Supplementary Materials for Optimization for Cooperative Target Tracking by Multiple UAVs

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1 Proof of Theorem

1.1 Proof of Theorem 1.

Theorem 1. If X_α^* is the α approximate optimal solution of (2), X_α^* is also the α approximate optimal solution of (1).

$$\begin{array}{l} \min f(\mathrm{X}) = EX \\ \text{s.t.} \left\{ X \in (\mu_i(X))_\alpha \; (A_i X = l_i + (1-\alpha)p_i),\, i = 1,2,...,N \right. \end{array}, \tag{1}$$

$$\begin{array}{l} \min f(X) = EX \\ \text{s.t.} \left\{ A_i X = l_i + (1-\alpha)p_i,\, i = 1,2,...,N \right. \end{array}. \tag{2}$$

Proof: Let $X_{\alpha 1}^F$,$X_{\alpha 2}^F$ be the sets of the α feasible solution of (1) and (2), respectively. $(X'_{\alpha 1})^F$ and $(X'_{\alpha 2})^F$ represent the sets of the α efficient solution. If $\forall X_\alpha \in X_{\alpha 2}^F$, $\forall X_\alpha \in (X'_{\alpha 2})^F \Leftrightarrow A_i X_\alpha = l_i + (1-\alpha)p_i \Leftrightarrow X_\alpha \in (X'_{\alpha 1})^F$, that is, $X_{\alpha 1}^F$=$X_{\alpha 2}^F$. Let $X'_\alpha \in (X'_{\alpha 2})^F$, $\forall\, X''_\alpha \in (X'_{\alpha 2})^F, (X''_\alpha \neq X'_\alpha)$ satisfies $EX''_\alpha \geq EX'_\alpha$, $X'_\alpha \in (X'_{\alpha 1})^F$ can be deduced from **Definition 3**(in the original paper). Thus, $(X'_{\alpha 1})^F$=$(X'_{\alpha 2})^F$. If $X_\alpha^* \in (X'_{\alpha 2})^F$ is the α approximate optimal solution of (2), $\Phi(X_\alpha^*) = \max \Phi(X_\alpha)$. According to **Definition 4**(in the original paper), Φ is also the approximate optimal function of (1), therefore X_α^* is the α approximate optimal solution of (1). □

1.2 Proof of Theorem 2.

Theorem 2. The time complexity of primary allocation in cooperative tracking method is $O(nN_t{}^2)$.

Proof: In the function DComplexScenario()(in the Section 3), the time complexity of dealing the transformation of the UAVs formation in an optimal observation position is $O(N_t)$. The time complexity of function DOcclude() (in the Section 3) is $O(N_t(N_t+1)/2) = O(N_t{}^2)$. The calculation process of function DChange() (in the Section 3) is similar to that of DOcclude()(in the Section 3). For all the transformation, the time complexity in the worst situation is $O(N_t{}^2)$. Therefore, the overall time complexity of cooperative tracking is calculated as follows: $O(n(N_t + N_t + N_t(N_t+1)/2 + N_t(N_t+1)/2)) = O(nN_t{}^2)$. □

Algorithm 1: Cooperative Target Tracking by Multiple UAVs

Input: Satisfaction α, Threshold θ_1,θ_2, Current location of multi-UAVs: $(x'_{it}, y'_{it})\;\; i = 1,2,...,N$, The number of UAVs N, Recognition rate of multi-UAVs $TRR_{it}\;\; i = 1,2,...,N$;
Output: Adjustment location of multi-UAVs $(x_{it}, y_{it})\;\; i = 1,2,...,N$;
for $t = 1$ *to* n **do**
  $s_1 \leftarrow 0$; $s_2 \leftarrow 0$;$s_3 \leftarrow 0$;
  for $i = 1$ *to* N **do**
    if $TRR_{it} \leq \theta_1$ **then**
      $s_1 \leftarrow s_1 + 1$;
    else if $TRR_{it} > \theta_1$ *and* $TRR_{it} \leq \theta_2$ **then**
      $s_2 \leftarrow s_2 + 1$;
    else
      $s_3 \leftarrow s_3 + 1$;
  if $s_1 = \;= N$ **then**
    DComplexScenario();
  else if $s_2 \geq 1$ *and* $s_3 == 0$ **then**
    DOcclusion();
  else if $s_3 \geq 1$ **then**
    DChange();
  else
    $x_{it} \leftarrow x'_{it}$, $y_{it} \leftarrow y'_{it}$;
  $t \leftarrow t + 1$;

Algorithm 2: DComplexScenario()

$j \leftarrow 0$; $TR \leftarrow 0$;
for $i = 1$ *to* N **do**
  Calculate offset vector of UAV_i;
  if TR<TRR_i **then**
    TR=TRR_i;
    j = i;
for $i = 1$ *to* N **do**
  Take UAV_j as the center, ensure the offset direction of the UAV_i;
  Transform the UAVs formation by: $x_{it} \leftarrow x'_{it} + \Delta\bar{x}_{it}$, $y_{it} \leftarrow y'_{it} + \Delta\bar{y}_{it}$;

2 Implementation of Cooperative Target Tracking by Multiple UAVs

The implementation of our model is shown in Algorithm 1.

Each UAV obtains a TRR by tracking algorithm when the target is moving, as UAVs have different shooting angles relative to the target. There are many uncertain factors such as complex scenarios, occlusion and angle change in the tracking process that affect the tracking results of multiple UAVs, thus we set two thresholds to judge the current tracking status. When $s_1 = \;= N$,

Algorithm 3: DOcclusion()

$I \leftarrow 1$;
for $i = 1$ *to* N **do**
  Add the elastic constraint terms $\tilde{l}_i$, $\tilde{d}_i$;
  Add the minimum safety distance constraint terms e_w, r_w $(w = 1, 2, \ldots \frac{N(N+1)}{2})$;
Build cooperative model $f(x_t)$, $f(y_t)$ respectively;
for $i = 1$ *to* N **do**
  Solve model with elastic constraints : $x_{it} \leftarrow x'_{it} + \Delta x_{it}$, $y_{it} \leftarrow y'_{it} + \Delta y_{it}$;

Algorithm 4: DChange()

$I \leftarrow 0$;
for $i = 1$ *to* N **do**
  Add strict constraint terms l_i, d_i;
  Add the minimum safety distance constraint terms e_w, r_w $(w = 1, 2, \ldots \frac{N(N+1)}{2})$;
Build cooperative model $f(x_t)$, $f(y_t)$ by (13), (14) respectively;
for $i = 1$ *to* N **do**
  Solve model by K-T condition: $x_{it} \leftarrow x'_{it} + \Delta x_{it}$, $y_{it} \leftarrow y'_{it} + \Delta y_{it}$;

$DComplexScenario()$ is employed to solve the issue that the target is in a complex scenario, we select the UAV with the maximum TRR value as center, and adjust the UAV formation. If $s_2 \geq 1$ and $s_3 == 0$, we deem that the target is partially occluded, then utilize $DOcclusion()$ which add the elastic constrain terms and safety distance to our model to cope with this issue. If $s_3 \geq 1$, the direction of the target is considered to change in the course of movement. We describe the solution as shown in $DChange()$. UAV tracking in this status is less affected than occlusion, so we add the strict constrain terms and safety distance to address it.

3 Experimental Parameter Setting

We assume that the target move uniformly in the tracking process. The safety distance among UAVs is 100m. The flight altitude is 200m. The monitoring period is 1s, that is, our model monitors and adjusts the flight positions of UAVs per second to obtain the optimal tracking accuracy. Inter-frame offset is used for initial cooperative flight control of multi-UAV, is concrete is:

$$\left\{ \begin{array}{l} {x'}_{it} = {x'}_{i(t-1)} + v\cos(\phi) \\ {y'}_{it} = {y'}_{i(t-1)} + v\sin(\phi) \\ \quad \phi = \arctan(\bar{y}_i/\bar{x}_i) \\ \quad \bar{x}_i = \frac{1}{J}\sum\limits_{j=1}^{J} {x'}_{i(t-j)} \\ \quad \bar{y}_i = \frac{1}{J}\sum\limits_{j=1}^{J} {y'}_{i(t-j)} \end{array} \right. , \tag{3}$$

where (x'_{it}, y'_{it}) is the location of UAV_i at time t. ϕ is the drift angle, and J represents the number of adjacent sampling frames. To satisfy the timeliness requirement of UAV tracking, we select DSST [1] with high real-time performance as tracking algorithm. The tracking robustness of this algorithm on UAV has been proved by Mueller et al. [2]. Its running speed is 25fps. In the elastic constraints, $\alpha = 0.6$.

3.1 Selection of UAV Count for Cooperative Tracking

To guarantee that multi-UAV has the ability to cooperatively complete tracking tasks, we need to limit the number of UAVs due to the delay caused by target recognition and information interaction. In our simulation system, information transmission among UAVs adopts ZigBee, which is a low-power and short-range wireless transmission protocol. Delay T is denoted as:

$$\begin{array}{l} T = \sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} t_{ij}, i \neq j \\ t_{ij} = s_{ij}/\tilde{b}_{ij} \end{array} . \tag{4}$$

where s is the amount of data, and $\tilde{b}$ represents the information transmission rate. In an ideal circumstance, the transmission rate of ZigBee is 250kbps. In the real situation, the rate will be greatly reduced (e.g. 20kbps) because of obstruction, air humidity, etc. The amount of data transmitted among UAVs is around 1.5kb in our system, which mainly includes UAV ID, geographical location, target recognition rate, etc.. Hence, the interaction delay between two UAVs is 0.011s. Our model needs to detect and adjust the UAVs$'$ positions timely. So the interaction delay T should be less than 0.5s, and the number of UAVs can reach up to 6-7. Considering the stability of cooperative flight and the cost-profit of single target tracking, we choose $N = 3$ in our system.

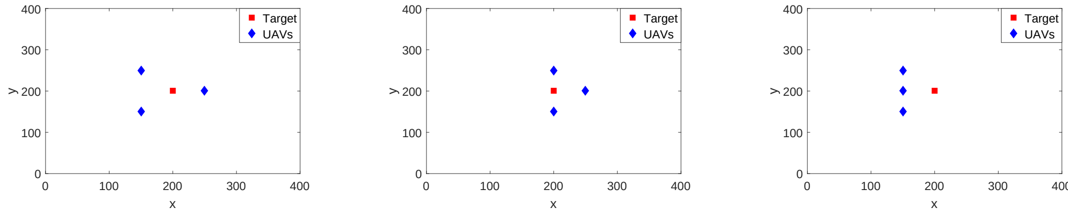
3.2 Selection of Initial Position

The initial positions of UAVs have an essential influence on tracking accuracy, especially when the target is moving because the target might be occluded by objects. In our experiment, we focus on selecting the optimal positions to maximize the accuracy of cooperative tracking. Theoretically, there are many initial position relationships between UAVs and target according to the difference of formation and the observation angle [2]. We mainly consider three typical locations (in Fig.1) to reduce the complexity of the problem. The surround tracking is shown in Fig.1 (a), which means that the target is in the center, and the UAVs are around it. Fig.1 (b) shows that two UAVs are on either side of the target and another one is in front of the target. Fig.1 (c) indicates that all UAVs behind the target for tracking. These three types of UAVs positions are named surround tracking, semi-surround tracking, and following tracking, respectively.

To select appropriate initial positions which can obtain the optimal effect of cooperative tracking for multiple UAVs, we try to test the tracking performance of UAVs in different positions shown in Fig.1, respectively, the comparison results are presented in Fig.2.

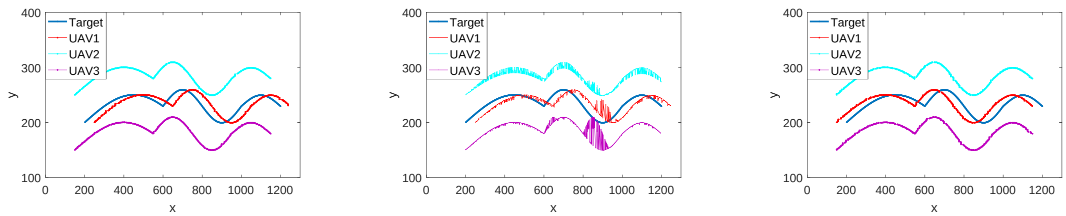
The results in Fig.2 (a) and Fig.2 (d) correspond to the situation that the UAVs are surround tracking, exactly as shown in Fig.1 (a). Fig.2 (a) displays the motion trajectories of target and multiple UAVs. It can be expressly observed that motion trajectories of the three UAVs are stable and highly consistent with the target when the target is in the formation center of multiple UAVs. Fig.2 (d) demonstrates the trajectory errors of UAVs obtained from our model in this case. We can see that the error of UAV1 is clearly greater than those of UAV2 and UAV3, yet the errors of UAV2 and UAV3 always maintain a low level over time, and the average trajectory error of three UAVs is around 1.5m.

Fig.2 (b) and Fig.2 (e) show the trajectories and position errors of UAVs in a semi-surrounded tracking condition, respectively. From Fig.2 (b), we can observe that there are noticeable fluctuations in trajectories of the three UAVs. The explanation of this

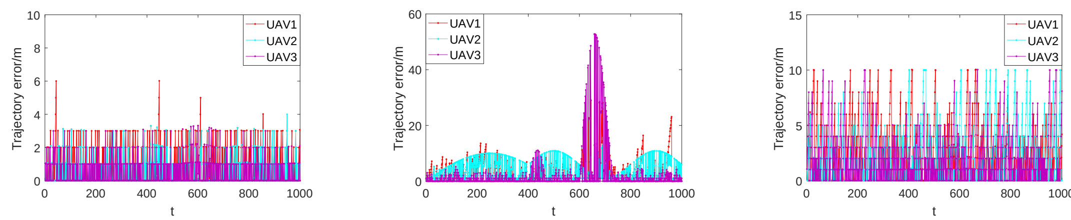


(a) Surround Tracking (b) Semi-Surround Tracking (c) Following Tracking

Fig. 1. The initial positions of UAVs formation.



(a) Motion Trajectory of Surround Tracking (b) Motion Trajectory of Semi-Surround Tracking (c) Motion Trajectory of Following Tracking



(d) Trajectory Error of Surround Tracking (e) Trajectory Error of Semi-Surround Tracking (f) Trajectory Error of Following Tracking

Fig. 2. The comparisons of multi-UAV cooperative tracking in three initial position conditions.

phenomenon is that the direction of the target has changed greatly in the course of movement, and UAV3 has larger undulation than other two UAVs, which indicates that it is unstable in tracking process. The average trajectory error of the three UAVs is more than 15m. Thus, we can conclude that the accuracy of cooperative tracking is poor if semi-surround tracking is used, this is because the initial positions of two UAVs are parallel to the target, one of them in the formation loses the target if the direction of the target rotates violently. Although our model adjusts the search formation of multi-UAV, their offsets are larger due to the limitation of initial position, which results in larger observation errors in this stage.

Similar to the two descriptions above, the motion trajectories of UAVs in following tracking condition are shown in Fig.2 (c), and homologous trajectory errors are exhibited in Fig.2 (f). When the target is in front of the multi-UAV formation, our model also maintains a high tracking accuracy with an average trajectory error of about 6m, but there are slight fluctuations in their trajectories. Also we observe that the errors of these three UAVs in Fig.2 (f) are obviously greater than that in Fig.2 (d). In contrast, with the surround tracking, after adjusting the multi-UAV formation with our model, the target is always within the scope of monitoring. Therefore, when multiple UAVs adopt surround tracking pattern, the cooperative tracking is able to gain better effect. Based on this situation, we use this scheme to track cooperatively in the simulation system.

3.3 Selection of Recognition Threshold

In our cooperative model, we denote recognition threshold θ_1 and θ_2 to compare with the real recognition rate and analyze the environment in which the target is located. Hence, we can see that θ_1 and θ_2 are crucial for cooperative tracking. Fig.3 shows average errors of θ_1 and θ_2 in surround tracking.

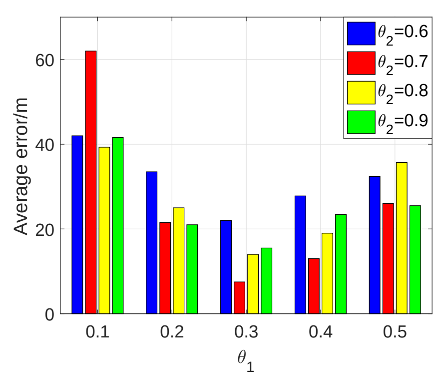


Fig. 3. Average errors of θ_1 and θ_2 in surround tracking.

θ_1 denotes the decision of losing target in the tracking process, $\theta_1 \in [0.1, 0.5]$. θ_2 is the judgment of the occlusion or rotation, $\theta_2 \in [0.6, 0.9]$. It can be seen from Fig.3 that tracking has the best performance with an average error of about 3-4m when $\theta_1 = 0.3$, $\theta_2 = 0.7$. But if $\theta_1 = 0.1$, the performance of tracking is poor (e.g. losing target), and the average error is more than 40 m. This is because the recognition rate of UAV is inaccurate due to target turning or occlusion. If the model only fine-tunes multi-UAV without adjusting the search range, the cooperative tracking will drift. As time goes on, the errors will increase continuously, then lead to tracking failure. Therefore, we set $\theta_1 = 0.3$, and $\theta_2 = 0.7$ in our system.

References

[1] M. Danelljan, G. Häger, F. Khan, and M. Felsberg, "Accurate Scale Estimation for Robust Visual Tracking," *Proc. British Machine Vision Conf. (BMVC)*, pp. 1-11, 2014.

[2] M. Mueller, N. Smith, and B. Ghanem, "A Benchmark and Simulator for UAV Tracking," *Proc. 14th European Conf. Computer Vision(ECCV)*, pp. 445-461, 2016.