# Ultra high-pass Fourier filtering

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(Received 7 July 2000; accepted 20 October 2000)

We demonstrate a spatial Fourier filtering experiment that goes beyond the conventional low- and high-pass filtering and is not described in the Fourier optics books. Blocking nearly all the light in the Fourier plane of a 4-f filtering system is shown to produce at the output plane an image that is substantially different from the one corresponding to high-pass filtering. It is verified that the presented effect is due to the very high spatial frequency components created by the film emulsion scattering. This experiment can be performed in any undergraduate level optics laboratory with the same setup normally used for the Fourier filtering. 
[DOI: 10.1119/1.1344168]

## I. INTRODUCTION

Spatial Fourier filtering is one of the basic Fourier optics techniques widely used in many fundamental optical experiments like holography, interferometry, optical information processing, etc.[1–8] Even in its simplest version (i.e., amplitude spatial filtering) this experiment allows one to understand key points in the image formation theory and is therefore widely taught in many undergraduate level courses in optics. In the 4-f spatial Fourier filtering, a two dimensional transparency is kept in the front focal plane of a lens and a modification of the spatial frequency spectrum is performed by filters placed in the back focal plane of the lens. An image corresponding to the altered spectrum is then reconstructed in the back focal plane of a second lens. Both purely amplitude or complex (including phase modification) spatial filtering can be performed. While amplitude filtering of images in the Fourier plane can readily be demonstrated, the phase filtering and related experiments like pattern recognition (though it is seemingly simple as well) require equipment of higher level and may not be easy to realize in an undergraduate level laboratory. Then, it appears to be important to add new points in studying the Fourier filtering with simple setup.

In this paper we demonstrate some interesting features of the spatial Fourier filtering of film transparencies that have been, to our knowledge, overlooked in the literature. We show that the behavior of the high-pass frequency filtering of film transparencies does not follow the well accepted picture of low- and high-pass filtering: when a large diameter blocking mask is inserted in the Fourier plane, a reconstruction of the ''body'' of the object (instead of the expected edge enhancement) is observed. This intriguing observation is attributed to scattering from the film emulsion. An experimental verification of the proposed explanation as well as simple theoretical treatment are presented.

## II. THE 4-f SPATIAL FILTERING SYSTEM

The experimental setup we use, shown in Fig. 1, is based on the well known two lens 4-f system. Theoretical description of the properties of such a system can be found in any of the Fourier optics books.[7,8] Here we will just recall several important formulae giving the relation between the input transparency (placed at the front focal plane P1 of the first convex lens L1) and the complex amplitude distribution at the subsequent positions of the 4-f system. Consider a plane monochromatic wave (expanded and collimated laser beam) illuminating an input transparency, placed in the plane P1. The complex amplitude distribution of the light field immediately after the transparency is given by

$$U_1(x_1,y_1)=t_0(x_1,y_1), \tag{1}$$

where $t_0(x_1,y_1)$ is the amplitude transmittance function of the transparency. Assuming that Fresnel approximation is valid for the propagation P1-L1 and L1-P2, as well as that the lens introduces a quadratic phase term, it can easily be shown that the amplitude distribution at the back focal plane of L1 is given by

$$U_2(\mu,\nu)=\frac{1}{i\lambda f}\int\int U_1(x_1,y_1)\times\exp[-i2\pi(x_1\mu+y_1\nu)]dx_1dy_1. \tag{2}$$

Equation (2) illustrates the remarkable property of the positive lens to perform an exact Fourier transform of the input wave amplitude distribution, which can be written as:

$$U_2(\mu,\nu)=F.T.(U_1(x_1,y_1))=T_0(\mu,\nu), \tag{3}$$

$T_0(\mu,\nu)$ being the Fourier transform of the input transparency.

We will recall the physical meaning of the Fourier transform, performed by the lens: the angular spectrum of the light field at the input plane P1 is converted into a spatial distribution in the lens focal plane, i.e., the coordinates $\mu$ and $\nu$ in this plane correspond to the spatial frequencies of the input image (this concept is discussed in depth by Goodman[8]). By the insertion of a filter with variable transmission $M(\mu,\nu)$ in this plane, one can modify the distribution of the transmitted light:

$$U_2'(\mu,\nu)=M(\mu,\nu)U_2(\mu,\nu). \tag{4}$$

As seen from Fig. 1, in the 4-f system the plane P2 coincides with the front focal plane of the second positive lens, resulting in a successive Fourier transform. The amplitude distribution at the output plane P3 can then be calculated by Fourier transforming the modified spatial frequency distribution

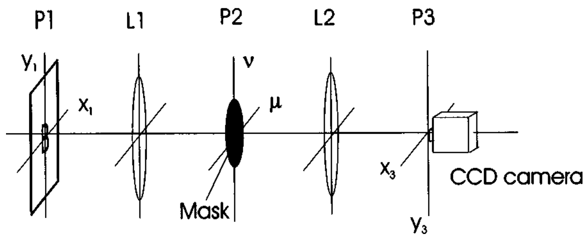


Fig. 1. ''4-f'' spatial filtering system.

(4). Using the convolution and inverse Fourier transform theorems (see, e.g., Ref. 8, Chap. 2.1.3) it is easy to show that the filtered image at the output of the 4-f system is given by

$$U_3(x_3,y_3)=m(x_3,y_3)\otimes U_1(x_3,y_3), \tag{5}$$

where $m(x_3,y_3)$ is the Fourier transform of the filter function. Note that in the absence of filter, i.e., $M(\mu,\nu)=1$, $m(x_3,y_3)$ becomes a delta function and one obtains an inverted replica of the original object.

As it has been already mentioned, by choosing simple objects and filter functions, it is possible to perform several classical demonstrations, like Abbé-Porter filtering (input is a mesh and filter is a slit), removal of speckle-like noise from an image, etc. Here we will concentrate on low- and high-pass filtering.

## III. DESCRIPTION OF THE SPATIAL FILTERING EXPERIMENT

A practical realization of the 4-f system can be built in nearly every undergraduate level optics laboratory. In our setup, a 1 mW (Spindler & Hoyer) He–Ne laser was used as a source. The laser beam, after being spatially filtered, expanded 20 times, and collimated, illuminates the input transparency. This transparency was a photographically produced image of the letter **B** of about 1 mm height. We recommend using a laser printer created image of larger size, e.g., 10–20 mm, which can then be reduced photographically. The lenses used for the 4-f imaging system ($L_1$ and $L_2$) are doublets having equal focal lengths of 250 mm and diameters of 40 mm (Spindler & Hoyer). A CCD camera (Panasonic WV BL600) is kept at the inverse Fourier transform plane. This camera is connected to a PC for image capturing using a cheap frame grabber (DT2853, Data Translation). A set of neutral density filters in front of the CCD camera is used for adjusting the image intensity. The photograph in Fig. 2(a) shows the image in the P3 plane in the absence of spatial filtering.

In the next figure [Fig. 2(b)], we show the Fourier spectrum of the letter **B** registered by the camera positioned in the P2 plane. The lower frequency part of the Fourier transform is overexposed to allow visualization of the higher frequencies. It should be noted here that the visible size of the Fourier transform depends on the sensitivity of the CCD camera. We emphasize that the low spatial frequencies of the input image contribute mainly to the central part and high spatial frequencies (coming from sharp features of the object) are located far from the center.

A low-pass filtering can be performed by transmitting only the central spot of the Fourier spectrum. Here we demonstrate the result of inserting a mask having a 1 mm circular

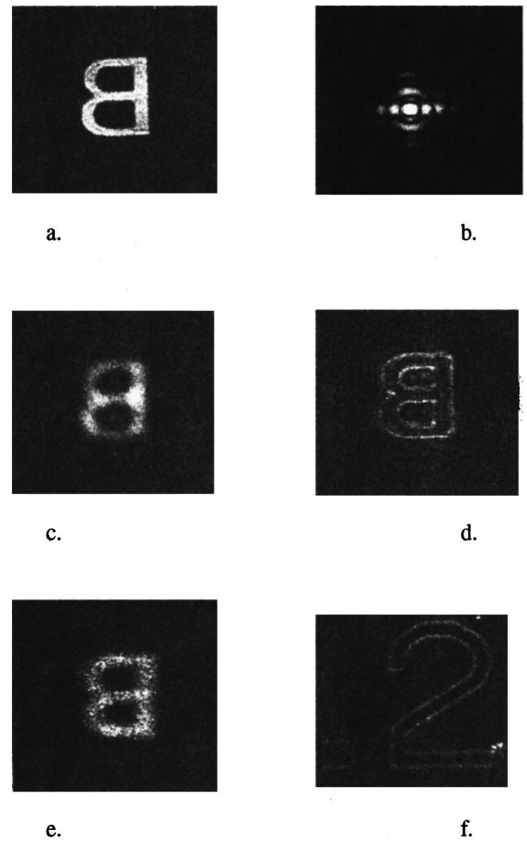


Fig. 2. (a) Restored image at output plane of the system in the absence of spatial filtering. (b) Fourier transform of the input transparency (letter **B**) obtained at the focal plane of the first positive lens. (c) Restored image after conventional low-pass filtering. (d) Restored image after conventional high-pass filtering. (e) Restored image after ultra-high-pass filtering by a large disk. (f) Restored image after ultra-high-pass filtering with an input transparency (number ''**2**'') not containing emulsion.

hole in the center, i.e., all spatial frequencies higher than the one corresponding to 1 mm are blocked. Figure 2(c) shows the photograph of the letter **B** obtained in such a fashion. The loss of sharp features on the restored image is evident. On the contrary, a high-pass filtering of the input transparency was performed by blocking the central spot using a 1 mm disc. As is seen from Fig. 2(d), an ''edge enhancement'' is observed. If the disc diameter is increased to 2 and 3 mm, the effect of ''edge enhancement'' becomes even more pronounced, and the intensity of the image drops down considerably. Now we come to the most interesting point. What will happen if the disc diameter is increased even more and completely covers the Fourier transform image seen on Fig. 2(b)? The expectation is that the image will disappear, or, if a faint image could still be seen, the latter would only contain edges. The observed result is, however, strikingly different. As an example, we show in Fig. 2(e) the image obtained when a black disk of diameter 30 mm is inserted in the Fourier plane. It is seen that the edges are lost; instead, the ''body'' of the letter is restored and appears to have a granular structure. The intensity of this image is of course much lower (about two orders of magnitude) than that of the original one. Even when a 35 mm disk is used, this kind of image is still obtained, with the size of granularities appearing larger. We emphasize again that the disk seems to completely stop the Fourier spectrum of our object.

The explanation of the observed effect lies in the fact that the processed image is written on a photographic emulsion which is actually composed of a large number of transparent or absorbing scattering particles (emulsion grains). These give rise to very high spatial frequencies that we will furthermore refer to as ultra-high frequencies, to emphasize that they are found beyond the range of spatial frequencies that belong to the processed image. The interesting point is that using only this highly scattered light, a clear image of the full object is obtained.

We have experimentally verified that the origin of the effect is really the scattering from the photographic emulsion. The input transparency was replaced by another one (number ''**2**''), taken from a test chart (Edmund Scientific). This object is made by metal deposition on a glass surface. Figure 2(f) shows the restored image after applying the same 30 mm disc at the Fourier plane. Contrary to the result obtained before, an edge enhancement effect is observed in this case.

## IV. THEORETICAL EXPLANATION

A strict theoretical explanation of our observations requires taking into account the angular distribution of the Mie scattering and the statistics of the emulsion grains and goes beyond the purpose of this paper. Instead, we present a simple explanation in terms of Fourier optics, aimed at understanding, rather than rigorously describing, the observed behavior. Let us modify the input transparency [Eq. (1)] in the following way:

$$U_1(x_1,y_1)=t_0(x_1,y_1)s(x_1,y_1), \tag{6}$$

where the term $s(x_1,y_1)$ represents an additional modulation term coming from the granularity of the emulsion. At this point we only know that this is a rapidly varying phase function, with respect to which the input transparency is slowly varying. The complex amplitude distribution in the Fourier plane can be written as:

$$U_2(\mu,\nu)=[T_0(\mu,\nu)\otimes S(\mu,\nu)]. \tag{7}$$

It can be shown that in general the above expression contains a zeroth-order term coming from the directly transmitted light, as well high-order terms originating from the diffraction on the fine structure $s(x_1,y_1)$. To understand this point, let us consider a one dimensional simplified case, supposing that the superfine structure of the transparency contains equal size equidistant grains that introduce a sinusoidal phase modulation of the type

$$s(x_1)=\exp\left[i\frac{v}{2}\sin\left(\frac{2\pi}{d}x_1\right)\right], \tag{8}$$

where $d$ and $v$ are the spatial period of the grating and its modulation depth, respectively. For easier Fourier transforming, following Ref. 8, Chap. 4, we can use the following representation of (8):

$$s(x_1)=\sum_{q=-\infty}^{\infty} J_q\left(\frac{v}{2}\right)\exp\left[i\left(\frac{2\pi}{d}qx_1\right)\right], \tag{9}$$

where $J_q$ is the $q$th-order Bessel function of the first kind. The Fourier transform of Eq. (8) is then found to be

$$S_f(\mu)=\sum_{q=-\infty}^{\infty} J_q\left(\frac{v}{2}\right)\delta\left(\mu-\frac{q}{d}\right). \tag{10}$$

Here we note that $\mu$ is the spatial frequency and is linked to the real space distance $x$ in the Fourier plane by the simple relation $\mu=x/\lambda f$. The index $q$ corresponds to different diffraction orders, and their relative contribution depends on the depth of modulation $v$. In our case $v$ is small ($v\ll1$), so only the coefficients $J_0(v/2)$ and $J_{\pm1}(v/2)$ have significant values (see, e.g., Ref. 9, Chap. 21.8) and will be considered. Thus,

$$S_f(\mu)=J_0\left(\frac{v}{2}\right)\delta(\mu)+J_1\left(\frac{v}{2}\right)\delta\left(\mu-\frac{1}{d}\right)+J_{-1}\left(\frac{v}{2}\right)\delta\left(\mu+\frac{1}{d}\right). \tag{11}$$

Substituting the last expression in (7) and using the ''shifting'' property of the $\delta$ function, we obtain

$$U_2(\mu,\nu)=J_0\left(\frac{v}{2}\right)T(\mu,\nu)+J_1\left(\frac{v}{2}\right)T\left(\mu-\frac{1}{d},\nu\right)+J_{-1}\left(\frac{v}{2}\right)T\left(\mu+\frac{1}{d},\nu\right). \tag{12}$$

It is seen that in addition to the ''normal'' Fourier transform of the object centered on the optical axis [first term in Eq. (12)], the presence of the fine sinusoidal structure superimposed on the object (6) creates two replicas shifted by $1/d$. Since, as we assumed, the period $d$ of the fine structure is much smaller than any sharp feature of the original image, the terms in (12) will be well separated and not overlapping. It is now easier to define the mask that can be used to perform ultra-high-pass filtering: it should block $T(\mu)$ and pass the shifted replicas. We therefore use a disk with radius $\rho$ having a transmission function $M(\mu,v)$ given by

$$M(\mu,\nu)=\mathrm{circ}\frac{\sqrt{\mu^2+\nu^2}}{\rho}=\begin{cases}0 & \text{for } (\mu^2+\nu^2)^{1/2}<\rho\\ 1 & \text{for } \rho<(\mu^2+\nu^2)^{1/2}.\\ \frac{1}{2} & \text{for } \rho=(\mu^2+\nu^2)^{1/2}\end{cases} \tag{13}$$

The mask is placed in the Fourier plane so the complex amplitude of the light field immediately after the Fourier plane is obtained by multiplying Eq. (12) by the transmission function Eq. (13). Recalling that the radius $\rho$ is chosen to be larger than the extent of $T(\mu)$, it is seen that this multiplication will give

$$\begin{aligned}U_2'(\mu,\nu)&=U_2(\mu,\nu)M(\mu,\nu)\\&=J_1\left(\frac{v}{2}\right)T\left(\mu-\frac{1}{d},\nu\right)+J_{-1}\left(\frac{v}{2}\right)T\left(\mu+\frac{1}{d},\nu\right).\end{aligned} \tag{14}$$

The image at the output of the system is obtained after an inverse Fourier transform (performed by the second positive lens, as explained earlier):

$$\begin{aligned}U_3(x_3,y_3)&=F^{-1}[U_2(\mu,\nu)]\\&=Ct(x_3,y_3)\exp(-j4\pi x_3/d),\end{aligned} \tag{15}$$

where we have used the shift property of the Fourier transform and $C$ is a constant. So, in this idealized case the input image intensity will be exactly restored at the output of the system. The main difference with the real case is that the granularity of our transparency does not contain a single spatial frequency; it can be considered as a superposition of gratings with spatial frequencies distributed in a certain range. Thus, in the Fourier plane there will be a nearly continuous distribution of terms, similar to those in Eq. (11). All those that pass the filter will contribute to the output image. It must be noted that the effective Fourier filter consists not only of the disk with transmission given by Eq. (13) (3 cm in diameter in our case), but it also includes the outer frame of the lens (4 cm) which limits the maximum spatial frequency transmitted by the system. So, one part of the widely spread terms mentioned above are cancelled and an effective band-pass filtering is performed. This explains the granularity observed in the ultra-high-pass filtered image [Fig. 2(e)]. We have verified that increasing even more the disk diameter (i.e., decreasing the width of this band-pass filter) increases the visible size of the granularity.

Based on the above arguments, we can estimate an upper limit for the size $d$ of the emulsion grains: it should be small enough so that the ''generated'' spatial frequencies are higher than those blocked by the mask. Assuming that grains are circular and recalling that the Fourier transform of a disc of diameter $d$ is an Airy pattern having a radius of the central lobe of $1.22\lambda f/d$ (see, e.g., Ref. 8, Eq. 4-32), we conclude that only grains of size $d < 1.22\lambda f/\rho$ can contribute to the ultra-high-pass filtering. Substituting here the values for our setup, namely $f = 250\,\mathrm{mm}$, $\rho = 15\,\mathrm{mm}$, and $\lambda = 0.633\,\mu\mathrm{m}$, we find that $d < 13\,\mu\mathrm{m}$. We note that $d$ depends on the type of emulsion and the development process.

## V. CONCLUSIONS

In this paper we have demonstrated some new observations on Fourier filtering at the ultra-high-frequency range and it is shown that the image thus obtained is due to the scattered light originating from the film emulsion of the input transparency. It is shown that this experiment can be done with the basic experimental setup described in most Fourier optics textbooks, i.e., the 4-f system. The relatively simple theoretical explanation presented here is understandable by students having basic Fourier optics knowledge. Further studies of the subject can be motivated by possible applications in real time image processing, e.g., for mapping of defects on transmitting or reflecting objects.


## ACKNOWLEDGMENTS

This work was done at the Laser Laboratory, Sincrotrone—Trieste, Italy. CSN was supported by the Associateship Scheme of the Abdus Salam International Center for Theoretical Physics, Treste, Italy. The authors acknowledge Professor G. Denardo for his untiring support and encouragement in the course of this work.



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### THE WRONG LINE

[Chadwick] meant to read mathematics. The entrance interviews were held publicly in a large, crowded hall. Chadwick got into the wrong line. He had already begun to answer the lecturer's questions when he realized he was being questioned for a physics course. Since he was too timid to explain, he decided that the physics lecturer impressed him and he would read for physics.

Richard Rhodes, *The Making of the Atomic Bomb* (Simon & Schuster, New York, 1986), p. 155.