varepsilon_0\chi^{(3)} [(\mathbf{E}_{\omega_1}^3 e^{i\omega_1 t} + c.c.) + \dots] \quad \text{THG} \quad (2.21)$$

$$+ \frac{3}{4}\varepsilon_0\chi^{(3)} \left[ \frac{1}{2}(\mathbf{E}_{\omega_1}^2 \mathbf{E}_{\omega_2} e^{i(2\omega_1+\omega_2)t} + c.c.) + \dots \right] \quad \text{FWM} \quad (2.22)$$

$$+ \frac{3}{4}\varepsilon_0\chi^{(3)} \left[ \frac{1}{2}(\mathbf{E}_{\omega_1}^2 \mathbf{E}_{\omega_2}^* e^{i(2\omega_1-\omega_2)t} + c.c.) + \dots \right] \quad \text{FWM} \quad (2.23)$$

$$+ \frac{6}{4}\varepsilon_0\chi^{(3)} \left[ \frac{1}{2}(\mathbf{E}_{\omega_1} \mathbf{E}_{\omega_2} \mathbf{E}_{\omega_3} e^{i(\omega_1+\omega_2+\omega_3)t} + c.c.) + \dots \right] \quad \text{FWM} \quad (2.24)$$

$$+ \frac{6}{4}\varepsilon_0\chi^{(3)} \left[ \frac{1}{2}(\mathbf{E}_{\omega_1} \mathbf{E}_{\omega_2} \mathbf{E}_{\omega_3}^* e^{i(\omega_1+\omega_2-\omega_3)t} + c.c.) + \dots \right] \quad \text{FWM} \quad (2.25)$$

In above equations,  $\dots$  stands for all possible permutation terms contributed by frequencies  $\omega_1, \omega_2, \omega_3$ . The abbreviation on the right side represent for

- **SPM, self-phase modulation**

SPM adds an intensity-dependent term except the linear polarization, leading to a broadening of the pulse spectrum.

Note the  $\chi^{(3)}$  is complex, thus the imaginary part may contribute to another intensity-dependent absorption mechanics, which is usually depicted in the *two-photon absorption* (TPA). The free carriers excited by TPA in further change the temporally both the absorption coefficient and the refractive index of material.

$$n = n_0 + n_2 I + i \frac{\lambda}{4\pi} (\alpha_0 + \alpha_2 I) \quad (2.26)$$

where the  $I$  is the intensity,  $n_2$  is the Kerr coefficient and  $\alpha_0, \alpha_2$  are related with TPA-induced free carrier absorption (FCA) and free carrier index (FCI) change, both interrelated with third-order susceptibility

$$n_2 = \frac{1}{cn_0^2\varepsilon_0} \frac{3}{4} \text{Re}\{\chi^{(3)}\} \quad (2.27)$$

$$\alpha_2 = \frac{-\omega}{c^2 n_0^2 \varepsilon_0} \frac{3}{2} \text{Im}\{\chi^{(3)}\} \quad (2.28)$$

A figure of merit (FOM) is often used to compare the magnitude of Kerr coefficient  $n_2$  with the strength of the TPA coefficient  $\alpha_2$

$$\text{FOM} = \frac{1}{\lambda} \frac{n_2}{\alpha_2} \quad (2.29)$$

- **XPM, cross-phase modulation**

XPM can be seen the first signal index influenced by a second signal. And the coefficient of XPM is twice as strong as the SPM coefficient.

- **THG, third-harmonic generation**

Like SHG, THG generated a new frequency with is one-third of input frequency.

- **FWM, four wave mixing**

In FWM process, more than three frequencies are involved. Nevertheless, ?? and ?? contain two identical wave, sometime calles as degenerate four wave mixing (DFWM). And ?? and ?? is a truly four wave process. Similar to the relation between SPM and XPM, the non-degenerate FWM is naturally twice stronger.

Traditionally, following the terminology in laser field, in DFWM, the  $\omega_1$  square term ?? is labeled as pump frequency, and another two frequencies are referred to signal and idler frequency.

Finally, it worth mentioning that in all THG and FWM processes, phase matching condition is required that

## 2.4 INTEGRATED NONLINEAR DEVICE

## Chapter 3

# Phase match condition for spontaneous four wave mixing in a ring cavity

### 3.1 CHROMATIC DISPERSION

### 3.2 BROADBAND PHASE MATCHING CONDITION

### 3.3 DISPERSION COMPENSATION AND ENGINEERING USING SLOT STRUCTURE

### 3.4 EFFECTS OF MODE CROSSING

### 3.5 FOUR WAVE MIXING

Assume in a ring resonator, to satisfy both energy conservation and momentum conservation,

$$\begin{aligned}\beta_i + \beta_s &= 2\beta_p \\ \omega_i + \omega_s &= 2\omega_p\end{aligned}$$

meanwhile,  $\beta = m\frac{2\pi}{L}$ , and  $L$  is the length of the resonator. Thus,

$$m_i + m_s = 2m_p \tag{3.1}$$

Hence, *the momentum conservation agrees with mode number equidistant*. Thus, we choose the frequency domain to estimate the phase mismatch. Expand the frequency into Taylor series at  $\omega_0$  to the propagation constant  $\beta$ ,

$$\begin{aligned}
\omega_\mu &= \omega_0 + \frac{d\omega}{d\beta}(\beta_\mu - \beta_0) + \frac{1}{2} \frac{d^2\omega}{d\beta^2}(\beta_\mu - \beta_0)^2 + \dots \quad (3.2) \\
&= \omega_0 + \frac{d\omega}{d\beta} \frac{2\pi}{L} \mu + \frac{d^2\omega}{d\beta^2} \left( \frac{2\pi}{L} \mu \right)^2 + \dots \\
&= \omega_0 + D_1 \mu + D_2 \mu^2 + D_3 \mu^3 + \dots
\end{aligned}$$

where  $D_1/2\pi = 1/v_g L$  is the *free spectrum range*, and

$$\text{as} = a$$



# **Appendix A**

## **Supple**