Unit 9 Homework: Large-Sample Regression Theory

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What Makes a Successful Video Game?

The file `video_games.csv` contains data on 1212 video games that were on sold in 2011. It was compiled by Joe Cox, an economist at the University of Portsmouth.

Three key variables are as follows:

Variable	Meaning
Metrics.Sales	The total sales, measured in millions of dollars.
Metrics.Review.Score	Metacritic review score, an indicator of quality, out of 100.
Length.Completionists.Average	The mean time that players reported completing everything in the game, in hours.

You can find an explanation of other variables at https://think.cs.vt.edu/corgis/csv/video_games/.

You want to fit a regression predicting `Metrics.Sales`, with `Metrics.Review.Score` and `Length.Completionists.Average` as predictors.

0. Rename the variables that you are going to use to something sensible – variable names that have both periods and capital letters are not sensible. :fire: Better would be, for example changing `Metrics.Sales` to just `sales`.

```
dat<-read.csv('video_games.csv')
df<-dat %>%
  select('Metrics.Sales', 'Metrics.Review.Score', 'Length.Completionists.Average')%>%
  rename(sales = Metrics.Sales, score = Metrics.Review.Score,
         length = Length.Completionists.Average )
```

1. Examining the data, and using your background knowledge, evaluate the assumptions of the large-sample linear model.

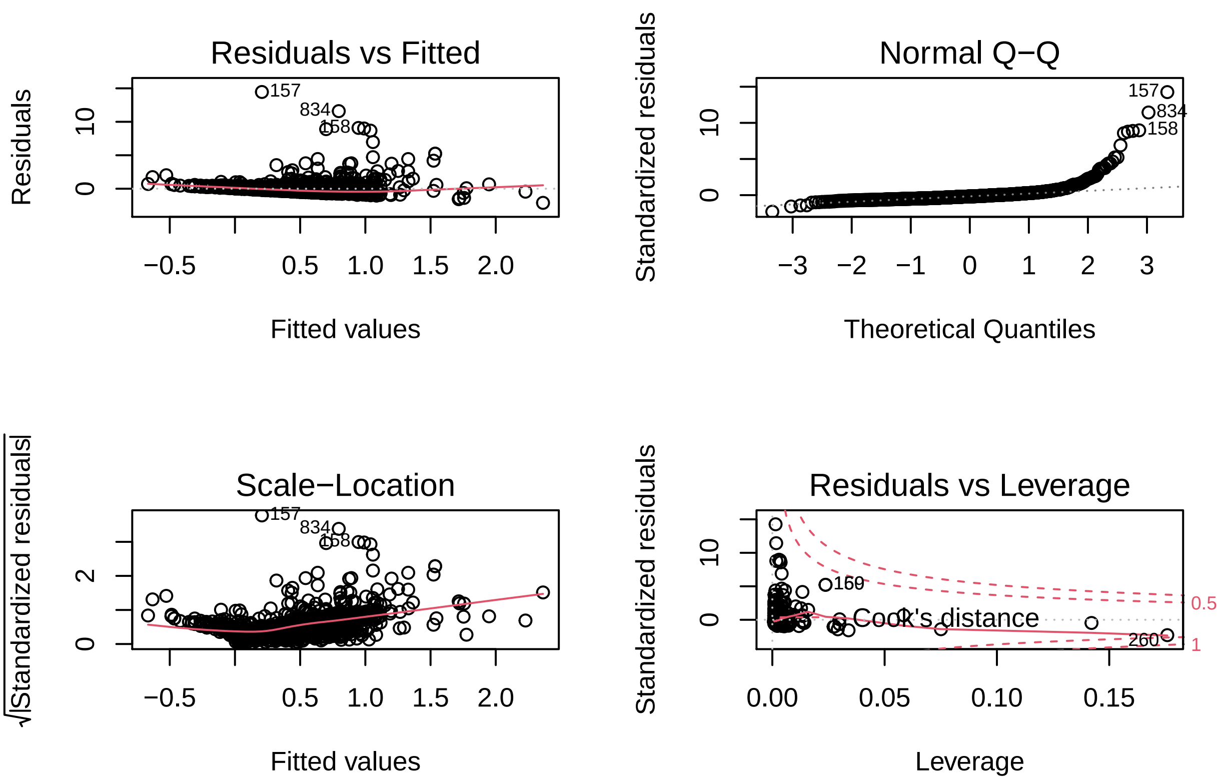
Examining the data:

```
summary(df)
```

```
##      sales             score           length
##  Min.   : 0.0100   Min.   :19.00   Min.   :  0.00
##  1st Qu.: 0.0900   1st Qu.:60.00   1st Qu.:  0.00
##  Median : 0.2100   Median :70.00   Median :  6.00
##  Mean   : 0.5032   Mean   :68.83   Mean   : 19.81
##  3rd Qu.: 0.4600   3rd Qu.:79.00   3rd Qu.: 21.55
##  Max.   :14.6600   Max.   :98.00   Max.   :683.13
```

To evaluate the assumptions of the large-sample linear model, I have made 4 regression plots along with the interpretations to them. In Linear model the sample size rule of thumb is that the regression analysis requires at least 20 cases per independent variable in the analysis. Linear model has five key assumptions:

- Linear relationship
- Multivariate normality
- No or little multicollinearity
- No auto-correlation
- Homoscedasticity



1. Fig.1 is interpretation of residual vs fitted regression plot: This scatter plot shows the distribution of residuals (errors) vs fitted values (predicted values). It reveals various useful insights including outliers.

If there exist any pattern (may be, a parabolic shape) in this plot, consider it as signs of non-linearity in the data. Here in Fig.1, a funnel shape is evident in the plot, I consider it as the signs of non constant variance i.e. heteroskedasticity.

2. Fig.2 is normal q-q plot regression interpretation: This q-q or quantile-quantile is a scatter plot which helps to validate the assumption of normal distribution in a data set. Using this plot we can infer if the data comes from a normal distribution. If yes, the plot would show fairly straight line. Absence of normality in the errors can be seen with deviation in the straight line. Here in Fig.2, the plot show a fairly straight line, confirm multivariate normality.

3. Fig.3 is scale location regression plot: This plot is also used to detect homoskedasticity (assumption of equal variance). It shows how the residual are spread along the range of predictors. Its similar to residual vs fitted value plot except it uses standardized residual values. Ideally, there should be no discernible pattern in the plot. This would imply that errors are normally distributed. But, here in the case of Fig.3, the plot shows discernible pattern (link a funnel shape), it would imply non-normal distribution of errors.

4. Fig.4 is residual vs leverage regression plot interpretation: Cook's distance attempts to identify the points which have more influence than other points. Such influential points tends to have a sizable impact of the regression line. In other words, adding or removing such points from the model can completely change the model statistics. Therefore, in this plot, the large values marked by cook's distance might require further investigation. For influential observations which are nothing but outliers, if not many, we can remove those rows.

2. Whether you consider the large-sample linear model sufficiently valid or not, proceed to fit the linear model using `lm()`.

```
library(gvlma)

fit <- lm(df$sales ~ df$score + df$length)
summary(fit)
```

```
##
## Call:
## lm(formula = df$sales ~ df$score + df$length)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.1125 -0.4223 -0.1852  0.0918 14.4534
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.0919732  0.1592648  -6.856 1.12e-11 ***
## df$score     0.0223899  0.0023092   9.696  < 2e-16 ***
## df$length    0.0027297  0.0006416   4.255 2.25e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.015 on 1209 degrees of freedom
## Multiple R-squared:  0.1022, Adjusted R-squared:  0.1007
## F-statistic: 68.79 on 2 and 1209 DF,  p-value: < 2.2e-16
```

This section of Summary() shows information including Residuals; Coefficients; Std. Error; t value; and Pr(>|t|). But I focus on the bottom of the summary output:

- Residual Standard Error is 1.015: Residual Standard Error is measure of the quality of a linear regression fit. Theoretically, every linear model is assumed to contain an error term E. Due to the presence of this error term, we are not capable of perfectly predicting our response variable sales from the predictor. The Residual Standard Error is the average amount that the response variable will deviate from the true regression line. In our example, the actual sales can deviate from the true regression line by approximately 1.015, on average.

- Multiple R-Squared is 0.1022: Also called the coefficient of determination, this is an oft-cited measurement of how well the model fits to the data. While there are many issues with using it alone (see Anscombe's quartet) , it's a quick and pre-computed check for the model. R-Squared subtracts the residual error from the variance in Y. The bigger the error, the worse the remaining variance will appear. Numerator doesn't have to be positive. If the model is so bad, it can actually end up with a negative R-Squared. Here in my above analysis, the Multiple R-squared is 0.1022.

- Adjusted R-Squared is 0.1007: Multiple R-Squared works great for simple linear (one variable) regression. However, in most cases, the model has multiple variables. The more variables we add, the more variance we're going to explain. So adjusted R-Squared normalizes Multiple R-Squared by taking into account how many samples we have and how many variables we're using. Notice how k is in the denominator.Here in my above analysis, the Multiple R-squared is 0.1007.

- F-Statistic is 68.79 on 2 and 1209 DF: F-statistic is a indicator of whether there is a relationship between our predictor and the response variables. The further the F-statistic is from 1 the better it is. However, how much larger the F-statistic needs to be depends on both the number of data points and the number of predictors. Generally, when the number of data points is large, an F-statistic that is only a little bit larger than 1 is already sufficient to reject the null hypothesis (H0 : There is no relationship between sales and score and length). The reverse is true as if the number of data points is small, a large F-statistic is required to be able to ascertain that there may be a relationship between predictor and response variables. In our example the F-statistic is 68.79 which is relatively larger than 1 given the size of our data, meaning reject the null hypothesis. In addition, p-value: $<$ 2.2e-16 is also sufficient to reject the null hypothesis.

Answer: based on the above observation, my conclusion is the large-sample linear model is not sufficiently valid.

3. Examine the coefficient for Metrics.Review.Score and give an interpretation of what it means.

```
summary(fit)
```

```
##
## Call:
## lm(formula = df$sales ~ df$score + df$length)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.1125 -0.4223 -0.1852  0.0918 14.4534
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.0919732  0.1592648  -6.856 1.12e-11 ***
## df$score     0.0223899  0.0023092   9.696  < 2e-16 ***
## df$length    0.0027297  0.0006416   4.255 2.25e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.015 on 1209 degrees of freedom
## Multiple R-squared:  0.1022, Adjusted R-squared:  0.1007
## F-statistic: 68.79 on 2 and 1209 DF,  p-value: < 2.2e-16
```

Interpreting Regression Coefficients of Score:

- In my test, the coefficient of Score is a positive number, indicating that as the value of the Score increases, the mean of the Sales also tends to increase. The sign of a regression coefficient tells whether there is a positive or negative correlation between independent variable and the dependent variable. A positive coefficient indicates that as the value of the independent variable increases, the mean of the dependent variable also tends to increase. A negative coefficient suggests that as the independent variable increases, the dependent variable tends to decrease.
- The estimation of the coefficient of Score is 0.0223899. It signifies how much the mean of the dependent variable changes given a one-unit shift in the independent variable while holding other variables in the model constant. This property of holding the other variables constant is crucial because it allows to assess the effect of each variable in isolation from the others.
- The t value is the coefficient divided by its standard error. The standard error is an estimate of the standard deviation of the coefficient, the amount it varies across cases. It can be thought of as a measure of the precision with which the regression coefficient is measured. If a coefficient is large compared to its standard error, then it is probably different from 0. Regression software compares the t statistic on variable with values in the Student's t distribution to determine the P value. The Student's t distribution describes how the mean of a sample with a certain number of observations is expected to behave.
- The P-values is $< 2e\text{-}16$, it works with coefficients together, to tell the relationships in the model are statistically significant.
- The coefficients in the model are estimates of the actual population parameters. To obtain unbiased coefficient estimates that have the minimum variance, and to be able to trust the p-values, the model must satisfy the seven classical assumptions of OLS linear regression.

4. Perform a hypothesis test to assess whether video game quality has a relationship with total sales. Please use `vcovHC` from the `sandwich` package with the default options ("HC3") to compute robust standard errors. To conduct the test, use `coeftest` from the `lmtest` package.

```
library(lmtest)

fit <- lm(df$sales ~ df$score + df$length)

library(sandwich)
result<-coeftest(fit, vcov. = vcovHC, type = "HC3")

print(result)
```

```
##
## t test of coefficients:
##
##               Estimate Std. Error t value  Pr(>|t|)
## (Intercept) -1.0919732  0.1806027 -6.0463 1.972e-09 ***
## df$score     0.0223899  0.0028686  7.8053 1.278e-14 ***
## df$length    0.0027297  0.0011670  2.3391   0.01949 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Answer: The P-values is 1.278e-14, it tells the video game quality has a statistically significant relationship with total sales.

5. How many more sales does your model predict for a game one standard-deviation higher than the mean review, vs. a game one standard-deviation lower than the mean review, holding all else equal? Answer this in two different ways:

(a) Compute the standard deviation of the review score, and multiply the appropriate model coefficient by two-times this standard deviation.

```
model <- lm(sales ~ score + length , data = df)
model
```

```
##
## Call:
## lm(formula = sales ~ score + length, data = df)
##
## Coefficients:
## (Intercept)        score       length
##    -1.09197      0.02239      0.00273
```

```
a <- sd(df$score)
b <- sd(df$length)
c <- a * 2 * 0.0224
cat("result is ", c)
```

```
## result is  0.5804407
```

(b) Use the `predict` function with the model that you have estimated. You can read the documentation for `predict.lm` which is the predict method for linear model objects (the type that you have fit here). Include a data frame (that has the same variable names as the data frame that you fitted the model against) in the `newdata` argument to `predict`. This data frame should have two rows and two columns. The column for the reviews should change from $\mu - \sigma$ to $\mu + \sigma$; the column for the play time should be set to a constant, sensible level (perhaps the μ of this variable).

```
x<-df$score
x1 = mean(x) + sd(x)
x2 = mean(x) - sd(x)
y <-mean(df$length)
```

```
## mu + sigma of score is : 81.78465

## mu - sigma of score is : 55.87212

## mu of length is : 19.80822

## New data frame is

##      score   length
## 1 81.78465 19.80822
## 2 55.87212 19.80822

##         fit       lwr      upr
## 1 0.7932500 -1.198935 2.785435
## 2 0.2130701 -1.779114 2.205255
```

```
## result is  0.5801799
```

Answer: holding all else equal, 0.58 more sales does my model predict for a game one standard-deviation higher than the mean review, vs. a game one standard-deviation lower than the mean review.

5. **Optional:** Open the attached paper by Joe Cox, and read section 3. Which assumption did the author focus on, and why do you think that is?

The assumption that the author focus on is heavy-tailed distributions or long-tailed nature of the unit sales variable. The "long-tailed nature" is to emphasize "individualization", "customer power", that is, the sales defendant on many factors include a number of variables that could theoretically explain variations in unit sales. in practical, to our case, the sales is to make very little money, but to make a lot of people's money. If you want to subdivide the sales market into very small, then you will find that the accumulation of these small markets will bring about an obvious long tail effect.

In the paper, the Figure 2 shows a graphical representation of the dependent variable, where the heavily tailed nature of the distribution is obvious. "This supports various claims from the literature that a minority of titles are responsible for a disproportionately high proportion of total sales, with the top 10% of titles observed to be responsible for over 54% of total unit sales. Just under 28% of titles in the sample sold fewer than 100 000 copies. The long-tailed nature of the variable can be demonstrated via a comparison of the mean and median value, where the median title sold 190 000 copies against a mean of 480 000." Four parameters are reported on the heavy-tailed distribution in Table 1, these being a the tail index, b the skewness parameter, g the scale parameter and d the location parameter. "As the estimated value of b is exactly equal to 1, this implies that the quantile estimators should be used in favor of the maximum likelihood estimators. Of these quantile estimations, only the 'S0' parameterisations are reported, where these are based on the Zolotarev's (1983) M for an alpha stable distribution with skewness = b, as well as an intuitive interpretation relative to other parameters."

Note: Maximum score on any homework is 100%