An Extension of the Ramsauer-Townsend Experiment in a Xenon Thyratron

G. A. WOOLSEY
Department of Physics
University of New England
Armidale, Australia
(Received 13 November 1970)

An experiment on the Ramsauer–Townsend effect in xenon using a 2D21 thyratron, which was originally devised by Kukolich, has been extended to allow for effects due to contact potential and electron emission energy. As well as improving the accuracy of the experiment, this extension serves as a useful method of introducing the concepts of contact potential and electron energy distribution.

Kukolich[1] has devised an experiment which simply and neatly demonstrates the Ramsauer–Townsend effect in xenon using a 2D21 thyratron. We are now using this experiment in our second year laboratory and finding that it gives very satisfactory results. The experimental data obtained for the probability of elastic collision P_c agree well with that in the literature.[2,3] A discrepancy exists, however, in that the maximum and minimum (the Ramsauer–Townsend effect) which occur in the curve of P_c plotted against electron energy are each displaced by a few tenths of an electron volt below the accepted values. Kukolich attributed this discrepancy to a contact potential difference between the cathode and shield of the thyratron. Analysis of the discrepancy reveals, however, that it is due to a combination of a contact potential and the emission energy of the thermionic electrons from the cathode.

Without any additions to the apparatus, an extension to Kukolich's experiment can be made to determine values for the contact potential and mean thermionic electron energy. Not only does this extension to the experiment provide for more accurate data on the Ramsauer–Townsend effect, but it also serves as a useful introduction to the concepts of contact potential and Maxwellian energy distribution and how these may be investigated in the laboratory.

The thyratron and its circuit as used by Kukolich are shown in Fig. 1. The electrons from the cathode are accelerated through the lower aperture in the shield and are scattered in the central field-free region. Unscattered electrons pass through the upper aperture to be collected by the plate. Plate and shield currents are obtained by measuring the voltages V_p and V_s. The elastic collision probability is then given by[1]

$$P_c = -p/l[\ln(I_p I_s^* / I_s I_p^*)].$$

I_p and I_p^* are the plate currents at xenon pressures of p and zero, respectively, and I_s and I_s^*

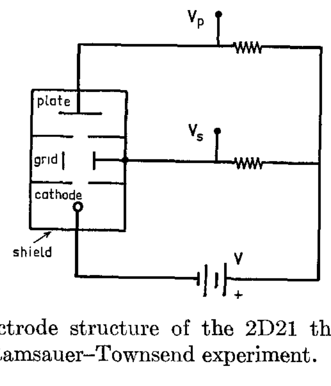

FIG. 1. Electrode structure of the 2D21 thyratron and circuit for Ramsauer–Townsend experiment.

are the corresponding shield currents (xenon is removed from the thyratron by submerging the top in liquid nitrogen). l is the distance from the first aperture to the plate.

By varying the voltage V, P_c may be determined as a function of electron energy. Electron collision probability data are normally plotted with electron momentum as the abscissa in units of $(\text{volt})^{1/2}$. In the original experiment, electron momentum was expressed as the square root of the measured potential difference between cathode and shield, $(V-V_s)^{1/2}$. However, the quantity $(V-V_s)^{1/2}$ does not give a true measure of the momentum of the electrons entering the scattering region at the first aperture for the following reasons.

The nickel of the shield has a higher work function than the barium oxide surface of the cathode, so that since the two electrodes are in contact via the gas, there is a net flow of electrons to the shield when they are at the same temperature and no potential difference is applied between them. Consequently, a small negative potential is required on the shield with respect to the cathode in order to balance this flow. An electron leaving the cathode surface therefore has experienced a total accelerating voltage of $(V-V_s+V_c)$ on reaching the shield, where V_c is referred to as the contact potential between the electrodes.

In addition, each electron leaves the cathode with a certain initial energy. If the mean emission energy of the thermionic electrons is $e\bar{V}$ electron volts, then this must be added to the energy acquired between cathode and shield $e(V-V_s+V_c)$, to give the true energy of the electrons as they enter the first aperture of the thyratron, i.e., the curve of P_c vs electron momentum must be plotted as P_c vs $(V-V_s+V_c+\bar{V})^{1/2}$.

For thermionic electrons, the distribution of energies is close to Maxwellian,[4] i.e., the number of electrons, $N(\epsilon)$, with energy ϵ is proportional to $\exp(-\epsilon/kT)$, where $3kT/2$ represents their mean energy, i.e.,

$$N(\epsilon) \propto \exp(-3V/2\bar{V}),$$

where $\epsilon=eV$. It can readily be shown[4] that, if such a group of electrons is collected by an electrode which is at a retarding potential V_p with respect to the electron source, the current collected is given by

$$i=i_0 \exp(-3V_p/2\bar{V}),$$

where i_0 is the current when $V_p=0$ and all the electrons can reach the collector. A curve of i vs V_p should, therefore, be a straight line whose slope allows $\bar{V}$ to be determined.

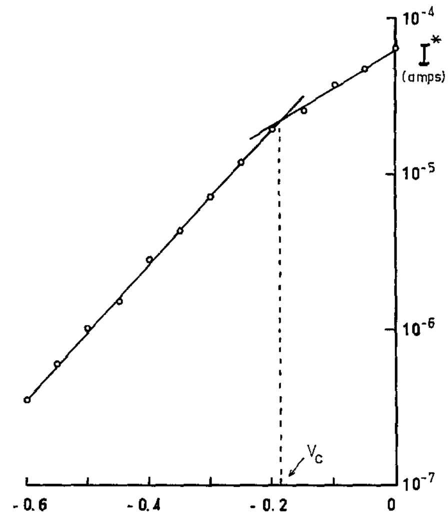

Fig. 2. Logarithm of the electron current to the shield I_s^* as a function of the potential between shield and cathode.

Such a curve can be obtained experimentally using the circuit of Fig. 1. The thyratron screen current I_s^* is measured as the potential on the shield with respect to the cathode is varied from zero through increasingly negative values, until I_s^* is reduced to zero, i.e., until all electrons are repelled from the shield. For this part of the experiment, the top of the thyratron is immersed in liquid nitrogen to eliminate scattering, and V is reversed, so that the potential of the shield with respect to the cathode is now $-(V+V_s)$. Readings should be taken at 0.05-V intervals.

A curve of $\log I_s^*$ vs $(V+V_s)$ yields a straight line, typically as shown in Fig. 2, which turns over due to saturation of the electron current when the potential difference between shield and cathode is zero. Note that the latter condition does not occur when the measured potential difference is zero, because of the presence of the contact potential between cathode and shield.

The difference in potential between the turn-over and zero points is a measure of the contact potential V_c. A number of sets of data similar to that presented in Fig. 2 were obtained with the filament voltage of the 2D21 thyratron at 4 V, and yielded average values for V_c and $\bar{V}$ of 0.2 ± 0.04 V and 0.15 ± 0.03 V, respectively. V_c is best obtained from the intersection point of the extrapolated linear sections of the curve. This is the method used to determine plasma potentials from logarithmic Langmuir probe characteristics,[5] which are derived from measurements of plasma electron current to a collector of varying potential immersed in the plasma.

When a curve was plotted of collision probability P_c as a function of corrected electron momentum $(V-V_s+V_c+\bar{V})^{1/2}$, the usual minimum and maximum in P_c occurred at 0.9 $(\mathrm{V})^{1/2}$ and 2.4 $(\mathrm{V})^{1/2}$. Within experimental error these values agree with those quoted in the literature.

It is felt that this addition to Kukolich's experiment is a valuable one. The student is taught the value of extending his initial investigation to allow for effects which may introduce significant error, and of course, he then has the satisfaction of obtaining results which are in closer agreement with established data. The experiment also serves as an introduction to the concepts of contact potential and energy distribution, and how these may be investigated in the laboratory.

[1] S. G. Kukolich, Amer. J. Phys. **36**, 701 (1968).
[2] R. B. Brode, Rev. Mod. Phys. **5**, 257 (1933).
[3] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford U. P., London, 1965), 3rd ed., Chap. 18.
[4] N. Cusack, *The Electrical and Magnetic Properties of Solids* (Longmans, London, 1958), Chap. 3.
[5] R. H. Huddlestone and S. L. Leonard, *Plasma Diagnostic Techniques* (Academic, New York, 1965), Chap. 4.

Solution of the Mathieu Equation in the WKB Approximation

M. J. RICHARDSON
Department of Physics
State University of New York at Stony Brook
Stony Brook, New York 11790
(Received 20 October 1970)

The Schrödinger equation for a particle in a one-dimensional sinusoidal potential is the Mathieu equation. The usual solution of this equation is in terms of a stability analysis and is given by power series expansions. In this paper, the Mathieu equation is discussed as a Schrödinger equation, and stability is interpreted in terms of allowed and forbidden energy bands. It is shown how simple physical arguments and use of the WKB approximation lead easily to the stability diagram given but not derived in most texts.

I. INTRODUCTION

To grasp the content of a differential equation it is often useful to analyze the physical problem from which the equation came. If the concepts arising there are familiar one can comprehend better the nature of the solutions and invent approximation methods more easily. This, of course, is not compulsory and one can always discuss the equation on its own merit. For a physicist "understanding a solution" often means that its idiosyncrasies are visualized in terms of simple physical explanations; for a mathematician this may be heresy.

The study of second-order differential equations with periodic coefficients (the Hill equation) arose originally through the stability analysis of celestial orbits. The same equation also arises in quantum theory, being the time independent Schrödinger equation for one-dimensional motion in a periodic potential. The Mathieu equation, occurring originally in the study of the vibrations