Robótica Aérea
Módulo III, Tema 2: Sensor fusion
Tema 2.1: Bayes filter

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Contenidos

Probablistic robotics

- The standard MULTISENSOR DATA FUSION methods employed in robotics are based on **probabilistic methods**.
- PROBABILISTIC ROBOTICS intends to address the **uncertainties in robot perception and action**.
- Instead of relying on a single "best guess", probabilistic algorithms represent information by **probability distributions over a whole space of guesses**.
- By doing so, they can **represent ambiguity and degree of belief** in a mathematically sound way. **Estimation and control choices** are made **robustly by taking into account of the uncertainty**.
- Probabilistic methods used in robotics are generally based on **Bayes' rule** (Teorema de Bayes) for **combining prior information (información previa) and observation information**.
- We shall illustrate this concept with **mobile robot localization** as an example - the problem of *estimating a robot's coordinates relative to an external reference frame*.
- The **belief** $bel(x)$ (creencia) of its location is represented by a **probability density function** (función de densidad de probabilidad) over the space of all locations x.

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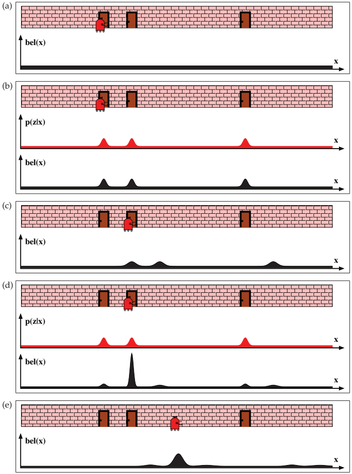

Figure: Mobile robot **localization** using probabilistic methods [Thrun, 2005].

- Prior to sensor measurements, the belief $bel(x)$ has a **uniform distribution** over all locations.
- After the first sensor measurement z, the robot realizes that it is **in front of a door**.
  - For a **given x position**, the **probability of the sensor detecting a door at** x is given by $p(z|x)$, **three bell-shape distributions** in front of three indistinguishable doors.
  - The robot then **updates the belief** $bel(x)$ accordingly, including **three distinct hypotheses** which are equally plausible given the sensor data.
  - The robot also assigns **small but non-zero probabilities in front of walls**, to account for the **possibilities of errors in its assessment of seeing a door**. The ability to maintain **low-probability hypotheses** is essential for attaining **robustness**.

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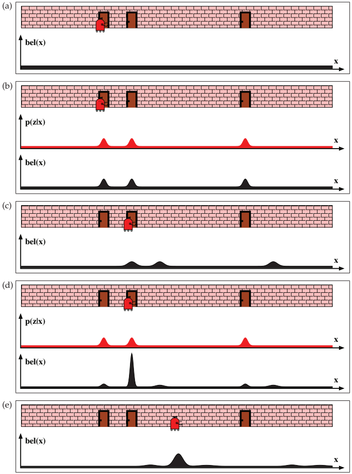

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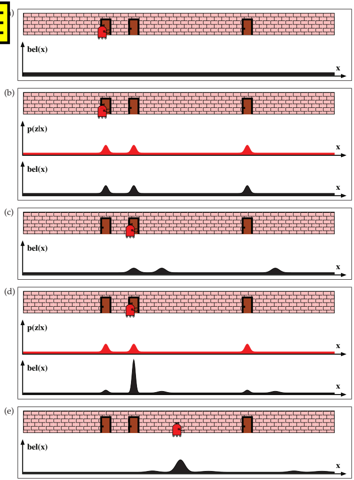

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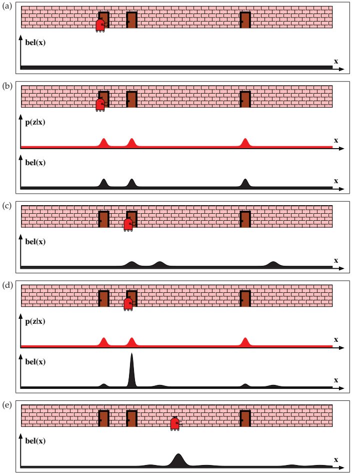

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- After **moving**, the **belief** has been **shifted** in the direction of motion. The **larger spread** of $bel(x)$ reflects the **uncertainty in robot motion**.
- The sensor detects a **second door**.
  - The probability of the sensor detecting a door, $p(z|x)$, is the **same as the first sensor measurement**.
  - However, for the belief $bel(x)$, based on prior information and current sensor measurements, the probabilistic algorithm now can place **most of the probability near one of the doors**. The robot is now **quite confident** about where it is.
- The probabilistic algorithm **updates the belief** $bel(x)$ **recursively** as the robot moves. At each iteration, there are two steps:
  - PREDICTION/PROPAGATION (propagación) by integrating equations of motion with control inputs
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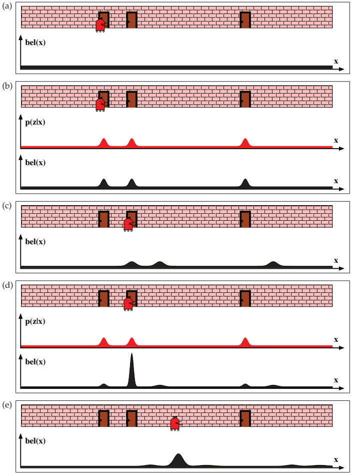

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Discrete probability distribution

$$\sum_x p(x) = \sum_{i=1}^{n} P(X = x_i) =$$

- DISCRETE RANDOM VARIABLE (variable aleatoria discreta) X can have a countable number of values in $\{x_1, x_2, \cdots, x_n\}$.
- PROBABILITY MASS FUNCTION (PMF, función de masa de probabilidad) of X, $p(x) = P(X = x_i)$, denotes the **probability** of X having a certain **value** x_i.
- For instance, a **fair coin** (moneda) gives $p(head, cara) = p(tail, cruz) =$.
- Discrete probability **sums up to** .

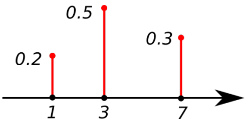

Figure: The probability mass function of a discrete probability distribution: $p(1) = 0.2$, $p(3) = 0.5$, $p(7) = 0.3$ (source, ©Oleg Alexandrov).

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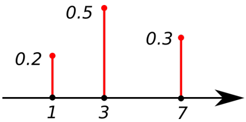

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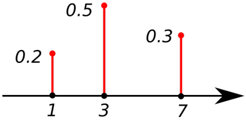

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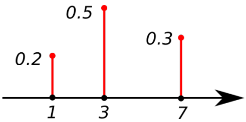

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Continuous probability distribution

$$P(X \in [a, b]) = \int_a^b p(x)\,\mathrm{d}x, \quad \int_{-\infty}^{\infty} p(x)\,\mathrm{d}x =$$

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- PROBABILITY DENSITY FUNCTION (PDF, función de densidad de probabilidad) $p(x)$ is the probability "per unit length".
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  - the probability for X to have a **particular value** x.
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- An example in thermodynamics: **1D Maxwellian velocity distribution function**.

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- **Integration** of the PDF over the **whole continuous space**, i.e., zeroth moment $\int_{-\infty}^{\infty} x^0 p(x)\,\mathrm{d}x$, equals to **one**.
- There can be $p(x) > 1$ **for a continuous PDF**, but **NOT for a discrete PMF**. For instance, Dirac Delta function with infinite value at origin and zero elsewhere, $\int_{-\infty}^{\infty} \delta(x)\,\mathrm{d}x = 1$.
- An example in thermodynamics: **1D Maxwellian velocity distribution function**.

1D Maxwellian velocity distribution function

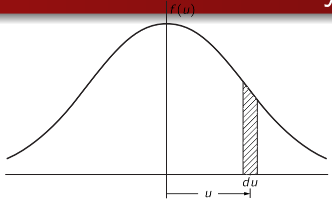

Figure: A **velocity distribution function**. The shaded area is $f(u)\,\mathrm{d}u$. (source, ©R. Feynman).

$$f_M(u) = \sqrt{\frac{m}{2\pi k_B T}} \exp\left(-\frac{mu^2/2}{k_B T}\right)$$

- A gas in **thermal equilibrium at rest** has the mean velocity $\bar{u} = \quad$.
- The measures the **random jiggling motion** of the atoms. With k_B the Boltzmann constant, $k_B T$ has the unit of energy J.
- The **possibility (fraction) of particles having velocities within a range** $\mathrm{d}u$ about u, $[u, u + \mathrm{d}u]$, is .

- For the zeroth moment, you can prove $\int_{-\infty}^{\infty} u^0 f_M(u)\,\mathrm{d}u = \quad$ by using Gaussin integral $\int_{-\infty}^{\infty} exp(-x^2)\,\mathrm{d}x = \sqrt{\pi}$.
- If the number density of this gas is n (in m^{-3}), the **number density of particles having velocities within a range** $\mathrm{d}u$ about u, $[u, u + \mathrm{d}u]$, is $nf_M(u)\,\mathrm{d}u$ and we have $\int_{-\infty}^{\infty} nf_M(u)\,\mathrm{d}u = n\int_{-\infty}^{\infty} f_M(u)\,\mathrm{d}u = \quad$.

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- The measures the average energy of the particle, which can be derived from the second moment as $\int_{-\infty}^{\infty} \frac{mu^2}{2} f_M(u)\,\mathrm{d}u = \quad$.

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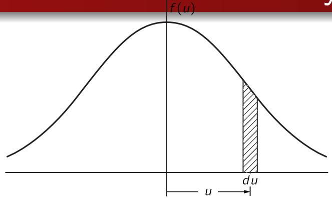

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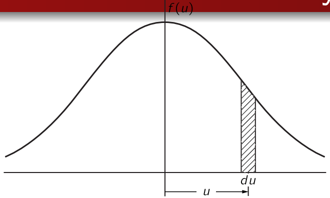

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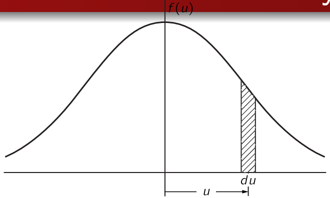

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$$f_M(u) = \sqrt{\frac{m}{2\pi k_B T}} \exp\left(-\frac{mu^2/2}{k_B T}\right)$$

- A gas in **thermal equilibrium at rest** has the mean velocity $\bar{u} = 0$.
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- For the zeroth moment, you can prove $\int_{-\infty}^{\infty} u^0 f_M(u)\,\mathrm{d}u =$ by using Gaussin integral $\int_{-\infty}^{\infty} exp(-x^2)\,\mathrm{d}x = \sqrt{\pi}$.
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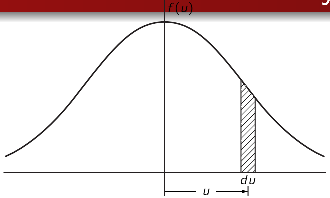
first moment $\int_{-\infty}^{\infty} u^1 f_M(u)\,\mathrm{d}u =$

.

- The measures the average energy of the particle, which can be derived from the second moment as $\int_{-\infty}^{\infty} \frac{mu^2}{2} f_M(u)\,\mathrm{d}u =$.

1D Maxwellian velocity distribution function



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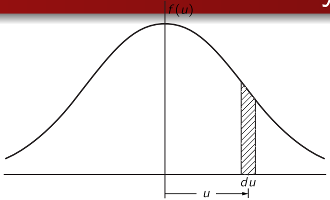

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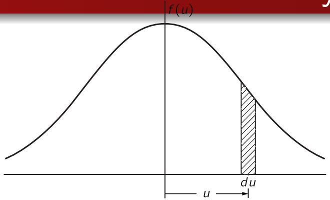

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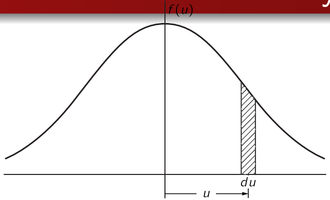

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Joint probability

The JOINT PROBABILITY (probabilidad conjunta) of two random variables X and Y describes the probability that both events $X = x$ and $Y = y$ occur together.

$$p(x, y) = P(X = x, Y = y)$$

	$X = 0$	$X = 1$
$Y = 0$	30	20
$Y = 1$	10	40

$$P(X = 0, Y = 0) = \quad \%$$
$$P(X = 0, Y = 1) = \quad \%$$
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Conditional probability

The CONDITIONAL PROBABILITY (probabilidad condicionada) of $X = x$ given $Y = y$ (x dado y) describes the probability that the event $X = x$ occurs **given the occurrence of the other event** $Y = y$ as a condition. In other words, it is the probability of $X = x$ conditioned on $Y = y$.

$$p(x|y) = P(X = x|Y = y)$$

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$Y = 0$	30	20
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$P(X = 0, Y = 1) = 10\%$ $\quad$ $P(X = 0|Y = 1) =$ %

$P(X = 1, Y = 0) = 20\%$ $\quad$ $P(X = 1|Y = 0) =$ %

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$$P(X = 1, Y = 0) = 20\% \qquad P(X = 1|Y = 0) = \frac{20}{30 + 20} = 40\%$$

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Total probability

	$X=0$	$X=1$
$Y=0$	30	20
$Y=1$	10	40

$P(X=0,Y=0)=30\%$ $\quad P(X=0|Y=0)=\dfrac{30}{30+20}=60\%$ $\quad P(X=0)=\quad\%$

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Joint probability $\quad p(x,y)=$

Total probability $\quad p(x)=$

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Total probability $\quad p(x) =$

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Joint probability $p(x,y)=p(x|y)p(y)$

Total probability $p(x)=$

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Joint probability $\quad p(x,y) = p(x|y)p(y)$

Total probability $\quad p(x) = \int_{-\infty}^{\infty} p(x,y)\,\mathrm{d}y = \int_{-\infty}^{\infty} p(x|y)p(y)\,\mathrm{d}y$

Joint, conditional, total probability

$p(x,y)$	x_1	x_2	x_3	x_4	$p(y)$
y_1	$1/8$	$1/16$	$1/32$	$1/32$	
y_2	$1/16$	$1/8$	$1/32$	$1/32$	
y_3	$1/16$	$1/16$	$1/16$	$1/16$	
y_2	$1/4$	0	0	0	
$p(x)$					

- Joint probability $p(x_2, y_2) = 1/8$.
- Total probability $p(y_2)$?
- Conditional probability $p(x_2|y_2)$?
- $\sum_{x_i} p(x_i) = \sum_{y_i} p(y_i)$?
- $\sum_{x_i} p(x_i|y_2)$?

Joint, conditional, total probability

$p(x,y)$	x_1	x_2	x_3	x_4	$p(y)$
y_1	1/8	1/16	1/32	1/32	1/4
y_2	1/16	1/8	1/32	1/32	1/4
y_3	1/16	1/16	1/16	1/16	1/4
y_2	1/4	0	0	0	1/4
$p(x)$	1/2	1/4	1/8	1/8	

- Joint probability $p(x_2, y_2) = 1/8$.
- Total probability $p(y_2)$?

$$p(y_2) = \sum_{x_i} p(y_2, x_i) = 1/16 + 1/8 + 1/32 + 1/32 = 1/4$$

- Conditional probability $p(x_2|y_2)$?
- $\sum_{x_i} p(x_i) = \sum_{y_i} p(y_i)$?
- $\sum_{x_i} p(x_i|y_2)$?

Joint, conditional, total probability

$p(x,y)$	x_1	x_2	x_3	x_4	$p(y)$
y_1	$1/8$	$1/16$	$1/32$	$1/32$	$1/4$
y_2	$1/16$	$1/8$	$1/32$	$1/32$	$1/4$
y_3	$1/16$	$1/16$	$1/16$	$1/16$	$1/4$
y_2	$1/4$	0	0	0	$1/4$
$p(x)$	$1/2$	$1/4$	$1/8$	$1/8$	

- Joint probability $p(x_2, y_2) = 1/8$.
- Total probability $p(y_2)$?

$$p(y_2) = \sum_{x_i} p(y_2, x_i) = 1/16 + 1/8 + 1/32 + 1/32 = 1/4$$

- Conditional probability $p(x_2|y_2)$? $p(x_2|y_2) = \frac{1/8}{1/4} = 1/2$.
- $\sum_{x_i} p(x_i) = \sum_{y_i} p(y_i)$?
- $\sum_{x_i} p(x_i|y_2)$?

Joint, conditional, total probability

$p(x,y)$	x_1	x_2	x_3	x_4	$p(y)$
y_1	1/8	1/16	1/32	1/32	1/4
y_2	1/16	1/8	1/32	1/32	1/4
y_3	1/16	1/16	1/16	1/16	1/4
y_2	1/4	0	0	0	1/4
$p(x)$	1/2	1/4	1/8	1/8	$\sum = 1$

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y_2	$1/16$	$1/8$	$1/32$	$1/32$	$1/4$
y_3	$1/16$	$1/16$	$1/16$	$1/16$	$1/4$
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Bayes' rule

$$p(x, y) = p(x|y)p(y) = p(y|x)p(x) \quad \Longrightarrow \quad \boxed{p(H|D) = \frac{p(D|H)p(H)}{p(D)}}$$

- In Bayesian probability theory, there are two events, being **hypothesis** H (e.g., position x) and **observation data** D (e.g., measurement z from the sensor to detect a door).
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Bayes' rule

$$p(H|D) = \frac{p(D|H)p(H)}{p(D)}$$

- The , describes **how likely it is to make a observation**. It can be obtained by

- To calculate the posterior probability distribution $p(H|D)$ for **any value** of H, . For this reason, a **constant** normalizer coefficient η is normally used in Bayes' rule, which guarantees $\Sigma_x p(x|y) = 1$.

$$\boxed{p(H|D) = \eta p(D|H)p(H),}$$

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$$\boxed{p(x|y,\ )} = \frac{p(x,y,\ )}{p(y,\ )} = \frac{p(y|x,\ )p(x,\ )}{p(y,\ )} \boxed{= \frac{p(y|x,\ )p(x|\ )}{p(y|\ )}}$$

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Independent, conditionally independent

- Two random variables X and Y are INDEPENDENT if *the occurrence of event* $X = x$ *does not affect the probability of occurrence of event* $Y = y$. The **joint probability** writes

$$p(x,y) = p(x)p(y) \ .$$

Equivalently, two random variables X and Y are **independent** if *knowing the value of* Y *does not change the probabilities of* X. The **conditional probability** writes

$$p(x|y) = \frac{p(x,y)}{p(y)} = \frac{p(x)p(y)}{p(y)} = p(x) \ .$$

- Two random variables X and Y are CONDITIONALLY INDEPENDENT given a third **conditioning random variable** Z if and only if they are **independent** in their **conditional probability** distribution given Z.

$$p(x,y|\ \ ) = p(x|\ \ )p(y|\ \ ) \ .$$

Equivalently, **conditional independence** implies that *a variable* Y *carries no information about a variable* X *if the value of conditioning variable* Z *is known*.

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- Generally, for conditional independence and (absolute) independence, **one condition does not imply the other.**

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- Total probability, $p(x) =$.
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hypothesis , observation data , prior probability distribution , posterior probability distribution , likehood function , normalizer .

- Bayes' rule with multiple conditions

- Totally independent, and .
- Conditionally independent, and .

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State, measurement, and control

- STATE is denoted by x,
- ENVIRONMENT MEASUREMENT DATA is denoted by z,
- CONTROL DATA is denoted by u,
- The specific variables included in x, u, and z **depend on the context**.
- TIME

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- The specific variables included in $\boldsymbol{x}$, $\boldsymbol{u}$, and $\boldsymbol{z}$ **depend on the context**.
- TIME is generally defined **discretely** at instant t_k, $k = 0, 1, 2 \cdots$ (0 for the initial instant).

Dynamic Bayes network

- At the instant t_k, the following quantities are defined
  - x_k,
  - $x_{0:k}$,
  - u_k,
  - $u_{1:k}$,
  - z_k,
  - $z_{1:k}$,

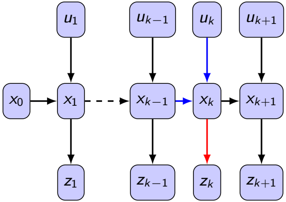

Figure: The **dynamic Bayes network** characterizes the evolution of controls, states, and measurements.

- The **evolution** of is governed by **probabilistic distributions**.

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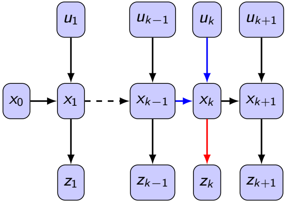

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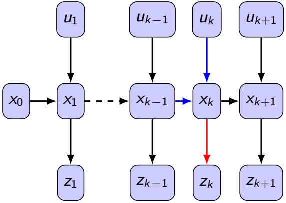

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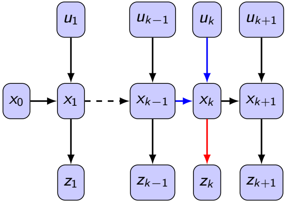

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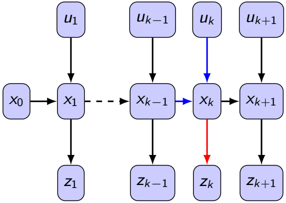

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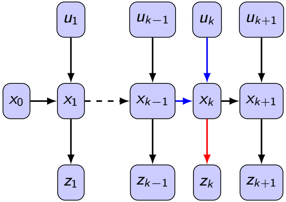

Figure: The **dynamic Bayes network** characterizes the evolution of controls, states, and measurements.

- The **evolution** of states and measurements is governed by **probabilistic distributions**.

the **evolution of state** is characterized by

$$p(x_k| \qquad\qquad\qquad) .$$

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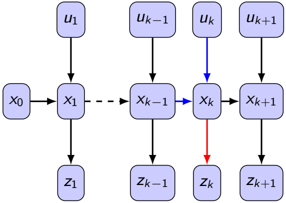

Figure: The **dynamic Bayes network** characterizes the evolution of controls, states, and measurements.

- The **evolution** of states and measurements is governed by **probabilistic distributions**.
  - Since the emergence of state x_t might be **conditioned on all past states, measurements, and controls**, the **evolution of state** is characterized by

$$p(x_k | x_{0:k-1}, z_{1:k-1}, u_{1:k}) \ .$$

  - The **evolution of measurement** z_k is characterized by

$$p(z_k | \quad) \ .$$

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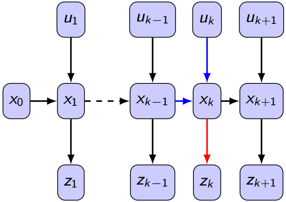

Figure: The **dynamic Bayes network** characterizes the evolution of controls, states, and measurements.

- The **evolution** of states and measurements is governed by **probabilistic distributions**.
  - Since the emergence of state x_t might be **conditioned on all past states, measurements, and controls**, the **evolution of state** is characterized by

$$p(x_k | x_{0:k-1}, z_{1:k-1}, u_{1:k}) \ .$$

  - The **evolution of measurement** z_k is characterized by

$$p(z_k | x_{0:k}, z_{1:k-1}, u_{1:k}) \ .$$

Markov assumption

The MARKOV ASSUMPTION postulates that **past and future data are independent if one knows the current state x_k.**

the state transition probability (probabilidad de transición de estado) characterizes the **state evolution** as

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the measurement probability (probabilidad de medición) characterizes the **measurement evolution** as

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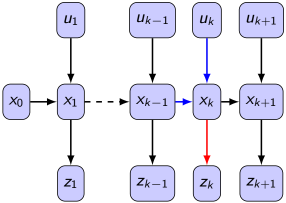

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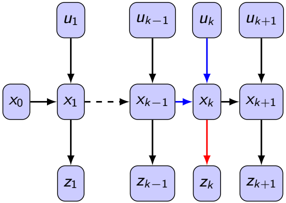

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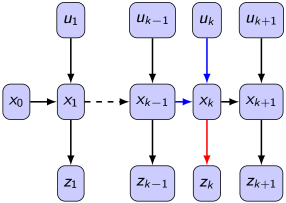

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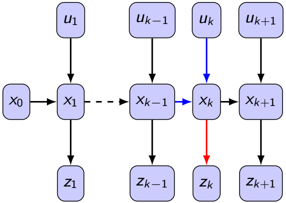

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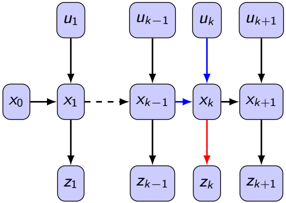

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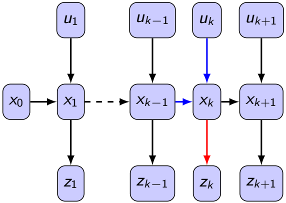

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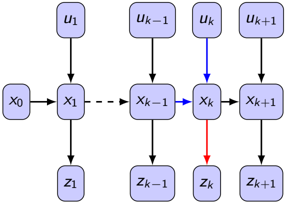

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Probabilistic filtering

- FILTERING (técnicas de filtrado) is concerned with the sequential process of maintaining a probabilistic model for a **state** that evolves over time and is periodically observed by a sensor. It forms the basis for many problems in **tracking and navigation**.
- The general filtering problem can be formulated in **Bayesian form**.
- The robot has an internal belief of its **true state**. BELIEFS are represented by conditional probabilities of **conditioned** on the available

- PREDICTION (predicción) $\overline{bel}(\quad)$: the probability distribution of , , conditioned on and .

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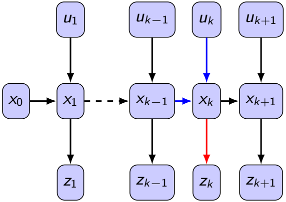

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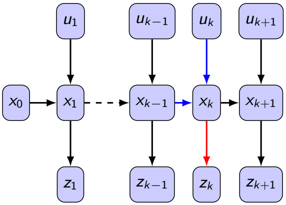

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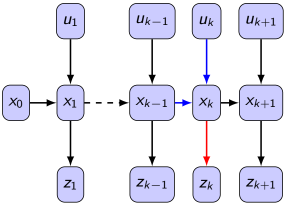

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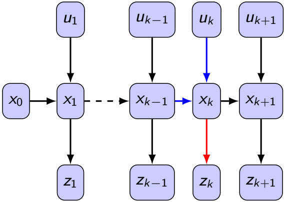

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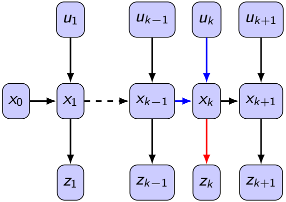

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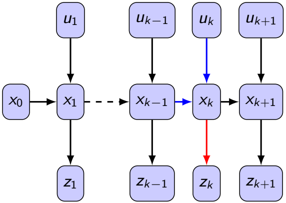

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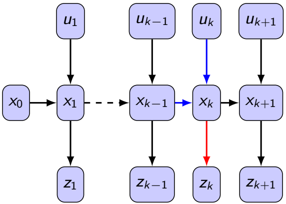

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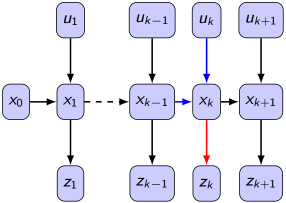

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Bayes filter: inputs and outputs

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- The above **pseudo-algorithm** depicts one **iteration** of the **recursive Bayes Filter**. At each iteration, the **current belief** $bel(x_k)$ is calculated from the belief $bel(x_{k-1})$ obtained in the **previous iteration**.
- Inputs
  - $bel(x_{k-1})$,
  - u_k,
  - z_k,
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  - u_k, the **control** at t_k;
  - z_k, the **measurement** at t_k.
- Outputs (return):
  - $bel(x_k)$, the **updated belief** at t_k.

Bayes filter: prediction/propagation step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
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4: **end for**
5: **return** $bel(x_k)$

PREDICTION/PROPAGATION step (predicción/propogación): the , $\overline{bel}(x_k)$, is given by the **integration** of the multiplication of , $p(x_k | u_k, x_{k-1})$, and , $bel(x_{k-1})$, over , x_{k-1}.

$$\overline{bel}(x_k) \xlongequal{\text{prediction def.}} p(x_k | z_{1:k-1}, u_{1:k})$$

$$\xlongequal{\text{total probability}}$$

$$\xlongequal{\text{conditional probability}}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{conditional independence } u_k, x_{k-1}}$$

$$\xlongequal{\text{belief def.}}$$

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$$\overline{bel}(x_k) \overset{\text{prediction def.}}{=\!=\!=} p(x_k|z_{1:k-1}, u_{1:k})$$

$$\overset{\text{total probability}}{=\!=\!=}$$

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$$\overset{\text{belief def.}}{=\!=\!=}$$

Bayes filter: prediction/propagation step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

PREDICTION/PROPAGATION step (predicción/propogación): the prediction, $\overline{bel}(x_k)$, is given by the **integration** of the multiplication of state transition probability, $p(x_k|u_k, x_{k-1})$, and , $bel(x_{k-1})$, over , x_{k-1}.

$$\overline{bel}(x_k) \xlongequal{\text{prediction def.}} p(x_k|z_{1:k-1}, u_{1:k})$$

$$\xlongequal{\text{total probability}}$$

$$\xlongequal{\text{conditional probability}}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{conditional independence } u_k, x_{k-1}}$$

$$\xlongequal{\text{belief def.}}$$

Bayes filter: prediction/propagation step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
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$$\overline{bel}(x_k) \xlongequal{\text{prediction def.}} p(x_k|z_{1:k-1}, u_{1:k})$$

$$\xlongequal{\text{total probability}}$$

$$\xlongequal{\text{conditional probability}}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{conditional independence } u_k, x_{k-1}}$$

$$\xlongequal{\text{belief def.}}$$

Bayes filter: prediction/propagation step

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1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
4: **end for**
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$$\xlongequal{\text{conditional probability}}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{conditional independence } u_k, x_{k-1}}$$

$$\xlongequal{\text{belief def.}}$$

Bayes filter: prediction/propagation step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

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$$\overline{bel}(x_k) \xlongequal{\text{prediction def.}} p(x_k|z_{1:k-1}, u_{1:k})$$

$$\xlongequal{\text{total probability}} \int p(x_k, x_{k-1}|z_{1:k-1}, u_{1:k})\, \mathrm{d}x_{k-1}$$

$$\xlongequal{\text{conditional probability}}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{conditional independence } u_k, x_{k-1}}$$

$$\xlongequal{\text{belief def.}}$$

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Bayes filter: prediction/propagation step

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$$\overline{bel}(x_k) \xlongequal{\text{prediction def.}} p(x_k|z_{1:k-1}, u_{1:k})$$

$$\xlongequal{\text{total probability}} \int p(x_k, x_{k-1}|z_{1:k-1}, u_{1:k})\,\mathrm{d}x_{k-1}$$

$$\xlongequal{\text{conditional probability}} \int p(x_k|x_{k-1}, z_{1:k-1}, u_{1:k})p(x_{k-1}|z_{1:k-1}, u_{1:k})\,\mathrm{d}x_{k-1}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{conditional independence } u_k, x_{k-1}}$$

$$\xlongequal{\text{belief def.}}$$

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Bayes filter: prediction/propagation step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
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3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

PREDICTION/PROPAGATION step (predicción/propogación): the prediction, $\overline{bel}(x_k)$, is given by the **integration** of the multiplication of state transition probability, $p(x_k|u_k, x_{k-1})$, and belief at t_{k-1}, $bel(x_{k-1})$, over all possible state at t_{k-1}, x_{k-1}.

$$\overline{bel}(x_k) \overset{\text{prediction def.}}{=\!=\!=} p(x_k|z_{1:k-1}, u_{1:k})$$

$$\overset{\text{total probability}}{=\!=\!=} \int p(x_k, x_{k-1}|z_{1:k-1}, u_{1:k})\, \mathrm{d}x_{k-1}$$

$$\overset{\text{conditional probability}}{=\!=\!=} \int p(x_k|x_{k-1}, z_{1:k-1}, u_{1:k}) p(x_{k-1}|z_{1:k-1}, u_{1:k})\, \mathrm{d}x_{k-1}$$

$$\overset{\text{Markov assumption}}{=\!=\!=} \int p(x_k|x_{k-1}, u_k) p(x_{k-1}|z_{1:k-1}, u_{1:k})\, \mathrm{d}x_{k-1}$$

$$\overset{\text{conditional independence } u_k, x_{k-1}}{=\!=\!=}$$

$$\overset{\text{belief def.}}{=\!=\!=}$$

Bayes filter: prediction/propagation step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

PREDICTION/PROPAGATION step (predicción/propogación): the prediction, $\overline{bel}(x_k)$, is given by the **integration** of the multiplication of state transition probability, $p(x_k|u_k, x_{k-1})$, and belief at t_{k-1}, $bel(x_{k-1})$, over all possible state at t_{k-1}, x_{k-1}.

$$
\begin{aligned}
\overline{bel}(x_k) &\xlongequal{\text{prediction def.}} p(x_k|z_{1:k-1}, u_{1:k}) \\
&\xlongequal{\text{total probability}} \int p(x_k, x_{k-1}|z_{1:k-1}, u_{1:k})\, \mathrm{d}x_{k-1} \\
&\xlongequal{\text{conditional probability}} \int p(x_k|x_{k-1}, z_{1:k-1}, u_{1:k}) p(x_{k-1}|z_{1:k-1}, u_{1:k})\, \mathrm{d}x_{k-1} \\
&\xlongequal{\text{Markov assumption}} \int p(x_k|x_{k-1}, u_k) p(x_{k-1}|z_{1:k-1}, u_{1:k})\, \mathrm{d}x_{k-1} \\
&\xlongequal{\text{conditional independence } u_k, x_{k-1}} \int p(x_k|x_{k-1}, u_k) p(x_{k-1}|z_{1:k-1}, u_{1:k-1})\, \mathrm{d}x_{k-1} \\
&\xlongequal{\text{belief def.}}
\end{aligned}
$$

Bayes filter: prediction/propagation step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1})bel(x_{k-1})\,\mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

PREDICTION/PROPAGATION step (predicción/propogación): the prediction, $\overline{bel}(x_k)$, is given by the **integration** of the multiplication of state transition probability, $p(x_k|u_k, x_{k-1})$, and belief at t_{k-1}, $bel(x_{k-1})$, over all possible state at t_{k-1}, x_{k-1}.

$$
\begin{aligned}
\overline{bel}(x_k) &\xlongequal{\text{prediction def.}} p(x_k|z_{1:k-1}, u_{1:k}) \\
&\xlongequal{\text{total probability}} \int p(x_k, x_{k-1}|z_{1:k-1}, u_{1:k})\,\mathrm{d}x_{k-1} \\
&\xlongequal{\text{conditional probability}} \int p(x_k|x_{k-1}, z_{1:k-1}, u_{1:k})p(x_{k-1}|z_{1:k-1}, u_{1:k})\,\mathrm{d}x_{k-1} \\
&\xlongequal{\text{Markov assumption}} \int p(x_k|x_{k-1}, u_k)p(x_{k-1}|z_{1:k-1}, u_{1:k})\,\mathrm{d}x_{k-1} \\
&\xlongequal{\text{conditional independence } u_k, x_{k-1}} \int p(x_k|x_{k-1}, u_k)p(x_{k-1}|z_{1:k-1}, u_{1:k-1})\,\mathrm{d}x_{k-1} \\
&\xlongequal{\text{belief def.}} \int p(x_k|u_k, x_{k-1})bel(x_{k-1})\,\mathrm{d}x_{k-1}
\end{aligned}
$$

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Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, dx_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

MEASUREMENT UPDATE/CORRECTION step (corrección): the , $bel(x_k)$, is given by the **multiplication** of , η, , $p(z_k|x_k)$, and , $\overline{bel}(x_k)$.

$$bel(x_k) \xlongequal{\text{belief def.}} p(x_k|z_{1:k}, u_{1:k})$$

$$\xlongequal{\text{Bayes' rule}}$$

$$\xlongequal{\text{Normalizer}}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{prediction def.}}$$

Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

Measurement update/correction step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of ,
η, , $p(z_k|x_k)$, and
, $\overline{bel}(x_k)$.

$$bel(x_k) \xlongequal{\text{belief def.}} p(x_k|z_{1:k}, u_{1:k})$$

$$\xlongequal{\text{Bayes' rule}}$$

$$\xlongequal{\text{Normalizer}}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{prediction def.}}$$

Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1})bel(x_{k-1})\,\mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, , $p(z_k|x_k)$, and , $\overline{bel}(x_k)$.

$$bel(x_k) \xlongequal{\text{belief def.}} p(x_k|z_{1:k}, u_{1:k})$$
$$\xlongequal{\text{Bayes' rule}}$$
$$\xlongequal{\text{Normalizer}}$$
$$\xlongequal{\text{Markov assumption}}$$
$$\xlongequal{\text{prediction def.}}$$

Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

for all possible values of state x_k **do**
  PREDICTION: $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
  UPDATE: $bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
end for
return $bel(x_k)$

MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, measurement probability, $p(z_k | x_k)$, and , $\overline{bel}(x_k)$.

$$
\begin{aligned}
bel(x_k) &\overset{\text{belief def.}}{=\!=\!=} p(x_k | z_{1:k}, u_{1:k}) \\
&\overset{\text{Bayes' rule}}{=\!=\!=} \\
&\overset{\text{Normalizer}}{=\!=\!=} \\
&\overset{\text{Markov assumption}}{=\!=\!=} \\
&\overset{\text{prediction def.}}{=\!=\!=}
\end{aligned}
$$

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Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

$$bel(x_k) \xlongequal{\text{belief def.}} p(x_k | z_{1:k}, u_{1:k})$$

$$\xlongequal{\text{Bayes' rule}}$$

$$\xlongequal{\text{Normalizer}}$$

$$\xlongequal{\text{Markov assumption}}$$

$$\xlongequal{\text{prediction def.}}$$

MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, measurement probability, $p(z_k | x_k)$, and prediction, $\overline{bel}(x_k)$.

Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
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3: UPDATE: $bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, measurement probability, $p(z_k|x_k)$, and prediction, $\overline{bel}(x_k)$.

$$bel(x_k) \overset{\text{belief def.}}{=\!=} p(x_k | z_{1:k}, u_{1:k})$$

$$\overset{\text{Bayes' rule}}{=\!=} \frac{p(z_k | x_k \qquad)\, p(x_k \qquad)}{p(z_k | \qquad)}$$

$$\overset{\text{Normalizer}}{=\!=}$$

$$\overset{\text{Markov assumption}}{=\!=}$$

$$\overset{\text{prediction def.}}{=\!=}$$

Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, measurement probability, $p(z_k|x_k)$, and prediction, $\overline{bel}(x_k)$.

$$
\begin{aligned}
bel(x_k) &\xlongequal{\text{belief def.}} p(x_k|z_{1:k}, u_{1:k}) \\
&\xlongequal{\text{Bayes' rule}} \frac{p(z_k|x_k, z_{1:k-1}, u_{1:k})p(x_k|z_{1:k-1}, u_{1:k})}{p(z_k|z_{1:k-1}, u_{1:k})} \\
&\xlongequal{\text{Normalizer}} \eta p(z_k|x_k, z_{1:k-1}, u_{1:k})p(x_k|z_{1:k-1}, u_{1:k}) \\
&\xlongequal{\text{Markov assumption}} \\
&\xlongequal{\text{prediction def.}}
\end{aligned}
$$

$$
\eta = \frac{1}{p(z_k|z_{1:k-1}, u_{1:k})}
$$

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Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
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3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
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MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, measurement probability, $p(z_k|x_k)$, and prediction, $\overline{bel}(x_k)$.

$$
\begin{aligned}
bel(x_k) &\xlongequal{\text{belief def.}} p(x_k|z_{1:k}, u_{1:k}) \\
&\xlongequal{\text{Bayes' rule}} \frac{p(z_k|x_k, z_{1:k-1}, u_{1:k})p(x_k|z_{1:k-1}, u_{1:k})}{p(z_k|z_{1:k-1}, u_{1:k})} \\
&\xlongequal{\text{Normalizer}} \eta p(z_k|x_k, z_{1:k-1}, u_{1:k})p(x_k|z_{1:k-1}, u_{1:k}) \\
&\xlongequal{\text{Markov assumption}} \\
&\xlongequal{\text{prediction def.}}
\end{aligned}
$$

$$
\eta = \frac{1}{p(z_k|z_{1:k-1}, u_{1:k})} = \frac{1}{\int p(z_k|x_k, z_{1:k-1}, u_{1:k})p(x_k|z_{1:k-1}, u_{1:k})\, \mathrm{d}x_k}
$$

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Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
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3: UPDATE: $bel(x_k) = \eta p(z_k|x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, measurement probability, $p(z_k|x_k)$, and prediction, $\overline{bel}(x_k)$.

$$
\begin{aligned}
bel(x_k) &\xlongequal{\text{belief def.}} p(x_k|z_{1:k}, u_{1:k}) \\
&\xlongequal{\text{Bayes' rule}} \frac{p(z_k|x_k, z_{1:k-1}, u_{1:k}) p(x_k|z_{1:k-1}, u_{1:k})}{p(z_k|z_{1:k-1}, u_{1:k})} \\
&\xlongequal{\text{Normalizer}} \eta p(z_k|x_k, z_{1:k-1}, u_{1:k}) p(x_k|z_{1:k-1}, u_{1:k}) \\
&\xlongequal{\text{Markov assumption}} \eta p(z_k|x_k) p(x_k|z_{1:k-1}, u_{1:k}) \\
&\xlongequal{\text{prediction def.}}
\end{aligned}
$$

$$
\eta = \frac{1}{p(z_k|z_{1:k-1}, u_{1:k})} = \frac{1}{\int p(z_k|x_k, z_{1:k-1}, u_{1:k}) p(x_k|z_{1:k-1}, u_{1:k}) \, \mathrm{d}x_k}
$$

Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, measurement probability, $p(z_k|x_k)$, and prediction, $\overline{bel}(x_k)$.

$$
\begin{aligned}
bel(x_k) &\overset{\text{belief def.}}{=\!=\!=} p(x_k | z_{1:k}, u_{1:k}) \\
&\overset{\text{Bayes' rule}}{=\!=\!=} \frac{p(z_k | x_k, z_{1:k-1}, u_{1:k}) p(x_k | z_{1:k-1}, u_{1:k})}{p(z_k | z_{1:k-1}, u_{1:k})} \\
&\overset{\text{Normalizer}}{=\!=\!=} \eta p(z_k | x_k, z_{1:k-1}, u_{1:k}) p(x_k | z_{1:k-1}, u_{1:k}) \\
&\overset{\text{Markov assumption}}{=\!=\!=} \eta p(z_k | x_k) p(x_k | z_{1:k-1}, u_{1:k}) \\
&\overset{\text{prediction def.}}{=\!=\!=} \eta p(z_k | x_k) \overline{bel}(x_k)
\end{aligned}
$$

$$
\eta = \frac{1}{p(z_k | z_{1:k-1}, u_{1:k})} = \frac{1}{\int p(z_k | x_k, z_{1:k-1}, u_{1:k}) p(x_k | z_{1:k-1}, u_{1:k}) \, \mathrm{d}x_k}
$$

Bayes filter: update/correction step

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of state x_k **do**
2: PREDICTION: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
3: UPDATE: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

MEASUREMENT UPDATE/CORRECTION step (corrección): the belief, $bel(x_k)$, is given by the **multiplication** of normalizer, η, measurement probability, $p(z_k|x_k)$, and prediction, $\overline{bel}(x_k)$.

$$
\begin{aligned}
bel(x_k) &\xlongequal{\text{belief def.}} p(x_k|z_{1:k}, u_{1:k}) \\
&\xlongequal{\text{Bayes' rule}} \frac{p(z_k|x_k, z_{1:k-1}, u_{1:k})p(x_k|z_{1:k-1}, u_{1:k})}{p(z_k|z_{1:k-1}, u_{1:k})} \\
&\xlongequal{\text{Normalizer}} \eta p(z_k|x_k, z_{1:k-1}, u_{1:k})p(x_k|z_{1:k-1}, u_{1:k}) \\
&\xlongequal{\text{Markov assumption}} \eta p(z_k|x_k)p(x_k|z_{1:k-1}, u_{1:k}) \\
&\xlongequal{\text{prediction def.}} \eta p(z_k|x_k)\overline{bel}(x_k)
\end{aligned}
$$

$$
\begin{aligned}
\eta &= \frac{1}{p(z_k|z_{1:k-1}, u_{1:k})} = \frac{1}{\int p(z_k|x_k, z_{1:k-1}, u_{1:k})p(x_k|z_{1:k-1}, u_{1:k}) \, \mathrm{d}x_k} \\
&= \frac{1}{\int p(z_k|x_k)\overline{bel}(x_k) \, \mathrm{d}x_k}
\end{aligned}
$$

Robot and door state: model

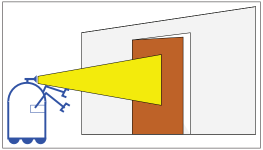

Figure: A mobile robot estimating the state of a door [Thrun, 2005].

- We use the example of a **robot estimating the state of a door** to illustrate Bayes filter algorithm.
- **State**
- **Measurement**
- **Control**

- The robot is assumed to have the following initial beliefs,
- The **noise** of the sensor is assumed to be characterized by the following measurement probabilities,

Robot and door state: model

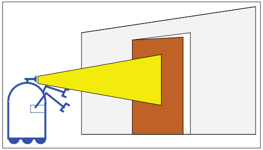

Figure: A mobile robot estimating the state of a door [Thrun, 2005].

- We use the example of a **robot estimating the state of a door** to illustrate Bayes filter algorithm.
- **State**

$$X_k = \{\text{open}, \text{closed}\}$$

- **Measurement**

$$Z_k = \{\text{s\_open}, \text{s\_closed}\}$$

- **Control**

$$U_k = \{\text{push}, \text{nothing}\}$$

- The robot is assumed to have the following initial beliefs,
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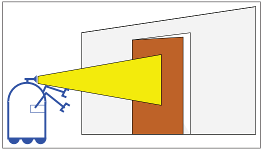
Robot and door state: model



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$$U_k = \{\text{push}, \text{nothing}\}$$

- The robot is assumed to have the following initial beliefs,

$$bel(X_0 = \text{open}) = 0.5\ ,$$
$$bel(X_0 = \text{closed}) = 0.5\ .$$

The robot **does not know** the state of the door, thus **assigning equal prior probability** for the two possible door state values.

- The **noise** of the sensor is assumed to be characterized by the following measurement probabilities,

Robot and door state: model

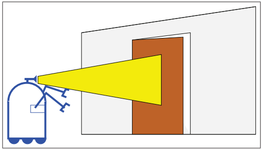

Figure: A mobile robot estimating the state of a door [Thrun, 2005].

- We use the example of a **robot estimating the state of a door** to illustrate Bayes filter algorithm.
- **State**

$$X_k = \{\text{open}, \text{closed}\}$$

- **Measurement**

$$Z_k = \{\text{s\_open}, \text{s\_closed}\}$$

- **Control**

$$U_k = \{\text{push}, \text{nothing}\}$$

- The robot is assumed to have the following initial beliefs,

$$bel(X_0 = \text{open}) = 0.5 \ ,$$
$$bel(X_0 = \text{closed}) = 0.5 \ .$$

The robot **does not know** the state of the door, thus **assigning equal prior probability** for the two possible door state values.

- The **noise** of the sensor is assumed to be characterized by the following measurement probabilities,

$$P(Z_k = \text{s\_open} | X_k = \text{open}) = 0.6 \ ,$$
$$P(Z_k = \text{s\_closed} | X_k = \text{open}) = 0.4 \ ,$$
$$P(Z_k = \text{s\_open} | X_k = \text{closed}) = 0.2 \ ,$$
$$P(Z_k = \text{s\_closed} | X_k = \text{closed}) = 0.8 \ .$$

The sensor is **relatively reliable** in detecting a **closed** door (error probability 0.2), but not an open door (error probability 0.4).

Robot and door state: model

- The following state transition probabilities are assumed for two types of controls.
  - The robot uses its manipulator to **push** the door open:
  - The robot **does nothing**:

Robot and door state: model

- The following state transition probabilities are assumed for two types of controls.
  - The robot uses its manipulator to **push** the door open:

$$P(X_k = \text{open} | U_k = \text{push}, X_{k-1} = \text{open}) = 1 \ ,$$
$$P(X_k = \text{closed} | U_k = \text{push}, X_{k-1} = \text{open}) = 0 \ ,$$
$$P(X_k = \text{open} | U_k = \text{push}, X_{k-1} = \text{closed}) = 0.8 \ ,$$
$$P(X_k = \text{closed} | U_k = \text{push}, X_{k-1} = \text{closed}) = 0.2 \ .$$

    If the door is **closed**, the robot has 80% *chance of success to open it*. If the door is already **open**, it remains *open*.
  - The robot **does nothing**:

Robot and door state: model

- The following state transition probabilities are assumed for two types of controls.
  - The robot uses its manipulator to **push** the door open:

$$P(X_k = \text{open} | U_k = \text{push}, X_{k-1} = \text{open}) = 1 \ ,$$
$$P(X_k = \text{closed} | U_k = \text{push}, X_{k-1} = \text{open}) = 0 \ ,$$
$$P(X_k = \text{open} | U_k = \text{push}, X_{k-1} = \text{closed}) = 0.8 \ ,$$
$$P(X_k = \text{closed} | U_k = \text{push}, X_{k-1} = \text{closed}) = 0.2 \ .$$

    If the door is **closed**, the robot has 80% *chance of success to open it*. If the door is already **open**, it remains *open*.
  - The robot **does nothing**:

$$P(X_k = \text{open} | U_k = \text{nothing}, X_{k-1} = \text{open}) = 1 \ ,$$
$$P(X_k = \text{closed} | U_k = \text{nothing}, X_{k-1} = \text{open}) = 0 \ ,$$
$$P(X_k = \text{open} | U_k = \text{nothing}, X_{k-1} = \text{closed}) = 0 \ ,$$
$$P(X_k = \text{closed} | U_k = \text{nothing}, X_{k-1} = \text{closed}) = 1 \ .$$

    The door **remains** its original state.

Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\quad \overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: $\quad bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- At $k = 1$, the robot **does nothing** (u_1 = nothing) and it **senses an open door** (z_1 = s_open).
- **Prediction**: $\overline{bel}(x_1) = \sum_{x_0} p(x_1|u_1, x_0) bel(x_0)$.
- Since the robot does nothing and we are **sure** about the consequences, the prediction is the same as previous belief, $\overline{bel}(x_1) = bel(x_0)$.

Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
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- **Prediction**: $\overline{bel}(x_1) = \sum_{x_0} p(x_1 | u_1, x_0) bel(x_0)$.
  $\overline{bel}(X_1 = \text{open})$

$\overline{bel}(X_1 = \text{closed})$

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Robot and door state: Bayes filter

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for all possible values of the state x_k **do**
  $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
  $bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
end for
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- **Prediction**: $\overline{bel}(x_1) = \sum_{x_0} p(x_1 | u_1, x_0) bel(x_0)$.

$$\overline{bel}(X_1 = \text{open}) = P(X_1 = \qquad | U_1 = \qquad, X_0 = \text{open}) \times bel(X_0 = \text{open})$$
$$+ P(X_1 = \qquad | U_1 = \qquad, X_0 = \text{closed}) \times bel(X_0 = \text{closed})$$

$$\overline{bel}(X_1 = \text{closed})$$

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Robot and door state: Bayes filter

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4: **end for**
5: **return** $bel(x_k)$

- At $k = 1$, the robot **does nothing** ($u_1 =$ nothing) and it **senses an open door** ($z_1 =$ s_open).
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$$\begin{aligned}\overline{bel}(X_1 = \text{open}) = & \ P(X_1 = \text{open} | U_1 = \text{nothing}, X_0 = \text{open}) \times bel(X_0 = \text{open}) \\ & + P(X_1 = \text{open} | U_1 = \text{nothing}, X_0 = \text{closed}) \times bel(X_0 = \text{closed})\end{aligned}$$

$$\overline{bel}(X_1 = \text{closed})$$

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Robot and door state: Bayes filter

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- **Prediction**: $\overline{bel}(x_1) = \sum_{x_0} p(x_1|u_1, x_0) bel(x_0)$.

$$\begin{aligned}\overline{bel}(X_1 = \text{open}) &= P(X_1 = \text{open}|U_1 = \text{nothing}, X_0 = \text{open}) \quad \times bel(X_0 = \text{open})\\ &+ P(X_1 = \text{open}|U_1 = \text{nothing}, X_0 = \text{closed}) \quad \times bel(X_0 = \text{closed})\\ &= 1 \times 0.5\end{aligned}$$

$\overline{bel}(X_1 = \text{closed})$

- Since the robot does nothing and we are **sure** about the consequences, the prediction is the same as previous belief, $\overline{bel}(x_1) = bel(x_0)$.

Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
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- **Prediction**: $\overline{bel}(x_1) = \sum_{x_0} p(x_1|u_1, x_0) bel(x_0)$.

$$\begin{aligned}\overline{bel}(X_1 = \text{open}) &= P(X_1 = \text{open}|U_1 = \text{nothing}, X_0 = \text{open}) && \times\, bel(X_0 = \text{open})\\ &+ P(X_1 = \text{open}|U_1 = \text{nothing}, X_0 = \text{closed}) && \times\, bel(X_0 = \text{closed})\\ &= 1 \times 0.5 + 0 \times 0.5\end{aligned}$$

$\overline{bel}(X_1 = \text{closed})$

- Since the robot does nothing and we are **sure** about the consequences, the prediction is the same as previous belief, $\overline{bel}(x_1) = bel(x_0)$.

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Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
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- **Prediction**: $\overline{bel}(x_1) = \sum_{x_0} p(x_1 | u_1, x_0) bel(x_0)$.

$$\begin{aligned}
\overline{bel}(X_1 = \text{open}) &= P(X_1 = \text{open} | U_1 = \text{nothing}, X_0 = \text{open}) && \times\, bel(X_0 = \text{open}) \\
&+ P(X_1 = \text{open} | U_1 = \text{nothing}, X_0 = \text{closed}) && \times\, bel(X_0 = \text{closed}) \\
&= 1 \times 0.5 + 0 \times 0.5 = 0.5
\end{aligned}$$

$$\begin{aligned}
\overline{bel}(X_1 = \text{closed}) &= P(X_1 = \text{closed} | U_1 = \text{nothing}, X_0 = \text{open}) && \times\, bel(X_0 = \text{open}) \\
&+ P(X_1 = \text{closed} | U_1 = \text{nothing}, X_0 = \text{closed}) && \times\, bel(X_0 = \text{closed}) \\
&= 0 \times 0.5 + 1 \times 0.5 = 0.5
\end{aligned}$$

- Since the robot does nothing and we are **sure** about the consequences, the prediction is the same as previous belief, $\overline{bel}(x_1) = bel(x_0)$.

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Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
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- **Prediction**: $\overline{bel}(x_1) = \sum_{x_0} p(x_1|u_1, x_0) bel(x_0)$.

$$\begin{aligned}\overline{bel}(X_1 = \text{open}) &= P(X_1 = \text{open}|U_1 = \text{nothing}, X_0 = \text{open}) \times bel(X_0 = \text{open})\\ &+ P(X_1 = \text{open}|U_1 = \text{nothing}, X_0 = \text{closed}) \times bel(X_0 = \text{closed})\\ &= 1 \times 0.5 + 0 \times 0.5 = 0.5\end{aligned}$$

$$\begin{aligned}\overline{bel}(X_1 = \text{closed}) &= P(X_1 = \text{closed}|U_1 = \text{nothing}, X_0 = \text{open}) \times bel(X_0 = \text{open})\\ &+ P(X_1 = \text{closed}|U_1 = \text{nothing}, X_0 = \text{closed}) \times bel(X_0 = \text{closed})\\ &= 0 \times 0.5 + 1 \times 0.5 = 0.5\end{aligned}$$

- Since the robot does nothing and we are **sure** about the consequences, the prediction is the same as previous belief, $\overline{bel}(x_1) = bel(x_0)$.

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Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\quad \overline{bel}(x_k) = \int p(x_k \mid u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: $\quad bel(x_k) = \eta p(z_k \mid x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- **Update**: $bel(x_1) = \eta P(Z_1 = \text{s\_open} | x_1) \overline{bel}(x_1)$.
- The goal of normalization: $bel(X_1 = \text{open}) + bel(X_1 = \text{closed}) = 1$.

Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\quad \overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
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5: **return** $bel(x_k)$

- **Update**: $bel(x_1) = \eta P(Z_1 = \text{s\_open} | x_1) \overline{bel}(x_1)$.

$$bel(X_1 = \text{open}) = \eta P(Z_1 = \text{s\_open} | X_1 = \quad\quad) \times \overline{bel}(X_1 = \quad\quad)$$
$$bel(X_1 = \text{closed}) = \eta P(Z_1 = \text{s\_open} | X_1 = \quad\quad) \times \overline{bel}(X_1 = \quad\quad)$$

- The goal of normalization: $bel(X_1 = \text{open}) + bel(X_1 = \text{closed}) = 1$.

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Robot and door state: Bayes filter

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1: **for** all possible values of the state x_k **do**
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4: **end for**
5: **return** $bel(x_k)$

- **Update**: $bel(x_1) = \eta P(Z_1 = \text{s\_open} | x_1) \overline{bel}(x_1)$.

$$P(Z_1 = \text{s\_open} | X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open})$$
$$P(Z_1 = \text{s\_open} | X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed})$$

$$bel(X_1 = \text{open}) = \eta P(Z_1 = \text{s\_open} | X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open})$$
$$bel(X_1 = \text{closed}) = \eta P(Z_1 = \text{s\_open} | X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed})$$

- The goal of normalization: $bel(X_1 = \text{open}) + bel(X_1 = \text{closed}) = 1$.

Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\quad \overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
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4: **end for**
5: **return** $bel(x_k)$

- **Update**: $bel(x_1) = \eta P(Z_1 = \text{s\_open} | x_1) \overline{bel}(x_1)$.

$$P(Z_1 = \text{s\_open} | X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open}) = 0.6 \times 0.5 = 0.3$$
$$P(Z_1 = \text{s\_open} | X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed}) = 0.2 \times 0.5 = 0.1$$

$$bel(X_1 = \text{open}) = \eta P(Z_1 = \text{s\_open} | X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open})$$
$$bel(X_1 = \text{closed}) = \eta P(Z_1 = \text{s\_open} | X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed})$$

- The goal of normalization: $bel(X_1 = \text{open}) + bel(X_1 = \text{closed}) = 1$.

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Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: $bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- **Update**: $bel(x_1) = \eta P(Z_1 = \text{s\_open} | x_1) \overline{bel}(x_1)$.

$$P(Z_1 = \text{s\_open} | X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open}) = 0.6 \times 0.5 = 0.3$$
$$P(Z_1 = \text{s\_open} | X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed}) = 0.2 \times 0.5 = 0.1$$
$$\eta = 1/(0.3 + 0.1) = 2.5$$
$$bel(X_1 = \text{open}) = \eta P(Z_1 = \text{s\_open} | X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open})$$
$$bel(X_1 = \text{closed}) = \eta P(Z_1 = \text{s\_open} | X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed})$$

- The goal of normalization: $bel(X_1 = \text{open}) + bel(X_1 = \text{closed}) = 1$.

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Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

for all possible values of the state x_k **do**

$\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$

$bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$

end for

return $bel(x_k)$

- **Update**: $bel(x_1) = \eta P(Z_1 = \text{s\_open} | x_1) \overline{bel}(x_1)$.

$$P(Z_1 = \text{s\_open} | X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open}) = 0.6 \times 0.5 = 0.3$$

$$P(Z_1 = \text{s\_open} | X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed}) = 0.2 \times 0.5 = 0.1$$

$$\eta = 1/(0.3 + 0.1) = 2.5$$

$$bel(X_1 = \text{open}) = \eta P(Z_1 = \text{s\_open} | X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open}) = 0.75$$

$$bel(X_1 = \text{closed}) = \eta P(Z_1 = \text{s\_open} | X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed}) = 0.25$$

- The goal of normalization: $bel(X_1 = \text{open}) + bel(X_1 = \text{closed}) = 1$.

Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\quad \overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: $\quad bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- **Update**: $bel(x_1) = \eta P(Z_1 = \text{s\_open}|x_1)\overline{bel}(x_1)$.

$$P(Z_1 = \text{s\_open}|X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open}) = 0.6 \times 0.5 = 0.3$$

$$P(Z_1 = \text{s\_open}|X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed}) = 0.2 \times 0.5 = 0.1$$

$$\eta = 1/(0.3 + 0.1) = 2.5$$

$$bel(X_1 = \text{open}) = \eta P(Z_1 = \text{s\_open}|X_1 = \text{open}) \times \overline{bel}(X_1 = \text{open}) = 0.75$$

$$bel(X_1 = \text{closed}) = \eta P(Z_1 = \text{s\_open}|X_1 = \text{closed}) \times \overline{bel}(X_1 = \text{closed}) = 0.25$$

- The goal of normalization: $bel(X_1 = \text{open}) + bel(X_1 = \text{closed}) = 1$.

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Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
3: $bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- At $k = 2$, the robot **pushes** the door (u_2 = push) and it **senses an open door** (z_2 = s_open).
- **Prediction**:
- **Normalizer**:
- **Update**:

Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\quad \overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
3: $\quad bel(x_k) = \eta p(z_k | x_k) \overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- At $k = 2$, the robot **pushes** the door (u_2 = push) and it **senses an open door** (z_2 = s_open).
- **Prediction**: $\overline{bel}(X_2 = \text{open}) = 1 \times 0.75 + 0.8 \times 0.25 = 0.95$, $\overline{bel}(X_2 = \text{closed}) = 0 \times 0.75 + 0.2 \times 0.25 = 0.05$.
- **Normalizer**:
- **Update**:

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Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1})bel(x_{k-1})\,\mathrm{d}x_{k-1}$
3: $bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- At $k = 2$, the robot **pushes** the door ($u_2 =$ push) and it **senses an open door** ($z_2 =$ s_open).
- **Prediction**: $\overline{bel}(X_2 = \text{open}) = 1 \times 0.75 + 0.8 \times 0.25 = 0.95$, $\overline{bel}(X_2 = \text{closed}) = 0 \times 0.75 + 0.2 \times 0.25 = 0.05$.
- **Normalizer**:

$$p(Z_2 = \text{s\_open}|X_2 = \text{open}) \times \overline{bel}(X_1 = \text{open}) = 0.6 \times 0.95 = 0.57$$
$$p(Z_2 = \text{s\_open}|X_2 = \text{closed}) \times \overline{bel}(X_1 = \text{closed}) = 0.2 \times 0.05 = 0.01$$
$$\eta = 1/0.58 \approx 1.724$$

- **Update**:

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Robot and door state: Bayes filter

Recursive_Bayes_filter($bel(x_{k-1}), u_k, z_k$ **)**

1: **for** all possible values of the state x_k **do**
2: $\quad \overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: $\quad bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- At $k = 2$, the robot **pushes** the door ($u_2 =$ push) and it **senses an open door** ($z_2 =$ s_open).
- **Prediction**: $\overline{bel}(X_2 = \text{open}) = 1 \times 0.75 + 0.8 \times 0.25 = 0.95$, $\overline{bel}(X_2 = \text{closed}) = 0 \times 0.75 + 0.2 \times 0.25 = 0.05$.
- **Normalizer**:

$$p(Z_2 = \text{s\_open}|X_2 = \text{open}) \times \overline{bel}(X_1 = \text{open}) = 0.6 \times 0.95 = 0.57$$
$$p(Z_2 = \text{s\_open}|X_2 = \text{closed}) \times \overline{bel}(X_1 = \text{closed}) = 0.2 \times 0.05 = 0.01$$
$$\eta = 1/0.58 \approx 1.724$$

- **Update**: $bel(X_2 = \text{open}) \approx 0.983$, $bel(X_2 = \text{closed}) \approx 0.017$.

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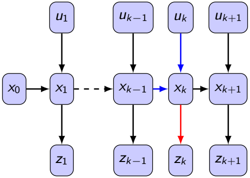
Glossary



, $\boldsymbol{x}$; , $\boldsymbol{z}$; , $\boldsymbol{u}$.

- Probabilistic distributions

$p(x_k \| x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \| x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \|$ $)$
$p(z_k \| x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \| x_k)$.	$bel(x_k) = p(x_k \|$ $)$

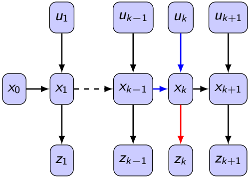
- Markov assumption

- Bayes filter

$$\textbf{prediction}: \ \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\, \mathrm{d}x_{k-1}$$

$$\textbf{update}: \ bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

$p(x_k \vert x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \vert x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \vert \qquad)$
$p(z_k \vert x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \vert x_k)$.	$bel(x_k) = p(x_k \vert \qquad)$

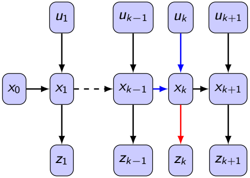
- Markov assumption

- Bayes filter

$$\textbf{prediction}: \ \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\, \mathrm{d}x_{k-1}$$

$$\textbf{update}: \ bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

evolution		
$p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \mid x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \mid \qquad)$
evolution		
$p(z_k \mid x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \mid x_k)$.	$bel(x_k) = p(x_k \mid \qquad)$

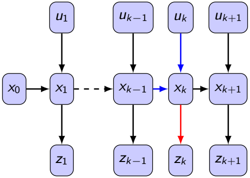
- Markov assumption

- Bayes filter

$$\textbf{prediction}: \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\, \mathrm{d}x_{k-1}$$

$$\textbf{update}: bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution		
$p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \mid x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \mid \qquad)$
evolution		
$p(z_k \mid x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \mid x_k)$.	$bel(x_k) = p(x_k \mid \qquad)$

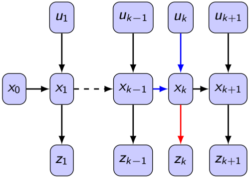
- Markov assumption

- Bayes filter

$$\textbf{prediction}: \ \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\, \mathrm{d}x_{k-1}$$

$$\textbf{update}: \ bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution		
$p(x_k \vert x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \vert x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \vert \qquad)$
Measurement evolution		
$p(z_k \vert x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \vert x_k)$.	$bel(x_k) = p(x_k \vert \qquad)$

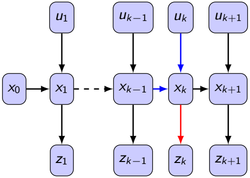
- Markov assumption

- Bayes filter

$$\textbf{prediction}: \; \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\, \mathrm{d}x_{k-1}$$

$$\textbf{update}: \; bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	
$p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \mid x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \mid \qquad)$
Measurement evolution		
$p(z_k \mid x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \mid x_k)$.	$bel(x_k) = p(x_k \mid \qquad)$

- Markov assumption

- Bayes filter

$$\textbf{prediction}: \ \overline{bel}(x_k) = \int \qquad\qquad bel(x_{k-1})\,\mathrm{d}x_{k-1}$$

$$\textbf{update}: \ bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

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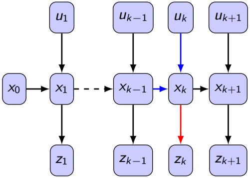
Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	
$p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \mid x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \mid \qquad)$
Measurement evolution	Measurement probability	
$p(z_k \mid x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \mid x_k)$.	$bel(x_k) = p(x_k \mid \qquad)$

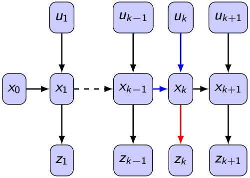
- Markov assumption

- Bayes filter

$$\textbf{prediction}: \ \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\,\mathrm{d}x_{k-1}$$

$$\textbf{update}: \ bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	Prediction
$p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \mid x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \mid \qquad)$
Measurement evolution	Measurement probability	
$p(z_k \mid x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \mid x_k)$.	$bel(x_k) = p(x_k \mid \qquad)$

- Markov assumption

- Bayes filter

$$\textbf{prediction}: \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\,\mathrm{d}x_{k-1}$$

$$\textbf{update}: bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

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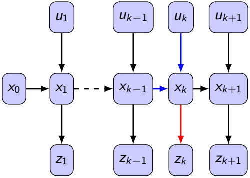
Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	Prediction
$p(x_k \vert x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \vert x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \vert z_{1:k-1}, u_{1:k})$
Measurement evolution	Measurement probability	
$p(z_k \vert x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \vert x_k)$.	$bel(x_k) = p(x_k \vert \qquad)$

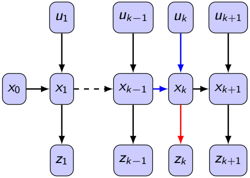
- Markov assumption

- Bayes filter

$$\textbf{prediction}: \ \overline{bel}(x_k) = \int \qquad\qquad bel(x_{k-1})\, \mathrm{d}x_{k-1}$$

$$\textbf{update}: \ bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	Prediction
$p(x_k \vert x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \vert x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \vert z_{1:k-1}, u_{1:k})$
Measurement evolution	Measurement probability	Belief
$p(z_k \vert x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \vert x_k)$.	$bel(x_k) = p(x_k \vert \qquad)$

- Markov assumption
- Bayes filter

$$\textbf{prediction}: \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\, \mathrm{d}x_{k-1}$$

$$\textbf{update}: bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

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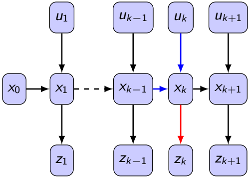
Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	Prediction
$p(x_k \| x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \| x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \| z_{1:k-1}, u_{1:k})$
Measurement evolution	Measurement probability	Belief
$p(z_k \| x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \| x_k)$.	$bel(x_k) = p(x_k \| z_{1:k}, u_{1:k})$

- Markov assumption

- Bayes filter

$$\textbf{prediction}: \overline{bel}(x_k) = \int \qquad bel(x_{k-1})\,\mathrm{d}x_{k-1}$$

$$\textbf{update}: bel(x_k) = \eta \qquad \overline{bel}(x_k)$$

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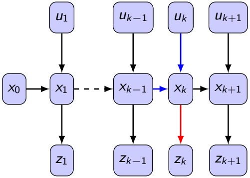
Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	Prediction
$p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \mid x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \mid z_{1:k-1}, u_{1:k})$
Measurement evolution	Measurement probability	Belief
$p(z_k \mid x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \mid x_k)$.	$bel(x_k) = p(x_k \mid z_{1:k}, u_{1:k})$

- Markov assumption

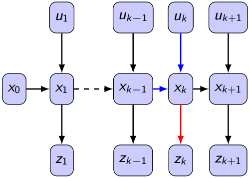
$$p(x_k \mid x_{k-1}, u_k) \approx p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$$
$$p(z_k \mid x_k) \approx p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$$

- Bayes filter

prediction: $\overline{bel}(x_k) = \int \qquad bel(x_{k-1})\,\mathrm{d}x_{k-1}$

update: $bel(x_k) = \eta \qquad \overline{bel}(x_k)$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	Prediction
$p(x_k \vert x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \vert x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \vert z_{1:k-1}, u_{1:k})$
Measurement evolution	Measurement probability	Belief
$p(z_k \vert x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \vert x_k)$.	$bel(x_k) = p(x_k \vert z_{1:k}, u_{1:k})$

- Markov assumption

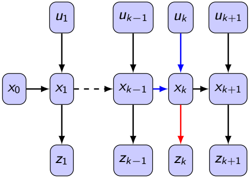
$$p(x_k|x_{k-1}, u_k) \approx p(x_k|x_{0:k-1}, u_{1:k}, z_{1:k-1})$$
$$p(z_k|x_k) \approx p(x_k|x_{0:k-1}, u_{1:k}, z_{1:k-1})$$

- Bayes filter

prediction: $\overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$

update: $bel(x_k) = \eta \qquad \overline{bel}(x_k)$

Glossary



- State, $\boldsymbol{x}$; Measurement, $\boldsymbol{z}$; Control, $\boldsymbol{u}$.
- Probabilistic distributions

State evolution	State transition probability	Prediction
$p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$	$p(x_k \mid x_{k-1}, u_k)$	$\overline{bel}(x_k) = p(x_k \mid z_{1:k-1}, u_{1:k})$
Measurement evolution	Measurement probability	Belief
$p(z_k \mid x_{0:k}, u_{1:k}, z_{1:k-1})$	$p(z_k \mid x_k)$.	$bel(x_k) = p(x_k \mid z_{1:k}, u_{1:k})$

- Markov assumption

$$p(x_k \mid x_{k-1}, u_k) \approx p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$$
$$p(z_k \mid x_k) \approx p(x_k \mid x_{0:k-1}, u_{1:k}, z_{1:k-1})$$

- Bayes filter

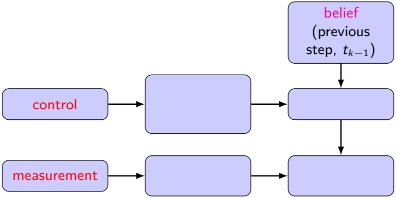
prediction: $\overline{bel}(x_k) = \int p(x_k \mid u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$

update: $bel(x_k) = \eta p(z_k \mid x_k) \overline{bel}(x_k)$

Bayes filter algorithm

Recursive_Bayes_filter($bel(x_{k-1})$, u_k, z_k **)**

```
for all possible values of the state x_k do
end for
return
```



- Define state x_k, control u_k, and measurement z_k.
- Assume initial belief $bel(x_0)$ at t_0.
- Provide control history $u_{1:k}$ and obtain measurement history $z_{1:k}$.
- Model

  and

- Carry out the **recursive** Bayes filter algorithm to **update** the belief $bel(x_k)$ at each time step t_k.

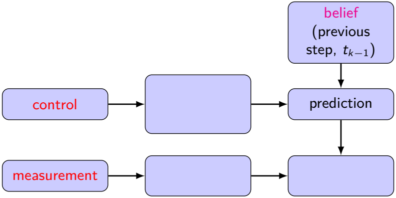
- Kalman filter is a kind of Gaussian filters that constitute the earliest tractable implementations of the Bayes filter for continuous spaces of random variables. All Gaussian filters share the basic idea that **beliefs are represented by normal distributions**.

Bayes filter algorithm

Recursive_Bayes_filter($bel(x_{k-1})$, u_k, z_k **)**

```
for all possible values of the state x_k do
    \overline{bel}(x_k) = ∫                    bel(x_{k-1}) dx_{k-1}

end for
return
```



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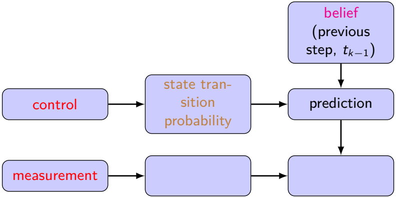
Bayes filter algorithm

Recursive_Bayes_filter($bel(x_{k-1})$, u_k, z_k **)**

1: **for** all possible values of the state x_k **do**
2: $\quad \overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
4: **end for**
5: **return**



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- Provide control history $u_{1:k}$ and obtain measurement history $z_{1:k}$.
- Model state transition probability $p(x_k | u_k, x_{k-1})$ and
- Carry out the **recursive** Bayes filter algorithm to **update** the belief $bel(x_k)$ at each time step t_k.

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Bayes filter algorithm

Recursive_Bayes_filter($bel(x_{k-1})$, u_k, z_k **)**

for all possible values of the state x_k **do**
    $\overline{bel}(x_k) = \int p(x_k | u_k, x_{k-1}) bel(x_{k-1}) \, \mathrm{d}x_{k-1}$
    $bel(x_k) = \eta \qquad \overline{bel}(x_k)$
end for
return

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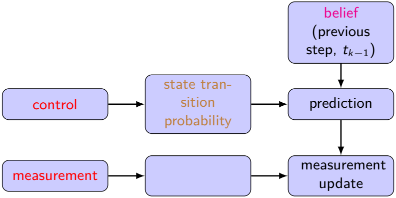

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Bayes filter algorithm

Recursive_Bayes_filter($bel(x_{k-1})$, u_k, z_k **)**

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2: $\quad \overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: $\quad bel(x_k) = \eta p(z_k|x_k) \overline{bel}(x_k)$
4: **end for**
5: **return**

- Define state x_k, control u_k, and measurement z_k.
- Assume initial belief $bel(x_0)$ at t_0.
- Provide control history $u_{1:k}$ and obtain measurement history $z_{1:k}$.
- Model state transition probability $p(x_k|u_k, x_{k-1})$ and measurement probability $p(z_k|x_k)$.
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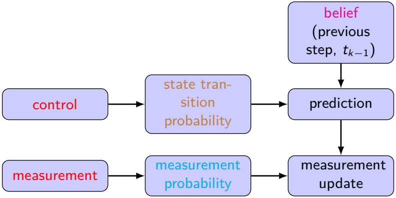

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Bayes filter algorithm

Recursive_Bayes_filter($bel(x_{k-1})$, u_k, z_k **)**

for all possible values of the state x_k **do**

$\quad \overline{bel}(x_k) = \int p(x_k \mid u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$

$\quad bel(x_k) = \eta p(z_k \mid x_k) \overline{bel}(x_k)$

end for

return $bel(x_k)$

- Define state x_k, control u_k, and measurement z_k.
- Assume initial belief $bel(x_0)$ at t_0.
- Provide control history $u_{1:k}$ and obtain measurement history $z_{1:k}$.
- Model state transition probability $p(x_k \mid u_k, x_{k-1})$ and measurement probability $p(z_k \mid x_k)$.
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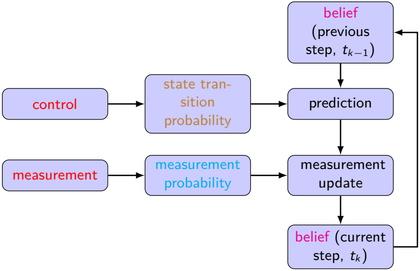

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Bayes filter algorithm

Recursive_Bayes_filter($bel(x_{k-1})$, u_k, z_k **)**

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2: $\quad \overline{bel}(x_k) = \int p(x_k|u_k, x_{k-1}) bel(x_{k-1})\, \mathrm{d}x_{k-1}$
3: $\quad bel(x_k) = \eta p(z_k|x_k)\overline{bel}(x_k)$
4: **end for**
5: **return** $bel(x_k)$

- Define state x_k, control u_k, and measurement z_k.
- Assume initial belief $bel(x_0)$ at t_0.
- Provide control history $u_{1:k}$ and obtain measurement history $z_{1:k}$.
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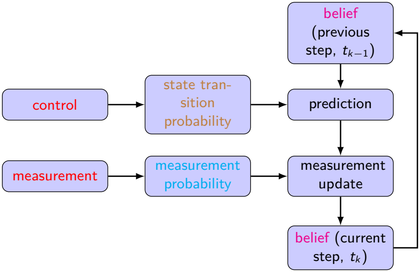

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Acrónimos

KF	Kalman Filter
PDF	Probability Distribution Function
PMF	Probability Mass Function

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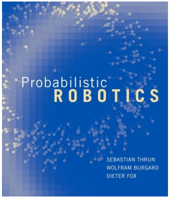

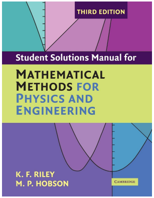

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