10 GeV/c. For comparison purposes figure 12 includes also the observations of MacKeown *et al* (1966a), Said (1966), Ashton *et al* (1966) and Palmer and Nash (1969). It can be concluded from the mean values of R_π shown in figure 12 that the K/π ratio is reasonably constant over all production energies at approximately $0{\cdot}3 \pm 0{\cdot}15$. It must be remembered that the interpretation relies upon a knowledge of the vertical muon spectrum and in so far as this is uncertain so also is the above conclusion. The errors in the vertical spectrum are certainly such as to not rule out the possibility that the muons have a 100% pion parentage.

The Nagoya group (Kamiya *et al* 1970) have operated a near horizontal solid iron spectrograph incorporating neon flash-tubes having an MDM of 1930 ± 30 GeV/c, making eventual measurements upon 28 149 muon tracks. Kamiya *et al* state that their measured spectrum is fully consistent with the predictions of the pion parentage model of Ashton *et al* (1966).

Measurements have been made of the cosmic ray muon spectra at large zenith angles using a spectrograph incorporating optical spark chambers by Asbury *et al* (1970), the MDM of the system being 830 GeV/c. Absolute values of the energy spectra up to 200 GeV/c were obtained at zenith angles of 75°, 80° and 85°. Some 20 139 events being analysed in the course of the experiment. The differential intensity spectrum at 80° obtained by Asbury *et al* (1970) is shown in figure 13, again compared with theory based upon a pure pion parentage and other *ad hoc* mixtures of parent particles. Clearly the data are not inconsistent with a pure pion parentage for the muons.

4.5. Conclusions on the vertical and horizontal spectrum measurements

The experimental data on the vertical and horizontal spectra have not yet reached sufficient accuracy to enable a precise determination of the K/π ratio to be made, although they do indicate that the present ideas on cosmic ray propagation in the atmosphere are basically correct. At the present time the near horizontal muon spectrum is known to a greater degree of accuracy than is the vertical spectrum and it is evident that more attention should now be paid to vertical measurements.

In so far as the experimental uncertainties allow, a comparison of the spectra in the two directions can be made, with the result that K/$\pi \simeq 0{\cdot}3 \pm 0{\cdot}15$ appears to fit most of the data. This value relates to muons in the range 100–1000 GeV, that is, primary proton energies of 1000–10 000 GeV. Comparison can be made with the value of about 0·15 found at ISR energies ($E_p \simeq 1500$ GeV) and it can be concluded that there is no evidence for a rapid increase in the K/π ratio with increasing interaction energy.

It is considered that when data are available from the new spectrographs being brought into service (table 1) some useful conclusions should be possible.

5. The muon intensity at intermediate angles

5.1. The muon spectrum as a function of zenith angle

There is, of course, no difference in the physical processes occurring at the various angles but a tendency has developed whereby very large angle data for high energy muons have been used to study the K/π ratio and data at intermediate angles have been used, largely at low energies, to examine the effect of the Earth's magnetic field.

The Earth's field gives rise to an east–west asymmetry, due principally to the fact that the primary cosmic rays are mostly positively charged, and an east–west effect of the charge ratio as reported by Moroney and Parry (1954). The variation of atmospheric thickness with zenith angle leads to modifications to the muon spectrum in both the low and high energy regions. The preceding sections of the chapter have primarily been concerned with the high energy end of the spectrum, and it has been seen that due to the air density being less at large zenith angles than in the near vertical direction for the regions where most interactions occur, the pions produced by primaries incident at large angles have a greater chance to decay than to undergo nuclear collisions and this gives rise to an enhancement in the intensity at higher energies. At the low energy end of the muon spectrum a considerable reduction in intensity with increasing zenith angle occurs due to the greater absorber thickness and to the greater decay probabilities of the muons themselves.

Allen and Apostolakis (1961) used their emulsion spectrograph to estimate the integral intensity of muons above 1 GeV/c over the range of zenith angle 65°–85° as shown in figure 14. Subsequently Coates and Nash (1962) measured the muon spectrum at angles to the zenith of 30° and 45°; the best fit differential spectrum of the Nottingham workers is given in figure 15. In this figure is shown also the spectrum of Moroney and Parry (1954). It is seen that the spectra of the two groups for a zenith angle of 30° differ in form. The spectrum of Moroney and Parry was obtained using the spectrograph of Caro *et al* (1951) the results of which, in the near vertical direction, are not consistent with those of other workers. It is therefore likely that the spectrum of Moroney and Parry includes the same instrumental uncertainty as that of Caro *et al* (1951), possibly due to errors in the corrections applied for magnetic cut-off and the scattering out of the muons from the instrument.

Observations at 45° to the zenith have also been reported by Allkofer and Andresen

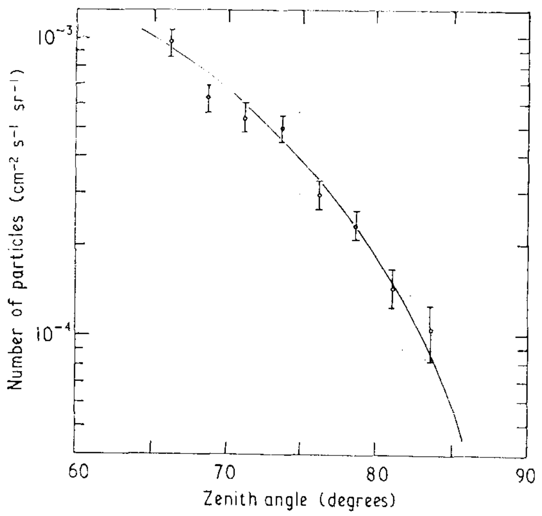

Figure 14. The variation with zenith angle of the integral rate of cosmic rays at sea level above 1 GeV/c according to Allen and Apostolakis (1961).

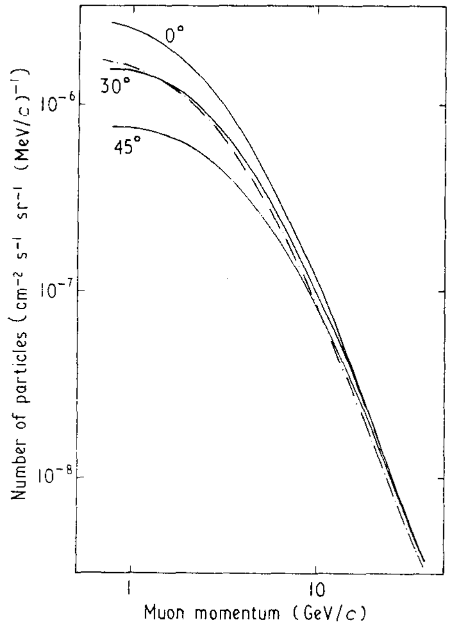

Figure 15. The best fit spectra of Coates and Nash (1962) at 0°, 30° and 45° to the zenith compared with the spectrum of Moroney and Parry (1954)—broken curve, at 30°.

(1967). There is some disagreement between the results of Coates and Nash (1962) and those of Allkofer and Andresen, the disagreement taking the form of an enhancement in the intensity below 2 GeV/*c* as reported by Allkofer and Andresen for observations from the west. The spectra of Allkofer and Andresen from the east are in satisfactory agreement with the data of Judge and Nash (1965b).

The range of zenith angles 10° to 75° has not been the subject of much work, most workers confining their attention to either the near vertical direction or the near horizontal direction. It has, however, been established by many workers that the intensity of the hard component integrated over all energies can be written in the form $I(\theta)=I_0 \cos^n \theta$, where I_0 is the vertical intensity and n is a constant, approximately 2, over the range of zenith angle 0°–75°. At zenith angles larger than 75° the air density at high altitudes varies rapidly with zenith angle and, as mentioned earlier, this affects the relative probabilities for pion decay and interaction to such an extent that the above simple $\cos^2 \theta$ relation could not be expected to continue.

5.2. *The variation of the muon spectrum with geomagnetic latitude*

Owing to the magnetic field of the Earth, there is a cut-off rigidity appropriate to each point and direction on the Earth's surface. For particles incident vertically the cut-off rigidity at the geomagnetic latitude, λ, is given by

$$R_0(\lambda)=14{\cdot}9 \cos^4 \lambda\,(1+0{\cdot}018 \sin \lambda)^2 \text{ GV}$$

(Jory 1956) and from this it can be seen that variations of the muon spectrum with λ would be expected for momenta up to several GeV/*c*.