blockchain linear logic cheat sheet

much logic, such formal, wow

Denis Erfurt& Jack Ek
DappHub
denis@dapphub.com

Abstract

LL Sequent Calculus

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, ?A}\,? \qquad \frac{\vdash \Gamma}{\vdash \Gamma, ?A}\,\mathrm{w} \qquad \frac{\vdash \Gamma, ?A, ?A}{\vdash \Gamma, ?A}\,\mathrm{c}$$

$$\frac{\vdash ?A_1, ..., ?A_n, B}{\vdash ?A_1, ..., ?A_n, !B}\,! \qquad \frac{\vdash \Gamma, A \quad \vdash \Gamma, B}{\vdash \Gamma, A \& B}\,\& \qquad \frac{\vdash \Gamma, A}{\vdash \Gamma, A \oplus B}\,\oplus_1$$

$$\frac{\vdash \Gamma, B}{\vdash \Gamma, A \oplus B}\,\oplus_2 \qquad \frac{\vdash \Gamma, A \quad \vdash B\Delta}{\vdash \Gamma, A \otimes B\Delta}\,\otimes \qquad \frac{\vdash \Gamma, A, B}{\vdash \Gamma, A \parr B}\,\parr$$

ILL Sequent Calculus

$$\frac{\Delta, A \vdash C \quad \Delta, B \vdash C}{\Delta, A \oplus B \vdash C}\,\oplus\mathrm{L} \qquad -\otimes\mathrm{L}$$

$$\frac{\Delta \vdash A_i}{\Delta \vdash A_0 \oplus A_1}\,\oplus R_i \qquad -\otimes\mathrm{R}$$

Negation and De-Morgan's laws

$$(A^\perp)^\perp \equiv A \qquad (A \otimes B)^\perp \equiv A^\perp \parr B^\perp \qquad (A \& B)^\perp \equiv A^\perp \oplus B^\perp$$

$$(A \parr B)^\perp \equiv A^\perp \otimes B^\perp \qquad (A \oplus B)^\perp \equiv A^\perp \& B^\perp$$

something

	conjunction	disjunction
multiplicative	$\otimes$	$\parr$
additive	$\&$	$\oplus$

positive : $\otimes, \oplus, \exists$
negative : $\&, \parr, \forall$

Structural Rules

distributivity

$$A \otimes (B \oplus C) \multimap (A \otimes B) \oplus (A \otimes C) \qquad A \parr (B \& C) \multimap (A \parr B) \& (A \parr C)$$

RuleZer

$$\frac{}{?A \vdash ?A}\,Id \qquad \frac{}{\Gamma \vdash \Delta}\,Prem \qquad \frac{}{\Gamma \vdash \Delta}\,Partial$$

RuleCut

$$\frac{\Theta, A \vdash \Delta \quad \Gamma \vdash A}{\Gamma, \Theta \vdash \Delta}\,Cut$$

RuleStruct

$$\frac{(\Gamma_1, \Delta_1), (\Gamma_2, \Delta_2) \vdash \Theta}{(\Gamma_1, \Gamma_2), (\Delta_1, \Delta_2) \vdash \Theta}\,PL \qquad \frac{\Theta \vdash (\Delta_1, \Gamma_1), (\Delta_2, \Gamma_2)}{\Theta \vdash (\Delta_1, \Delta_2), (\Gamma_1, \Gamma_2)}\,PR \qquad \frac{\Gamma \vdash (\Delta_1, \Delta_2), \Delta_3}{\Gamma \vdash \Delta_1, (\Delta_2, \Delta_3)}\,AR$$

$$\frac{\Gamma \vdash \Delta_1, (\Delta_2, \Delta_3)}{\Gamma \vdash (\Delta_1, \Delta_2), \Delta_3}\,AR \qquad \frac{(\Delta_1, \Delta_2), \Delta_3 \vdash \Gamma}{\Delta_1, (\Delta_2, \Delta_3) \vdash \Gamma}\,AL \qquad \frac{\Delta_1, (\Delta_2, \Delta_3) \vdash \Gamma}{(\Delta_1, \Delta_2), \Delta_3 \vdash \Gamma}\,AL$$

$$\frac{\cdot, \Gamma \vdash \Delta}{\Gamma \vdash \Delta}\,IL_L \qquad \frac{\Gamma \vdash \Delta}{\cdot, \Gamma \vdash \Delta}\,IL_L \qquad \frac{\Gamma, \cdot \vdash \Delta}{\Gamma \vdash \Delta}\,IL_R$$

$$\frac{\Gamma \vdash \Delta}{\Gamma, \cdot \vdash \Delta}\,IL_R \qquad \frac{\Gamma \vdash \cdot, \Delta}{\Gamma \vdash \Delta}\,IR_L \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \cdot, \Delta}\,IR_L$$

$$\frac{\Gamma \vdash \Delta, \cdot}{\Gamma \vdash \Delta}\,IR_R \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, \cdot}\,IR_R$$

RuleU

$$\frac{\Gamma, (A, B) \vdash C}{\Gamma, A \otimes B \vdash C}\,\otimes_L \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\,\multimap_R$$

RuleBin

$$\frac{\Delta \vdash B \quad \Gamma \vdash A}{\Gamma, \Delta \vdash A \otimes B}\,\otimes_R \qquad \frac{\Delta, B \vdash C \quad \Gamma \vdash A}{(\Gamma, \Delta), A \multimap B \vdash C}\,\multimap_L$$