Inference for SRL Report

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I. Probabilistic Inference Using Weighted Model Counting

A. PGM to CNF

Table I shows the semantics of the domain variables used for those tasks.
Tables II and III show the logical variables used for encoding the Bayesian Network.
Table IV represents the encoded Bayesian Network using ENC1 and table V contains the corresponding weights.
Likewise, table VI represents the encoded Bayesian Network using ENC2 and table VII contains the corresponding weights.

Table I. Variables and domain semantics

Variable	Domain
B = Burglary	b1 = theres is a burglary b2 = theres is no burglary
E = Earthquake	e1 = there is a heavy earthquake e2 = there is a mild earthquake e3 = there is no earthquake
A = Alarm	a1 = alarm rings a2 = alarm does not ring
J = John	j1 = John calls j2 = John does not call
M = Mary	m1 = Mary calls m2 = Mary does not call

Table II. Logical variables using ENC1

Network variables	Indicator Variable	CTP
B	λ_{b1}, λ_{b2}	θ_{b1}, θ_{b2}
E	λ_{e1}, λ_{e2}, λ_3	θ_{e1}, θ_{e2}, θ_3
A	λ_{a1}, λ_{a2}	$\theta_{a1\|b1,e1}$, $\theta_{a1\|b1,e2}$,$\theta_{a1\|b1,e3}$, $\theta_{a1\|b2,e1}$, $\theta_{a1\|b2,e2}$, $\theta_{a1\|b2,e3}$, $\theta_{a2\|b1,e1}$, $\theta_{a2\|b1,e2}$,$\theta_{a2\|b1,e3}$, $\theta_{a2\|b2,e1}$, $\theta_{a2\|b2,e2}$, $\theta_{a2\|b2,e3}$
J	λ_{j1}, λ_{j2}	$\theta_{j1\|a1}$, $\theta_{j2\|a1}$, $\theta_{j1\|a2}$, $\theta_{j2\|a2}$
M	λ_{m1}, λ_{m2}	$\theta_{m1\|a1}$, $\theta_{m2\|a1}$,$\theta_{m1\|a2}$, $\theta_{m2\|a2}$

Table III. Logical variables using ENC2

Variables	Indicator Variable	CTP
B	λ_{b1}, λ_{b2}	ρ_{b1}
E	λ_{e1}, λ_{e2}, λ_3	ρ_{e1}, ρ_{e2}
A	λ_{a1}, λ_{a2}	$\rho_{a1\|b1,e1}$, $\rho_{a1\|b1,e2}$, $\rho_{a1\|b1,e3}$, $\rho_{a1\|b2,e1}$, $\rho_{a1\|b2,c2}$, $\rho_{a1\|b2,e3}$
J	λ_{j1}, λ_{j2}	$\rho_{j1\|a1}$, $\rho_{j1\|a2}$
M	λ_{m1}, λ_{m2}	$\rho_{m1\|a1}$, $\rho_{m1\|a2}$

Table IV. CNF REPRESENTATION OF BAYESIAN NETWORK USING ENC1

Variables	CNF	
B	$\lambda_{b1} \vee \lambda_{b2}$ $\neg\lambda_{b1} \vee \neg\lambda_{b2}$	$\lambda_{b1} \Leftrightarrow \theta_{b1}$ $\lambda_{b2} \Leftrightarrow \theta_{b2}$
E	$\lambda_{e1} \vee \lambda_{e2} \vee \lambda_{e3}$ $\neg\lambda_{e1} \vee \neg\lambda_{e2}$ $\neg\lambda_{e1} \vee \neg\lambda_{e3}$ $\neg\lambda_{e2} \vee \neg\lambda_{e3}$	$\lambda_{e1} \Leftrightarrow \theta_{e1}$ $\lambda_{e2} \Leftrightarrow \theta_{e2}$ $\lambda_{e3} \Leftrightarrow \theta_{e3}$
A	$\lambda_{a1} \vee \lambda_{a2}$ $\neg\lambda_{a1} \vee \neg\lambda_{a2}$	$\lambda_{a1} \wedge \lambda_{b1} \wedge \lambda_{e1} \Leftrightarrow \theta_{a1\|b1,e1}$ $\lambda_{a1} \wedge \lambda_{b1} \wedge \lambda_{e2} \Leftrightarrow \theta_{a1\|b1,e2}$ $\lambda_{a1} \wedge \lambda_{b1} \wedge \lambda_{e3} \Leftrightarrow \theta_{a1\|b1,e3}$ $\lambda_{a1} \wedge \lambda_{b2} \wedge \lambda_{e1} \Leftrightarrow \theta_{a1\|b2,e1}$ $\lambda_{a1} \wedge \lambda_{b2} \wedge \lambda_{e2} \Leftrightarrow \theta_{a1\|b2,e2}$ $\lambda_{a1} \wedge \lambda_{b2} \wedge \lambda_{e3} \Leftrightarrow \theta_{a1\|b2,e3}$ $\lambda_{a2} \wedge \lambda_{b1} \wedge \lambda_{e1} \Leftrightarrow \theta_{a2\|b1,e1}$ $\lambda_{a2} \wedge \lambda_{b1} \wedge \lambda_{e2} \Leftrightarrow \theta_{a2\|b1,e2}$ $\lambda_{a2} \wedge \lambda_{b1} \wedge \lambda_{e3} \Leftrightarrow \theta_{a2\|b1,e3}$ $\lambda_{a2} \wedge \lambda_{b2} \wedge \lambda_{e1} \Leftrightarrow \theta_{a2\|b2,e1}$ $\lambda_{a2} \wedge \lambda_{b2} \wedge \lambda_{e2} \Leftrightarrow \theta_{a2\|b2,e2}$ $\lambda_{a2} \wedge \lambda_{b2} \wedge \lambda_{e3} \Leftrightarrow \theta_{a2\|b2,e3}$
J	$\lambda_{j1} \vee \lambda_{j2}$ $\neg\lambda_{j1} \vee \neg\lambda_{j2}$	$\lambda_{j1} \wedge \lambda_{a1} \Leftrightarrow \theta_{j1\|a1}$ $\lambda_{j1} \wedge \lambda_{a2} \Leftrightarrow \theta_{j1\|a2}$ $\lambda_{j2} \wedge \lambda_{a1} \Leftrightarrow \theta_{j2\|a1}$ $\lambda_{j2} \wedge \lambda_{a2} \Leftrightarrow \theta_{j2\|a2}$
M	$\lambda_{m1} \vee \lambda_{m2}$ $\neg\lambda_{m1} \vee \neg\lambda_{m2}$	$\lambda_{m1} \wedge \lambda_{a1} \Leftrightarrow \theta_{m1\|a1}$ $\lambda_{m1} \wedge \lambda_{a2} \Leftrightarrow \theta_{m1\|a2}$ $\lambda_{m2} \wedge \lambda_{a1} \Leftrightarrow \theta_{m2\|a1}$ $\lambda_{m2} \wedge \lambda_{a2} \Leftrightarrow \theta_{m2\|a2}$

Table V. WEIGHTS ASSOCIATION USING ENC1

Weights	Value
W(θ_{b1})	0.7
W(θ_{b2})	0.3
W(θ_{e1})	0.01
W(θ_{e2})	0.19
W(θ_{e3})	0.80
W($\theta_{a1\|b1,e1}$)	0.90
W($\theta_{a1\|b1,e2}$)	0.85
W($\theta_{a1\|b1,e3}$)	0.80
W($\theta_{a1\|b2,e1}$)	0.30
W($\theta_{a1\|b2,e2}$)	0.10
W($\theta_{a1\|b2,e3}$)	0.00
W($\theta_{a2\|b1,e1}$)	0.10
W($\theta_{a2\|b1,e2}$)	0.15
W($\theta_{a2\|b1,e3}$)	0.20
W($\theta_{a2\|b2,e1}$)	0.70
W($\theta_{a2\|b2,e2}$)	0.90
W($\theta_{a2\|b2,e3}$)	1.00
W($\theta_{j1\|a1}$)	0.80
W($\theta_{j1\|a2}$)	0.10
W($\theta_{j2\|a1}$)	0.20
W($\theta_{j2\|a2}$)	0.90
W($\theta_{m1\|a1}$)	0.80
W($\theta_{m1\|a2}$)	0.10
W($\theta_{m2\|a1}$)	0.20
W($\theta_{m2\|a2}$)	0.90

Table VI. CNF REPRESENTATION OF BAYESIAN NETWORK USING ENC2

Variables	CNF	
B	$\lambda_{b1} \vee \lambda_{b2}$ $\neg\lambda_{b1} \vee \neg\lambda_{b2}$	$\rho_{b1} \Rightarrow \lambda_{b1}$ $\neg\rho_{b1} \Rightarrow \lambda_{b2}$
E	$\lambda_{e1} \vee \lambda_{e2} \vee \lambda_{e3}$ $\neg\lambda_{e1} \vee \neg\lambda_{e2}$ $\neg\lambda_{e1} \vee \neg\lambda_{e3}$ $\neg\lambda_{e2} \vee \neg\lambda_{e3}$	$\rho_{e1} \Rightarrow \lambda_{e1}$ $\neg\rho_{e1} \wedge \rho_{e2} \Rightarrow \lambda_{e2}$ $\neg\rho_{e1} \wedge \neg\rho_{e2} \Rightarrow \lambda_{e3}$
A	$\lambda_{a1} \vee \lambda_{a2}$ $\neg\lambda_{a1} \vee \neg\lambda_{a2}$	$\lambda_{b1} \wedge \lambda_{e1} \wedge \rho_{a1\|b1,e1} \Rightarrow \lambda_{a1}$ $\lambda_{b1} \wedge \lambda_{e2} \wedge \rho_{a1\|b1,e2} \Rightarrow \lambda_{a1}$ $\lambda_{b1} \wedge \lambda_{e3} \wedge \rho_{a1\|b1,e3} \Rightarrow \lambda_{a1}$ $\lambda_{b2} \wedge \lambda_{e1} \wedge \rho_{a1\|b2,e1} \Rightarrow \lambda_{a1}$ $\lambda_{b2} \wedge \lambda_{e2} \wedge \rho_{a1\|b2,e2} \Rightarrow \lambda_{a1}$ $\lambda_{b2} \wedge \lambda_{e3} \wedge \rho_{a1\|b2,e3} \Rightarrow \lambda_{a1}$ $\lambda_{b1} \wedge \lambda_{e1} \wedge \neg\rho_{a1\|b1,e1} \Rightarrow \lambda_{a2}$ $\lambda_{b1} \wedge \lambda_{e2} \wedge \neg\rho_{a1\|b1,e2} \Rightarrow \lambda_{a2}$ $\lambda_{b1} \wedge \lambda_{e3} \wedge \neg\rho_{a1\|b1,e3} \Rightarrow \lambda_{a2}$ $\lambda_{b2} \wedge \lambda_{e1} \wedge \neg\rho_{a1\|b2,e1} \Rightarrow \lambda_{a2}$ $\lambda_{b2} \wedge \lambda_{e2} \wedge \neg\rho_{a1\|b2,e2} \Rightarrow \lambda_{a2}$ $\lambda_{b2} \wedge \lambda_{e3} \wedge \neg\rho_{a1\|b2,e3} \Rightarrow \lambda_{a2}$
J	$\lambda_{j1} \vee \lambda_{j2}$ $\neg\lambda_{j1} \vee \neg\lambda_{j2}$	$\lambda_{a1} \wedge \rho_{j1\|a1} \Rightarrow \lambda_{j1}$ $\lambda_{a2} \wedge \rho_{j1\|a2} \Rightarrow \lambda_{j1}$ $\lambda_{a1} \wedge \neg\rho_{j1\|a1} \Rightarrow \lambda_{j2}$ $\lambda_{a2} \wedge \neg\rho_{j1\|a2} \Rightarrow \lambda_{j2}$
M	$\lambda_{m1} \vee \lambda_{m2}$ $\neg\lambda_{m1} \vee \neg\lambda_{m2}$	$\lambda_{a1} \wedge \rho_{m1\|a1} \Rightarrow \lambda_{m1}$ $\lambda_{a2} \wedge \rho_{m1\|a2} \Rightarrow \lambda_{m1}$ $\lambda_{a1} \wedge \neg\rho_{m1\|a1} \Rightarrow \lambda_{m2}$ $\lambda_{a2} \wedge \neg\rho_{m1\|a2} \Rightarrow \lambda_{m2}$

Table VII. WEIGHTS ASSOCIATION USING ENC2

Weights	Value
W(ρ_{b1})	0.7
W($\neg\rho_{b1}$)	0.3
W(ρ_{e1})	0.01
W(ρ_{e2})	0.19
W($\neg\rho_{e1}$)	1-0.01 = 0.99
W($\neg\rho_{e2}$)	1-0.19 = 0.81
W($\rho_{a1\|b1,e1}$)	0.90
W($\neg\rho_{a1\|b1,e1}$)	1-0.90=0.10
W($\rho_{a1\|b1,e2}$)	0.85
W($\neg\rho_{a1\|b1,e2}$)	1-0.85=0.15
W($\rho_{a1\|b1,e3}$)	0.80
W($\neg\rho_{a1\|b1,e3}$)	1-0.80=0.20
W($\rho_{a1\|b2,e1}$)	0.30
W($\neg\rho_{a1\|b2,e1}$)	1-0.30=0.70
W($\rho_{a1\|b2,e2}$)	0.10
W($\neg\rho_{a1\|b2,e2}$)	1-0-10=0.90
W($\rho_{a1\|b2,e3}$)	0
W($\neg\rho_{a1\|b2,e3}$)	1-0=1
W($\rho_{j1\|a1}$)	0.80
W($\neg\rho_{j1\|a1}$)	1-0.80=0.20
W($\rho_{j1\|a2}$)	0.10
W($\neg\rho_{j1\|a2}$)	1-0.10=0.90

B. SRL to CNF

First the program must be grounded, while taking into account **Q** and **E**. In this case the evidence set **E** is empty (there is no evidence available). The grounding process of the queries will be described step-by-step in listings 1 and 2. If only **query(path(1,5))** was considered, then **edge(5,6)** and **edge(2,6)** would have been irrelevant. With the inclusion of **query(path(1,6))** all edges become relevant.

```
% grounding path(1,5) becomes:
path(1,5) :- edge(1,3), 5 \== 3, path(3,5).
path(1,5) :- edge(1,2), 5 \== 2, path(2,5).
% grounding path(3,5)
path(3,5) :- edge(3,4), 5 \== 4, path(4,5).
% grounding path(4,5).
path(4,5) :- edge(4,5).
% grounding path(2,5)
path(2,5) :- edge(2,5).
% putting the results together (and resolving the inequalities) gives:
path(1,5) :- edge(1,3), edge(3,4), edge(4,5).
path(1,5) :- edge(1,2), edge(2,5).
```

Listing 1: Grounding of **path(1,5)**

```
% grounding path(1,6) becomes:
path(1,6) :- edge(1,3), 6 \== 3, path(3,6).
path(1,6) :- edge(1,2), 6 \== 2, path(2,6).
% grounding path(3,6)
path(3,6) :- edge(3,4), 6 \== 4, path(4,6).
% grounding path(4,6).
path(4,6) :- edge(4,5), 6 \== 5, path(5,6).
% grounding path(5,6)
path(5,6) :- edge(5,6).
% grounding path(2,6)
path(2,6) :- edge(2,6).
path(2,6) :- edge(5,6), 6 \== 5, path(5,6).
% path(5,6) has already been grounded
% putting the results together (and resolving the inequalities) gives:
path(1,6) :- edge(1,3), edge(3,4), edge(4,5), edge(5,6).
path(1,6) :- edge(1,2), edge(2,5), edge(5,6).
path(1,6) :- edge(1,2), edge(2,6).
```

Listing 2: Grounding of **path(1,6)**

The second step is to find an equivalent CNF of the ground program. Given the grounded rules **w :- r** and **w :- s**, the equivalent CNF contains the following three clauses: $\neg r \vee w$, $\neg s \vee w$ and $\neg w \vee s \vee r$. In our case r and s both are conjunctions, so De Morgans law is used to write the first two clauses. For the last clause, all permutations of the combinations of the elements $\neg w$, r and s are considered. For **path(1,5)** this yields $2 * 3 = 6$ combinations. For **path(1,6)** there are $2 * 3 * 4 = 24$ combinations. The CNF is shown in table VIII.

Table VIII. CNF REPRESENTATION OF THE GROUND RULES

Variables	CNF
$path_{15}$	$path_{15} \vee \neg\ edge_{13} \vee \neg\ edge_{34} \vee \neg\ edge_{45}$ $path_{15} \vee \neg\ edge_{12} \vee \neg\ edge_{25}$ $(\neg path_{15} \vee edge_{12} \vee edge_{13}) \wedge (\neg path_{15} \vee edge_{12} \vee edge_{34}) \wedge (\neg path_{15} \vee edge_{12} \vee edge_{45}) \wedge$ $(\neg path_{15} \vee edge_{25} \vee edge_{13}) \wedge (\neg path_{15} \vee edge_{25} \vee edge_{34}) \wedge (\neg path_{15} \vee edge_{25} \vee edge_{45})$
$path_{16}$	$path_{16} \vee \neg\ edge_{13} \vee \neg\ edge_{34} \vee \neg\ edge_{45} \vee \neg\ edge_{56}$ $path_{16} \vee \neg\ edge_{12} \vee \neg\ edge_{25} \vee \neg\ edge_{56}$ $path_{16} \vee \neg\ edge_{12} \vee \neg\ edge_{26}$ $(\neg path_{16} \vee edge_{13} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{13} \vee edge_{12} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{13} \vee edge_{25} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{13} \vee edge_{25} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{13} \vee edge_{56} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{13} \vee edge_{56} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{34} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{34} \vee edge_{12} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{34} \vee edge_{25} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{34} \vee edge_{25} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{34} \vee edge_{56} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{34} \vee edge_{56} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{45} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{45} \vee edge_{12} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{45} \vee edge_{25} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{45} \vee edge_{25} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{45} \vee edge_{56} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{45} \vee edge_{56} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{56} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{56} \vee edge_{12} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{56} \vee edge_{25} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{56} \vee edge_{25} \vee edge_{26}) \wedge$ $(\neg path_{16} \vee edge_{56} \vee edge_{56} \vee edge_{12}) \wedge (\neg path_{16} \vee edge_{56} \vee edge_{56} \vee edge_{26})$

The final step is to obtain a weighted CNF. Since there's no evidence in our example, the CNF remains the same as shown in table VIII. Table IX displays the weighted literals. The weights for $path_{15}$, $path_{16}$, $\neg path_{15}$ and $\neg path_{16}$ equal 1 because they're defined in clauses. The weight of any world ω can be calculated as the product of the weight of all literals in ω. For example, the world $path_{15}, edge_{12}, edge_{25}, edge_{13}, edge_{34}, \neg edge_{45}$ has the weight $0.6 * 0.4 * 0.1 * 0.3 * 0.2 = 0.00144$.

Table IX. WEIGHTED LITERALS

Variables	Weight	Variables	Weight
$edge_{12}$	0.6	$\neg edge_{12}$	0.4
$edge_{13}$	0.1	$\neg edge_{13}$	0.9
$edge_{25}$	0.4	$\neg edge_{25}$	0.6
$edge_{26}$	0.3	$\neg edge_{26}$	0.7
$edge_{34}$	0.3	$\neg edge_{34}$	0.7
$edge_{45}$	0.8	$\neg edge_{45}$	0.2
$edge_{56}$	0.2	$\neg edge_{56}$	0.8
$path_{15}$	1	$\neg path_{15}$	1
$path_{16}$	1	$\neg path_{16}$	1