

Family Tax Benefits and Child Care Subsidy: A Macroeconomic Analysis

Darapheak Tin Chung Tran

Research School of Economics,
Australian National University

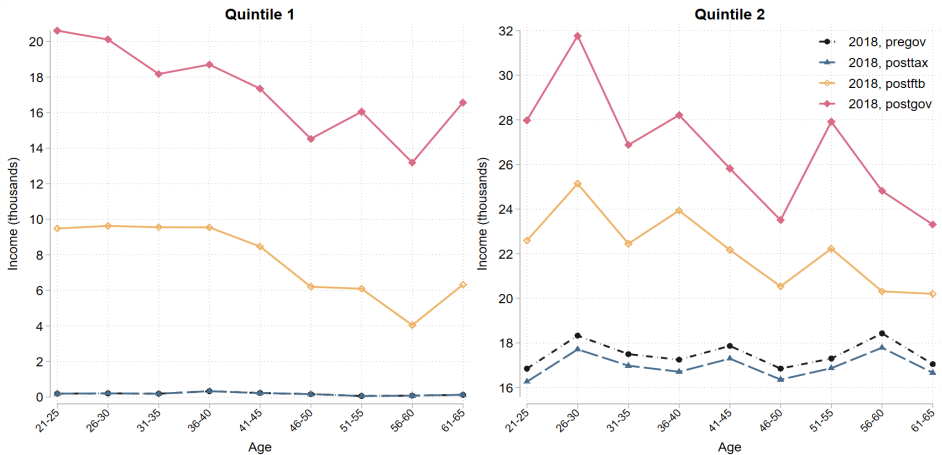
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Motivation

1. Sizeable child-related transfers (2-2.5% of GDP in the past decade).

Family Tax Benefits for lower income households (parents of dependent children)



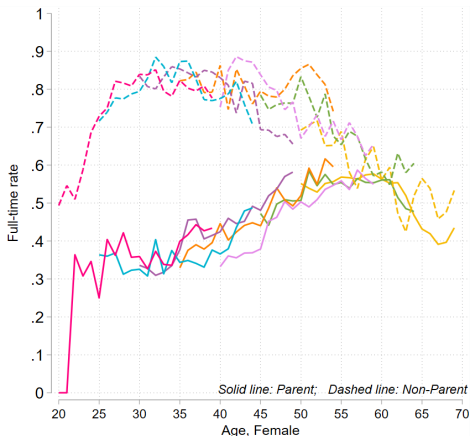
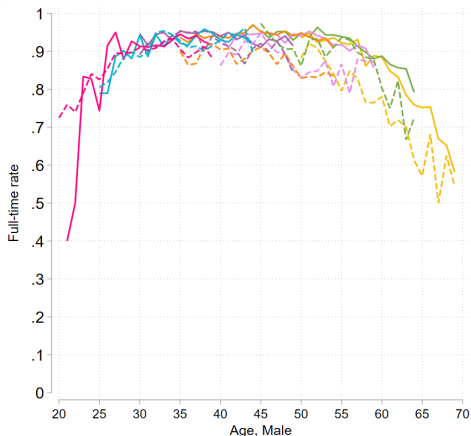
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 - ▶ *Similarities:*
Both programs test age and number of children, marital status, and income.
 - ▶ *Differences:*
FTB is **not** conditional on labour participation, but CCS has an activity test.

Age-profile of full-time participation by cohort, gender, and parenthood



See [full time rate profiles by gender](#).

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 - ▶ *Differences:*
FTB is **not** conditional on labour participation, but CCS has an activity test.
3. Interaction between transfer schemes with opposing effects and household demographic characteristics.

Our study

We develop a GE heterogeneous-agent OLG framework featuring:

- ▶ *Family structure,*
- ▶ *Endogenous female human capital,*
- ▶ *Time and monetary costs of children, and*
- ▶ *Detailed transfer programs.*

Our goal is to address the following questions:

1. What does a world without one or both of these child-related transfers look like? Which is better and from which perspective (*efficiency vs. welfare*)? - **our focus today**
2. Are there more optimal alternatives ? - future work

Summary of findings (1)

1. Partially offsetting effects over life cycle

- ▶ FTB discourages full-time work despite the presence of CCS (low-skilled and single mothers respond strongly)
- ▶ However, eliminating both programs → less participation for young mothers (i.e., CCS's work incentive effect is dominant)

2. Transfer programs interact. E.g., Removing the FTB:

- ▶ increases labor supply → higher CCS spending
- ▶ increases savings against earnings and longevity risk → lower age-pension spending due to assets test
- ▶ → affects tax

Summary of findings (2)

3. Understanding the impact of a policy reform requires a holistic view

3.1 From macro perspective:

- ▶ **remove FTB:** +16.7% female LFP, +1.1% GDP
- ▶ **remove FTB and CCS:** -3.6% female LFP, -3.3% GDP
- ▶ **remove CCS:** -15.7% female LFP, -3.48% GDP

3.2 From welfare perspective:

- ▶ **remove CCS:** -1% welfare
- ▶ **remove FTB:** -5.5% welfare (single mothers lose, though have CCS that allows for lower cost of work)
- ▶ **remove FTB and CCS:** -51.5% welfare (single mothers lose; no support and work is too costly)

4. Within the confines of this model, either cutting FTB or raising CCS creates some gains at a relatively low cost.

Model overview

A small open economy calibrated to Australia (2012-2018):

1. Households

- ▶ heterogeneous in age (j), family structure (λ), asset (a), human capital (h), education (θ), transitory shocks (ϵ^m, ϵ^f);
- ▶ time and monetary costs of children;
- ▶ make decision on joint consumption c , savings a^+ and female labor supply $\ell \in \{0, 1, 2\}$;

2. A representative firm with Cobb-Douglas technology;

3. Government commits to balance the budget every period:

- ▶ income tax, corporate income tax, consumption tax, borrowing;
- ▶ general expenditure, age pension, FTB, CCS, debt;

4. Goods and factor markets clear.

Demographics (1)

1. Time-invariant population growth rate (n) and survival probabilities (ψ_j^m and ψ_j^f);
2. Populated by three household types:
 - ▶ Married parents, $\lambda = 0$
 - ▶ Single childless men, $\lambda = 1$
 - ▶ Single mothers, $\lambda = 2$
3. Households are born as workers at $j = 1$, retire at $j = 45$ and can live to the maximum age of $j = J = 80$;

Demographics (2)

4. Low education (θ_L) households have children earlier;
5. Child spacing is identical for all parents;
6. Children are exogenous and fully determined by household age, j :
 - ▶ the k^{th} child is born to households aged $j = b_{k,\theta}$;
 - ▶ the k^{th} child is dependent for 18 years from $j = b_{k,\theta}$ to $j = b_{k,\theta} + 17$;

Households (working age): Costs of working for women

If a woman works, she incurs:

1. A time cost, χ , and time cost per child, $\chi_{c,jc}$:

$$l_j^f = \begin{cases} 1 & \text{if } \ell = 0 \\ 0 < \left(1 - n_{j,\lambda,\ell=1}^f\right) < 1 & \text{if } \ell = 1 \\ 0 < \left(1 - n_{j,\lambda,\ell=2}^f - \chi - \chi_{c,jc} \times nc_j\right) < 1 & \text{if } \ell = 2 \end{cases}$$

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2. A **formal childcare cost per child** κ_j that is decreasing in the age of children:

3. A loss of a portion or all of the means-tested FTB benefits.

Households (working age): Benefits of working for women

However, if she works, she gains:

1. Labor income

$$y_j = wn_j\theta h_j\epsilon_j$$

$$\ln(\epsilon_j) = \rho \times \ln(\epsilon_{j-1}) + v_j; \quad v_j \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

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2. Child care subsidy s_j per child;

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2. Child care subsidy s_j per child;

3. Human capital accumulation for the next period that evolves according to a law of motion:

$$\log(h_j) = \log(h_{j-1}) + (\xi_1 + \xi_2 \times (j-1)) \mathbf{1}_{\{\ell_{j-1} \neq 0\}} - \delta_\ell \mathbf{1}_{\{\ell_{j-1} = 0\}} \quad (1)$$

Key Macro Variables: Model vs. Data

Moments	Benchmark economy	Data	Source
<i>Targeted</i>			
Capital, K/Y	3.2	3-3.3	ABS (2012-2018)
Savings, S/Y	6.5%	5-8%	ABS (2013-2018)
Mother's labor participation, LFP	63%	65-70%	HILDA (2012-2018)*
Mother's full time rate, FT	23%	26-28% ($40\% \times LFP$)	HILDA (2012-2018)*
Consumption Tax, T^C/Y	4.26%	4.50%	APH Budget Review
Company Tax, T^K/Y	4.25%	4.25%	APH Budget Review
Age Pension, P/Y	3.31%	3.20%	ABS (2012-2018)
Gini coefficient (male aged 21)	0.35	0.35	
<i>Non-targeted</i>			
Consumption, C/Y	53.23%	54-58%	ABS (2012-2018)
Investment, I/Y	32.30%	24-28%	ABS (2013-2018)
Income tax, T^I/Y	12.11%	11%	APH Budget Review
Tax revenue to output	20.35%	25%	ABS(2012-2018)
Child-related transfers (FTB + CCS)	2.75%	2%	ABS (2012-2018)
Gini coefficient (working age male)	0.3766	0.45	PC (2018)

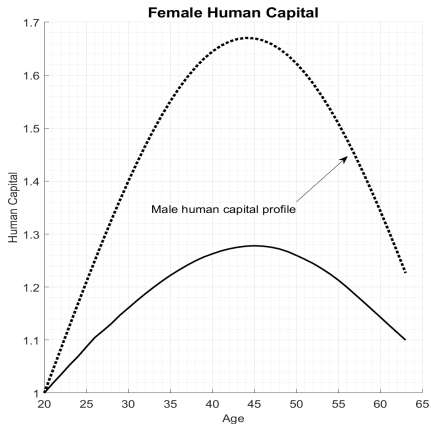
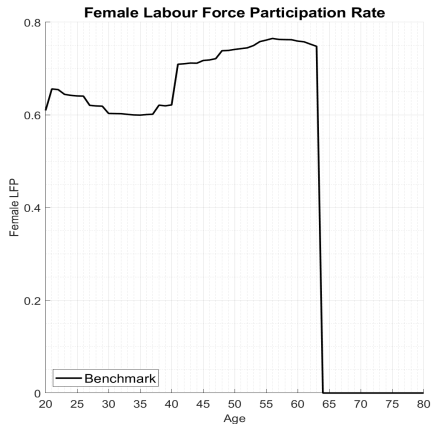
Results: Benchmark vs. Experiments

	Pre-reform	Remove FTB	Remove CCS	Remove both
	Benchmark values	Change	Change	Change
Income (Y)	1.13	1.01%	-3.48%	-3.05%
Consumption (C)	0.60	1.43%	-3.26%	-2.31%
Savings (S)	0.07	16.03%	-1.41%	18.99%
Female LFP	63.55%	10.58 p.p.	-10.00 p.p.	-2.31 p.p.
Female FT rate	24.02%	11.18 p.p.	-4.55 p.p.	0.26 p.p.
Income tax rate	19.77	-1.72%	-0.70%	-4.22%
FTB expense	0.018306	—	10.89%	—
CCS expense	0.013	79.23%	—	—
Pension	0.0382	-3.14%	-3.93%	-8.12%
HEV (newborn)	0	-5.5021%	-1.00%	-51.46%**

Table: Stationary equilibria comparison

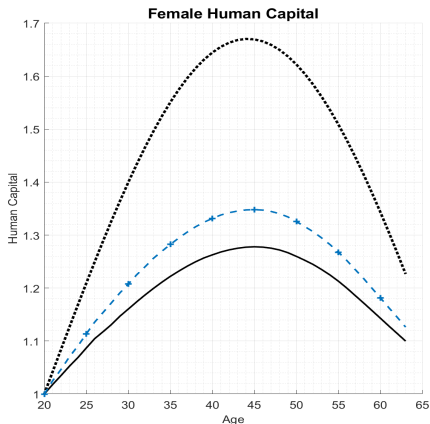
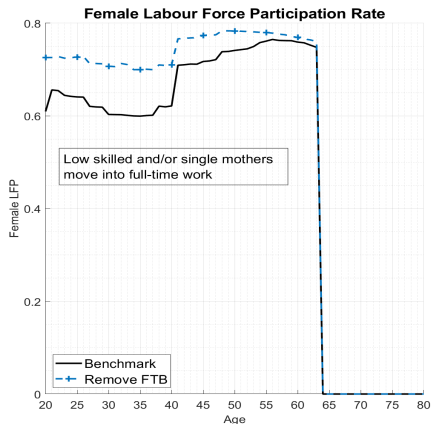
**Turning off household types, the loss is only 2%. See also [experiment 2](#).

LFP and human capital of mothers: Benchmark

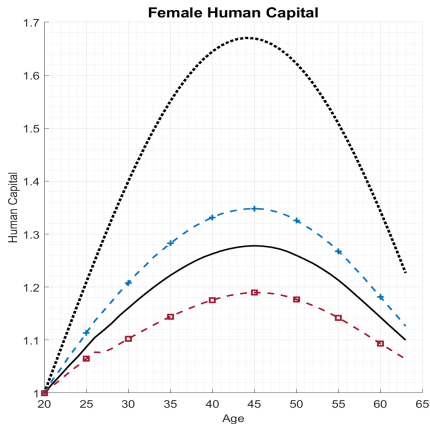
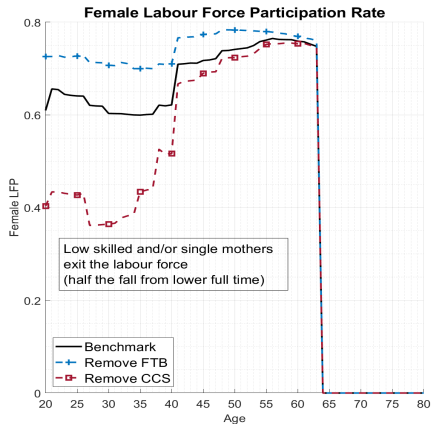


See [female labour force participation rate](#). estimates using HILDA.

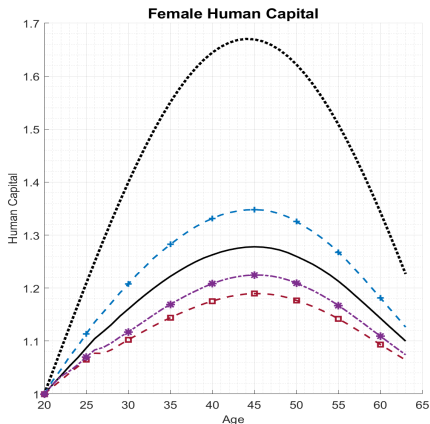
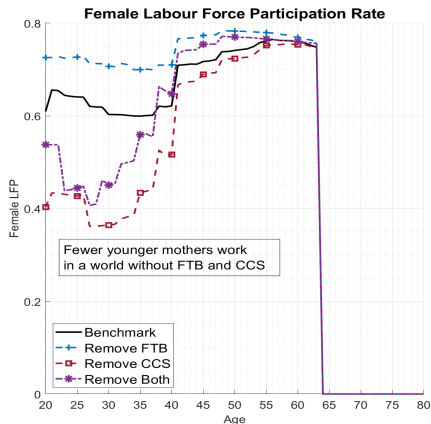
LFP and human capital of mothers: Remove FTB



LFP and human capital of mothers: Remove CCS



LFP and human capital of mothers: Remove FTB and CCS

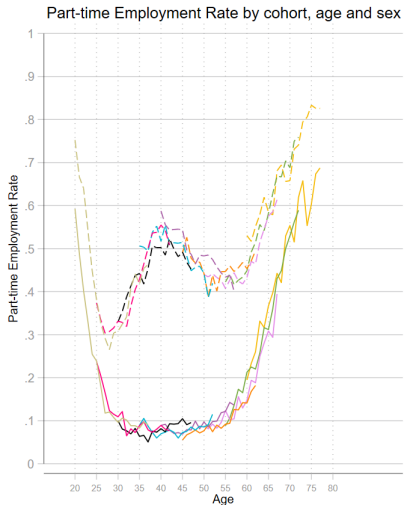
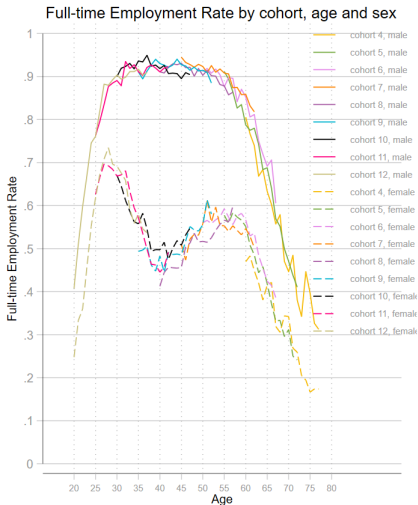


Future plan

Planned expansion:

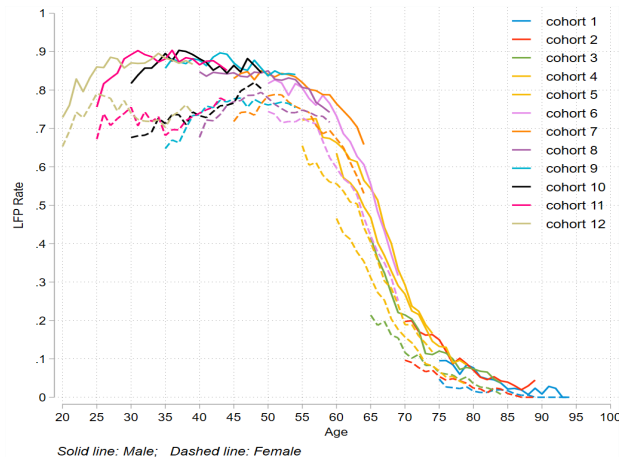
1. More labor options (permanent and casual employments);
2. Endogenize intensive margin of labour supply;
3. Richer income process (See [De Nardi et al. \(2020\)](#));
4. More detailed policy experiments;
5. Optimal transfer policy.

Full time employment rate by gender



ltg

Labour force participation rate by gender



Literature

Tax-Transfer in heterogeneous agent models with family structure:

1. Joint-filing income tax
 - ▶ For proportional and separate filing income tax in the US ([Guner et al., 2012a,b](#)) and in US and 10 EU countries ([Bick and Fuchs-Schundeln, 2017](#))
2. Spousal and survival benefits
 - ▶ For elimination (US) ([Kaygusuz, 2015](#); [Nishiyama, 2019](#); [Borella et al., 2020](#))*
3. Child-related transfers
 - ▶ Expansion requires stronger evidence (US) ([Guner et al., 2020](#))
 - ▶ Negative childcare price elasticity of labour supply (AU) ([Doiron and Kalb, 2004](#))*
4. Old age pension
 - ▶ For (at least) partial means-tested (US) ([Feldstein, 1987](#); [Braun et al., 2017](#))
 - ▶ Balancing insurance and incentive effects of means-tested Age Pension (AU) ([Tran and Woodland, 2014](#))

Demographics (2)

As in [Nishiyama \(2019\)](#), the household type evolves according to Markov transition probabilities:

$\pi_{h_{j+1} h_j}$	$\lambda_{j+1} = 0$	$\lambda_{j+1} = 1$	$\lambda_{j+1} = 2$
$\lambda_j = 0$	$\psi_{j+1,m}\psi_{j+1,f}$	$\psi_{j+1,m}(1 - \psi_{j+1,f})$	$(1 - \psi_{j+1,m})\psi_{j+1,f}$
$\lambda_j = 1$	0	$\psi_{j+1,m}$	0
$\lambda_j = 2$	0	0	$\psi_{j+1,f}$

Table: Transition probabilities of household type

Households: Preferences (1)

Households born at time t maximize expected intertemporal utility:

$$\max_{c_j, l_j^f} \sum_{j=1}^J \beta^{j-1} \left(\prod_{s=1}^{j-1} \pi_{\lambda_{s+1}|\lambda_s} \right) u(c_j, l_j^m, l_j^f, \lambda_j, nc_j) \quad (2)$$

- ▶ β - discount factor;
- ▶ ψ - time-invariant survival probabilities;
- ▶ λ - household type (by marital status)
- ▶ c - joint consumption;
- ▶ l^i - leisure time of $i \in m, f$;

[◀ Back to Model Summary](#)

Households: Preferences (2)

The periodic utility functions at age j are:

$$u(c, l^m, l^f, \lambda = 1, 0) = \frac{\left[\left(\frac{c}{ces(1,0)}\right)^\nu (l^m)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1 - \frac{1}{\gamma}} \quad (3)$$

$$u(c, l^m, l^f, \lambda = 2, nc) = \frac{\left[\left(\frac{c}{ces(2,nc)}\right)^\nu (l^f)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1 - \frac{1}{\gamma}} \quad (4)$$

$$u(c, l^m, l^f, \lambda = 0, nc) = \frac{\left[\left(\frac{c}{ces(0,nc)}\right)^\nu (l^m)^{1-\nu}\right]^{1-\frac{1}{\gamma}} + \left[\left(\frac{c}{ces(0,nc)}\right)^\nu (l^f)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1 - \frac{1}{\gamma}} \quad (5)$$

- ▶ Spouses are perfectly altruistic towards one another;
- ▶ $ces(\lambda, nc) = \sqrt{\mathbf{1}_{\{\lambda \neq 1\}} + \mathbf{1}_{\{\lambda \neq 2\}} + nc}$ - square root consumption equivalence scale;
- ▶ γ - intertemporal elasticity of substitution;
- ▶ ν - taste for consumption relative to leisure.

More on children...

5. Households have full information on children (e.g., arrival time, costs and benefits if work, etc);
6. No informal child care available;
7. Childcare quality and cost are identical;
8. Children leave home at 18 years old. This marks the end of the link between parents and their children;
9. No bequest motive.

◀ Back to Main Section

[Bick \(2016\)](#) finds that child care support does not increase the fertility rate in Germany. Discussed in [Guner et al. \(2020\)](#), evidence on child care quality is mixed. Marriage/divorce and education decisions are more likely impacted. ↻ 🔍 🔗

Households: Endowments

Labour income for $i \in \{m, f\}$ in working age $j = 1$ to $j = J_R = 45$:

$$y_{j,\lambda}^i = wn_{j,\lambda}^i e_{j,\lambda}^i$$

- ▶ w - wage rate;
- ▶ n - exogenous labour hours ($n = 1 - l$);
- ▶ e - earning ability:

Where

$$e_{j,\lambda}^m = \bar{e}_j(\theta, h_{j,\lambda}^m) \times \epsilon_j^m$$

- ▶ *Deterministic*: θ - permanent education; h - human capital;
- ▶ *Stochastic*: ϵ - transitory shocks.

Retirees receive means-tested pension ***pen***($y_{j,\lambda}^m + y_{j,\lambda}^f$, a_j).

Households (working age): Men

Men always works and receives labor income:

$$y_{j,\lambda}^m = wn_{j,\lambda}^m \theta h_{j,\lambda}^m \epsilon_j^m$$

n^m and h^m are exogenous.

The transitory shocks follow an *AR1* process:

$$\overbrace{\ln(\epsilon_j^m)}^{=\eta_j^m} = \rho^m \times \overbrace{\ln(\epsilon_{j-1}^m)}^{=\eta_{j-1}^m} + v_j^m; \quad v_j^m \sim \mathcal{N}(0, \sigma_v^2) \quad (6)$$

Dynamic Optimization Problem: **Working households**

$V(z_j)$ denotes the value function for a household aged j with state $z_j = \{\lambda_j, a_j, h_{j,\lambda,\ell}^f, \theta, \eta_j^m, \eta_j^f\}$ for $j < J_R$.

$$V(z_j) = \max_{c_j, \ell_j, a_{j+1}} \{ u(c_j, l_j^m, l_j^f, \lambda_j, nc_j) + \beta \sum_{\Lambda} \int_{S^2} V(z_{j+1}) d\Pi(\lambda_{j+1}, \eta_{j+1}^m, \eta_{j+1}^f | \lambda_j, \eta_j^m, \eta_j^f) \} \quad (7)$$

s.t.

$$(1 + \tau^c)c_j + a_{j+1} + \mathbf{1}_{\{\lambda \neq 1, \ell_j > 0\}} [wn_{j,\lambda}^f \sum_{i=1}^{nc_j} (1 - s_{j,i}) \kappa_{j,i}] \quad (8)$$

$$= (1 + r)a_j + y_{j,\lambda} + \mathbf{1}_{\{\lambda \neq 1\}} (nc_j \times tr_j^A + tr_j^B) + beq_j - tax_j$$

$$c_j > 0 \quad (9)$$

$$a_{j+1} \geq 0 \quad (10)$$

$$l_j^f = 1 \quad \text{if } \lambda = 1 \quad (11)$$

$$0 < l_j^f < 1 - n_{j,\lambda,\ell}^f - \mathbf{1}_{\{\ell=1\}} (\chi + \chi_{c,jc} \times nc_j) \quad \text{if } \lambda = 0 \text{ or } \lambda = 2 \quad (12)$$

Dynamic Optimization Problem: **Retirees**

Retiree's state vector is $z_j = \{a_j, \lambda_j\}$

- ▶ No labour income, no children;
- ▶ Pension is independent of labour earnings history but dependent on household type.

$$V(z_j) = \max_{c_j, a_{j+1}} \left\{ u(c_j, \lambda_j) + \beta \sum_{\Lambda} V(z_{j+1}) d\Pi(\lambda_{j+1}|\lambda_j) \right\} \quad (13)$$

s.t.

$$(1 + \tau^c)c_j + a_{j+1} = (1 + r)a_j + pen_j - tax_j \quad (14)$$

$$c_j > 0 \quad (15)$$

$$a_{j+1} \geq 0 \text{ and } a_{J+1} = 0 \quad (16)$$

Technology

- ▶ A firm with Cobb-Douglas production and labour-augmenting technology A (with constant growth rate g):

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

- ▶ Firm maximizes profit according to:

$$\max_{K_t, L_t} (1 - \tau_t^k)(Y_t - w_t A_t L_t) - (r_t + \delta)K_t \quad (17)$$

- Firm's FOC yields:

$$r_t = (1 - \tau_t^k) \alpha \frac{Y_t}{K_t} - \delta \quad (18)$$

$$w_t = (1 - \alpha) \frac{Y_t}{A_t L_t} \quad (19)$$

Government: Family Tax Benefit part A (1)

The FTB part A is paid per dependent child.

There are 3 pairs of key parameters:

1. **Max and base payments per child:** $\{tr_j^{A1}; tr_j^{A2}\};$
2. **Income thresholds for max and base payments:**
 $\{\bar{y}_{max}^{tr}; \bar{y}_{base}^{tr}\};$
3. **Taper rates for max and base payments:** $\{\omega_{A1}; \omega_{A2}\}$

Government: Family Tax Benefit part A (2)

Let $y_{j,\lambda} = y_{j,\lambda}^m + y_{j,\lambda}^f + r a_j$. The benefit received per child, tr_j^A , is:

$$tr_j^A = \begin{cases} tr_j^{A1} & \text{if } y_{j,\lambda} \leq \bar{y}_{max}^{tr} \\ \max \left\{ tr_j^{A2}, \quad tr_j^{A1} - \omega_{A1} (y_{j,\lambda} - \bar{y}_{max}^{tr}) \right\} & \text{if } \bar{y}_{max}^{tr} < y_{j,\lambda} < \bar{y}_{base}^{tr} \\ \max \left\{ 0, \quad tr_j^{A2} - \omega_{A2} (y_{j,\lambda} - \bar{y}_{base}^{tr}) \right\} & \text{if } y_{j,\lambda} \geq \bar{y}_{base}^{tr}, \end{cases} \quad (22)$$

Government: Family Tax Benefit part B (1)

The FTB part B is paid per household to provide additional support to single parents and single-earner parents with limited means.

There are 3 pairs of key parameters:

1. **Two max payments** *for households with children aged $[0, 4]$ or $[5, 18]$: $\{tr_j^{B1}; tr_j^{B2}\}$;*
2. **Separate income thresholds** *for y_{pe} and y_{se} : $\{\bar{y}_{pe}^{tr}; \bar{y}_{se}^{tr}\}$;*
3. **A taper rate** *based on y_{se} : ω_B*

Where

- ▶ $y_{pe} = \max(y_{j,\lambda}^m, y_{j,\lambda}^f)$ is the primary earner's income
- ▶ $y_{se} = \min(y_{j,\lambda}^m, y_{j,\lambda}^f)$ is the secondary earner's income

Government: Family Tax Benefit part B (2)

The benefit received per household, tr_j^B , is given by:

$$tr_j^B = \quad (23)$$

$$\begin{cases} \Upsilon_1 \times tr_j^{B1} + \Upsilon_2 \times tr_j^{B2} & \text{if } y_{pe} \leq \bar{y}_{pe}^{tr} \text{ and } y_{se} \leq \bar{y}_{se}^{tr} \\ \Upsilon_1 \times \max \{0, tr_j^{B1} - \omega_B(y_{se} - \bar{y}_{se}^{tr})\} \\ + \Upsilon_2 \times \max \{0, tr_j^{B2} - \omega_B(y_{se} - \bar{y}_{se}^{tr})\} & \text{if } y_{pe} \leq \bar{y}_{pe}^{tr} \text{ and } y_{se} > \bar{y}_{se}^{tr} \end{cases}$$

Where

- ▶ $\Upsilon_1 = \mathbf{1}_{\{nc_{[0,4],j} \geq 1\}}$
- ▶ $\Upsilon_2 = \mathbf{1}_{\{nc_{[0,4],j} = 0 \text{ and } (nc_{[5,15],j} \geq 1 \text{ or } nc_{[16,18]_{AS},j} \geq 1)\}}$

Government: Child Care Subsidy (1)

The Child Care Subsidy (CCS) assists households with the cost of formal care for **children aged 13 or younger**.

The rate of subsidy depends on

1. **Statutory rates:** $sr = \{0.85, 0.5, 0.2, 0\}$;
2. **Income thresholds:** \bar{y}_i^{sr} for $i \in \{1, 2, 3, 4, 5\}$;
3. **Hour thresholds** of recognized activities;
4. A **taper rate**, ω_C^i , on household income y_{hh}

Government: Child Care Subsidy (2)

The rate of subsidy, s_j , is:

$$s_j = \Psi(y_{j,\lambda}, n_{j,\lambda}^m, n_{j,\lambda}^f) \times \quad (24)$$

$$\begin{cases} sr_1 & \text{if } y_{j,\lambda} \leq \bar{y}_1^{sr} \\ \max\{sr_2, sr_1 - \omega_c^1\} & \text{if } \bar{y}_1^{sr} < y_{j,\lambda} < \bar{y}_2^{sr} \\ sr_2 & \text{if } \bar{y}_2^{sr} \leq y_{j,\lambda} < \bar{y}_3^{sr} \\ \max\{sr_3, sr_2 - \omega_c^3\} & \text{if } \bar{y}_3^{sr} \leq y_{j,\lambda} < \bar{y}_4^{sr} \\ sr_3 & \text{if } \bar{y}_4^{sr} \leq y_{j,\lambda} < \bar{y}_5^{sr} \\ sr_4 & \text{if } y_{j,\lambda} \geq \bar{y}_5^{sr} \end{cases}$$

Where

$$\blacktriangleright \omega_C^i = \frac{y_{j,\lambda} - \bar{y}_i^{sr}}{\$3,000}$$

- ▶ Let $n_j^{min} = \min\{n_{j,\lambda}^m, n_{j,\lambda,\ell}^f\}$. The adjustment factor is

$$\Psi(y_{j,\lambda}, n_{j,\lambda}^m, n_{j,\lambda}^f) = 0 + 0.24_{\{y_{j,\lambda} \leq AU\$70,015, n_j^{min} \leq 8\}} + 0.36_{\{8 < n_j^{min} \leq 16\}} \\ + 0.72_{\{16 < n_j^{min} \leq 48\}} + 1_{\{n_j^{min} > 48\}}$$

Government: Old Age Pension (1)

Pension is funded by the general government budget.

Pension is available to households aged $j \geq J_R$ and is means-tested (*income and assets tests*).

Income test:

$$\mathcal{P}^y(y_{j,\lambda}) = \begin{cases} p^{\max} & \text{if } y_{j,\lambda} \leq \bar{y}_1^p \\ \max \{0, p^{\max} - \omega_y (y_j^p - \bar{y}_1^p)\} & \text{if } y_{j,\lambda} > \bar{y}_1^p, \end{cases} \quad (25)$$

Asset test:

$$\mathcal{P}^a(a_j) = \begin{cases} p^{\max} & \text{if } a_j \leq \bar{a}_1 \\ \max \{0, p^{\max} - \omega_a (a_j - \bar{a}_1)\} & \text{if } a_j > \bar{a}_1, \end{cases} \quad (26)$$

Government: Old Age Pension (2)

The amount of pension benefit claimable, pen_j , is the minimum of (25) and (26). That is,

$$pen_j = \begin{cases} \min \{ \mathcal{P}^a(a_j), \mathcal{P}^y(y_{j,\lambda}) \} & \text{if } j \geq J_P \text{ and } \lambda = 0 \\ \frac{2}{3} \min \{ \mathcal{P}^a(a_j), \mathcal{P}^y(y_{j,\lambda}) \} & \text{if } j \geq J_P \text{ and } \lambda = 1, 2 \\ 0 & \text{otherwise} \end{cases} \quad (27)$$

Government: Budget

Government at time t collects taxes (T_t^C, T_t^K, T_t^I) and issue bond $(B_{t+1} - B_t)$ to meet its debt obligation $(r_t B_t)$ and its commitment to three spending programs:

- ▶ General government purchase, G_t ;
- ▶ Family transfers (FTB + CCS), Tr_t ;
- ▶ Old age pension, P_t .

The fiscal budget balance equation is therefore

$$(B_{t+1} - B_t) + T_t^C + T_t^K + T_t^I = G_t + Tr_t + P_t + r_t B_t. \quad (28)$$

Competitive Equilibrium: Measure of Households

Let $\phi_t(z)$ and $\Phi_t(z)$ denote the population growth- and mortality-unadjusted population density and cumulative distributions, respectively, and Ω_t denotes the vector of parameters at time t .

Initial distribution of newborns:

$$\begin{aligned}\int_{\Lambda \times A \times H \times \Theta \times S^2} d\Phi_t(\lambda, a, h, \theta, \eta_m, \eta_f) &= \int_{\Lambda \times \Theta \times S^2} d\Phi_t(\lambda, 0, 0, \theta, \eta_m, \eta_f) = 1, \text{ and} \\ \phi_t(\lambda, 0, 0, \theta, \eta_m, \eta_f) &= \pi(\lambda) \times \pi(\theta) \times \pi(\eta_m) \times \pi(\eta_f).\end{aligned}$$

The population density $\phi_t(z)$ evolves according to:

$$\begin{aligned}\phi_{t+1}(z^+) &= \int_{\Lambda \times A \times H \times \Theta \times S^2} \mathbf{1}_{\{a^+ = a^+(z, \Omega_t), h^+ = h^+(z, \Omega_t)\}} \times \pi(\lambda^+ | \lambda) \\ &\quad \times \pi(\eta_m^+ | \eta_m) \times \pi(\eta_f^+ | \eta_f) d\Phi_t(z) \quad (29)\end{aligned}$$

Competitive Equilibrium: **Aggregation (Households)**

Given the optimal decisions $\{c(z, \Omega_t), \ell(z, \Omega_t), a(z, \Omega_t)\}_{j=1}^J$, the share of alive households ($\mu_{j,t}$) and the distribution of households $\phi_t(z)$ at time t , we arrive at:

$$C_t = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} c(z, \Omega_t) \mu_{j,t} d\Phi_t(z) \quad (30)$$

$$A_t = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} a(z, \Omega_t) \mu_{j,t} d\Phi_t(z) \quad (31)$$

$$LFP_t = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} \mathbf{1}_{\{\ell(z, \Omega_t) \neq 0\}} \mu_{j,t} d\Phi_t(z). \quad (32)$$

$$LM_t = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} h_{j,\lambda}^m e^{\theta + \eta_m} \mu_{j,t} d\Phi_t(z) \quad (33)$$

$$LF_t = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} \mathbf{1}_{\{\ell(z, \Omega_t) \neq 0\}} h_{j,\lambda,\ell}^f e^{\theta + \eta_f} \mu_{j,t} d\Phi_t(z). \quad (34)$$

Competitive Equilibrium: **Aggregation (Government)**

Given the optimal decisions $\{c(z, \Omega_t), \ell(z, \Omega_t), a(z, \Omega_t)\}_{j=1}^J$, government policy parameters, the share of alive households $(\mu_{j,t})$ and the distribution of households $\phi_t(z)$ at time t , we arrive at:

$$T_t^C = \tau_t^C C_t \quad (35)$$

$$T_t^K = \tau_t^K (Y_t - w_t A_t L_t) \quad (36)$$

$$T_t^I = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} tax_j \mu_{j,t} d\Phi_t(z). \quad (37)$$

$$Tr_t = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} (ftba_j + ftbb_j + ccs_j) \mu_{j,t} d\Phi_t(z) \quad (38)$$

$$\mathcal{P}_t = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} pen_j \mu_{j,t} d\Phi_t(z). \quad (39)$$

Competitive Equilibrium: Definition (1)

Given the household, firm and government policy parameters, the demographic structure, the world interest rate, a steady state equilibrium is such that:

1. The collection of individual household decisions $\{c_j, \ell_j, a_{j+1}\}_{j=1}^J$ solve the household problem (7) and (13);
2. The firm chooses labor and capital inputs to solve the profit maximization problem (18);
3. The government budget constraint (28) is satisfied;
4. The markets for capital and labour clear:

$$K_t = A_t + B_t + B_{F,t} \quad (40)$$

$$L_t = LM_t + LF_t \quad (41)$$

Competitive Equilibrium: Definition (2)

5. Goods market clears:

$$Y_t = C_t + I_t + G_t + NX_t \quad (42)$$

$$NX_t = (1 + n)B_{F,t+1} - (1 + r)B_{F,t}$$

$$B_{F,t} = A_t - K_t - B_t$$

Where

- ▶ $I_t = (1 + n)K_{t+1} - (1 - \delta)K_t$ is investment
- ▶ NX_t is the trade balance, and
- ▶ $B_{F,t}$ is the foreign capital required to clear the capital market.

Competitive Equilibrium: Definition (3)

6. The total lump-sum bequest transfer, BQ_t , is the total assets left by all deceased households at time t :

$$BQ_t = \sum_{j=1}^J \int_{\Lambda \times A \times H \times \Theta \times S^2} (1 - \psi_{j,\lambda})(1 + r_t)a(z, \Omega_t) d\Phi_t(z). \quad (43)$$

Bequest to each surviving household aged j at time t is

$$beq_{j,t} = \left[\frac{b_{j,t}}{\sum_{j=1}^J b_{j,t} m_{j,t}} \right] BQ_t \quad (44)$$

Assuming bequest is uniform among alive working-age agents, then $b_{j,t} = \frac{1}{JR-1}$ if $j < JR$ and $b_{j,t} = 0$ otherwise. Thus,

$$beq_{j,t} = \frac{BQ_t}{\sum_{j=1}^{JR-1} m_{j,t}} \quad (45)$$

Summary: Externally Calibrated Parameters (1)

Parameter	Value	Target (2012-2018)
<i>Demographics</i>		
Lifespan	$J = 80$	Age 21-100
Retirement	$J_R = 45$	Age 65
Population growth	$n = 1.6\%$	Average (ABS)
Survival probabilities	ψ_m, ψ_f	Average (Aus. Life Tables, ABS)
Measure of newborns by type	$\{\pi(\lambda_0), \pi(\lambda_1), \pi(\lambda_2)\} = \{0.70, 0.14, 0.16\}$	HILDA 2010-2018
<i>Technology</i>		
Labour augmenting tech. growth	$g = 1.3\%$	Average per capita growth rate (World Bank)
Output share of capital	$\alpha = 0.4$	Output share of capital for Australia
Real interest rate	$r = 4\%$	Average (World Bank)
<i>Households</i>		
Relative risk aversion	$\sigma = \frac{1}{\gamma} = 3$	standard values 2.5-3.5
Work hours	$n_{m,\lambda}, n_{f,\lambda}$	Age-profiles of avg. labour hours (HILDA)
Male human capital profile	h_λ^m	Age-profile of hourly wages for married men

Summary: Externally Calibrated Parameters (2)

Parameter	Value	Target
<i>Permanent shocks</i>		
Value	$\{\theta_L, \theta_H\}$ $= \{0.745, 1.342\}$	College-HS wage premium of 1.8 (HILDA, 2012-2018)
Measure of $\{\theta_L, \theta_H\}$ type households	$\{\pi(\theta_L), \pi(\theta_H)\}$ $= \{0.7, 0.3\}$	College to high school ratio (2018, ABS)
<i>Fiscal Policy</i>		
Consumption tax	$\tau_c = 8\%$	$\tau_c \frac{C_0}{Y_0} = 4.5\%$; $\frac{C_0}{Y_0} = 56.3\%$
Company profit tax	$\tau^k = 10.625\%$	$\tau^k \left(\frac{Y - WL}{Y} \right) = 4.25\%$; $\frac{WL}{Y} = \alpha$
Gov't debt-to-GDP	$\frac{B}{Y} = 20\%$	Average (CEIC data, 2012-2018)
Gov't general purchase	$\frac{G}{Y} = 14\%$	Net of FTB, CCS and Age Pension (WDI and AIHW)
FTB, CCS and pension parameters		HILDA Tax-Benefit model

Summary: Internally Calibrated Parameters (1)

Parameter	Value	Target
<i>Households</i>		
Discount factor	$\beta = 0.99$	Saving ratio 5% – 8% (ABS, 2013-2018)
Taste for consumption	$\nu = 0.365$	LFP rate for mothers = 65-70%
Time cost of non-mother's FT work	$\chi = 0.14$	Mother's full time rate = 24%
Extra time cost of mother's full time work	$\{\chi_{c,jc}=[0,6], \chi_{c,jc}=[7,12]\}$ $\{0.025, 0.005\}$	Age-profile of full time share
Female human capital accumulation	$(\xi_{1,\lambda,\ell}; \xi_{2,\lambda,\ell})$	Age-profile of hourly wages of male counterpart (if $\ell > 0$ every period)
Female human capital depreciation	$\delta_h = 0.074$	Peak married male-female wage gap 30% (HILDA)
<i>Transitory shocks, ϵ</i>		
Persistence	$\rho = 0.98$	Literature
Variance of shocks	$\sigma_\epsilon^2 = 0.0145$	$GINI_{j=1,m} = 0.35$
<i>Fiscal policy</i>		
Progressive income tax	$\lambda = 0.7237, \tau = 0.2$	Tran and Zakariyya (2021)
Maximum pension	$pen^{max} = 30\% \times Y_m$	Pension/GDP = 3.2% (ABS, 2012-2018)

Calibration: Demographics (1)

1. Since child-related transfers are concentrated during child-bearing and raising age, we set one model period to correspond to 1 year of life to better capture behavioural responses;
2. Time-invariant n , ψ_m and ψ_m induce an unchanging population structure in every period t (see [share of survivors](#)).

Calibration: Demographics (2)

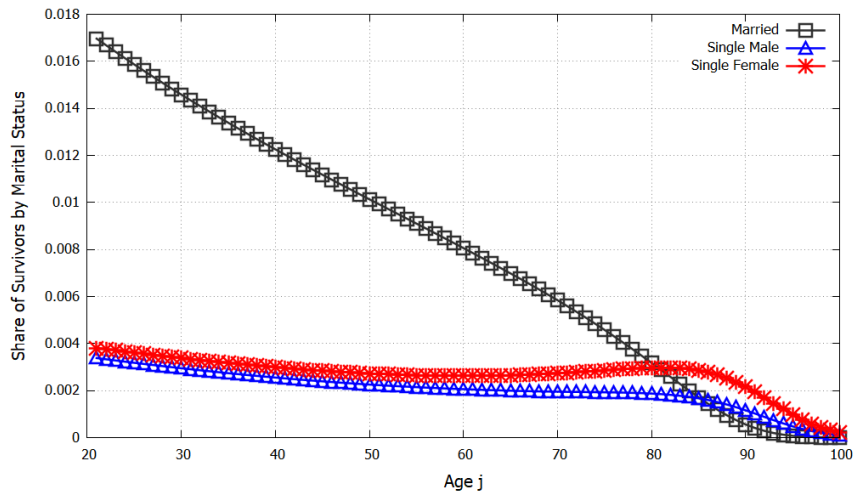


Figure: Share of survivors over life cycle

Calibration: Endowment (Deterministic) (1)

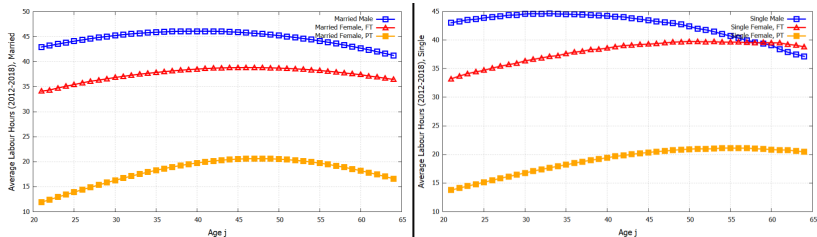


Figure: Age profiles of average labor hours

Calibration: Endowment (Deterministic) (2)

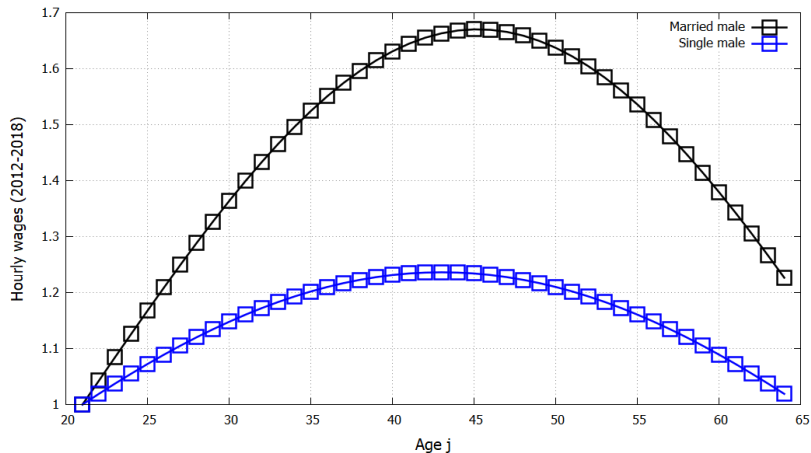


Figure: Age profiles of male hourly wages

Calibration: Endowment (Deterministic, Female)

We calibrate the female human capital accumulation rate that their human capital profiles match those of their male counterparts:

- ▶ if the wife works without time off over life cycle, and
- ▶ assuming ex-ante assortative matching of couples in terms of skills.

Our estimates are:

- ▶ Married mothers working full time:
 $(\xi_{1,\lambda=0,\ell=1}, \xi_{2,\lambda=0,\ell=1}) = (0.0450, -0.00175)$
- ▶ Married mothers working part time:
 $(\xi_{1,\lambda=0,\ell=2}, \xi_{2,\lambda=0,\ell=2}) = (0.0350, -0.00135)$
- ▶ Single mothers working full time:
 $(\xi_{1,\lambda=2,\ell=1}, \xi_{2,\lambda=2,\ell=1}) = (0.0206, -0.00088)$
- ▶ Single mothers working part time:
 $(\xi_{1,\lambda=2,\ell=2}, \xi_{2,\lambda=2,\ell=2}) = (0.0179, -0.00060)$

Calibration: Endowment (Deterministic, Children)

Children:

1. Assign *first and second child births* to
 - ▶ type θ_H households aged $\{28, 31\}$;
 - ▶ type θ_L households aged $\{21, 24\}$ (See **LSAC** and **AIHW** reports)
2. Child care service fee is \$12.5/*hour* or 48% of age 21 married male hourly wage.
3. Based on approximates from child care service and school fees, parents pay
 - ▶ 100% of the fee for child aged 0-2;
 - ▶ 80% for child aged 3-5;
 - ▶ 60% for child aged 6-11;
 - ▶ 40% for child aged 12-17.

Calibration: Endowment (Stochastic income process)

We calibrate the AR1 stochastic process, η^i , for $i \in \{m, f\}$ as follows:

- Discretized into 5 grid points:

$$\eta^i = \{0.29813, 0.54601, 1, 1.83146, 3.35424\}$$

- Transition probabilities obtained via Rouwenhorst method:

$$\begin{bmatrix} 0.9606 & 0.0388 & 0.0006 & 0 & 0 \\ 0.0097 & 0.9609 & 0.0291 & 0.0003 & 0 \\ 0.0001 & 0.0194 & 0.9610 & 0.0194 & 0.0001 \\ 0 & 0.0003 & 0.0291 & 0.9609 & 0.0097 \\ 0 & 0 & 0.0006 & 0.0388 & 0.9606 \end{bmatrix}$$

Calibration: Endowment (Stochastic income process)

- ▶ Persistence: $\rho = 0.98$;
- ▶ Variance of the innovation to shocks: $\sigma_{\epsilon}^2 = 0.0145$ to achieve a Gini coefficient of age 21 male wage distribution of 0.35;
- ▶ The set-up results in $\text{GINI} = 0.3766$ for wage distribution of work-age male population (not targeted).

Lorenz Curve (male wages at aged 21 and 22)

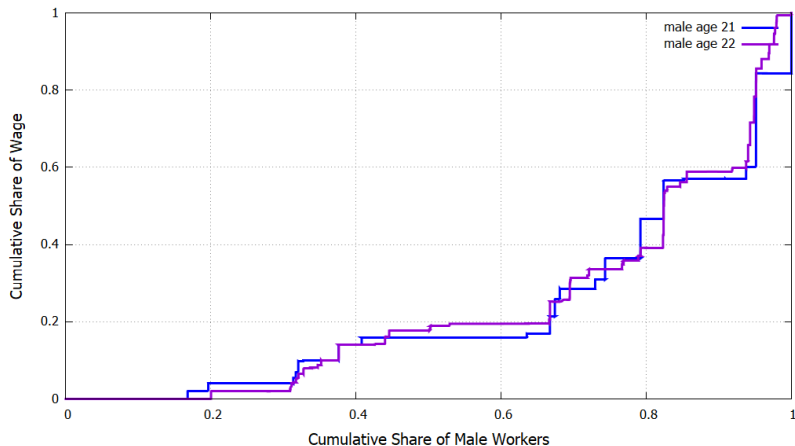


Figure: Lorenz curves of the distributions of married male wages at age 21 and 22

Lorenz Curve (male wages at working age)

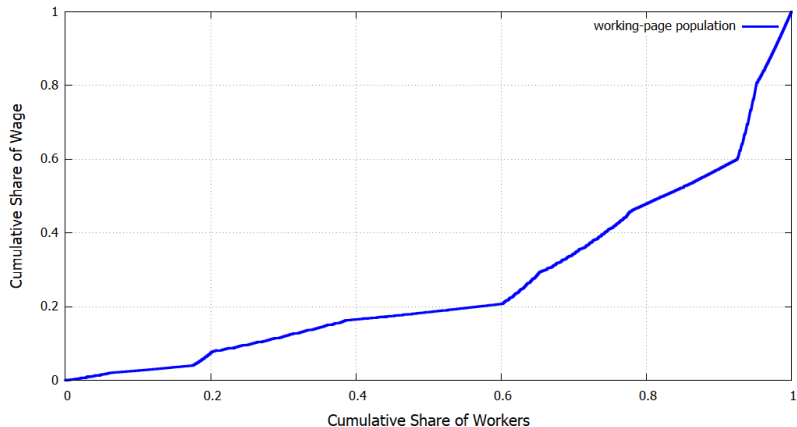


Figure: Lorenz curve of the wage distribution of the working-age male population (accounting for human capital, education and transitory shocks over the life cycle)

Benchmark: Life cycle profiles

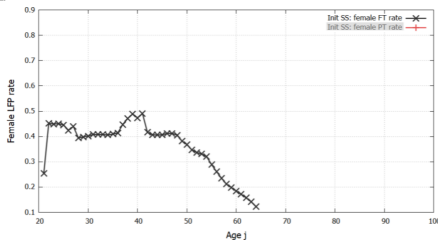
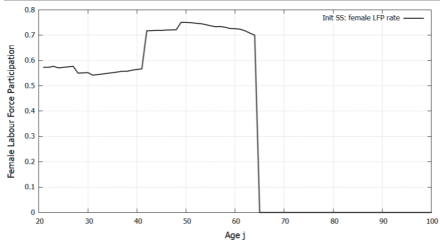
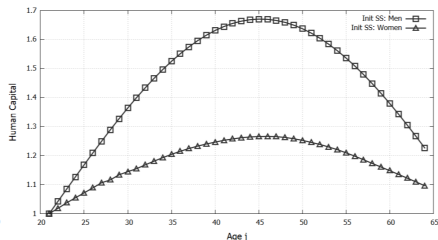
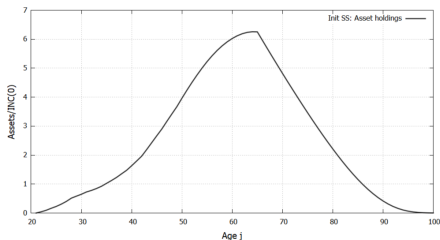


Figure: **Top left:** Assets-to-income ratio. **Top right:** Married male and female human capital. **Bottom left:** Female labor force participation rate. **Bottom right:** Female full time share of employment.

Results: Experiment set 2

	+50% FTB	+50% CCS	-50% FTB	-50% CCS
	Change	Change	Change	Change
Income (Y)	-1.11%	1.33%	1.09%	0.69%
Consumption (C)	-1.81%	2.05%	2.30%	-0.90%
Savings (S)	-3.75%	1.13%	4.22%	-2.11%
Female LFP	-3.48 p.p.	7.61 p.p.	6.83 p.p.	-4.31 p.p.
Female FT rate	-1.85 p.p.	4.29 p.p.	5.51 p.p.	-1.95 p.p.
Income tax rate	1.48%	0.61%	-1.15%	-1.08%
FTB expense	49.90%	-14.24%	-49.34%	4.88%
CCS expense	-12.12%	75.38%	37.40%	-60.00%
Pension	-0.31%	1.31%	0.11%	1.31%
HEV (newborn)	0.83%	0.1%	-1.00%	-0.3467%

Table: Changes relative to benchmark values

Computing the Steady State: Algorithm (1)

We solve the benchmark model (*small open economy*) for its initial balanced-growth path steady state equilibrium.

1. Parameterize the model and discretize assets on $[a_{min}, a_{max}]$ such that:
 - ▶ Number of grid points, $N_A = 70$;
 - ▶ $a_{min} = 0$ (No-borrowing constraint);
 - ▶ The grid is fairly dense near a_{min} so households are not restricted by an all-or-nothing decision;
 - ▶ a_{max} is sufficiently large so that (i) *households are not bound by a_{max}* , and (ii) *there is enough room for upward movement induced by new policy regimes*.

and for human capital grids on $[h_{min}^f, h_{max}^f]$:

- ▶ Number of grid points, $N_H = 25$;
- ▶ $h_{min}^f = h_{j=21}^m = 1$;
- ▶ $h_{max}^f = h_{j=50}^m = 1.546$;

Computing the Steady State: Algorithm (2)

2. Guess K_0 and L_0 , endogenous government policy variables, and w_m , taking $r = r^w$ as given;
3. Solve the firm's problem for (w_m, w_f) ;
4. Given the factor prices (w_m, w_f, r) and the initial steady state vector of parameters (Ω_0) , solve the household problem for decision rules on $\{a^+, c, l^f\}$ by backward induction (from $j = J$ to $j = 1$) using *value function iteration*;

Computing the Steady State: Algorithm (3)

5. Starting from a known distribution of newborns, compute the measure of households across states by forward induction, using
 - ▶ the computed decision rules,
 - ▶ ψ ,
 - ▶ η and its [Markov transition probabilities](#), and
 - ▶ the law of motion of female human capital (1).
6. Accounting for the share of alive agents, sum across states for aggregate variables: A , C , L , T and Tr . Update L , K , I and Y (convex update). Solve for endogenous government policy variables.

Computing the Steady State: Algorithm (4)

7. Given the updated variables, compute the goods market convergence criterion for a small open economy:

$$Y = C + I + G + NX$$

- ▶ $B_F = A - K - B$;
 - ▶ $NX = (1 + r)B_{F,t} - (1 + n)(1 + g)B_{F,t+1}$;
 - ▶ $NX < 0$ implies a capital account surplus (increase in foreign indebtedness).
8. Return to step 3 until the convergence criterion is satisfied.

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