In the description of two common methods for solving recurrences below, I followed the book by Anany Levitin "Introduction to the Design and Analysis of Algorithms" (ISBN 0-201-74395-7).

(A) Method of "backward substitution", or "telescoping.

The idea of this method will be clear from the way it applies to solving the following recurrence:

$$T(n) = T(n-1) + n\,;\ T(1) = 1\,.$$

$T(n) = T(n-1) + n = \{\text{substitute } T(n-1) = T(n-2) + (n-1)\ \} =$

$= [T(n-2) + (n-1)] + n = T(n-2) + (n-1) + n =$

$\{\text{substitute } T(n-2) = T(n-3) + n - 2\ \} = [T(n-3) + (n-2)] + (n-1) + n) =$

$= T(n-3) + (n-2) + (n-1) + n = \ \ldots$

After a couple of such operations, we see an emerging pattern, which makes it possible to predict the general formula:
$T(n) = T(n-i) + (n-i+1) + \ldots + (n-2) + (n-1) + n$. Since an initial condition is specified for $n = 1$, we have to substitute $i = n-1$ in the pattern's formula to get the ultimate result: $T(n) = T(n-i) + (n-i+1) + \ldots + (n-2) + (n-1) + n =$

$= 1 + 2 + \ \ldots + (n-2) + (n-1) + n = \dfrac{1+n}{2} \cdot n\,.$

So, the solution to the recurrence is: $T(n) = \dfrac{1+n}{2} \cdot n$. If we need only the order of growth of the solution, $T(n) \in \Theta(n^2)$.

See more examples in the class notes.

(B) Master Theorem.

Assume that a problem's instance of size n is divided into several instances of size n/b, with a of them to be solved. (Here a and b are constants; $a \ge 1$ and $b > 1$.) Assuming that size n is a power of b, we get the following recurrence for the running time $T(n)$:

$$(*) \qquad T(n) = aT(n/b) + f(n),$$

where $f(n)$ is a function that accounts for the time spent on dividing the problem into smaller ones and on combining their solutions. Then, if $f(n) \in \Theta(n^d)$ and $d \ge 0$, the order of growth of $T(n)$ may be determined by Master Theorem according to the table below:

Relationship between a, b and d.	Order of growth of $T(n)$ (solution to the recurrence (*))
$a < b^d$	$T(n) \in \Theta(n^d)$
$a = b^d$	$T(n) \in \Theta(n^d \log n)$
$a > b^d$	$T(n) \in \Theta(n^{\log_b a})$