Aggregate Implications of Child-Related Transfers with Means Testing

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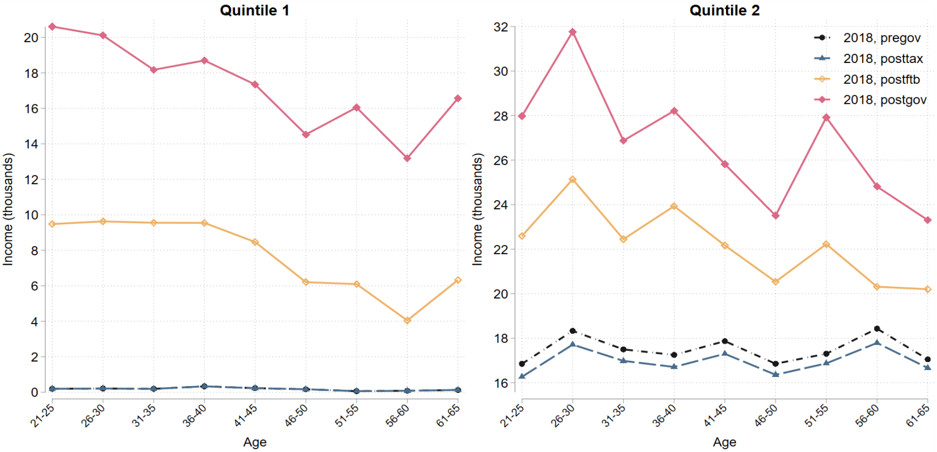
Motivation

1. Substantial child-related transfers (2–2.5% of GDP over the past decade).

Family Tax Benefits for lower income households (parents of dependent children)



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   - *Differences*:
     FTB is **NOT** conditional on labour participation, but CCS has a work hour test.

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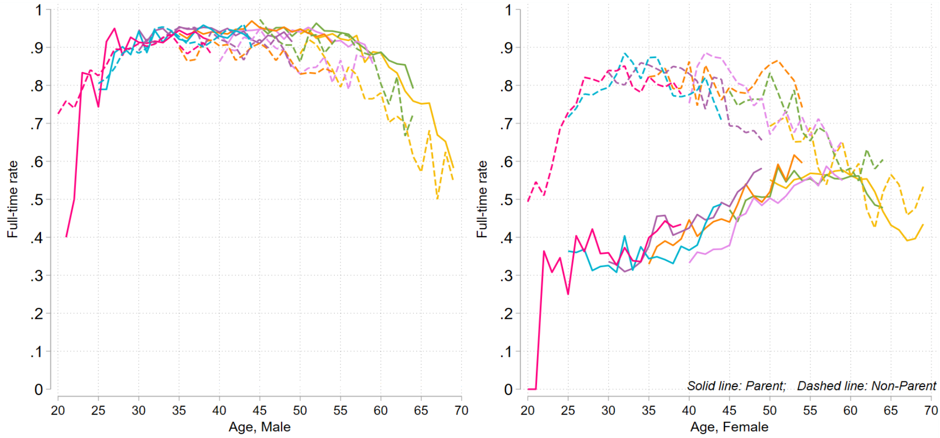
3. Large differences in the age-profiles of labor supply and wages between mothers and everybody else (i.e., fathers and non-parents).

Age-profile of full-time participation



See full time rate profiles by gender.

Our study

We construct a GE heterogeneous-agent OLG framework featuring:

- Family structure;
- Endogenous female human capital (learning-by-doing);
- Longevity and income risks;
- Time and monetary costs of children;
- Detailed child-related transfer programs.

Data: HILDA, ABS, World Bank.

See related literature on tax and transfer using heterogeneous agent model.

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Key questions:

1. What are the macroeconomic implications of means-testing child-related transfers with opposing eligibility criteria?
   - our focus today
2. Are there efficiency and/or welfare improvement reforms? If yes, what do they look like?

See related literature on tax and transfer using heterogeneous agent model.

Overview of counterfactual policy experiments

With *income tax as a budget-balancing tool*,

1. Reforming means-testing parameters
   - Experiment 1-5: vary FTB income test thresholds across a range of taper rates;
   - Experiment 6-10: vary CCS income test thresholds across a range of taper rates;
2. Reforming payment rates
   - Experiment 11-12: vary the FTB or CCS payment rates;
3. Elimination - our focus today
   - Experiment 13: eliminate either the FTB, the CCS, or both.

Key mechanisms

1. Transfer programs' interaction

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2. Life-cycle dynamics

   - Offsetting responses lower aggregate labor supply effect.
   - But, reforms affecting the young have irreversible effect on human capital even if labor supply increases at older age.
   - A small labor effect does not imply a small output effect.

Key mechanisms

3. Family structure

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     $\rightarrow$ more responsive to work incentive effect of reforms
     $\rightarrow$ smaller welfare change due a reform.

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     $\rightarrow$ smaller welfare change due a reform.

   - **Single mothers**: lack (i) economies of scale in consumption, (ii) family insurance, and (iii) self-insurance capacity.

     $\rightarrow$ tend to work part-time regardless
     $\rightarrow$ the presence of single mothers amplifies welfare effect of reforms.

Summary of lessons learned from policy experiments

1. Policy reforms require a holistic view.

   1.1 Efficiency perspective:

   - **remove FTB**: +10.6 p.p. LFP, +1% GDP
   - **remove FTB and CCS**: -2.3 p.p. LFP, -3.1% GDP
   - **remove CCS**: -10 p.p. LFP, -3.5% GDP (*means-testing*)

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2. Partially offsetting effects over life cycle.
   - FTB discourages full-time work despite the presence of CCS
   - However, eliminating both programs → less participation for young mothers (i.e., CCS's work incentive effect is dominant)

Summary of lessons learned from reforms

3. Multiple channels of improvement:
   - $\downarrow$ FTB or $\uparrow$ CCS payment rates lead to efficiency gains ($+7\%$ LFP, $+1\%$ GDP) at low or no welfare cost;
   - $\downarrow$ FTB taper rates (MTRs on earnings) generate gains ($+$4-5% LFP, $+$1.5-2% GDP, $+$0-0.7% welfare);
   - $\uparrow$ CCS income test thresholds generate efficiency gains ($+$0.6-1% LFP, $+$1-1.4% GDP, $+$0.05-0.1% welfare).

     (Similar to Guner et al. (2020))

*NOTE: Changes refer to either *doubling* or *halving*.

See detail on means-testing reforms.

Model overview

A small open economy calibrated to Australia (2012-2018):

1. Households
   - heterogeneous in age (j), family structure (λ), asset (a), human capital (h), education (θ), transitory shocks (ϵ^m, ϵ^f);
   - time and monetary costs of children;
   - longevity and earnings risks;
   - male labor supply is exogenous;
   - make decision on joint consumption c, savings a^+ and female labor supply $\ell \in \{0, 1, 2\}$;
2. A representative firm with Cobb-Douglas technology;
3. Government commits to balance the budget every period:
   - income tax, corporate income tax, consumption tax, borrowing;
   - general expenditure, age pension, FTB, CCS, debt;
4. Goods and factor markets clear.

Demographics (1)

1. Time-invariant population growth rate (n) and survival probabilities (ψ_j^m and ψ_j^f);
2. Populated by three household types:
   - Married parents, $\lambda = 0$
   - Single childless men, $\lambda = 1$
   - Single mothers, $\lambda = 2$
3. Households are born as workers at $j = 1$, retire at $j = 45$ and can live to the maximum age of $j = J = 80$;

Demographics (2)

4. Low education (θ_L) households have children earlier;

5. Child spacing is identical for all parents;

6. Children are exogenous and fully determined by household age, j:
   - the k^{th} child is born to households aged $j = b_{k,\theta}$;
   - the k^{th} child is dependent for 18 years from $j = b_{k,\theta}$ to $j = b_{k,\theta} + 17$;

See further details on children.

Households (working age): Costs of working for women

If a woman works, she incurs:

1. A **time cost**, χ, and **time cost per child**, χ_{c,j_c}, for full-time work:

$$l_j^f = \begin{cases} 1 & \text{if } \ell = 0 \\ 0 < 1 - n_{j,\lambda,\ell}^f < 1 & \text{if } \ell = 1 \\ 0 < 1 - n_{j,\lambda,\ell}^f - (\chi + \chi_{c,j_c} \times nc_j) < 1 & \text{if } \ell = 2 \end{cases}$$

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2. A **formal childcare cost per child** κ_j that is decreasing in the age of children:

3. A loss of a portion or all of the means-tested FTB benefits.

Households (working age): Benefits of working for women

However, if she works, she gains:

1. Labor income

$$y_j = wn_j\theta h_j\epsilon_j$$
$$\ln(\epsilon_j) = \rho \times \ln(\epsilon_{j-1}) + \upsilon_j; \qquad \upsilon_j \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

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3. Human capital accumulation for the next period:

$$log(h_j) = log(h_{j-1}) + [\xi_1 + \xi_2 \times (j-1)]\,\mathbf{1}_{\{\ell_{j-1} \neq 0\}} - \delta_\ell \mathbf{1}_{\{\ell_{j-1} = 0\}} \quad (1)$$

Key Macro Variables: Model vs. Data

Moments	Benchmark economy	Data	Source
Targeted			
Capital, K/Y	3.2	3-3.3	ABS (2012-2018)
Savings, S/Y	6.5%	5-8%	ABS (2013-2018)
Mother's labor participation, LFP	63%	65-70%	HILDA (2012-2018)*
Mother's full time rate, FT	23%	26-28% (40% $\times$ LFP)	HILDA (2012-2018)*
Consumption Tax, T^C/Y	4.26%	4.50%	APH Budget Review
Company Tax, T^K/Y	4.25%	4.25%	APH Budget Review
Age Pension, P/Y	3.31%	3.20%	ABS (2012-2018)
Gini coefficient (male aged 21)	0.35	0.35	
Non-targeted			
Consumption, C/Y	53.23%	54-58%	ABS (2012-2018)
Investment, I/Y	32.30%	24-28%	ABS (2013-2018)
Income tax, T^I/Y	12.11%	11%	APH Budget Review
Tax revenue to output	20.35%	25%	ABS(2012-2018)
Child-related transfers (FTB + CCS)	2.75%	2%	ABS (2012-2018)
Gini coefficient (working age male)	0.3766	0.45	PC (2018)

Results: FTB means-testing parameter reforms (1)

$\bar{y}^{tr}$ \ ω	$0.5 \times \bar{y}^{tr}$	$\bar{y}^{tr}$	$1.5 \times \bar{y}^{tr}$
LFP & FT rates			
$0.5 \times \omega$	5.88 p.p.	5.08 p.p.	3.71 p.p.
ω	5.72 p.p.	0.00 p.p.	2.22 p.p.
$1.5 \times \omega$	3.95 p.p.	-0.22 p.p.	-0.48 p.p.
Human Cap.			
$0.5 \times \omega$	2.72%	2.55%	1.41%
ω	2.70%	0.00%	0.54%
$1.5 \times \omega$	1.61%	-0.13%	-1.25%
Output			
$0.5 \times \omega$	2.21%	1.98%	1.58%
ω	2.47%	0.00%	1.29%
$1.5 \times \omega$	1.78%	0.82%	0.29%

Table: **Percentage point (*p.p.*) changes in LFP, and percentage (%) changes in human capital and output.**

Results: FTB means-testing parameters reforms (2)

ω \ $\bar{y}^{tr}$	$0.5 \times \bar{y}^{tr}$	$\bar{y}^{tr}$	$1.5 \times \bar{y}^{tr}$
$0.5 \times \omega$	-0.05%	0.41%	0.68%
ω	-0.44%	0.00%	0.50%
$1.5 \times \omega$	-0.79%	-0.01%	0.38%

Table: **Percentage (%) changes in welfare of newborns**

Results: CCS means-testing parameter reforms (1)

ω_{ccs} \ $\bar{y}^{sr}$	$0.5 \times \bar{y}^{sr}$	$\bar{y}^{sr}$	$1.5 \times \bar{y}^{sr}$
LFP & FT rates			
$1.5 \times \omega_{ccs}$	-2.46 p.p.	0.36 p.p.	0.75 p.p.
ω_{ccs}	-2.48 p.p.	0.00 p.p.	0.67 p.p.
$0.5 \times \omega_{csc}$	-2.67 p.p.	-1.86 p.p.	1.02 p.p.
Human Cap.			
$1.5 \times \omega_{ccs}$	-1.50%	0.11%	0.37%
ω_{ccs}	-1.53%	0.00%	0.34%
$0.5 \times \omega_{csc}$	-1.66%	-0.95%	0.70%
Output			
$1.5 \times \omega_{ccs}$	-0.22%	0.87%	0.99%
ω_{ccs}	-0.13%	0%	1.37%
$0.5 \times \omega_{csc}$	-0.15%	0.33%	1.26%

Table: **Percentage point (*p.p.*) changes in labor force participation (LFP) rate, and percentage (%) changes to aggregate human capital and output.**

Results: CCS means-testing parameter reforms (2)

$\bar{y}^{sr}$ / ω_{ccs}	$0.5 \times \bar{y}^{sr}$	$\bar{y}^{sr}$	$1.5 \times \bar{y}^{sr}$
$1.5 \times \omega_{ccs}$	-0.0154%	0.0893%	0.0519%
$1.25 \times \omega_{ccs}$	-0.0009%	0.0879%	0.0847%
ω_{ccs}	-0.0050%	0%	0.1007%
$0.75 \times \omega_{ccs}$	0.0131%	0.0383%	0.0801%
$0.5 \times \omega_{csc}$	-0.02%	0.02%	0.08%

Table: **Percentage (%) changes in welfare of newborns**

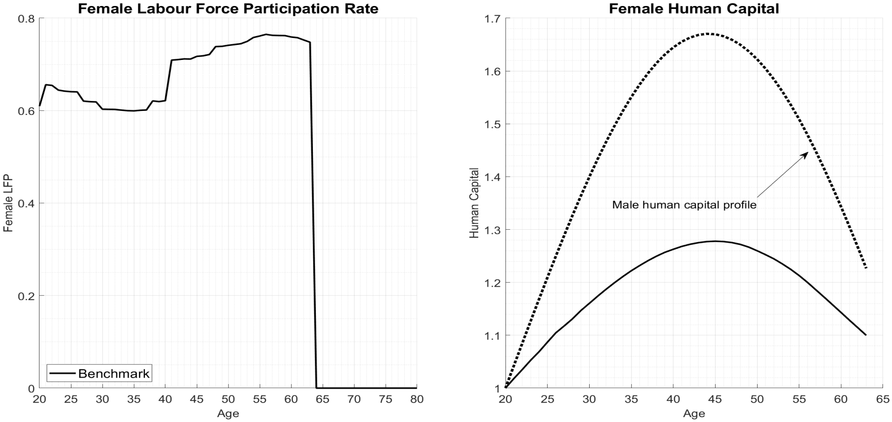
Results: Benchmark vs. Experiments

	Pre-reform	Remove FTB	Remove CCS	Remove both
	Benchmark values	Change	Change	Change
Income (Y)	1.13	1.01%	-3.48%	-3.05%
Consumption (C)	0.60	1.43%	-3.26%	-2.31%
Savings (S)	0.07	16.03%	-1.41%	18.99%
Female LFP	63.55%	10.58 p.p.	-10.00 p.p.	-2.31 p.p.
Female FT rate	24.02%	11.18 p.p.	-4.55 p.p.	0.26 p.p.
Income tax rate	19.77	-1.72 p.p.	-0.70 p.p.	-4.22 p.p.
FTB expense	0.0183	—	10.89%	—
CCS expense	0.013	79.23%	—	—
Pension	0.0382	-3.14%	-3.93%	-8.12%
HEV (newborn)	0	-5.5021%	-1.00%	-51.46%**

Table: Stationary equilibria comparison

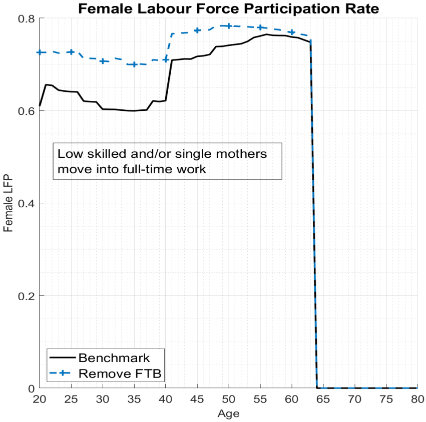
**Turning off household types, the loss is only 2%. See also experiment 2.

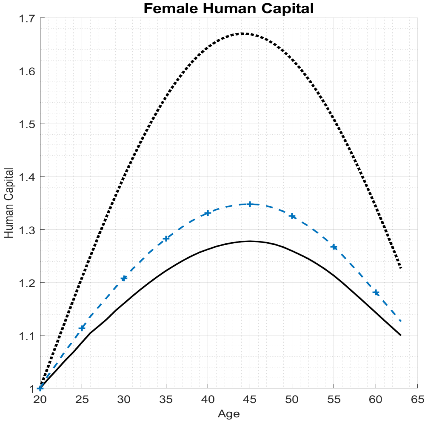
LFP and human capital of mothers: Benchmark



See female labour force participation rate. estimates using HILDA.

LFP and human capital of mothers: Remove FTB





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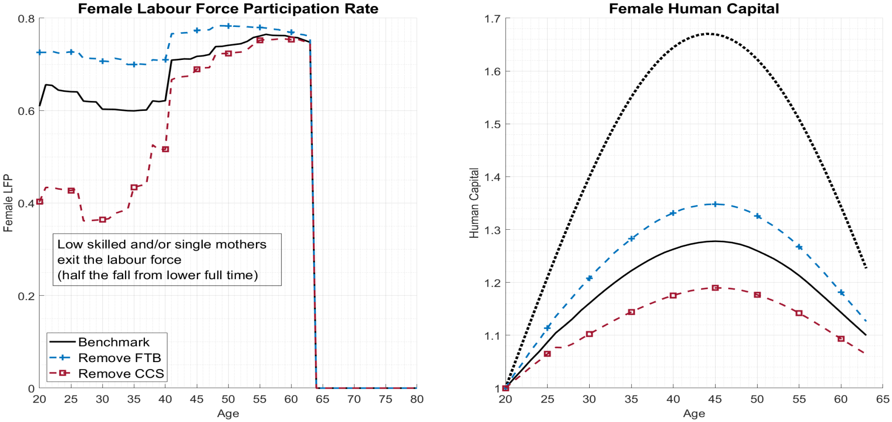
LFP and human capital of mothers: Remove CCS



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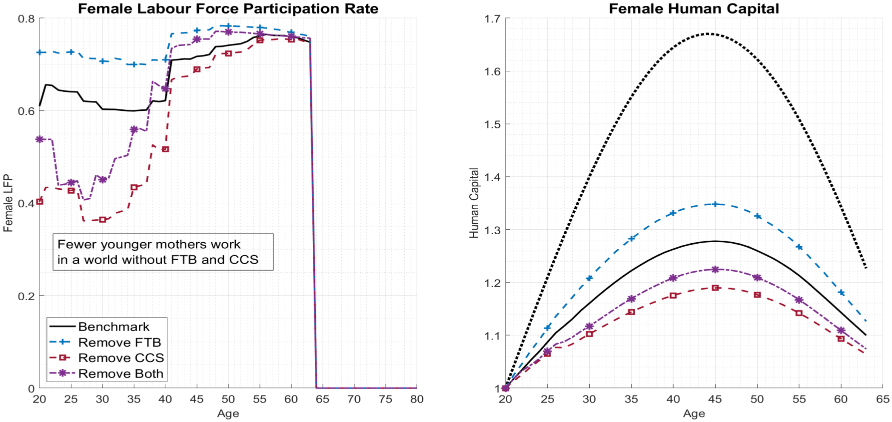
LFP and human capital of mothers: Remove FTB and CCS



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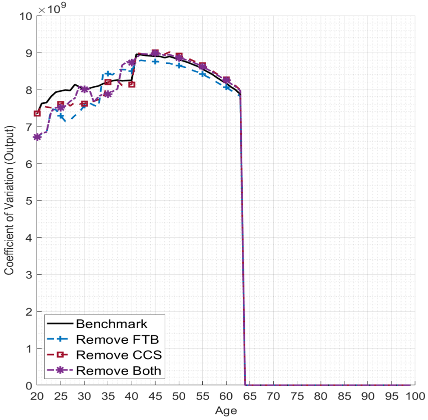
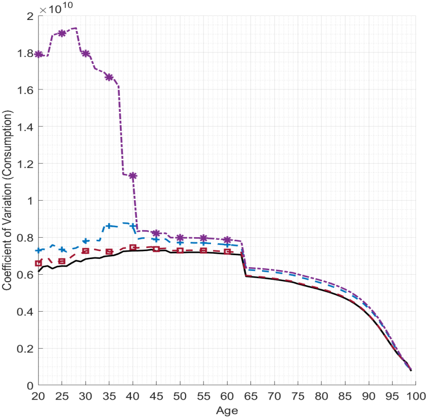
Reform effect on coefficients of variation of log output and log consumption





Conclusion

1. Experiments on key means-testing and payment rate parameters in an environment where transfers have opposing eligibility criteria;

2. A spotlight on the interplay between transfer programs;

3. An emphasis on the role of family structure and life-cycle dynamics in policy analyses;

4. Lessons from a unique configuration in Australia:
   - Efficiency and welfare improvement through multiple channels;
   - As in Guner et al. (2020), our findings suggest expanding the work conditional program;
   - In general, approaching a more universal setting (e.g., $\downarrow$ taper rates) is an improvement;

Caveats

We abstract from:

1. Labor market and political frictions;
2. Administrative overhead of a complex welfare system;
3. Intensive margin of female labor supply;
4. Endogenous male labor supply;
5. Child-less married household;
6. Fertility, education and marriage/divorce decisions;
7. Full welfare analysis along the transitional dynamics between two steady states.

Future plan

Planned expansion:

1. More labor options (permanent and casual employments);
2. Endogenize intensive margin of labour supply;
3. Richer income process (See De Nardi et al. (2020));
4. More detailed policy experiments;
5. Optimal transfer policy.

Welfare expenditure in Australia

Financial year	Welfare ($b)	Welfare-GDP (%)	Welfare-Revenue (%)
2010–11	140.19	8.43	34.04
2011–12	149.66	8.7	34.2
2012–13	153.24	8.89	33.62
2013–14	155.68	8.88	33.47
2014–15	165.13	9.41	35.15
2015–16	167.68	9.47	34.59
2016–17	165.76	8.95	33.02
2017–18	171.62	8.99	32
2018–19	174.24	8.8	31.18
2019–20	195.71	9.86	36.05

Note: $ value is expressed in 2019–20 prices.

Source: *Australian Institute of Health and Welfare*

Welfare expenditure to GDP (%) by target groups

Financial year	Families & children	Old people	Disabled	Unemployed	Others
2009-10	2.51	3.33	1.87	0.48	0.40
2010-11	2.39	3.33	1.94	0.44	0.34
2011-12	2.33	3.43	1.98	0.44	0.52
2012-13	2.31	3.57	2.00	0.49	0.52
2013-14	2.26	3.47	2.02	0.55	0.57
2014-15	2.33	3.79	2.09	0.59	0.61
2015-16	2.32	3.86	2.08	0.60	0.62
2016-17	2.02	3.72	2.01	0.57	0.63
2017-18	1.94	3.67	2.18	0.56	0.65
2018-19	1.81	3.63	2.22	0.49	0.64
2019-20	1.92	3.85	2.53	0.93	0.62

Source: Australian Institute of Health and Welfare

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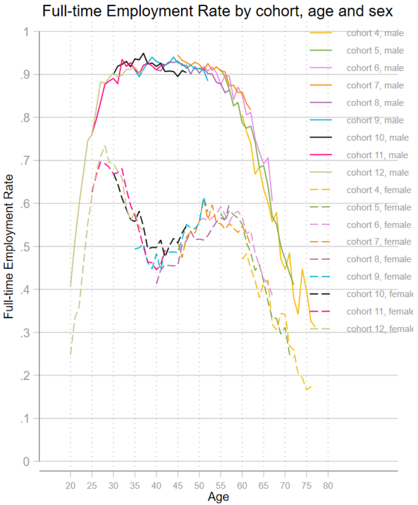
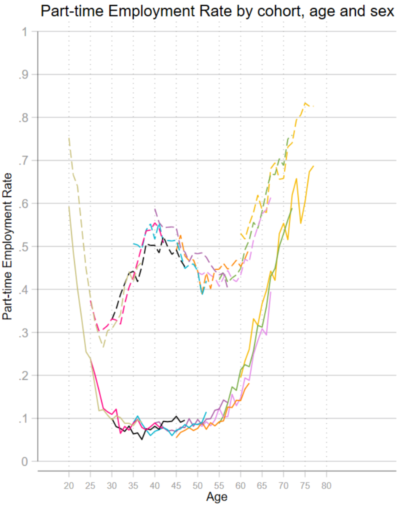
Full time employment rate by gender





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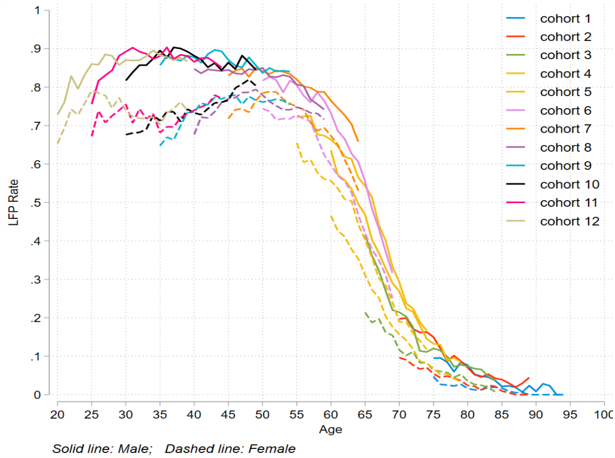
Labour force participation rate by gender



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Literature

Tax-Transfer in heterogeneous agent models with family structure:

1. Joint-filing income tax
   - For proportional and separate filing income tax in the US (Guner et al., 2012a,b) and in US and 10 EU countries (Bick and Fuchs-Schundeln, 2017)
2. Spousal and survival benefits
   - For elimination (US) (Kaygusuz, 2015; Nishiyama, 2019; Borella et al., 2020)*
3. Child-related transfers
   - Expansion requires stronger evidence (US) (Guner et al., 2020)
   - Negative childcare price elasticity of labour supply (AU) (Doiron and Kalb, 2004)*
4. Old age pension
   - For (at least) partial means-tested (US) (Feldstein, 1987; Braun et al., 2017)
   - Balancing insurance and incentive effects of means-tested Age Pension (AU) (Tran and Woodland, 2014)

Demographics (2)

As in Nishiyama (2019), the household type evolves according to Markov transition probabilities:

$\pi_{h_{j+1}\mid h_j}$	$\lambda_{j+1} = 0$	$\lambda_{j+1} = 1$	$\lambda_{j+1} = 2$
$\lambda_j = 0$	$\psi_{j+1,m}\psi_{j+1,f}$	$\psi_{j+1,m}(1-\psi_{j+1,f})$	$(1-\psi_{j+1,m})\psi_{j+1,f}$
$\lambda_j = 1$	0	$\psi_{j+1,m}$	0
$\lambda_j = 2$	0	0	$\psi_{j+1,f}$

Table: Transition probabilities of household type

Households: Preferences (1)

Households born at time t maximize expected intertemporal utility:

$$\max_{c_j, l_j^f} \sum_{j=1}^{J} \beta^{j-1} \left(\prod_{s=1}^{j-1} \pi_{\lambda_{s+1}|\lambda_s}, \right) u(c_j, l_j^m, l_j^f, \lambda_j, nc_j) \qquad (2)$$

- β - discount factor;
- ψ - time-invariant survival probabilities;
- λ - household type (by marital status)
- c - joint consumption;
- l^i - leisure time of $i \in m, f$;

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Households: Preferences (2)

The periodic utility functions at age j are:

$$u(c, l^m, l^f, \lambda = 1, 0) = \frac{\left[\left(\frac{c}{ces(1,0)}\right)^{\nu} (l^m)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} \tag{3}$$

$$u(c, l^m, l^f, \lambda = 2, nc) = \frac{\left[\left(\frac{c}{ces(2,nc)}\right)^{\nu} (l^f)^{1-\nu}\right]}{1-\frac{1}{\gamma}} \tag{4}$$

$$u(c, l^m, l^f, \lambda = 0, nc) = \frac{\left[\left(\frac{c}{ces(0,nc)}\right)^{\nu} (l^m)^{1-\nu}\right]^{1-\frac{1}{\gamma}} + \left[\left(\frac{c}{ces(0,nc)}\right)^{\nu} (l^f)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} \tag{5}$$

- Spouses are perfectly altruistic towards one another;
- $ces(\lambda, nc) = \sqrt{\mathbf{1}_{\{\lambda \neq 1\}} + \mathbf{1}_{\{\lambda \neq 2\}} + nc}$ - square root consumption equivalence scale;
- γ - intertemporal elasticity of substitution;
- ν - taste for consumption relative to leisure.

More on children...

5. Households have full information on children (e.g., arrival time, costs and benefits if work, etc);
6. No informal child care available;
7. Childcare quality and cost are identical;
8. Children leave home at 18 years old. This marks the end of the link between parents and their children;
9. No bequest motive.

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Bick (2016) finds that child care support does not increase the fertility rate in Germany. Discussed in Guner et al. (2020), evidence on child care quality is mixed. Marriage/divorce and education decisions are more likely impacted.

Households: Endowments

Labour income for $i \in \{m, f\}$ in working age $j = 1$ to $j = J_R = 45$:

$$\boldsymbol{y^i_{j,\lambda} = wn^i_{j,\lambda} e^i_{j,\lambda}}$$

- w - wage rate;
- n - exogenous labour hours $(n = 1 - l)$;
- e - earning ability:

Where

$$e^m_{j,\lambda} = \overline{e}_j \left(\theta, h^m_{j,\lambda}\right) \times \epsilon^m_j$$

- *Deterministic*: θ - permanent education; h - human capital;
- *Stochastic*: ϵ - transitory shocks.

Retirees receive means-tested pension $\boldsymbol{pen(y^m_{j,\lambda} + y^f_{j,\lambda},\ a_j)}$.

Households (working age): Men

Men always works and receives labor income:

$$y_{j,\lambda}^m \quad = w n_{j,\lambda}^m \theta h_{j,\lambda}^m \epsilon_j^m$$

n^m and h^m are exogenous.

The transitory shocks follow an *AR1* process:

$$\overbrace{\ln\left(\epsilon_j^m\right)}^{=\eta_j^m} = \rho^m \times \overbrace{\ln\left(\epsilon_{j-1}^m\right)}^{=\eta_{j-1}^m} + \upsilon_j^m; \qquad \upsilon_j^m \sim \mathcal{N}(0, \sigma_\upsilon^2) \qquad (6)$$

Dynamic Optimization Problem: **Working households**

Let $V(z_j)$ denotes the value function for a household aged j with state $z_j = \left\{\lambda_j, a_j, h^f_{j,\lambda,\ell}, \theta, \eta^m_j, \eta^f_j\right\}$ for $j < J_R$, and let $y_{j,\lambda} = \mathbf{1}_{\{\lambda\neq 2\}} y^m_{j,\lambda} + \mathbf{1}_{\{\lambda\neq 1,\ \ell_j>0\}} y^f_{j,\lambda} + ra_j$.

$$V(z_j) = \max_{c_j,\, \ell_j,\, a_{j+1}} \{\ u(c_j, l^m_j, l^f_j, \lambda_j, nc_j) + \tag{7}$$

$$\beta \sum_{\Lambda} \int_{S^2} V(z_{j+1})\, d\Pi(\lambda_{j+1}, \eta^m_{j+1}, \eta^f_{j+1} | \lambda_j, \eta^m_j, \eta^f_j)\}$$

s.t.

$$(1+\tau^c)c_j + (a_{j+1} - a_j) + \mathbf{1}_{\{\lambda\neq 1,\, \ell_j>0\}}[wn^f_{j,\lambda} \sum_{i=1}^{nc_j} (1 - sr_{j,i})\kappa_{j,i}] \tag{8}$$

$$= y_{j,\lambda} + \mathbf{1}_{\{\lambda\neq 1\}}(nc_j \times tr^A_j + tr^B_j) + beq_j - tax_j$$

$$c_j > 0 \tag{9}$$

$$a_{j+1} \geq 0 \tag{10}$$

$$l^m_j = 1 - n^m_{j,\lambda} \quad \text{if } \lambda = 0 \text{ or } \lambda = 1 \tag{11}$$

$$l^f_j = 1 - \mathbf{1}_{\{\ell>0\}} n^f_{j,\lambda,\ell} - \mathbf{1}_{\{\ell=1\}}(\chi + \chi_{c,j_c} \times nc_j) \quad \text{if } \lambda = 0 \text{ or } \lambda = 2 \tag{12}$$

Dynamic Optimization Problem: **Retirees**

Retiree's state vector is $z_j^R = \{a_j, \lambda_j\}$

- No labour income, no children;
- Pension is independent of labour earnings history but dependent on household type.

$$V(z_j^R) = \max_{c_j,\, a_{j+1}} \left\{ u(c_j, \lambda_j) + \beta \sum_{\Lambda} V(z_{j+1}^R)\, d\Pi(\lambda_{j+1}|\lambda_j) \right\} \quad (13)$$

s.t.

$$(1+\tau^c)c_j + (a_{j+1} - a_j) = ra_j + pen_j - tax_j \quad (14)$$

$$c_j > 0 \quad (15)$$

$$a_{j+1} \geq 0 \text{ and } a_{J+1} = 0 \quad (16)$$

Technology

- A firm with Cobb-Douglas production and labour-augmenting technology A (with constant growth rate g):

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

- Firm maximizes profit according to:

$$\max_{K_t, L_t} \quad (1-\tau_t^k)(Y_t - w_t A_t L_t) - (r_t + \delta)K_t \tag{17}$$

- Firm's FOC yields:

$$r_t = (1-\tau_t^k)\alpha \frac{Y_t}{K_t} - \delta \tag{18}$$

$$w_t = (1-\alpha)\frac{Y_t}{A_t L_t} \tag{19}$$

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Government: Tax system

Separate tax filing for $i \in \{m, f\}$ on $\widetilde{y}_j$

If *proportional*:

$$tax^i_j = \tau^w \times \widetilde{y}_j \tag{20}$$

If *progressive*:

$$tax^i_j = \max\left\{0, \;\; \widetilde{y}_j - \zeta \widetilde{y}_j^{1-\tau}\right\} \tag{21}$$

Where

- $\widetilde{y}_j = y^i_{j,\lambda} + \mathbf{1}_{\lambda=0}\frac{ra_j}{2} + \mathbf{1}_{\lambda\neq 0} ra_j$ is the taxable income
- ζ is a scaling parameter
- τ controls progressivity of the tax scheme:
  - $\tau \to \infty \implies tax^i_j \to y^i_{j,\lambda}$; i.e., tax everything;
  - $\tau = 0 \implies tax^i_j = (1-\zeta)y^i_{j,\lambda}$; i.e., $(1-\zeta)$ is a flat tax rate.

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Government: Family Tax Benefit part A (1)

The FTB part A is paid per dependent child.

There are 3 pairs of key parameters:

1. **Max** *and* **base** *payments per child*: $\{tr_j^{A1};\ tr_j^{A2}\}$;
2. **Income thresholds** *for max and base payments*: $\{\bar{y}^{tr}_{max};\ \bar{y}^{tr}_{base}\}$;
3. **Taper rates** *for max and base payments*: $\{\omega_{A1};\ \omega_{A2}\}$

Government: Family Tax Benefit part A (2)

Let $y_{j,\lambda} = y^m_{j,\lambda} + y^f_{j,\lambda} + ra_j$. The benefit received per child, $\boldsymbol{tr^A_j}$, is:

$$tr^A_j = \begin{cases} tr^{\mathrm{A1}}_j & \text{if } y_{j,\lambda} \leq \bar{y}^{tr}_{max} \\ \max\left\{tr^{\mathrm{A2}}_j, \quad tr^{\mathrm{A1}}_j - \omega_{A1}\left(y_{j,\lambda} - \bar{y}^{tr}_{max}\right)\right\} & \text{if } \bar{y}^{tr}_{max} < y_{j,\lambda} < \bar{y}^{tr}_{base} \\ \max\left\{0, \quad tr^{\mathrm{A2}}_j - \omega_{A2}\left(y_{j,\lambda} - \bar{y}^{tr}_{base}\right)\right\} & \text{if } y_{j,\lambda} \geq \bar{y}^{tr}_{base}, \end{cases} \tag{22}$$

Government: Family Tax Benefit part B (1)

The FTB part B is paid per household to provide additional support to single parents and single-earner parents with limited means.

There are 3 pairs of key parameters:

1. **Two max payments** *for households with children aged* $[0,4]$ *or* $[5,18]$: $\{tr_j^{\mathrm{B1}};\ tr_j^{\mathrm{B2}}\}$;
2. **Separate income thresholds** *for* y_{pe} *and* y_{se}: $\{\bar{y}_{pe}^{tr};\ \bar{y}_{se}^{tr}\}$;
3. *A* **taper rate** *based on* y_{se}: ω_B

Where

- $y_{pe} = \max(y_{j,\lambda}^{m},\ y_{j,\lambda}^{f})$ is the primary earner's income
- $y_{se} = \min(y_{j,\lambda}^{m},\ y_{j,\lambda}^{f})$ is the secondary earner's income

See FTB part B formula.

Government: Family Tax Benefit part B (2)

The benefit received per household, $\boldsymbol{tr_j^B}$, is given by:

$$tr_j^B = \tag{23}$$

$$\begin{cases} \Upsilon_1 \times tr_j^{\mathrm{B1}} + \Upsilon_2 \times tr_j^{\mathrm{B2}} & \text{if } y_{pe} \leq \bar{y}_{pe}^{tr} \text{and } y_{se} \leq \bar{y}_{se}^{tr} \\ \\ \Upsilon_1 \times \max\left\{0,\ tr_j^{\mathrm{B1}} - \omega_B\left(y_{se} - \bar{y}_{se}^{tr}\right)\right\} & \text{if } y_{pe} \leq \bar{y}_{pe}^{tr} \text{and } y_{se} > \bar{y}_{se}^{tr} \\ +\Upsilon_2 \times \max\left\{0,\ tr_j^{\mathrm{B2}} - \omega_B\left(y_{se} - \bar{y}_{se}^{tr}\right)\right\} & \end{cases}$$

Where

- $\Upsilon_1 = \mathbf{1}_{\{nc_{[0,4],j} \geq 1\}}$
- $\Upsilon_2 = \mathbf{1}_{\{nc_{[0,4],j}=0 \text{ and } (nc_{[5,15],j} \geq 1 \ \ or \ \ nc_{[16,18]_{AS},j} \geq 1)\}}$

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Government: Child Care Subsidy (1)

The Child Care Subsidy (CCS) assists households with the cost of formal care for **children aged 13 or younger**.

The rate of subsidy depends on

1. **Statutory rates**: $sr = \{0.85, 0.5, 0.2, 0\}$;
2. **Income thresholds**: $\bar{y}_i^{sr}$ *for* $i \in \{1, 2, 3, 4, 5\}$;
3. **Hour thresholds** *of recognized activities*;
4. *A* **taper rate**, ω_C^i, *on household income* y_{hh}

See CCS formula.

Government: Child Care Subsidy (2)

The rate of subsidy, $\boldsymbol{s_j}$, is:

$$s_j = \Psi(y_{j,\lambda}, n^m_{j,\lambda}, n^f_{j,\lambda}) \times \begin{cases} sr_1 & \text{if } y_{j,\lambda} \leq \bar{y}^{sr}_1 \\ max\{sr_2,\ sr_1 - \omega^1_c\} & \text{if } \bar{y}^{sr}_1 < y_{j,\lambda} < \bar{y}^{sr}_2 \\ sr_2 & \text{if } \bar{y}^{sr}_2 \leq y_{j,\lambda} < \bar{y}^{sr}_3 \\ max\{sr_3,\ sr_2 - \omega^3_c\} & \text{if } \bar{y}^{sr}_3 \leq y_{j,\lambda} < \bar{y}^{sr}_4 \\ sr_3 & \text{if } \bar{y}^{sr}_4 \leq y_{j,\lambda} < \bar{y}^{sr}_5 \\ sr_4 & \text{if } y_{j,\lambda} \geq \bar{y}^{sr}_5 \end{cases} \tag{24}$$

Where

- $\omega^i_C = \dfrac{y_{j,\lambda} - \bar{y}^{sr}_i}{\$3,000}$
- Let $n^{min}_j = min\{n^m_{j,\lambda}, n^f_{j,\lambda,\ell}\}$. The adjustment factor is

$$\Psi(y_{j,\lambda}, n^m_{j,\lambda}, n^f_{j,\lambda}) = 0 + 0.24_{\{y_{j,\lambda} \leq AU\$70{,}015,\ n^{min}_j \leq 8\}} + 0.36_{\{8 < n^{min}_j \leq 16\}} + 0.72_{\{16 < n^{min}_j \leq 48\}} + 1_{\{n^{min}_j > 48\}}$$

Goverment: Old Age Pension (1)

Pension is funded by the general government budget.

Pension is available to households aged $j \geq J_R$ and is means-tested (*income and assets tests*).

Income test:

$$\mathcal{P}^y\left(y_{j,\lambda}\right) = \begin{cases} p^{\max} & \text{if } y_{j,\lambda} \leq \bar{y}_1^p \\ \max\left\{0,\ p^{\max} - \omega_y\left(y_j^p - \bar{y}_1^p\right)\right\} & \text{if } y_{j,\lambda} > \bar{y}_1^p, \end{cases} \tag{25}$$

Asset test:

$$\mathcal{P}^a\left(a_j\right) = \begin{cases} p^{\max} & \text{if } a_j \leq \bar{a}_1 \\ \max\left\{0,\ p^{\max} - \omega_a\left(a_j - \bar{a}_1\right)\right\} & \text{if } a_j > \bar{a}_1, \end{cases} \tag{26}$$

Government: Old Age Pension (2)

The amount of pension benefit claimable, pen_j, is the minimum of (25) and (26). That is,

$$pen_j = \begin{cases} \min\left\{\mathcal{P}^a\left(a_j\right), \mathcal{P}^y\left(y_{j,\lambda}\right)\right\} & \text{if } j \geq J_P \text{ and } \lambda = 0 \\ \frac{2}{3}\min\left\{\mathcal{P}^a\left(a_j\right), \mathcal{P}^y\left(y_{j,\lambda}\right)\right\} & \text{if } j \geq J_P \text{ and } \lambda = 1, 2 \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

Government: Budget

Government at time t collects taxes (T_t^c, T_t^K, T_t^I) and issue bond ($B_{t+1} - B_t$) to meet its debt obligation ($r_t B_t$) and its commitment to three spending programs:

- General government purchase, G_t;
- Family transfers (FTB + CCS), Tr_t;
- Old age pension, P_t.

The fiscal budget balance equation is therefore

$$(B_{t+1} - B_t) + T_t^C + T_t^K + T_t^I = G_t + Tr_t + \mathcal{P}_t + r_t B_t. \quad (28)$$

Competitive Equilibrium: **Measure of Households**

Let $\phi_t(z)$ and $\Phi_t(z)$ denote the population growth- and mortality-unadjusted population density and cumulative distributions, respectively, and Ω_t denotes the vector of parameters at time t.

Initial distribution of newborns:

$$\begin{aligned} \int_{\Lambda \times A \times H \times \Theta \times S^2} d\Phi_t(\lambda, a, h, \theta, \eta_m, \eta_f) &= \int_{\Lambda \times \Theta \times S^2} d\Phi_t(\lambda, 0, 0, \theta, \eta_m, \eta_f) = 1, \quad \text{and} \\ \phi_t(\lambda, 0, 0, \theta, \eta_m, \eta_f) &= \pi(\lambda) \times \pi(\theta) \times \pi(\eta_m) \times \pi(\eta_f). \end{aligned}$$

The population density $\phi_t(z)$ evolves according to:

$$\begin{aligned} \phi_{t+1}(z^+) = \int_{\Lambda \times A \times H \times \Theta \times S^2} & \mathbf{1}_{\{a^+ = a^+(z, \Omega_t),\, h^+ = h^+(z, \Omega_t)\}} \times \pi(\lambda^+ | \lambda) \\ & \times \pi(\eta_m^+ | \eta_m) \times \pi(\eta_f^+ | \eta_f)\, d\Phi_t(z) \qquad (29) \end{aligned}$$

Competitive Equilibrium: **Aggregation (Households)**

Given the optimal decisions $\{c(z,\Omega_t),\ \ell(z,\Omega_t),\ a(z,\Omega_t)\}_{j=1}^{J}$, the share of alive households $(\mu_{j,t})$ and the distribution of households $\phi_t(z)$ at time t, we arrive at:

$$C_t = \sum_{j=1}^{J} \int_{\Lambda\times A\times H\times\Theta\times S^2} c(z,\Omega_t)\mu_{j,t}\, d\Phi_t(z) \tag{30}$$

$$A_t = \sum_{j=1}^{J} \int_{\Lambda\times A\times H\times\Theta\times S^2} a(z,\Omega_t)\mu_{j,t}\, d\Phi_t(z) \tag{31}$$

$$LFP_t = \sum_{j=1}^{J} \int_{\Lambda\times A\times H\times\Theta\times S^2} \mathbf{1}_{\{\ell(z,\Omega_t)\neq 0\}}\mu_{j,t}\, d\Phi_t(z). \tag{32}$$

$$LM_t = \sum_{j=1}^{J} \int_{\Lambda\times A\times H\times\Theta\times S^2} h^m_{j,\lambda} e^{\theta+\eta_m}\mu_{j,t}\, d\Phi_t(z) \tag{33}$$

$$LF_t = \sum_{j=1}^{J} \int_{\Lambda\times A\times H\times\Theta\times S^2} \mathbf{1}_{\{\ell(z,\Omega_t)\neq 0\}} h^f_{j,\lambda,\ell} e^{\theta+\eta_f}\mu_{j,t}\, d\Phi_t(z). \tag{34}$$

Competitive Equilibrium: **Aggregation (Government)**

Given the optimal decisions $\{c(z,\Omega_t),\ \ell(z,\Omega_t),\ a(z,\Omega_t)\}_{j=1}^{J}$, government policy parameters, the share of alive households $(\mu_{j,t})$ and the distribution of households $\phi_t(z)$ at time t, we arrive at:

$$
\begin{aligned}
T_t^C &= \tau_t^c C_t && (35)\\
T_t^K &= \tau_t^k (Y_t - w_t A_t L_t) && (36)\\
T_t^I &= \sum_{j=1}^{J} \int_{\Lambda\times A\times H\times\Theta\times S^2} tax_j \mu_{j,t}\, d\Phi_t(z). && (37)\\
Tr_t &= \sum_{j=1}^{J} \int_{\Lambda\times A\times H\times\Theta\times S^2} (ftba_j + ftbb_j + ccs_j)\ \mu_{j,t}\, d\Phi_t(z) && (38)\\
\mathcal{P}_t &= \sum_{j=1}^{J} \int_{\Lambda\times A\times H\times\Theta\times S^2} pen_j \mu_{j,t}\, d\Phi_t(z). && (39)
\end{aligned}
$$

Competitive Equilibrium: Definition (1)

Given the household, firm and government policy parameters, the demographic structure, the world interest rate, a steady state equilibrium is such that:

1. The collection of individual household decisions $\{c_j, \ell_j, a_{j+1}\}_{j=1}^{J}$ solve the household problem (7) and (13);
2. The firm chooses labor and capital inputs to solve the profit maximization problem (18);
3. The government budget constraint (28) is satisfied;
4. The markets for capital and labour clear:

$$K_t = A_t + B_t + B_{F,t} \tag{40}$$

$$L_t = LM_t + LF_t \tag{41}$$

Competitive Equilibrium: Definition (2)

5. Goods market clears:

$$Y_t = C_t + I_t + G_t + NX_t \tag{42}$$
$$NX_t = (1+n)(1+g)B_{F,t+1} - (1+r)B_{F,t}$$
$$B_{F,t} = A_t - K_t - B_t$$

Where

- $I_t = (1+n)(1+g)K_{t+1} - (1-\delta)K_t$ is investment
- NX_t is the trade balance, and
- $B_{F,t}$ is the foreign capital required to clear the capital market.

Competitive Equilibrium: Definition (3)

6. The total lump-sum bequest transfer, BQ_t, is the total assets left by all deceased households at time t:

$$BQ_t = \sum_{j=1}^{J} \int_{\Lambda \times A \times H \times \Theta \times S^2} (1 - \psi_{j,\lambda})(1 + r_t) a(z, \Omega_t) \, d\Phi_t(z). \tag{43}$$

Bequest to each surviving household aged j at time t is

$$beq_{j,t} = \left[\frac{b_{j,t}}{\sum_{j=1}^{J} b_{j,t} \, m_{j,t}} \right] BQ_t \tag{44}$$

Assuming bequest is uniform among alive working-age agents, then $b_{j,t} = \frac{1}{JR-1}$ if $j < JR$ and $b_{j,t} = 0$ otherwise. Thus,

$$beq_{j,t} = \frac{BQ_t}{\sum_{j=1}^{JR-1} m_{j,t}} \tag{45}$$

Summary: Externally Calibrated Parameters (1)

Parameter	Value	Target (2012-2018)
Demographics		
Lifespan	$J = 80$	Age 21-100
Retirement	$J_R = 45$	Age 65
Population growth	$n = 1.6\%$	Average (ABS)
Survival probabilities	$\psi_m,\ \psi_f$	Australian Life Tables (ABS)
Measure of newborns by type	$\{\pi(\lambda_0), \pi(\lambda_1), \pi(\lambda_2)\} = \{0.70, 0.14, 0.16\}$	HILDA 2010-2018
Technology		
Labour augmenting tech. growth	$g = 1.3\%$	Average per hour worked growth rate (World Bank)
Output share of capital	$\alpha = 0.4$	Output share of capital for Australia
Real interest rate	$r = 4\%$	Average (World Bank)
Households		
Relative risk aversion	$\sigma = \frac{1}{\gamma} = 3$	standard values 2.5-3.5
Work hours	$n_{m,\lambda},\ n_{f,\lambda}$	Age-profiles of avg. hours for employees (HILDA)
Male human capital profile	h^m_λ	Age-profile of hourly wages for married men

Summary: Externally Calibrated Parameters (2)

Parameter	Value	Target
Permanent shocks		
Value	$\{\theta_L, \theta_H\} = \{0.745, 1.342\}$	College-HS wage ratio of 1.8 (HILDA, 2012-2018)
Measure of $\{\theta_L, \theta_H\}$ type households	$\{\pi(\theta_L), \pi(\theta_H)\} = \{0.7, 0.3\}$	College-HS ratio (ABS, 2018)
Fiscal Policy		
Consumption tax	$\tau_c = 8\%$	$\tau_c \frac{C_0}{Y_0} = 4.5\%; \frac{C_0}{Y_0} = 56.3\%$
Company profit tax	$\tau^k = 10.625\%$	$\tau^k \left(\frac{Y - WL}{Y}\right) = 4.5\%; \frac{WL}{Y} = 1 - \alpha$
Gov't debt-to-GDP	$\frac{B}{Y} = 20\%$	Average (CEIC data, 2012-2018)
Gov't general purchase	$\frac{G}{Y} = 14\%$	Net of FTB, CCS and Age Pension (WDI and AIHW)
FTB, CCS and pension parameters		HILDA Tax-Benefit model

Summary: Internally Calibrated Parameters (1)

Parameter	Value	Target
Households		
Discount factor	$\beta = 0.99$	Saving ratio $5\% - 8\%$ (ABS, 2013-2018)
Taste for consumption	$\nu = 0.365$	LFP rate for mothers = 65-70%
Time cost of non-mother's FT work	$\chi = 0.14$	Mother's full time rate = 26-28%
Extra time cost of mother's full time work	$\{\chi_{c,j_c=[0,5]},\ \chi_{c,j_c=[6,11]}\} = \{0.0325,\ 0.005\}$	Age-profile of full time share
Female human capital accumulation	$(\xi_{1,\lambda,\ell};\ \xi_{2,\lambda,\ell})$	Age-profile of hourly wages of male counterpart (if $\ell > 0$ every period)
Female human capital depreciation	$\delta_h = 0.074$	Male-female wage gap at age 50 (HILDA)
Transitory shocks, ϵ		
Persistence	$\rho = 0.98$	Literature
Variance of shocks	$\sigma^2_\epsilon = 0.0145$	Gini of male wges at age 21, $GINI_{j=1,m} = 0.35$
Fiscal policy		
Income tax	20%	Budget-balancing variable
Maximum pension	$pen^{max} = 30\% \times Y_m$	$\frac{\mathcal{P}_t}{Y_t} = 3.2\%$ (ABS, 2012-2018)

Calibration: Demographics (1)

1. Since child-related transfers are concentrated during child-bearing and raising age, we set one model period to correspond to 1 year of life to better capture behavioural responses;

2. Time-invariant n, ψ_m and ψ_m induce an unchanging population structure in every period t (see share of survivors).

Calibration: Demographics (2)

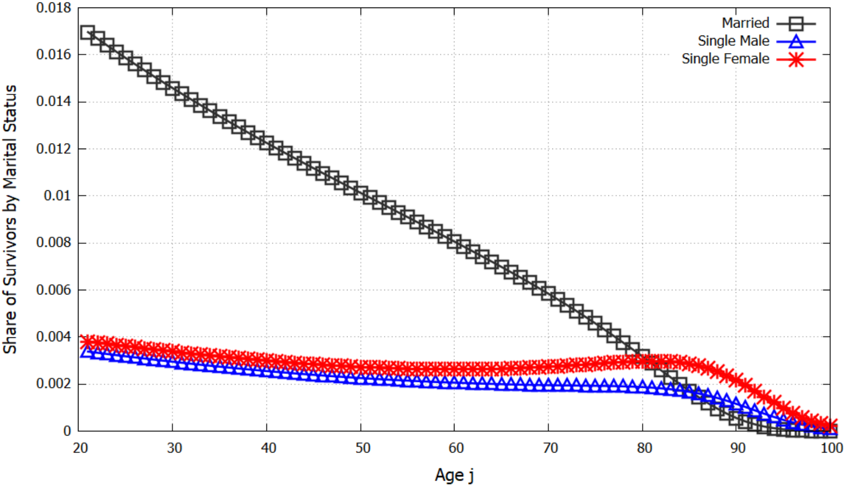

Figure: Share of survivors over life cycle

Calibration: Endowment (Deterministic) (1)

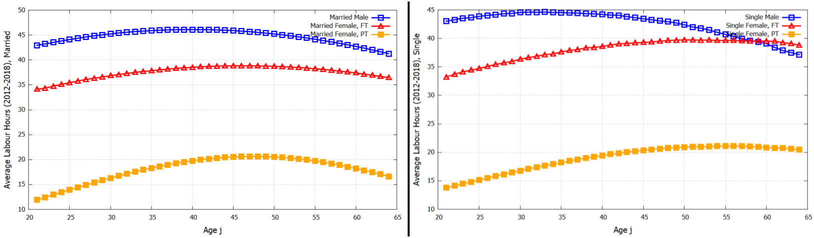

Figure: Age profiles of average labor hours

Calibration: Endowment (Deterministic) (2)

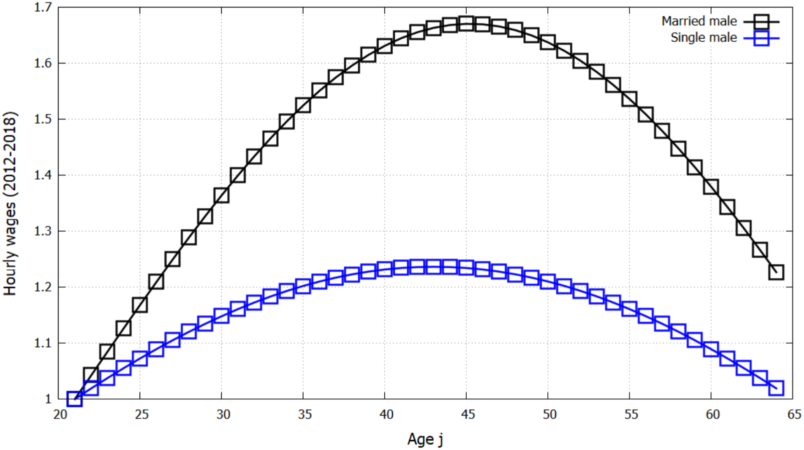

Figure: Age profiles of male hourly wages

Calibration: Endowment (Deterministic, Female)

We calibrate the female human capital accumulation rate that their human capital profiles match those of their male counterparts:

- if the wife works without time off over life cycle, and
- assuming ex-ante assortative matching of couples in terms of skills.

Our estimates are:

- Married mothers working full time: $(\xi_{1,\lambda=0,\ell=1}, \xi_{2,\lambda=0,\ell=1}) = (0.0450, -0.00175)$
- Married mothers working part time: $(\xi_{1,\lambda=0,\ell=2}, \xi_{2,\lambda=0,\ell=2}) = (0.0350, -0.00135)$
- Single mothers working full time: $(\xi_{1,\lambda=2,\ell=1}, \xi_{2,\lambda=2,\ell=1}) = (0.0206, -0.00088)$
- Single mothers working part time: $(\xi_{1,\lambda=2,\ell=2}, \xi_{2,\lambda=2,\ell=2}) = (0.0179, -0.00060)$

Calibration: Endowment (Deterministic, Children)

Children:

1. Assign *first and second child births* to
   - type θ_H households aged {28, 31};
   - type θ_L households aged $\{21, 24\}$ (See LSAC and AIHW reports)
2. Child care service fee is \$12.5/*hour* or 48% of age 21 married male hourly wage.
3. Based on approximates from child care service and school fees, parents pay
   - 100% of the fee for child aged 0-2;
   - 80% for child aged 3-5;
   - 60% for child aged 6-11;
   - 40% for child aged 12-17.

Calibration: Endowment (Stochastic income process)

We calibrate the AR1 stochastic process, η^i, for $i \in \{m, f\}$ as follows:

- Discretized into 5 grid points:

$$\eta^i = \{0.29813,\ 0.54601,\ 1,\ 1.83146,\ 3.35424\}$$

- Transition probabilities obtained via Rouwenhorst method:

$$\begin{bmatrix} 0.9606 & 0.0388 & 0.0006 & 0 & 0 \\ 0.0097 & 0.9609 & 0.0291 & 0.0003 & 0 \\ 0.0001 & 0.0194 & 0.9610 & 0.0194 & 0.0001 \\ 0 & 0.0003 & 0.0291 & 0.9609 & 0.0097 \\ 0 & 0 & 0.0006 & 0.0388 & 0.9606 \end{bmatrix}$$

Calibration: Endowment (Stochastic income process)

- Persistence: $\rho = 0.98$;
- Variance of the innovation to shocks: $\sigma_\epsilon^2 = 0.0145$ to achieve a Gini coefficient of age 21 male wage distribution of 0.35;
- The set-up results in $\text{GINI} = 0.3766$ for wage distribution of work-age male population (not targeted).

Lorenz Curve (male wages at aged 21 and 22)

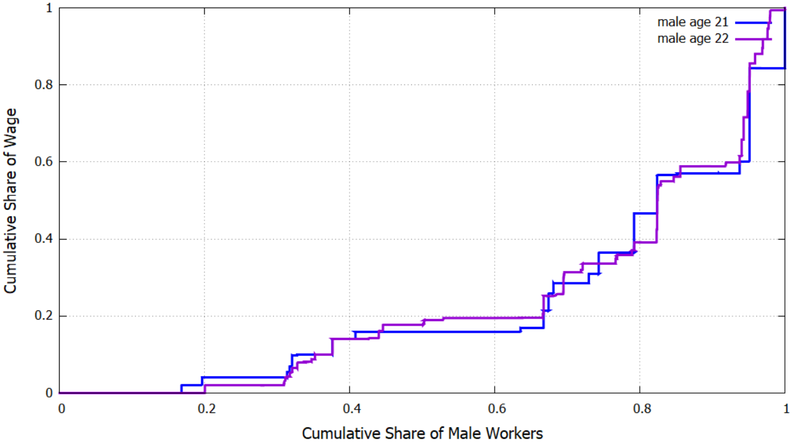

Figure: Lorenz curves of the distributions of married male wages at age 21 and 22

Lorenz Curve (male wages at working age)

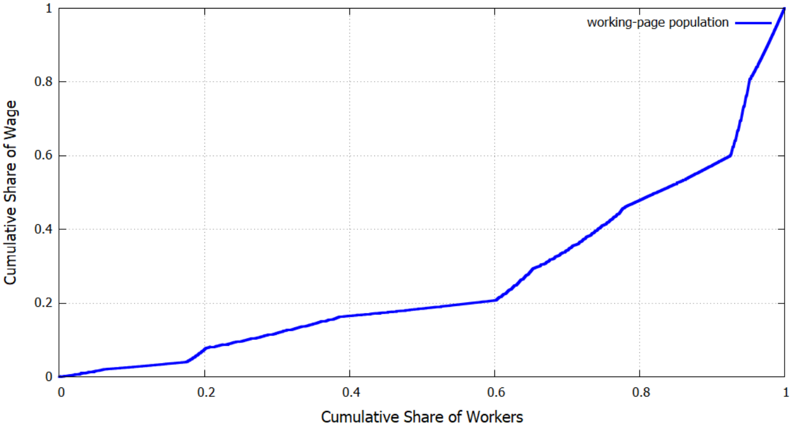

Figure: Lorenz curve of the wage distribution of the working-age male population (accounting for human capital, education and transitory shocks over the life cycle)

Benchmark: Life cycle profiles

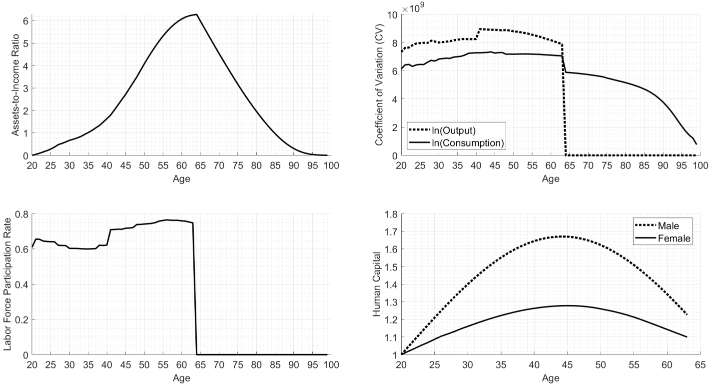

Figure: **Top left:** Assets-to-income ratio. **Top right:** CVs of log output and consumption. **Bottom left:** Female labor force participation rate. **Bottom right:** Male and female human capital.

Experiment set 1: Varying FTB means-testing parameters

	Status quo FTB payment rates, $\{\boldsymbol{tr_j^{A1}}, \boldsymbol{tr_j^{A2}}\}and\{\boldsymbol{tr_j^{B1}}, \boldsymbol{tr_j^{B2}}\}$		
ω \ $\bar{y}^{tr}$	$0.5 \times \bar{y}^{tr}$	$\bar{y}^{tr}$	$1.5 \times \bar{y}^{tr}$
$0.5 \times \omega$	exp 1.1	exp 1.2	exp 1.3
$0.75 \times \omega$	exp 2.1	exp 2.2	exp 2.3
ω	exp 3.1	exp 3.2	exp 3.3
$1.25 \times \omega$	exp 4.1	exp 4.2	exp 4.3
$1.5 \times \omega$	exp 5.1	exp 5.2	exp 5.3

Table: **Summary of policy experiments on FTB, keeping CCS unchanged**

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Experiment set 2: Varying CCS means-testing parameters

$\bar{y}^{sr}$ \ ω_{ccs}	Status quo CCS statutory subsidy rates, $\{sr_1, sr_2, sr_3, sr_4\}$		
	$0.5 \times \bar{y}^{sr}$	$\bar{y}^{sr}$	$1.5 \times \bar{y}^{sr}$
$1.5 \times \omega_{ccs}$	exp 6.1	exp 6.2	exp 6.3
$1.25 \times \omega_{ccs}$	exp 7.1	exp 7.2	exp 7.3
ω_{ccs}	exp 8.1	exp 8.2	exp 8.3
$0.75 \times \omega_{ccs}$	exp 9.1	exp 9.2	exp 9.3
$0.5 \times \omega_{csc}$	exp 10.1	exp 10.2	exp 10.3

Table: **Summary of policy experiments on CCS, keeping FTB unchanged**

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Experiment set 3: Varying payment rates

	FTB A	FTB B	CCS
Change FTB payment	$\{tr_j^{A1}, tr_j^{A2}\}$	$\{tr_j^{B1}, tr_j^{B2}\}$	
Experiment 11.1	+50%	+50%	–
Experiment 11.2	-50%	-50%	–
Change CCS rate			$\{sr_1, sr_2, sr_3, sr_4\}$
Experiment 12.1	–	–	+50%
Experiment 12.2	–	–	-50%
Complete removal	$\{tr_j^{A1}, tr_j^{A2}\}$	$\{tr_j^{B1}, tr_j^{B2}\}$	$\{sr_1, sr_2, sr_3, sr_4\}$
Experiment 13.1	-100%	-100%	–
Experiment 13.2	–	–	-100%
Experiment 13.3	-100%	-100%	-100%

Table: **Summary of policy experiments**

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Results: FTB means-testing parameter reforms (1)

ω \ $\bar{y}^{tr}$	$0.5 \times \bar{y}^{tr}$	$\bar{y}^{tr}$	$1.5 \times \bar{y}^{tr}$
LFP & FT rates			
$0.5 \times \omega$	5.88 (0.25)	5.08 (1.73)	3.71 (-0.01)
$0.75 \times \omega$	5.83 (3.19)	2.61 (1.47)	3.02 (-0.23)
ω	5.72 (3.57)	0 (0)	2.22 (-0.56)
$1.25 \times \omega$	5.56 (4.13)	-0.04 (-0.15)	-0.42 (-1.77)
$1.5 \times \omega$	3.95 (2.47)	-0.22 (-0.06)	-0.48 (-1.75)
Human Cap.			
$0.5 \times \omega$	2.72%	2.55%	1.41%
$0.75 \times \omega$	2.70%	1.56%	1.14%
ω	2.70%	0.00%	0.54%
$1.25 \times \omega$	2.70%	-0.08%	-1.24%
$1.5 \times \omega$	1.61%	-0.13%	-1.25%
Output			
$0.5 \times \omega$	2.21%	1.98%	1.58%
$0.75 \times \omega$	2.32%	1.72%	1.39%
ω	2.47%	0.00%	1.29%
$1.25 \times \omega$	2.26%	0.76%	0.34%
$1.5 \times \omega$	1.78%	0.82%	0.29%

Results: FTB means-testing parameters reforms (2)

ω \ $\bar{y}^{tr}$	$0.5 \times \bar{y}^{tr}$	$\bar{y}^{tr}$	$1.5 \times \bar{y}^{tr}$
$0.5 \times \omega$	-0.05%	0.41%	0.68%
$0.75 \times \omega$	-0.29%	0.29%	0.58%
ω	-0.44%	0.00%	0.50%
$1.25 \times \omega$	-0.73%	0.03%	0.38%
$1.5 \times \omega$	-0.79%	-0.01%	0.38%

Table: **Percentage (%) changes in welfare of newborns**

Results: CCS means-testing parameter reforms (1)

ω_{ccs} \ $\bar{y}^{sr}$	$0.5 \times \bar{y}^{sr}$	$\bar{y}^{sr}$	$1.5 \times \bar{y}^{sr}$
LFP & FT rates			
$1.5 \times \omega_{ccs}$	-2.46 (-2.12)	0.36 (0.06)	0.75 (0.68)
$1.25 \times \omega_{ccs}$	-2.49 (-2.19)	0.29 (-0.02)	0.74 (0.83)
ω_{ccs}	-2.48 (-2.16)	0 (0)	0.67 (0.55)
$0.75 \times \omega_{ccs}$	-2.50 (-2.33)	-1.67 (-0.71)	0.63 (0.71)
$0.5 \times \omega_{csc}$	-2.67 (-2.45)	-1.86 (-1.02)	1.02 (0.83)
Human Cap.			
$1.5 \times \omega_{ccs}$	-1.50%	0.11%	0.37%
$1.25 \times \omega_{ccs}$	-1.52%	0.08%	0.38%
ω_{ccs}	-1.53%	0%	0.34%
$0.75 \times \omega_{ccs}$	-1.56%	-0.81%	0.39%
$0.5 \times \omega_{csc}$	-1.66%	-0.95%	0.70%
Output			
$1.5 \times \omega_{ccs}$	-0.22%	0.87%	0.99%
$1.25 \times \omega_{ccs}$	-0.14%	0.86%	1.15%
ω_{ccs}	-0.13%	0%	1.37%
$0.75 \times \omega_{ccs}$	-0.05%	0.43%	1.21%
$0.5 \times \omega_{csc}$	-0.15%	0.33%	1.26%

Results: CCS means-testing parameter reforms (2)

$\bar{y}^{sr}$ / ω_{ccs}	$0.5 \times \bar{y}^{sr}$	$\bar{y}^{sr}$	$1.5 \times \bar{y}^{sr}$
$1.5 \times \omega_{ccs}$	-0.0154%	0.0893%	0.0519%
$1.25 \times \omega_{ccs}$	-0.0009%	0.0879%	0.0847%
ω_{ccs}	-0.0050%	0%	0.1007%
$0.75 \times \omega_{ccs}$	0.0131%	0.0383%	0.0801%
$0.5 \times \omega_{csc}$	-0.02%	0.02%	0.08%

Table: **Percentage (%) changes in welfare of newborns**

Results: Experiment set 2

	+50% FTB	+50% CCS	-50% FTB	-50% CCS
	Change	Change	Change	Change
Income (Y)	-1.11%	1.33%	1.09%	0.69%
Consumption (C)	-1.81%	2.05%	2.30%	-0.90%
Savings (S)	-3.75%	1.13%	4.22%	-2.11%
Female LFP	-3.48 p.p.	7.61 p.p.	6.83 p.p.	-4.31 p.p.
Female FT rate	-1.85 p.p.	4.29 p.p.	5.51 p.p.	-1.95 p.p.
Income tax rate	1.48 p.p.	0.61 p.p.	-1.15 p.p.	-1.08 p.p.
FTB expense	49.90%	-14.24%	-49.34%	4.88%
CCS expense	-12.12%	75.38%	37.40%	-60.00%
Pension	-0.31%	1.31%	0.11%	1.31%
HEV (newborn)	0.83%	0.1%	-1.00%	-0.3467%

Table: Changes relative to benchmark values

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Computing the Steady State: Algorithm (1)

We solve the benchmark model (*small open economy*) for its initial balanced-growth path steady state equilibrium.

1. Parameterize the model and discretize assets on $[a_{min}, a_{max}]$ such that:
   - Number of grid points, $N_A = 70$;
   - $a_{min} = 0$ (No-borrowing constraint);
   - The grid if fairly dense near a_{min} so households are not restricted by an all-or-nothing decision;
   - a_{max} is sufficiently large so that (i) *households are not bound by* a_{max}, and (ii) *there is enough room for upward movement induced by new policy regimes*.

   and for human capital grids on $[h^f_{min}, h^f_{max}]$:
   - Number of grid points, $N_H = 25$;
   - $h^f_{min} = h^m_{j=21} = 1$;
   - $h^f_{max} = h^m_{j=50} = 1.546$;

Computing the Steady State: Algorithm (2)

2. Guess K_0 and L_0, endogenous government policy variables, and w_m, taking $r = r^w$ as given;
3. Solve the firm's problem for $(w_m,\ w_f)$;
4. Given the factor prices (w_m, w_f, r) and the initial steady state vector of parameters (Ω_0), solve the household problem for decision rules on $\{a^+, c, l^f\}$ by backward induction (from $j = J$ to $j = 1$) using *value function iteration*;

$w_m = w_f = w$ in this basic version of our model.

Computing the Steady State: Algorithm (3)

5. Starting from a known distribution of newborns, compute the measure of households across states by forward induction, using
   - the computed decision rules,
   - ψ,
   - η and its Markov transition probabilities, and
   - the law of motion of female human capital (1).
6. Accounting for the share of alive agents, sum across states for aggregate variables: A, C, L, T and Tr. Update L, K, I and Y (convex update). Solve for endogenous government policy variables.

Computing the Steady State: Algorithm (4)

7. Given the updated variables, compute the goods market convergence criterion for a small open economy:

$$Y = C + I + G + NX$$

- $B_F = A - K - B$;
- $NX = (1 + r)B_{F,t} - (1 + n)(1 + g)B_{F,t+1}$;
- $NX < 0$ implies a capital account surplus (increase in foreign indebtedness).

8. Return to step 3 until the convergence criterion is satisfied.

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