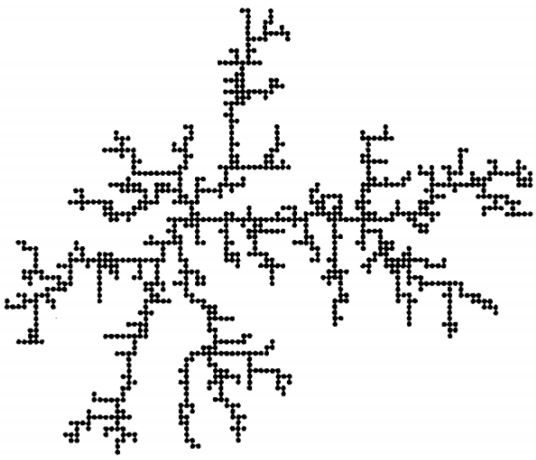
12. Diffusion Limited Aggregation

Many objects in nature grow by the random addition of sub-units. Examples include snow flakes, lightning, crack formation along a geological fault, and the growth of bacterial colonies. Although it might seem unlikely that such phenomena have much in common, the behavior observed in many models that have been developed in recent years gives us clues that these and many other natural phenomena can be understood in terms of a few unifying principles. One model that has provided much insight is known as diffusion limited aggregation or DLA. The model provides an example of how random motion can give rise to beautiful self-similar clusters.

The first step is to occupy a site with a seed particle. Next, a particle is released from the perimeter of a large circle whose center coincides with the seed. The particle undergoes a random walk, that is, diffuses, until it reaches a perimeter site of the seed and sticks. Then another random walker is released and allowed to walk until it reaches a perimeter site of one of the two particles in the cluster and sticks. The process is repeated many times (typically on the order of several thousand to several million) until a large cluster is formed. A typical DLA cluster is shown in the Figure below.



An example of a DLA cluster of 1000 particles on a square lattice.

Figure 1: DLA

- Write a program to generate diffusion limited aggregation clusters on a square lattice. Let each walker begin at a random site on a circle of radius $r = R_{max} + 2$, where R_{max} is the maximum distance of any cluster particle from the origin. To save computer time, assume that a walker that reaches a distance $2R_{max}$ from the seed site is removed and a new walker is placed at random on the circle of radius $r = R_{max} + 2$. Choose a lattice of linear dimension $L \geq 31$. Colour code the cluster sites according to their time of arrival, for example, choose the first 100 sites to be blue, the next 100 sites to be yellow, etc. Which parts of the cluster grow faster? If the clusters appear to be fractals, make a visual estimate of the fractal dimension. (Experts can make a visual estimate of D to within a few percent!)

- At $t = 0$ the four perimeter (growth) sites on the square lattice each have a probability $p_i = \frac{1}{4}$ of growing, that is, of becoming part of the cluster. At $t = 2$, the cluster has mass two and six perimeter sites. Identify the perimeter sites and convince yourself that their growth probabilities are not uniform. Do a Monte Carlo simulation and verify that two perimeter sites have $p_i = \frac{2}{9}$ and the other four have $p_i = \frac{5}{36}$.

- It is likely that your program generates DLA clusters inefficiently, because most of the CPU time is spent while the random walker is wandering far from the perimeter sites of the cluster.

There are several ways of overcoming this problem. One way is to let the walker take bigger steps the further the walker is from the cluster. For example, if the random walker is at a distance $R > R_{max}$, a step of length greater than or equal to $R - R_{max} - 1$ may be permitted if this distance is greater than one lattice unit. If the walker is very close to the cluster, the step length is one lattice unit. Other possible modifications are discussed by Meakin (see references). Modify your program and estimate the fractal dimension of diffusion limited clusters generated on a square lattice.

- Modify your program so that DLA clusters are generated on a triangular lattice. Do the clusters have the same visual appearance as on the square lattice? Estimate the fractal dimension and compare your estimate to your result for the square lattice.