On the Joint Optimal Design of Taxes and Child Benefits*

(Updated Regularly: Latest version)

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Abstract

While means-tested child benefit and progressive tax systems aim to support low-income families, my empirical analysis based on Australian household survey data (HILDA) reveals that their interaction creates high, non-linear effective marginal tax rates (EMTRs) for low-income mothers. Building on these findings, this paper examines the joint optimal design of tax and child benefit systems using a dynamic general equilibrium model of overlapping generations, calibrated to Australia (2012-2018). The model incorporates rich household heterogeneity in family structure, age and number of children, female human capital, and uninsurable earnings risks. I find that an optimal tax reform, focused solely on income tax, requires reducing tax progressivity, which encourages labor supply primarily among highly educated women. However, it also raises tax liabilities for low-income parents, thereby undermining the objectives of child benefit programs. A joint optimal system combines reduced tax progressivity with a universal lump-sum child benefit set at approximately 30% of median income. While the more proportional tax scheme benefits high-education parents, the transfer—free from means-testing and double the average baseline payment—compensates low-income parent households for the increased tax liabilities. This approach significantly improves parental and overall welfare but shifts more of the tax burden onto non-parent households, causing them to experience notable losses. A moderately scaled-back benefit, though not optimal, yields welfare gains for parents at a lower cost to non-parents. These findings highlight the importance of coordinating tax and child benefit policies to effectively support vulnerable parents while cautioning against equity losses for non-parents when policies are optimized solely for overall welfare.

JEL: E62, H24, H31

Keywords: Child Benefits; Means-Testing; Progressive Taxes; Female Labor Supply; Optimal Taxes and Transfers; Welfare; Welfare Decomposition; Dynamic General Equilibrium.

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1 Introduction

Means-tested child benefits—including direct cash transfers, child care subsidies, child tax credits, and other in-kind support—along with a progressive tax system, serve as important government insurance mechanisms for low-income families with dependent children. Means testing, often based on family income, helps ensure that limited public funds are directed to those most in need while promoting fiscal sustainability. However, my empirical documentation based on Australian household survey data (HILDA) reveals that the overlap between child benefit phase-outs and marginal tax rates (MTRs) creates persistently high, non-linear, and non-monotonic effective marginal tax rate (EMTR) schedules for mothers, potentially leading to significant work disincentives for women across earnings levels. Furthermore, the EMTR schedules differ across demographic and socioeconomic groups, resulting in heterogeneous effects on households.

This insurance-incentive trade-off is a central focus at the intersection of labor economics, macroeconomics, and public finance research. Prior studies have approached this issue from various perspectives. The optimal tax literature (e.g., Ramsey 1927, Mirrlees 1971, Atkinson and Stiglitz 1976, Diamond 1998, Saez 2002) addresses the challenge of raising public revenue using direct and indirect taxes while ensuring minimal inefficiency, mainly focusing on individual heterogeneity in abilities (or skills). More recent studies consider demographic differences, such as gender, marital, and parental status, to assess the impacts of tax reforms (e.g., Guner et al. 2012a, Guner et al. 2012b, Bick 2016, Bick and Fuchs-Schündeln 2018) or transfer reforms (e.g., Baker et al. 2008, Kaygusuz 2015, Nishiyama 2019, Borella et al. 2020) on female labor supply and household welfare. Nonetheless, they do not consider individual or joint system optimality. In this paper, I study the joint optimal design of tax and child benefit systems, focusing on their aggregate and distributional implications and accounting for household heterogeneity in education and family structure (gender, marital, and parental status). My contributions are twofold.

First, the presence of children imposes additional constraints on household consumption and leisure. Targeted child benefits may therefore address unique challenges faced by parents that tax reforms cannot. That is, because standalone tax reforms focus solely on redistribution along the income dimension, they may not accommodate the objectives of the child benefit system. Thus, by (i) considering optimality in both tax and child benefit systems and (ii) endogenizing their interaction, a key contribution of this paper is addressing how optimizing one policy impacts the objectives of the other and demonstrating how these systems can be jointly designed to achieve greater welfare improvements.

Second, joint optimal design studies have largely focused on the U.S. policy context and often combine a broad range of transfers (e.g., Guner et al. 2023 and Ferriere et al. 2023).[1] Since policy alternatives outside of the U.S., particularly child-related transfers, are less known, I also contribute to the literature by considering policy design in Australia, where lump-sum child benefits play a prominent role, serving over one million families with average benefit rates up to 40% of total income for low-income households. Specifically, Australia implements two primary means-tested child benefits based on family income—the Family Tax Benefit (FTB), a direct lump-sum transfer for families with dependent children, and the Child Care Subsidy (CCS), a subsidy for formal child care costs for working parents—within a moderately progressive tax regime.[2]

I begin by formulating a representative-agent, deterministic general equilibrium model to illustrate how means-tested benefits interact with other policies to affect aggregate outcomes such as female labor supply, output, and welfare. Informed by this analytical framework and empirical evidence, I then construct a dynamic general equilibrium model with overlapping generations of households making joint decisions on consumption, savings, and female labor supply (participation and hours). The model is rich in household heterogeneity,

[1] Guner et al. 2023 explore alternative systems to means-tested transfers for working-age households and progressive taxation, such as combining universal transfers with a proportional tax regime. Ferriere et al. 2023 optimize tax and transfer systems, represented by different parametric functions.

[2] The Australian system also incorporates fine-tuning instruments, such as multi-tier family income tests and demographic criteria (e.g., marital status, number and age of dependent children), to determine eligibility and benefit levels. Further details are provided in Appendix Section H.2. See also Tin and Tran 2024 for detailed discussion of the child-related transfer system in Australia.

featuring family structure, age and number of children, education, asset holdings, female human capital, and uninsurable idiosyncratic earnings risks. I define an optimal system as one that maximizes ex-ante welfare (under the veil of ignorance). Welfare changes are measured using Consumption Equivalent Variation (CEV) and grouped into consumption- and leisure-driven components. Adapting the approach of Bhandari et al. (2021), these are further decomposed into three effects: (i) *allocative efficiency*, reflecting changes in average consumption/leisure levels over the life cycle; (ii) *distributional (equity)*, capturing changes in ex-ante consumption/leisure shares relative to the population average; and (iii) *insurance*, representing changes in ex-post risks to consumption/leisure. This decomposition allows for a detailed analysis of the underlying factors driving household welfare.

I discipline the model using 2012-2018 macroeconomic aggregates and household-level microdata from Australia. Taking the calibrated model economy as a baseline, the paper examines a series of counterfactual reforms, each varying key policy parameters and adjusting income taxes to balance the government budget. The main findings are summarized as follows.[3]

In the first experiment, where the benchmark means-tested child benefit system is maintained, I find that an optimal tax regime requires lowering progressivity ($\tau^* = 0.1$) compared to the baseline level ($\tau = 0.2$).[4] By reducing the marginal tax rates for higher earnings, this income-focused reform promotes female labor supply, primarily among those with high education. Simultaneously, it increases work hours and consumption among young low-education single mothers, improving their consumption allocative efficiency (CEV_{CE}).[5] The overall welfare increases by 1.38%. However, by shifting tax liabilities to low-income households, the reform comes at the expense of other households, including low-education married parents. These results demonstrate the close connection between tax and child benefit systems: optimizing the tax system, even if beneficial on average, risks undermining the objectives of child benefit policies by disadvantaging some vulnerable parent groups.

The second experiment keeps tax progressivity at the status quo level ($\tau = 0.2$) while optimizing the child benefit system. The results indicate that an optimal system involves removing means-testing from the lump-sum child benefit (FTB). Under this reform, a universal lump-sum child benefit at 25% of median income in 2018 (approximately AUD $15,000$) is optimal. Since children impose greater constraints on low-income parents' consumption and leisure, this group, particularly single mothers, experiences substantial welfare gains from this reform. Overall welfare improves by 7.39%, a figure several times larger than the gains achieved under the optimal tax regime. However, while it multiplies the gains for parents, the universal benefit also amplifies welfare losses for non-parent households due to the increased tax burden, rendering the optimal child benefit system inequitable.

The third experiment assesses whether a joint design of taxes and child benefits can yield superior aggregate and distributional outcomes. The findings reveal that a joint optimal system integrate features from the individual optimal reforms, incorporating reduced tax progressivity ($\tau^* = 0.1$) and a universal lump-sum child benefit at 30% of median income in 2018 (around AUD $18,000$). Under this regime, the lower tax progressivity favors high-education households at the expense of low-education parents, who bear higher tax liabilities in lower income brackets. The joint optimal system compensates by increasing transfers to parents by 5 percentage points (*pp*) relative to the standalone optimal child benefits. The resulting improvements in overall and parental welfare surpass those of the individual reforms, thus underscoring the importance of joint system design to meet policy objectives. However, such a system places even larger welfare losses on non-parent households, as they face higher tax liabilities in lower income brackets from the tax reform and a greater overall tax burden to fund universal lump-sum child benefits from the child benefit reform.

[3] All other behavioral, technology, and policy parameters are held constant at their initial steady state values.

[4] The parametric tax function and definition of tax progressivity follow Feldstein (1969); Benabou (2000), and Heathcote et al. (2017), with detailed descriptions in Subsection 4.5.1.

[5] In this framework, low-education single mothers, who lack family insurance (spousal earnings) and face credit constraints (no-borrowing constraint assumption), rely heavily on self-insurance via labor supply and savings. Consequently, reforms that alleviate these households' self-insurance constraints can generate substantial payoffs by allowing them to work longer hours, earn and consume more, especially during their younger years when they have not yet accumulated sufficient wealth. In this case, this improvement outweighs the losses incurred by other households, thus leading to an overall welfare gain.

The counterfactual analyses offer several key insights. First, in this model, where parents constitute the majority and face multiple constraints from child-related costs, optimizing for overall welfare tends to favor policies that benefit parents, even when they disadvantage the minority non-parent households.[6] In this environment, an income-focused optimal tax policy, which excludes the child-related dimensions of households, results in smaller overall welfare improvements and undermines the objectives of child benefit programs. To meet both tax and child benefit objectives, a joint optimal system delivers generous transfers to low-income parents, offsetting the increased tax liabilities caused by the reduced tax progressivity that primarily benefits high-education parents.

Second, consistent with Tin and Tran (2024), I demonstrate that balancing child benefits with broader fiscal pressures is crucial. Failure to do so could harm not only society at large but also the intended beneficiaries. In other words, excessive transfers fail to outweigh the negative effects of the associated tax burden, to the extent that they deteriorate welfare for parents and non-parents alike. In contrast, a less generous scheme, while offering smaller gains, imposes lower costs on non-parents and may be more viable depending on the policy context.

Lastly, the analysis highlights the vulnerability of low-education parents, especially single mothers, to policy reforms. Structural constraints—such as limited family insurance, child-related costs, and lack of access to credit—make this group particularly susceptible to significant welfare changes as policy environments evolve. In many cases, the welfare outcomes of low-education single mothers are pivotal in shaping overall post-reform welfare, making it essential to explicitly consider their well-being in policy design.

Related literature. This research draws upon the foundation established by seminal works of Mirrlees (1971), Diamond (1998), and Saez (2001), which focus on the optimal design of non-linear income tax systems, balancing efficiency and equity. Mirrlees (1971) shows that the marginal tax rate (MTR) schedule should exhibit an inverted U-shape. Diamond (1998) and Saez (2001) argue for a U-shaped MTR schedule as an efficient means to redistribute income. More recent work by Ferriere et al. (2023), for example, extends this dialogue, advocating for a U-shaped effective marginal tax rate (EMTR) schedule in combined tax and transfer systems and for a higher joint system progressivity in the U.S.

This paper also relates to a strand of literature on female labor supply and fiscal reforms (e.g., see Baker et al. 2008; Guner et al. 2012a; Guner et al. 2012b; Bick 2016; Bick and Fuchs-Schündeln 2018; Borella et al. 2020; Borella et al. 2022; Borella et al. 2023; and Tin and Tran 2024, among others). Guner et al. (2012a) and Guner et al. (2012b), for instance, examine the disincentive effects of joint-taxation in the U.S. on female labor supply. Recent developments also delve into marriage-related social security (e.g., Kaygusuz 2015; Nishiyama 2019; Borella et al. 2020) and child benefits (e.g., Guner et al. 2020 for the U.S. and Tin and Tran (2024) for Australia). The decision to model family composition in this research is also motivated by Borella et al. (2023) and their earlier works (Borella et al. (2018, 2022)), which emphasize the importance of considering family structure in quantitative evaluations of fiscal reforms. For the case of Australia, Tin and Tran (2024) echo this sentiment by showing that modeling single mothers and their constraints can significantly influence policy recommendations.

Keane (2022) highlights that the frontier of optimal tax research involves dynamic stochastic general equilibrium models with overlapping generations of heterogeneous workers, incorporating endogenous wages, participation decisions, educational differences, and family structure. He notes that while many studies address subsets of these issues, none have tackled all of them. This paper fully integrate these elements into its framework, and therefore contributes to the quantitative literature.[7] At the same time, it fills a gap in the structural

[6]Child-related costs include explicit costs, such as time and monetary commitments, and implicit costs from reduced per capita consumption in larger households. Consequently, all else being equal, parent households typically experience lower per capita consumption and higher marginal utilities of consumption compared to their childless counterparts.

[7]Conesa et al. 2009 treat hours as a choice variable but not labor force participation. Blundell et al. 2016 account for the interaction between tax and child benefit systems within a dynamic life cycle model of female labor supply, human capital formation, and savings to identify optimal policy mixes, although they abstract from family structure. Guner et al. (2020) include all key elements to study tax and welfare systems in the U.S., though they do not focus on optimality. This paper aligns more closely with recent works, such as Guner et al. 2023. However, their study addresses a broader set of means-tested transfers

modeling of taxes and child benefits in Australia. In particular, the current model synthesizes insights from Guner et al. (2020), Borella et al. (2023), Ferriere et al. (2023), and Tin and Tran (2024). Methodologically, it extends the framework of Tin and Tran (2024) in multiple directions, by endogenizing female participation and work-hour decisions while retaining rich household heterogeneity, exploring the joint design of tax and child benefit systems, proposing an optimal system, and decomposing post-reform welfare changes to identify their driving forces.

Additionally, the study contributes to a broader literature on means-tested social insurance (e.g., Feldstein 1987; Hubbard et al. 1995; Neumark and Powers 2000; Tran and Woodland 2014; Braun et al. 2017), which generally shows that means-testing can distort incentives to work and save, but can also be useful for balancing the trade-off between insurance and incentive effects, thus improving welfare. The findings of this study, in a context where means-testing rules are complex and benefits are confined to a short period in a parent's life, suggest that universal lump-sum transfers can enhance welfare but come with equity trade-offs.

Lastly, by investigating the implications of the proposed optimal tax and child benefit systems on female labor supply in Australia, this paper complements the collection of empirical research on labor supply (e.g., Doiron and Kalb 2005; Gong and Breunig 2017; Hérault and Kalb 2022; Tran and Zakariyya 2022; Tin and Tran 2023), as well as adding to the growing body of quantitative studies on fiscal policies in Australia (e.g., Tran and Woodland 2014, Iskhakov and Keane 2021, Kudrna et al. 2022, and Tin and Tran 2024).

The paper hereinafter proceeds as follows. Section 2 presents stylized facts. Section 3 introduces a simple analytical model for intuition. Section 4 describes the quantitative model. Section 5 reports the internal and external calibration procedures, and the benchmark model's performance. Section 6 discusses the main results. Section 7 concludes. The Appendix provides supplementary results and statistics, detailed information on the child benefit programs, and the algorithm used to solve the model.

2 Progressive income taxes and means-tested child benefits in Australia

This section outlines the key institutional features of Australia's child benefit programs and presents selected empirical facts, including simulated effective marginal tax rate (EMTR) schedules, using data from the Household, Income and Labour Dynamics in Australia (HILDA) Survey, Restricted Release 20 (2001-2020). These serve as the empirical foundation for the subsequent analytical and quantitative analyses. Unless otherwise stated, all monetary values are expressed in 2018 Australian dollars (AUD).

In Australia, labor and capital income are taxed on an individual basis, while social security benefits are means-tested based on family income. Family assistance payments are a significant component of the benefit programs, constituting approximately 22% of total public transfers (or 2% of GDP) over the last two decades, second only to pensions, which account for 56%. Notwithstanding, because the Age Pension dominates pension expenditures, family payments become the primary transfer mechanism for working-age parents and play a pivotal role in redistribution.

Within the family assistance category, two means-tested child benefit programs—the Family Tax Benefit (FTB) and Child Care Subsidy (CCS)—are central, accounting for 70% of total family payments, according to the 2018-19 budget report.[8] The FTB consists of two parts: FTB Part A (FTB-A) and FTB Part B (FTB-B). Both provide direct lump-sum transfers to support low-income families with dependent children, with means-testing parameters, including payment amounts, income thresholds, and phase-out rates, vary based on factors such as the number and age of children and marital status. However, the FTB-A is paid per child and is means-tested on joint family income, while the FTB-B is paid per household to provide extra support to single parents and single-earner families, with eligibility determined by the primary earner's income and payments

for working-age households in the U.S. and mainly consider alternative policies for welfare improvements, while my focus is on assessing and jointly optimizing taxes and child benefits in Australia.

[8]This study excludes the Paid Parental Leave program, which represents a smaller share of family assistance expenditure.

adjusted based on the secondary earner's income. On the other hand, the CCS subsidizes formal child care costs for children up to 13 years of age. Like the FTB, the base subsidy rate is determined by means-testing family income, but the program's distinguishing feature is its activity test on the secondary earner's work hours to adjust the base subsidies.[9]

More importantly, the FTB and CCS are not mutually exclusive, and each program delivers benefits to approximately one million families, representing over 50% of families with children under 16 years old.[10] In addition to its extensive coverage, these programs provide substantial benefits, averaging between $8,000 and $10,000 per family. For households in the bottom quintile of the (pre-government) income distribution, in particular, the FTB payments alone can account for as much as 40% of gross income.[11] A detailed description of the two programs and related statistics is available in the Appendix and in earlier work by Tin and Tran (2024).

2.1 Joint effects on the level of tax-transfer progressivity

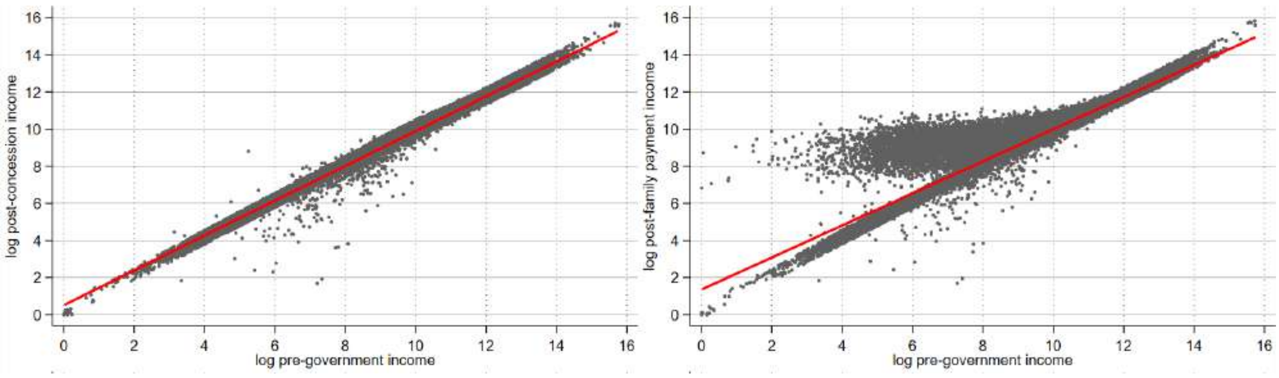

Figure 1: **Log post-concession income (left panel) and log post-family-payment income (right panel) by log pre-government income level.**
Notes: Pre-government income includes regular private market income and private transfers (excluding irregular flows such as severance payment and irregular private transfers). Post-concession income refers to private income after tax and concessions. Post-family-payment income is the sum of post-concession income and family payment (public transfers to family).

As evident in Figure 1, the comparison by log pre-government income of log post-concession income (left panel) with log post-family-payment income (right panel) reveals a flatter fitted line for the latter, driven by a large cluster of observations above the 45-degree line. That is, the average post-family-payment income for working parents in the lower income brackets is significantly higher than their post-tax-and-concessions income. This high level of progressivity marks the first defining characteristic of the combined tax and child benefit systems.[12]

2.2 Joint effects on effective marginal tax rate (EMTR)

The second defining characteristic of the joint systems is that their interplay significantly increases the EMTR for beneficiaries, beyond what a single program would be capable of. Furthermore, since means-testing is based

[9]In this paper, only labor supply is considered for the CCS activity test. In practice, households with secondary earners engaged in recognized activities—such as employment, training, or volunteering—for 48 hours or more per fortnight are eligible for the full base subsidy, which covers up to 85% of formal child care costs for low-income families.

[10]More precisely, as of June 2018, 1.4 million families were receiving FTB payments, 77% of whom received both FTB-A and FTB-B (AIHW report 2022). In the December quarter of 2018, the CCS covered 974,600 families (Child Care in Australia report 2018).

[11]Our estimates of average benefits are based on the HILDA survey data. For the FTB, the APH report on Social security and family assistance reports total expenses at around $17 billion in 2018. Given the 1.4 million recipients, this translates to a higher figure of $12,000 annual FTB payment per family.

[12]It is important to note, however, that the use of log transformation implies that the figure excludes transfers to those with zero or negative pre-government income.

on family income, this effect is especially pronounced for secondary earners, most of whom are women, whose family earnings fall within the benefit phase-out zones.

As illustrated by Figure 2—which shows a simulated EMTR schedule for a low-education young mother of two children, whose husband earns the median income (around $\$60,000$ in 2018)—her final EMTR (red line) hovers between 80% and 100% from the first dollar she earns, despite her earnings being within the zero-tax bracket.[13] To understand how the formation of her EMTR schedule, it is important to recognize that, even in the absence of taxes and transfers, her labor supply is naturally constrained by child care costs. While her marginal tax rate (MTR, black solid line) never exceeds 40%, the hourly formal child care fee causes her pre-transfer EMTR (green line) to rise to the point where she incurs a net loss for every dollar earned beyond the tax-free threshold.[14] The inclusion of means-tested child benefits introduces complex interactions that create nuanced marginal effects on the mother's labor supply. Specifically, at lower earnings level (below $\$50,000$), while the means-tested CCS (heavy-blue line) substantially reduces her EMTR, the FTB phase-outs undo much of this favorable work incentive effect by elevating the EMTR back to 100% or more (red line).[15]

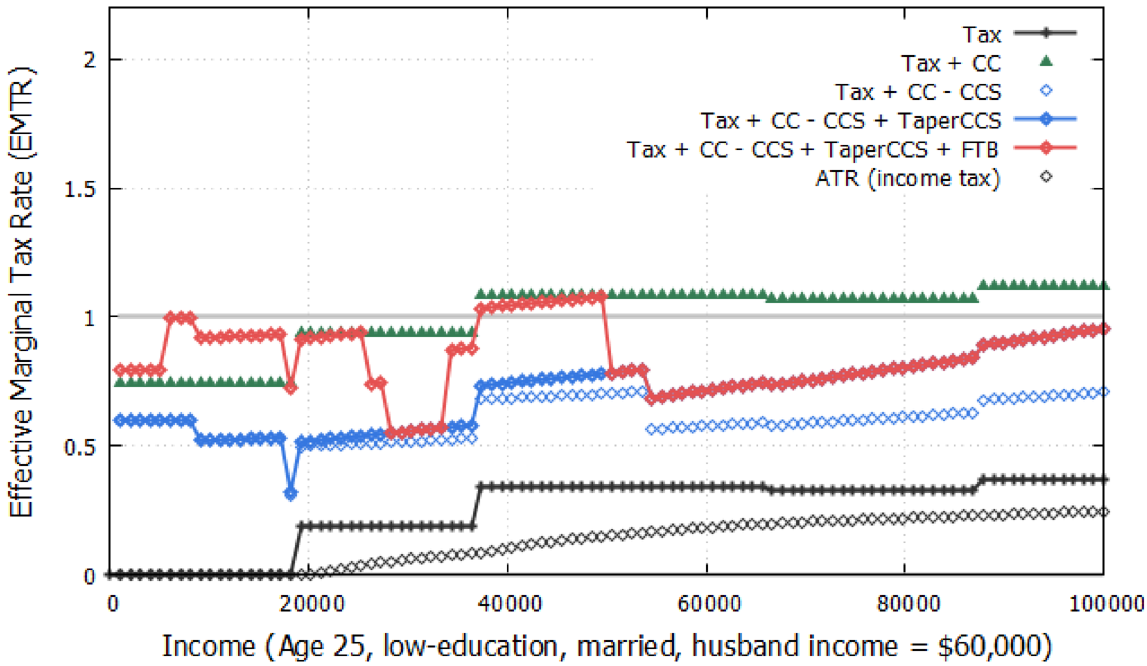

Figure 2: **Effective Marginal Tax Rate (EMTR) schedule of a representative low-education (at most high school) young married mother with two children, whose husband earns $\$60,000$ in 2018.**
Notes: These lines show the cumulative effects, stacked successively. The black dotted line is the average income tax rate (ATR). The black solid line is the marginal tax rate (MTR), including Low Income Tax Offset (LITO). The dotted green line is the EMTR when the marginal rate of the gross child care cost (CC) is added on top of the MTR. The light dotted blue line is the EMTR that incorporates the base subsidy rates of the CCS. The heavy solid blue line accounts for both the base subsidies and phase-out rates of the CCS. The solid red line is the total EMTR schedule when the FTB's phase-out rates are included.

The simulation also demonstrates that the tax and child benefit systems play a critical role in shaping the EMTR schedule faced by beneficiaries. In the absence of taxes, the FTB's phase-out acts as an implicit tax, raising the EMTR for a low-income mother. As her earnings grow, the combination of increasing MTRs and phase-out rates of subsidies (CCS) keeps her EMTR elevated, even after the FTB benefits are completely phased out. As seen in Figure 2, for earnings above $\$50,000$, the FTB phase-out rate is simply superseded by

[13] A formal expression of the EMTR is detailed in Equation (51) of the household problem in Subsection 4.7.

[14] In the simulation, hourly child care fees are fixed at $\$12.5$/hour. This accounts for a significant fraction of a low-skilled mother's hourly wage, which explains the sharp increase in her simulated pre-transfer EMTR schedule (green line). For high-skilled mothers, the impact of child care costs on their EMTR schedules is generally weaker.

[15] In fact, the CCS without means-testing (light-blue line) would have a substantially stronger EMTR reduction effect, particularly at higher income levels (weaker at lower incomes due to the work hour test). However, because the CCS is means-tested, its phase-out rate adds to the EMTR (raising the EMTR from light-blue to heavy-blue line), diluting the intended work incentive effect as the mother's earnings increase.

the CCS phase-out and MTRs. Therefore, the interaction between these programs results in a high, non-linear, and non-monotonic EMTR schedule for the recipients, with different elements dominating at various income levels. This constitutes the third characteristic of the joint systems.

The fourth characteristic is the heterogeneity in EMTR schedules across demographic and socioeconomic groups. As depicted in the left panel of Figure 3, a mother whose demographic traits are identical to her counterpart in Figure 1, except with a partner earning twice as much ($120,000), experiences a different EMTR schedule. Her husband's high earnings exceed the FTB income-test thresholds, thus eliminating the FTB phase-out effect (heavy-blue and red lines overlap). However, she still confronts high EMTRs due to the reduced CCS subsidy rate and the steeper CCS phase-out rate for higher-income families.

Conversely, in the right panel of Figure 3, a single mother with the same demographic traits (but without a partner's income) encounters an entirely different situation. Since her family income comes solely from her own earnings, the FTB phase-out does not begin until her income cross the first family-income test threshold of $52,706 (for maximum FTB payment). She also benefits from the full base CCS rate, which only begins to taper at earnings well above the median.[16] Her total EMTR generally hovers around 60%, which is smaller than that of her married counterpart. However, as her income rises, the combination of high MTRs and benefit phase-out rates gradually increases her EMTR. This highlights the different impacts that means-testing has based solely on marital status. As demonstrated, holding all other demographic and socioeconomic factors constant, a low-income single mother generally faces smaller distortions from means-testing compared to her married counterpart.

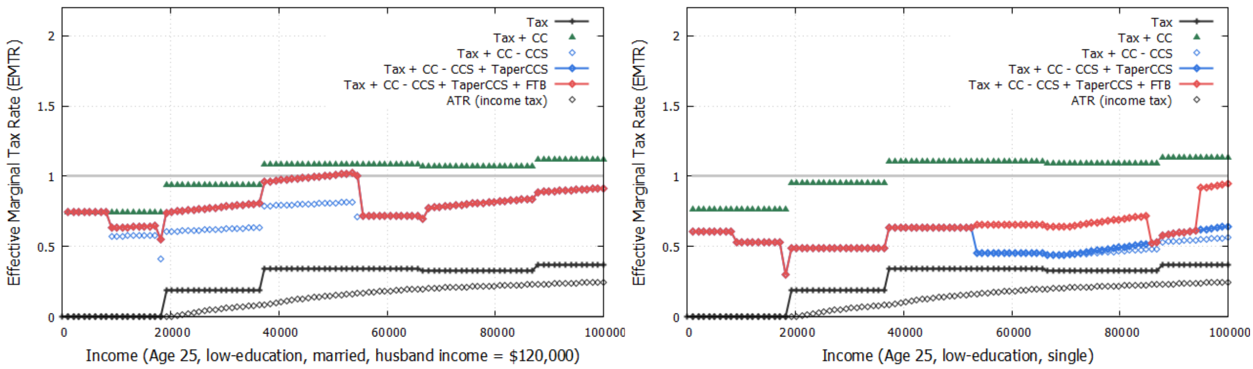

Figure 3: **Effective Marginal Tax Rate (EMTR) schedule of a representative low-education (at most high school degree) young married mother with two children**. **Left**: With husband earning $120,000. **Right**: Single mother.
Notes:
(*) These lines show the cumulative effects, stacked successively. The black dotted line is the average income tax rate (ATR). The black solid line is the marginal tax rate (MTR), including Low Income Tax Offset (LITO). The dotted green line is the EMTR when the marginal rate of the gross child care cost (CC) is added on top of the MTR. The light dotted blue line is the EMTR that incorporates the base subsidy rates of the CCS. The heavy solid blue line accounts for both the base subsidies and phase-out rates of the CCS. The solid red line is the total EMTR schedule when the FTB's phase-out rates are included;
(**) On the left panel, note how the red line (total EMTR) overlaps the blue line (EMTR without FTB). This suggests that the FTB phase-out rate has no effect on the EMTR.

The simulated case studies presented above underscore the role of taxes and means-tested child benefits in generating high progressivity. At the same time, they demonstrate the significant impact of the interaction between these systems on the EMTR schedules faced by parents, especially mothers.[17]

In summary, the interplay between progressive taxes and means-tested child benefits brings about four key effects: (i) strong redistribution; (ii) persistently high overall EMTR levels; (iii) non-linear and non-monotonic EMTR schedules; (iv) heterogeneity in EMTR schedules across socioeconomic and demographic groups. These findings warrant the paper's investigation into the optimal joint design of taxes and child benefits, as well as

[16] For a single mother earning below $50,000, taxes and child care costs increase her EMTR schedule, but the addition of the CCS reduces it. Here, the FTB plays no role at lower income levels, which explains the overlap between the heavy-blue line (EMTR without FTB) and red line (ETMR with FTB).

[17] Further discussion on EMTR variations over the parental life cycle can be found in Subsection A.2 of the Appendix.

their impacts on overall welfare, parental welfare, distribution, and key macroeconomic performance indicators such as female labor supply and output.

3 A simple model

In this section, I formulate a simple model of a representative parent household, firm, and government in a general equilibrium environment, where income tax is used to balance the public budget. First, the model illustrates how means-tested benefits and work subsidies—central features of the current child benefit system—interact and affect women's labor supply, household consumption, output, and overall welfare. Second, it demonstrates that, in a representative-agent, deterministic setting, the optimal outcome is always a distortion-free economy. In other words, provided the benchmark economy where distortions from means-testing and taxes are prevalent, the optimal policy is to eliminate these distortions, either by removing means-testing or by introducing counter-programs to offset the existing distortions. However, this outcome overlooks the distributional, insurance, and fiscal control roles of means-tested benefits, which are addressed more comprehensively in the quantitative framework discussed in Subsection 4.

Representative parent household

Consider a married parent household making static decisions on consumption c and female labor supply n to maximize joint utility, subject to a budget constraint. The husband's labor supply n_m is perfectly inelastic and earns a unit wage rate, with income taxed at a rate τ.

To derive a closed-form solution, I focus on the role of tax as a government budget-balancing tool and abstract from its distortionary effects on female labor supply. That is, suppose the mother's labor supply n falls within a tax-free zone, but she faces a child care fee κ.

The government wants to encourage female labor supply by offsetting κ. Thus, suppose further that the mother's earnings are subsidized at a rate s, emulating the Child Care Subsidy (CCS) that supports secondary earners. In addition, to reflect the real-world child care policies that aim to support low-income parents, I also assume that the household may be eligible for a means-tested child benefit (FTB).

Let $u(c, 1-n)$ denote a well-behaved utility function of consumption c and leisure $1-n$ satisfying standard properties: $u' > 0$, $u'' < 0$, $\lim_{x\to 0} u' = \infty$, $\lim_{x\to\infty} u' = 0$ for all its arguments $x \in \{c, 1-n\}$. The household's optimization problem is:

$$\max_{c,\, n} \quad \{u(c, 1-n)\} \tag{1}$$

subject to

$$c \quad = \quad (1-\tau)n_m + (1 - \overbrace{(\kappa - s)}^{\text{Net child care cost}})n + \overbrace{FTB(n)}^{\text{Means-tested transfer}} \tag{2}$$

where $FTB(n) = \max\{\min\{\bar{tr},\ \bar{tr} - \omega(n_m + n - \bar{y})\},\ 0\}$, with $\bar{tr}$ denoting the maximum payment, ω the phase-out rate, and $\bar{y}$ the family-income test threshold.

Representative firm

The single firm in the economy employs a basic technology that transforms labor linearly into output y. The firm does not differentiate between male and female labor, paying all workers at the unit wage rate, $w = 1$. The total output is:

$$y \quad = \quad n_m + n$$

Government

The government balances its budget by collecting income tax τn_m to finance general expenditures G and total transfers $sn + \bar{tr} - \omega(n_m + n - \bar{y})$. The government budget equation is:

$$\tau n_m \quad = \quad G + sn + \bar{tr} - \omega(n_m + n - \bar{y})$$

For simplicity, assume that the household derives no benefit from G.

3.1 First- and second-best allocations of female labor supply

Given the setup above, I compare two economies: (i) a first-best economy without distortions, and (ii) a second-best economy with work subsidies and means-tested child benefits.

For this purpose, I first re-formulate the household problem and government budget equation by assuming that the only policy the household faces is a lump-sum tax T. Next, I derive the distortion-free optimal labor supply n^* and consumption c^*, also known as the *first-best allocations of labor and consumption*, and the corresponding baseline efficiency and welfare.

The household problem is re-written as:

$$\max_{c,\, n} \quad \{u(c, 1-n)\} \tag{3}$$

subject to

$$c \quad = \quad n_m + (1-\kappa)n - T \tag{4}$$

The government budget equation simplifies to:

$$T \quad = \quad G \tag{5}$$

The optimal consumption-leisure trade-off condition is:

$$MRS_{c,1-n} = \frac{u'_c}{u'_{1-n}} \quad = \quad \frac{1}{1-\kappa} \tag{6}$$

Suppose the household's utility takes a Cobb-Douglas form $u(c, 1-n) = c^{\alpha}(1-n)^{1-\alpha}$, where $0 < \alpha < 1$ denotes the taste-for-consumption parameter.[18] From Equation (6), we get:

$$n \quad = \quad 1 - \frac{1-\alpha}{\alpha(1-\kappa)}c$$

Using (4) and the household's budget constraint (5), expressions for the first-best (female) labor and consumption allocations, n^* and c^*, can be derived:

$$n^* \quad = \quad \alpha - \frac{1-\alpha}{1-\kappa}(n_m - G^*) \tag{7}$$

$$c^* \quad = \quad \alpha(1 - \kappa + n_m - G^*) \tag{8}$$

where G^* denote distortion-free government spending. Equations (7) and (8) show that n^* and c^* are both increasing functions in α. The exogenous male income n_m is a positive income effect (IE) that reduces n^* and increases c^*, whereas G^*, a negative IE, does the opposite. While there is no distortionary tax in this economy, the child care fee κ behaves as a natural tax on the mother's labor supply, causing n^* and c^* to fall.

The aggregate output y^* is:

$$y^* \quad = \quad n_m + n^* \tag{9}$$

[18] The interdependence between c and $1-n$ therefore occurs via both the household's preference and the budget constraint.

Let $u^* := u(c^*, 1-n^*)$ represent the household utility associated with the first-best allocations. The welfare measure is obtained by substituting (7) and (8) into the Cobb-Douglas utility function. For ease of comparison with the utilities from the second-best economy, the welfare measure is expressed in log form:

$$ln(u^*) \quad = \quad \alpha ln(\alpha) + (1-\alpha)ln(1-\alpha) + \underbrace{ln(1-\kappa+n_m-G^*)}_{(a)\text{ Income effects}} - \underbrace{(1-\alpha)log(1-\kappa)}_{(b)\text{ Effect of }\kappa\text{ on leisure}} \tag{10}$$

Term (a) of the welfare equation (10) indicates that household utility increases with n_m and decreases with κ and G^*. Term (b) shows that in addition to its negative IE through the household's budget constraint, κ also leads to more leisure taken, causing an increase in utility weighted by the household's taste for leisure.

3.1.1 Second-best economy with means-tested child benefits

Wage distortions due to means-testing lead to deviations of optimal labor and consumption from their first-best allocations (7) and (8). To understand the implications stemming from such departures, I consider a case where family income falls in the phase-out zone of benefits.[19]

In this context, the household budget constraint becomes:

$$c \quad = \quad (1-\tau)n_m + (1-\kappa+s)n + \bar{tr} - \omega(n+n_m-\bar{y}) \tag{11}$$

Solving the first-order conditions yields:

$$MRS_{c,1-n} = \frac{u'_c}{u'_{1-n}} \quad = \quad \frac{1}{1-\kappa-\omega+s} \tag{12}$$

The government budget-clearing tax rate is

$$\tau \quad = \quad \frac{G+sn+\bar{tr}-\omega(n_m+n-\bar{y})}{n_m} \tag{13}$$

Substituting the Cobb-Douglas utility in (12), together with the household budget constraint (11) and the government budget-clearing tax rate (13), I derive expressions for the second-best allocations of labor supply n_ω and consumption c_ω:

$$n_\omega \quad = \quad \frac{\alpha(1-\kappa-\omega+s)-(1-\alpha)(n_m-G)}{1-\kappa-\alpha(\omega-s)} \tag{14}$$

$$c_\omega \quad = \quad \frac{\alpha(1-\kappa-\omega+s)(1-\kappa+n_m-G)}{1-\kappa-\alpha(\omega-s)} \tag{15}$$

The aggregate output is

$$y_\omega \quad = \quad n_m + n_\omega$$

Using (14) and (15), together with the first-best allocations (7) and (8), I then derive functions relating n_ω and c_ω to their first-best counterparts n^* and c^*:

$$n_\omega(n^*) \quad = \quad \frac{(1-\kappa)n^*-\alpha(\omega-s)+(1-\alpha)(G-G^*)}{1-\kappa-\alpha(\omega-s)} \tag{16}$$

$$c_\omega(c^*) \quad = \quad \frac{(1-\kappa-\omega+s)\left[c^*-\alpha(G-G^*)\right]}{1-\kappa-\alpha(\omega-s)} \tag{17}$$

Now, assume that $G = G^*$, meaning the government maintains the same level of general (non-transfer) spending

[19] Other scenarios, where family income lies outside of the phase-out zone and thus female labor supply is not distorted by the transfers, can be obtained by setting $\omega = 0$. However, they are not considered here.

as in the first-best economy. Equations (16) and (17) simplify to:

$$n_\omega(n^*) = \frac{(1-\kappa)n^* - \alpha(\omega - s)}{1-\kappa-\alpha(\omega - s)} \tag{18}$$

$$c_\omega(c^*) = \frac{(1-\kappa-\omega+s)c^*}{1-\kappa-\alpha(\omega - s)} \tag{19}$$

The budget-clearing tax (13) implies that the total transfers $sn + \bar{tr} - \omega(n_m + n - \bar{y})$ are financed by an equal increase in τn_m (lump-sum). The balanced public budget requirement, therefore, eliminates the positive IEs by the transfers that enter directly into the household's budget constraint. As a result, deviations from the first-best allocations are driven solely by marginal considerations, as demonstrated in Equations (18) and (19). In this setting, s and ω are the only two policy instruments that affect the second-best allocations. We can show that

$$\frac{\partial n_\omega}{\partial s} = \frac{\alpha(1-\kappa)(1-n^*)}{(1-\kappa-\alpha(\omega-s))^2} > 0 \quad ; \quad \frac{\partial n_\omega}{\partial \omega} = -\frac{\alpha(1-\kappa)(1-n^*)}{(1-\kappa-\alpha(\omega-s))^2} < 0 \tag{20}$$

$$\frac{\partial c_\omega}{\partial s} = \frac{(1-\alpha)(1-\kappa)c^*}{(1-\kappa-\alpha(\omega-s))^2} > 0 \quad ; \quad \frac{\partial c_\omega}{\partial \omega} = -\frac{(1-\alpha)(1-\kappa)c^*}{(1-\kappa-\alpha(\omega-s))^2} < 0 \tag{21}$$

For $1-\kappa-\alpha(\omega-s) \neq 0$, both n_ω and c_ω increase as the subsidy rate s increases and decreases as the phase-out rate ω increases. Since $y_\omega = n_m + n_\omega$, the economic output also increases with a higher s or a lower ω.[20]

The second-best welfare measure is obtained by substituting (14) and (15) into the Cobb-Douglas utility function. In logarithmic form:

$$\begin{aligned} ln(u_\omega) &= \alpha ln(\alpha) + (1-\alpha)ln(1-\alpha) \\ &\quad + ln(1-\kappa+n_m-G) + \alpha ln(1-\kappa-\omega+s) - ln\,(1-\kappa-\alpha(\omega-s)) \end{aligned} \tag{22}$$

Using (22) and the first-best welfare (10), we arrive at the following welfare gap equation between the first- and second-best economies:

$$\underbrace{ln(u_\omega) - ln(u^*)}_{\text{Welfare gap}} = \underbrace{ln\,(1-\kappa+n_m-G) - ln(1-\kappa+n_m-G^*)}_{(a)\text{ Relative strength of } IE} \tag{23}$$

$$+\underbrace{\alpha ln(1-\kappa-\omega+s) + (1-\alpha)ln(1-\kappa) - ln\,(1-\kappa-\alpha(\omega-s))}_{(b)\text{ Effects of wage distortion}} \tag{24}$$

If $G = G^*$ as per assumption above, then the expression simplifies to:

$$\underbrace{ln(u_\omega) - ln(u^*)}_{\Delta u} = \underbrace{\alpha ln(1-\kappa-\omega+s) + (1-\alpha)ln(1-\kappa) - ln\,(1-\kappa-\alpha(\omega-s))}_{(b)\text{ Effects of wage distortion}} \tag{25}$$

Let $\Delta u := ln(u_\omega) - ln(u^*)$. The only policy tools that influence the welfare gap are the subsidy rate s and the phase-out rate ω. The first derivatives of the welfare gap with respect to s and ω are:

$$\frac{\partial \Delta u}{\partial s} = \frac{\alpha}{1-\kappa-(\omega-s)} - \frac{\alpha}{1-\kappa-\alpha(\omega-s)} \tag{26}$$

$$\frac{\partial \Delta u}{\partial \omega} = \frac{\alpha}{1-\kappa-\alpha(\omega-s)} - \frac{\alpha}{1-\kappa-(\omega-s)} \tag{27}$$

Since $0 < \alpha < 1$, the signs of these derivatives depend on the sign of $\omega - s$.

$$\text{If } \omega - s > 0: \frac{\partial \Delta u}{\partial s} > 0 \quad \text{and} \quad \frac{\partial \Delta u}{\partial \omega} < 0 \tag{28}$$

$$\text{If } \omega - s = 0: \frac{\partial \Delta u}{\partial s} = 0 \quad \text{and} \quad \frac{\partial \Delta u}{\partial \omega} = 0 \tag{29}$$

$$\text{If } \omega - s < 0: \frac{\partial \Delta u}{\partial s} < 0 \quad \text{and} \quad \frac{\partial \Delta u}{\partial \omega} > 0 \tag{30}$$

[20]Note too that, $\frac{\partial x}{\partial s} = -\frac{\partial x}{\partial \omega}$ for $x \in \{n_\omega, c_\omega\}$. The first derivatives with respect to s and ω are identical in magnitude, indicating symmetric effects on labor supply and consumption.

In addition to the net-zero direct income effect of transfers due to the tax burden, these welfare conditions further highlight the importance of policy interaction. The second-best welfare response to any reform depends crucially on the status quo policy mix.

If the current policy is such that the benefit phase-out rate ω dominates the subsidy rate s, as in (28), increasing s or reducing ω enhances the second-best welfare u_ω relative to the first-best u^*. Conversely, if s dominates ω, as in (30), the opposite occurs. When $\omega = s$, as in (29), the marginal welfare effect from any reform is nil. This suggests that the first-best welfare u^* represents the maximum welfare attainable.[21]

This points to an important insight: *wage distortions, $|\omega - s|$, in either direction are welfare-deteriorating.* If one starts with the first-best economy, provided that the tax burden from transfers is fully borne by the recipients, further welfare improvements are not possible. Otherwise, a policy reform that minimizes distortions is always welfare improving. The optimal policy thus occurs in two scenarios: (i) the first-best economy where all policies are lump-sum, such that $\omega = s = 0$, or (ii) an alternative economy where policies perfectly balance each other, such that $|\omega - s| = 0$.

The welfare prospect is a stark contrast to the labor, consumption, and output outcomes. Equation (18) and its first derivatives (20) indicate that these variables can be exceed their first-best values by raising s relative to ω, albeit at the cost of welfare since more consumption and output necessitate more work and therefore less leisure.[22]

Assuming output serves as a proxy for efficiency, these results also suggest that efficiency-welfare trade-offs emerge only when a policy that improves output also increases the wage distortion, $|\omega - s|$. In other words, an efficiency-welfare improving policy is possible in the current environment in Australia where $|\omega - s| > 0$ (as evident in the simulated EMTR schedule from Figure 2 of Section 2). A reduction in ω (or alternatively an increase in s) promotes labor supply and output while enhancing welfare by correcting the wage distortion.[23]

These analytical findings highlight three key lessons that shape the quantitative framework in Section 4. First, they emphasize *the interplay between policies.* A child benefit policy can be welfare-enhancing or deteriorating depending on its interaction with other policies, including the tax system. Neglecting this interplay may alter conclusions in counterfactual reforms, potentially skewing policy recommendations.

Second, even in the absence of tax distortions, the findings draw attention to *the pivotal role of financing mechanisms for transfers.* In a partial equilibrium environment, when the financial needs of child benefit programs are not taken into account, their welfare contributions might be overstated. A comprehensive investigation must consider the general equilibrium effect via the tax channel, as tax burdens can counteract the positive IE of transfers.

Third, the conclusions of this analysis may be limited by the omission of the redistributive and insurance roles of child benefits. The theoretical model is built on a representative-agent foundation, where every household receives benefits and bears the resultant tax burden. In reality, taxes and benefits are unevenly distributed. Means-tested child benefits are targeted at low-income parents, whereas the burden of financing is spread across the working population. Empirical evidence from Figures 2 and 3 also suggests that policy-induced $EMTR$ schedules vary across socioeconomic and demographic groups. In addition, households face different constraints on their labor supply, such as the monetary and time costs of child care. These elements suggest that child benefits may impact welfare through their redistributive role, a mechanism not captured in the simplified model. Furthermore, as the analytical model is deterministic, it is silent on the potential welfare improvements from the insurance effect of child benefits against idiosyncratic earnings shocks. These considerations underscore *the importance of incorporating household heterogeneity and income uncertainty* in child benefit policy assessments.

[21] Note that, because the model is homogeneous in households and deterministic in the earnings process, welfare improvements must stem from a more efficient allocation of consumption and leisure.

[22] Since $y_\omega = n_m + n_\omega$, a caveat is that n_m is assumed to be perfectly inelastic to any increased tax rate to finance the subsidy s.

[23] This theoretical result aligns with the findings by Tin and Tran (2024), which show that reducing the phase-out rate of the CCS program improves labor supply, output, and welfare. In this paper, Section 6 demonstrates that a reduction in tax progressivity, which lowers the EMTR, produces similar effects.

Informed by these theoretical insights, I build a structural model with three core components to analyze the joint optimization of taxes and child benefits. First, the model includes the progressive tax structure and the two major child benefit programs, the FTB and CCS, to fully account for their interactions. Second, all counterfactual experiments are conducted within a general equilibrium environment with an endogenous income tax balancing the government budget. Lastly, the model is constructed on a heterogeneous-agent foundation with uninsurable income shocks to capture welfare changes through both redistribution and insurance channels.

4 A dynamic general equilibrium model

I study a small open economy model populated by a continuum of overlapping generations of households, a representative firm with constant returns to scale (CRS) technology, and a government who commits to balancing its budget every period. Time begins at $t = 0$ when the model economy is in an initial steady state, and ends at $t = T$. One model period corresponds to one year. The model is an extension of the structural framework established in Tin and Tran (2024).[24]

4.1 Demographics

Every period t, a new cohort of households aged $j = 1$ (equivalent to real age of 21) enters the economy. Each adult member of gender $i \in \{m, f\}$ in a household born at time t survives each subsequent period $t + j - 1$ with a time-invariant conditional probability $\psi_{j,i}$ and can live up to a maximum age $J = 80$ (i.e., $\psi_{J+1,i} = 0$). Individuals begin to work at $j = 1$ and retire at age $J_R = 45$. The initial total number of households at time $t = 0$ is normalized to one. The model population grows at a constant rate, g_N.[25]

Family structure. Households are assigned one of four family types at birth: married parents ($\lambda = 1$), married childless couples ($\lambda = 2$), single childless men ($\lambda = 3$), and single mothers ($\lambda = 4$). Married households comprise a husband and wife of identical age and education. The evolution of marital status depends solely on survival probabilities, meaning a married household becomes single if one spouse dies. Single households, on the other hand, remain single until death. The model does not account for divorce, marriage, or re-marriage after the initial assignment. Parenthood, defined as the state of having had a co-resident child, is a permanent status. Married childless couples ($\lambda = 2$) cannot become married parents ($\lambda = 1$), and vice versa. Additionally, all single women are assumed to be mothers, whereas single men are childless. The transition probabilities for family structure ($\pi_{\lambda_{j+1}|\lambda_j}$) are given by Table 1.

$\pi_{\lambda_{j+1}\|\lambda_j}$	$\lambda_{j+1} = 1$	$\lambda_{j+1} = 2$	$\lambda_{j+1} = 3$	$\lambda_{j+1} = 4$
$\lambda_j = 1$	$\psi_{j+1,m}\psi_{j+1,f}$	0	$\psi_{j+1,m}(1 - \psi_{j+1,f})$	$(1 - \psi_{j+1,m})\psi_{j+1,f}$
$\lambda_j = 2$	0	$\psi_{j+1,m}\psi_{j+1,f}$	$\psi_{j+1,m}(1 - \psi_{j+1,f})$	$(1 - \psi_{j+1,m})\psi_{j+1,f}$
$\lambda_j = 3$	0	0	$\psi_{j+1,m}$	0
$\lambda_j = 4$	0	0	0	$\psi_{j+1,f}$

Table 1: Transition probabilities of family structure

Children. I abstract from fertility choice. Children are exogenous and deterministic. They contribute neither to the utility of parents nor to the broader economy once they reach adulthood.[26] Married and single

[24]The new features introduced in this paper include: (i) more detailed representation of family composition (by incorporating childless couples); (ii) fully endogenized female labor supply decisions at both the intensive and extensive margins (Subsections 4.3 and 4.7); and (iii) a decomposition of welfare measures to identify the key drivers of welfare changes (Subsection 4.10). Furthermore, while Tin and Tran (2024) focus on child benefit reforms that can improve aggregate and distributional outcomes, this paper extends their analysis to propose a joint optimal design of taxes and child benefits that maximizes the overall ex-ante welfare.

[25]Population growth g_N and conditional survival probabilities ψ are included in the model to approximate the population structure. They serve as weighting factors in the aggregation of cohort-based variables.

[26]Children indirectly affect household utility through time costs, which impact leisure, and child care expenses, which affect the budget constraints of working parents. I also assume that children and population growth are detached, and resources allocated to a child's upbringing do not contribute to future labor productivity. Additionally, because fertility is exogenous, making children affect household utility, aside from the indirect effects, is not a necessary feature.

parent households have full information on the timing of children's arrival, non-pecuniary and pecuniary child care costs, the FTB transfer per child, the CCS rate per hour worked, and the human capital gains (or losses) if the mother works (or stays at home). For simplicity, child care quality and costs for a child aged j_c are exogenous and identical for all households. The per-hour child care service fee is a constant fraction κ of the market wage w. There is no informal care.

The number and age of children in a household are fully determined by the household age j and education θ. All parents have the same number of children, $\bar{nc} = 2$, over their lifetime. Child spacing is identical, although the timing of births varies by education. The firstborn arrives earlier for low-education (θ_L) households and later for high-education (θ_H) households. Thus, the k^{th} child is born to every parent household at age $j = b_{k,\theta}$ and remains dependent until the age of 18 (i.e., from $j = b_{k,\theta}$ to $j = b_{k,\theta}+17$). Afterwards, the child leaves home permanently, ending the parent-child link. With these simplifications, the number of children in a household of age j and education θ is given by $nc_{j,\theta} = \sum_{k=1}^{\bar{nc}} \mathbf{1}_{\{b_{k,\theta} \leq j \leq b_{k,\theta}+17\}}$.

4.2 Preferences

Household preferences are represented by a time-separable expected utility function

$$W(c_j, l_j^f) = \sum_{j=1}^{J} \beta^{j-1} \left(\prod_{s=1}^{j-1} \pi_{\lambda_{s+1}|\lambda_s} \right) u(c_j, l_j^m, l_j^f, \theta, \lambda_j)$$

where β is the time discount factor, c is the joint consumption, $l^m = 1 - n^m$ is the male leisure time, $l^f = 1 - n$ is the female leisure time, θ is the education level, and λ is the family type. I use n^m to denote the exogenous male labor supply, and n to denote the endogenous female labor supply at the intensive margin (instead of n^f). $W(c_j, l_j^f)$ is the total expected utility expressed as a function of the decision variables.

Suppressing the age subscript j to ease notation, the periodic household utility functions for different family types—married parents, married childless couples, single childless men, and single mothers—are as follows:

$$\begin{aligned}
u(c, l^m, l^f, \theta, \lambda = 1) &= \frac{\left[\left(\frac{c}{\iota_{1,\theta}}\right)^{\nu} (l^m)^{1-\nu}\right]^{1-\frac{1}{\gamma}} + \left[\left(\frac{c}{\iota_{1,\theta}}\right)^{\nu} (l^f)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} \\
u(c, l^m, l^f, \theta, \lambda = 2) &= \frac{\left[\left(\frac{c}{\iota_{2,\theta}}\right)^{\nu} (l^m)^{1-\nu}\right]^{1-\frac{1}{\gamma}} + \left[\left(\frac{c}{\iota_{2,\theta}}\right)^{\nu} (l^f)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} \\
u(c, l^m, \theta, \lambda = 3) &= \frac{\left[(c)^{\nu} (l^m)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} \\
u(c, l^f, \theta, \lambda = 4) &= \frac{\left[\left(\frac{c}{\iota_{4,\theta}}\right)^{\nu} (l^f)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}
\end{aligned}$$

where ν is the taste for consumption, γ is the elasticity of intertemporal substitution (EIS) and $\iota_{\lambda,\theta} = \sqrt{\mathbf{1}_{\{\lambda \neq 3\}} + \mathbf{1}_{\{\lambda \neq 4\}} + nc_\theta}$ is the consumption equivalence scale. While the model does not explicitly include children in the household utility functions, parents' concern for their children's welfare is partially reflected in their efforts to maximize per capita consumption in their household.

Consumption equivalence scale. Children increase household size, thereby reducing per capita consumption. I capture this effect using the square root consumption equivalence scale $\iota_{\lambda,\theta}$, formally defined as:

$$\iota_{\lambda,\theta} = \sqrt{\mathbf{1}_{\{\lambda \neq 3\}} + \mathbf{1}_{\{\lambda \neq 4\}} + nc_\theta}$$

where $\mathbf{1}_{\{x\}}$ is an indicator function with a logical argument x, and $\mathbf{1}_{\{\lambda \neq 3\}} + \mathbf{1}_{\{\lambda \neq 4\}} + nc_\theta$ calculates the household size (number of adults and children).

$\iota_{\lambda,\theta}$ reflects the economies of scale within households, as shared consumption (e.g., utilities and durable goods) means the cost of living does not increase linearly with each additional member. It also adjusts for household composition. For instance, a family of four (two parents and two children) requires more resources than a childless couple but not necessarily twice as much.[27]

4.3 Endowments

Married and single men. Male labor supply is exogenous. Men work full-time until retirement and earn labor income of $y^m_{j,\lambda} = wn^m_{j,\lambda}e^m_{j,\lambda,\theta}$, where w is the market wage, and $n^m_{j,\lambda}$ and $e^m_{j,\lambda,\theta}$ are exogenous work hours and earning ability, respectively. Their intensive margin of labor supply, $n^m_{j,\lambda} = 1 - l^m_{j,\lambda}$, is set at the normalized average work hours over the working age. Earning ability $e^m_{j,\lambda,\theta}$ is composed of a deterministic component $\bar{e}_j$ and a stochastic shock ϵ^m_j:

$$e^m_{j,\lambda,\theta} = \bar{e}\left(\theta, h^m_{j,\lambda,\theta}\right) \times \epsilon^m_j$$

where $\bar{e}\left(\theta, h^m_{j,\lambda,\theta}\right) = e^{\theta} h^m_{j,\lambda,\theta}$ is a non-linear function of education θ and male human capital $h^m_{j,\lambda,\theta}$. The stochastic component ϵ^m_j is modeled as a first-order autoregressive process

$$\overbrace{\ln\left(\epsilon^m_j\right)}^{=\eta^m_j} = \rho \times \overbrace{\ln\left(\epsilon^m_{j-1}\right)}^{=\eta^m_{j-1}} + \upsilon^m_j \tag{31}$$

with persistence parameter ρ, and white-noise disturbance $\upsilon^m_j \sim N\left(0, \sigma^2_\upsilon\right)$.

Married and single women. Female labor supply is endogenous. A household first chooses among three employment statuses for its female members: staying at home ($\ell = 0$), working part-time ($\ell = 1$), or working full-time ($\ell = 2$). Once the employment status is determined, her work hours n are decided accordingly.

The female labor supply decision process is detailed in Subsection 4.7. In brief, it involves balancing various work-related trade-offs to maximize household lifetime utility. These trade-offs affect female labor supply behavior, their susceptibility to the insurance and incentive effects of transfer schemes, and consequently, their responses to policy reforms in the counterfactual economies.

1. **Benefits of working**: If a woman works, she: (i) *earns labor income,* $y^f_j = wn_j e^f_{j,\theta,\ell}$; (ii) *accumulates human capital for the next period,* $h^f_{j+1,\theta,\ell}$; and (iii) *receives a subsidy of* sr_j *per dollar spent on child care*, provided she meets the CCS criteria outlined in Section 5.5. Her earning ability is

$$e^f_{j,\theta,\ell} = \bar{e}\left(\theta, h^f_{j,\theta,\ell}\right) \times \epsilon^f_j$$

where the deterministic part $\bar{e}\left(\theta, h^f_{j,\theta,\ell}\right)$ is determined by her education θ and human capital $h^f_{j,\theta,\ell}$. The stochastic component ϵ^f_j follows an autoregressive process:

$$\ln\left(\epsilon^f_j\right) = \rho \times \ln\left(\epsilon^f_{j-1}\right) + \upsilon^f_j \tag{32}$$

with persistence parameter ρ and innovation term σ^2_υ that governs a white-noise disturbance $\upsilon^f_j \sim N\left(0, \sigma^2_\upsilon\right)$, identical to those influencing male earnings. Unlike her male counterpart, however, female human capital $h^f_{j,\theta,\ell}$ evolves endogenously over her life cycle according to the law of motion (47). In short, working today not only generates immediate income but also enhances future earning ability, while staying at home results in depreciation of that ability.

[27] The consumption equivalence scale can be translated into the required income that equalizes per capita consumption levels between parent and non-parent households. For example, using the square root scale $\iota_{\lambda,\theta}$ to compare between childless couples and parents who have nc_θ children, a dollar to the former is equivalent to x dollars to the latter if $\frac{1}{\sqrt{2}} = \frac{x}{\sqrt{2 + nc_\theta}}$. This results in \$1.22 for couples with one child and \$1.41 for those with two children. While the square root scale is adopted in this model for ease of computation, these implied equivalent incomes are closely aligned with the average estimates for Australia in the Department of Social Services (DSS) report and for New Zealand by Chatterjee and Michelini (1998).

2. **Costs of working**: Labor force participation also comes with costs. If a woman works, she incurs (i) *formal child care costs per child,* κ_j; (ii) *a potential reduction or total loss of means-tested child benefits*, and (iii) *employment-specific (ℓ-specific) fixed time costs to her household leisure in addition to her work hours.* I also assume spouses are perfectly altruistic in both consumption and leisure, thus sharing the fixed time costs evenly. Specifically, at age j, a woman's employment status and work hours affect her leisure time l_j^f as follows

$$l_j^f = \begin{cases} 1 & \text{if staying at home } (\ell = 0) \\ 0 < 1 - n_j - \mathbf{1}_{\{\lambda=1,2\}} \dfrac{\chi_{\lambda,1}}{2} - \mathbf{1}_{\{\lambda=4\}} \chi_{\lambda,1} < 1 & \text{if working part-time } (\ell = 1) \\ 0 < 1 - n_j - \mathbf{1}_{\{\lambda=1,2\}} \dfrac{\chi_{\lambda,2}}{2} - \mathbf{1}_{\{\lambda=4\}} \chi_{\lambda,2} < 1 & \text{if working full-time } (\ell = 2). \end{cases} \tag{33}$$

where $\chi_{\lambda,\ell}$ for $\ell \in \{1, 2\}$ represents fixed time costs for households associated with part-time and full-time work, respectively. $\chi_{\lambda,\ell}$ varies between parents, $\lambda \in \{1, 4\}$, and non-parents, $\lambda = 2$. The fixed costs are modeled using a parametric function that decreases monotonically with age:

$$\chi_{\lambda,\ell}(j) = \frac{\chi^y_{\lambda,\ell}}{1 + e^{\chi^s_{\lambda,\ell}(j - \bar{j}_\lambda)}} \tag{34}$$

where $\chi^y_{\lambda,\ell} = \chi^{max}_{\lambda,\ell} \times (1 + e^{\chi^s_{\lambda,\ell}(1 - \bar{j}_\lambda)})$ governs the maximum fixed cost $\chi^{max}_{\lambda,\ell} = \chi_{\lambda,\ell}(1)$ at age $j = 1$ (i.e., the intercept of the fixed-cost profile). $\bar{j}_\lambda$ is the inflection point, and $\chi^s_{\lambda,\ell}$ controls the slope around $\bar{j}_\lambda$. A higher $\chi^s_{\lambda,\ell}$ results in an inverse Sigmoid age-profile of fixed costs that stays near its maximum value longer and declines more steeply around $\bar{j}_\lambda$.

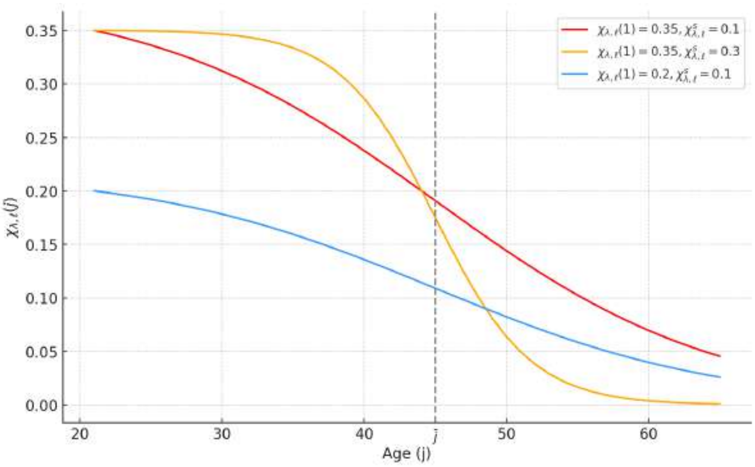

Figure 4: **Fixed cost function.**
Notes: The figure shows the age profiles of fixed cost to leisure for women for three different parameterizations.

The female labor supply decision therefore hinges on the interplay between these costs and benefits, including child care costs, the insurance and work incentive effects of the means-tested child benefits, human capital potential, and family insurance through a partner's labor earnings. These dynamics are further explored in the quantitative analysis of Section 6.

4.4 Technology

In every time period t, a representative firm with labor-augmenting technology A_t and a Cobb-Douglas production function $Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$ transforms capital K_t and total labor services L_t into output Y_t.A_t grows at a constant rate g_A. The firm pays a capital income tax τ_t^k and chooses its capital and labor inputs to maximize profit, taking the capital rental rate $q_t = r_t + \delta$ and wage rate w_t as given, where r_t is the real interest rate and δ denotes the depreciation rate of capital.

In this small open economy model, the free flow of foreign capital $B_{F,t}$ ensures that $r_t = r_w$, where r_w is a constant world interest rate (no-arbitrage condition). As a result, the real interest rate r_t, and thus wage rate w_t, are unchanged across steady states.

Suppressing the time subscript, the firm's problem is:

$$\max_{K,L}(1-\tau^k)(Y - wAL) - qK \tag{35}$$

The firm's first-order conditions are:

$$r = r_w = (1-\tau^k)\alpha\frac{Y}{K} - \delta \tag{36}$$

$$w_t = (1-\alpha)\frac{Y}{AL} \tag{37}$$

4.5 Fiscal policy

I model key features of the Australian fiscal system, including a progressive income tax system, two means-tested child benefit programs for families with children, and a means-tested Age Pension program for retirees.

4.5.1 Tax system

Progressive income tax. The government levies taxes on individual labor earnings.[28] I model a progressive tax scheme to capture the additional distortions (or lack thereof) that occur when taxes interact with child benefits at different income levels. For instance, in tax-free or low-tax low-income brackets, the add-on work disincentive effects from the FTB phase-out rate could be less consequential compared to its effects under a proportional tax scheme, whereas the opposite might hold true in high-income brackets.

The taxable income for an individual $i \in \{m, f\}$ at age j is $\widetilde{y}^i_{j,\lambda}$, representing the individual's total labor earnings. I approximate the tax schedule using a parametric tax function following Feldstein (1969); Benabou (2000), and Heathcote et al. (2017). Suppressing the family type λ subscript and gender i superscript, the individual income tax payment is given by:

$$tax_j = \max\left\{0, \widetilde{y}_j - \zeta\widetilde{y}_j^{1-\tau}\right\} \tag{38}$$

Here, tax_j denotes the tax payment, ζ is a scaling factor, and τ controls the progressivity of the tax system. At one extreme, if τ approaches infinity, tax_j approaches $\tilde{y}_j$, implying 100% of the taxable income is taxed. At the other extreme, if $\tau = 0$, then $tax_j = (1-\zeta)\tilde{y}_j$, making $(1-\zeta)$ a flat tax rate. As τ increases (or *decreases*), the marginal tax rate (MTR) and average tax rate (ATR) increase (or *decrease*) for a given income level. A non-negative tax restriction is applied to exclude government transfers in the form of negative income taxes.

ζ serves as the public budget balancing variable. Adjusting ζ shifts the overall tax schedule without changing the system's progressivity. As illustrated in Figure 5, a higher ζ reduces the tax burden across all income levels, shifting the tax schedule downward, and expanding the zero-tax income bracket. In turn, this decreases the MTR, especially for low-income households, for a give τ. Conversely, a lower ζ increases the overall tax burden, compressing the zero-tax income bracket, and raising the MTR.

[28] Australia runs a separate tax filing system, treating individuals, not households, as the basic unit for income tax purposes. In the current model, I abstract from capital earnings taxes and franking credits under Australia's dividend imputation system. Franking credits represent the way in which the corporate tax of a firm is recorded and later credited to households (shareholders). The underlying idea is to prevent double taxation. I assume that the representative firm pays corporate taxes τ^k and distributes fully franked dividends to households, exempting them from capital earnings tax. See the Parliamentary Budget Office (PBO) 2024 report on dividend imputation and franking credits for further details.

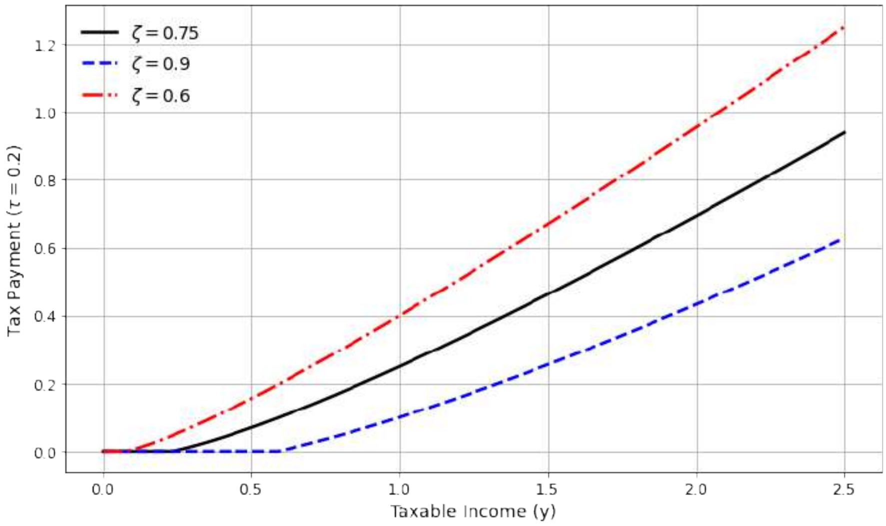

Figure 5: **Tax schedules for $\tau = 0.2$ and different parametrization of ζ.**

4.5.2 Transfer system

The government also runs a means-tested child benefit system to support families with dependent children through two main programs: the Family Tax Benefit (Part A and Part B) and the Child Care Subsidy. Below is a simplified overview of these programs. For more detailed information, I refer interested readers to Appendix Section H.2.

Family Tax Benefit Part A (FTB-A). The FTB-A is paid per dependent child. The claimable amount depends on the household combined taxable income, and the age and number of dependent children. Key policy parameters determining the levels, kinks, and slopes of the FTB-A schedule are: (i) maximum and base payments per child, tr_j^{A1} and tr_j^{A2}; (ii) joint income test thresholds for maximum and base payments, $\bar{y}_{max}^{tr}$ and $\bar{y}_{base}^{tr}$; and (iii) phase-out rates for maximum and base payments, ω_{A1} and ω_{A2}. Accordingly, the FTB-A benefit per child, tr_j^A, is given by:

$$tr_j^A = \begin{cases} tr_j^{\text{A1}} & \text{if } y_{j,\lambda} \leq \bar{y}_{max}^{tr} \\ \max\left\{tr_j^{\text{A2}},\ tr_j^{\text{A1}} - \omega_{A1}\left(y_{j,\lambda} - \bar{y}_{max}^{tr}\right)\right\} & \text{if } \bar{y}_{max}^{tr} < y_{j,\lambda} \leq \bar{y}_{base}^{tr} \\ \max\left\{0,\ tr_j^{\text{A2}} - \omega_{A2}\left(y_{j,\lambda} - \bar{y}_{base}^{tr}\right)\right\} & \text{if } y_{j,\lambda} > \bar{y}_{base}^{tr}, \end{cases} \tag{39}$$

where $y_{j,\lambda} = \mathbf{1}_{\{\lambda\neq 4\}} y_{j,\lambda}^m + \mathbf{1}_{\{\lambda\neq 3,\ell\neq 0\}} y_j^f + ra_j$ denotes household combined income.

Family Tax Benefit Part B (FTB-B). The FTB-B is paid per household as additional support to single parents and single-earner partnered parents with limited means. Similar to the FTB-A, it is a function of the age and number of dependent children. However, eligibility and payment amounts depend on marital status and separate income tests on primary and secondary earners' taxable incomes. Key policy parameters determining the levels, kinks, and slopes of the FTB-B schedule are: (i) two maximum payments for families with children aged below 5 or between 5 and 18, tr_j^{B1} and tr_j^{B2}; (ii) income test thresholds for primary and secondary earners, $\bar{y}_{pe}^{tr}$ and $\bar{y}_{se}^{tr}$; and (iii) a phase-out rate based on the secondary earner's taxable income, ω_B. Let $y_{pe} = \max(y_{j,\lambda}^m,\ y_j^f)$ and $y_{se} = \min(y_{j,\lambda}^m,\ y_j^f)$ denote the primary and secondary earners' taxable incomes, respectively. The FTB-B benefit per household, tr_j^B, is:

$$tr_j^B = \begin{cases} \Upsilon_1 \times tr_j^{\text{B1}} + \Upsilon_2 \times tr_j^{\text{B2}} & \text{if } y_{pe} \leq \bar{y}_{pe}^{tr} \text{and } y_{se} \leq \bar{y}_{se}^{tr} \\ \\ \Upsilon_1 \times \max\left\{0,\ tr_j^{\text{B1}} - \omega_B(y_{se} - \bar{y}_{se}^{tr})\right\} & \text{if } y_{pe} \leq \bar{y}_{pe}^{tr} \text{and } y_{se} > \bar{y}_{se}^{tr} \\ +\Upsilon_2 \times \max\left\{0,\ tr_j^{\text{B2}} - \omega_B(y_{se} - \bar{y}_{se}^{tr})\right\} & \end{cases} \tag{40}$$

where $\Upsilon_1 = \mathbf{1}_{\{nc_{[0,4],j} \geq 1\}}$ and $\Upsilon_2 = \mathbf{1}_{\{nc_{[0,4],j}=0 \text{ and } nc_{[5,18],j} \geq 1\}}$ are indicator variables representing whether a household aged j has dependent children in the specified age ranges $[a, b]$, and nc denotes the number of children.

Child care subsidy (CCS). The CCS subsidizes formal child care costs for children aged 13 or younger. Like the FTB, the CCS is means-tested based on family income and depends on the age and number of children. However, unlike the FTB, the CCS is also conditional on work.[29] Key parameters determining eligibility and subsidy rate per child include: (i) joint income test thresholds, $\{\bar{y}_1^{sr}, \bar{y}_2^{sr}, \bar{y}_3^{sr}, \bar{y}_4^{sr}, \bar{y}_5^{sr}\}$; (ii) fortnightly work hour test thresholds, $\{0, 8, 16, 48\}$; and (iii) phase-out rates, $\{\omega_c^1, \omega_c^3\}$. The base CCS rate per child, denoted by sr, for a household aged j is given by:

$$sr = \Psi(y_{j,\lambda}, n_{j,\lambda}^m, n_j) \times \begin{cases} sr_1 & \text{if } y_{j,\lambda} \leq \bar{y}_1^{sr} \\ max\{sr_2,\ sr_1 - \omega_c^1\} & \text{if } \bar{y}_1^{sr} < y_{j,\lambda} < \bar{y}_2^{sr} \\ sr_2 & \text{if } \bar{y}_2^{sr} \leq y_{j,\lambda} < \bar{y}_3^{sr} \\ max\{sr_3,\ sr_2 - \omega_c^3\} & \text{if } \bar{y}_3^{sr} \leq y_{j,\lambda} < \bar{y}_4^{sr} \\ sr_3 & \text{if } \bar{y}_4^{sr} \leq y_{j,\lambda} < \bar{y}_5^{sr} \\ sr_4 & \text{if } y_{j,\lambda} \geq \bar{y}_5^{sr}, \end{cases} \tag{41}$$

where $y_{j,\lambda} = \mathbf{1}_{\{\lambda \neq 4\}} y_{j,\lambda}^m + \mathbf{1}_{\{\lambda \neq 3, \ell \neq 0\}} y_j^f + ra_j$ is the joint family income, and ω_c^i is the phase-out rate. $\Psi(y_{j,\lambda}, n_{j,\lambda}^m, n_j)$ is the adjustment factor applied to the base subsidy rate through a test on the lower of the two spouses' work hours if married, or on individual work hours if single. Let $n_j^{min} = min\{n_{j,\lambda}^m, n_j\}$ be the household's minimum work hours. The adjustment factor is:

$$\Psi(y_{j,\lambda}, n_{j,\lambda}^m, n_j) = 0.24_{\{y_{j,\lambda} \leq AU\$70{,}015\ and\ n_j^{min} \leq 8\}} + 0.36_{\{8 < n_j^{min} \leq 16\}} + 0.72_{\{16 < n_j^{min} \leq 48\}} + 1_{\{n_j^{min} > 48\}}$$

Otherwise, $\Psi(y_{j,\lambda}, n_{j,\lambda}^m, n_j) = 0$.

Age pension. The Age pension is a means-tested benefit for retirees based on both income and assets tests, and is independent of contribution history. The pension becomes accessible to households once they reach the qualifying age, $j = J_R$. The pension benefit based on the assets test, denoted as $\mathcal{P}^a(a_j)$, is determined as follows:

$$\mathcal{P}^a(a_j) = \begin{cases} p^{\max} & \text{if } a_j \leq \bar{a}_1^P \\ \max\{0,\ p^{\max} - \omega_a(a_j - \bar{a}_1)\} & \text{if } a_j > \bar{a}_1^P \end{cases} \tag{42}$$

where p^{max} is the maximum pension payment, $\bar{a}_1^P$ is the assets test threshold, and ω_a is the phase-out rate for the assets test.

Similarly, the pension benefit according to the income test, denoted as $\mathcal{P}^y(y_{j,\lambda})$, is given by:

$$\mathcal{P}^y(y_{j,\lambda}) = \begin{cases} p^{\max} & \text{if } y_{j,\lambda} \leq \bar{y}_1^p \\ \max\{0,\ p^{\max} - \omega_y(y_{j,\lambda} - \bar{y}_1^p)\} & \text{if } y_{j,\lambda} > \bar{y}_1^p \end{cases} \tag{43}$$

where $\bar{y}_1^p$ is the income test threshold, and ω_y is the phase-out rate for the income test.

Given $\mathcal{P}^a(a_j)$ and $\mathcal{P}^y(y_{j,\lambda})$, the pension benefit, pen_j, received by a household is:

$$pen_j = \begin{cases} \min\{\mathcal{P}^a(a_j), \mathcal{P}^y(y_{j,\lambda})\} & \text{if } j \geq J_P \text{ and } \lambda = 1, 2 \\ \frac{2}{3}\min\{\mathcal{P}^a(a_j), \mathcal{P}^y(y_{j,\lambda})\} & \text{if } j \geq J_P \text{ and } \lambda = 2, 3 \\ 0 & \text{otherwise} \end{cases} \tag{44}$$

Government budget. At time t, the government collects taxes on consumption, corporate profits, and household income (T_t^C, T_t^K, T_t^I), and issues bonds $(B_{t+1} - B_t)$ to meet its debt obligation $(r_t B_t)$ and its commitment to three spending programs: (i) general government purchase (G_t), (ii) child benefits $(Tr_t =$

[29] In practice, the CCS assesses the number of hours spent on recognized activities, which comprise paid work (self-employment included), unpaid work in a family business, volunteering, and job-seeking activities, among others.

$FTB_t + CCS_t$), and (iii) the Age Pension ($\mathcal{P}_t$). The inter-temporal government budget constraint is:

$$T_t^C + T_t^K + T_t^I + (B_{t+1} - B_t) \quad = \quad G_t + Tr_t + \mathcal{P}_t + r_t B_t \tag{45}$$

4.6 Market structure

Markets are incomplete. Households cannot insure against idiosyncratic earnings and mortality risks by trading state-contingent assets. They can only hold one-period risk-free assets to insure against these risks, and are subject to a no-borrowing constraint, meaning asset holdings are always non-negative.

The model economy is a small open economy where the free flow of foreign capital ensures that the domestic interest rate is maintained at the constant world interest rate r^w. Additionally, the model abstracts from labor market frictions, assuming no search for employment and no adjustment costs when switching between part-time and full-time work.

4.7 The household problem

Households are heterogeneous in age $j \in \{1, 2, \ldots, J\}$, family type $\lambda \in \Lambda$ where $\Lambda = \{1, 2, 3, 4\}$, permanent education realized at birth $\theta \in \Theta$ where $\Theta = \{\theta_L, \theta_H\}$, female human capital $h^f_{j,\theta,\ell} \in H$ where $H = [h_{min}, h_{max}] \subset \mathcal{R}^+$, asset holdings $a_j \in A$ where $A = [a_{min}, a_{max}] \subset \mathcal{R}^+$, and transitory shocks to male and female labor income, ϵ^m_j and $\epsilon^f_j \in S$ where $S \subset \mathcal{R}$.

To simplify the description of the household problem below, age and time subscripts (j and t) are omitted where appropriate. Define $Z = \Lambda \times A \times H \times \Theta \times S \times S$ as the state space for households aged j. Let $z = \left\{\lambda_j, a_j, h^f_{j,\theta,\ell}, \theta, \eta^m_j, \eta^f_j\right\} \in Z$ be the current period state vector, and $z_+ = \{\lambda_{j+1}, a_{j+1}, h^f_{j+1,\theta,\ell}, \theta, \eta^m_{j+1}, \eta^f_{j+1}\} \in Z$ be the state vector of the next period.

4.7.1 Working-age households

The decision process of working-age households varies by family type λ. Married and single-mother households ($\lambda = \{1, 2, 4\}$) must decide on female labor supply, whereas single male households ($\lambda = 3$) do not. Specifically, their decision-making processes are as follows:

Working-age married and single-mother households. Married or single-mother households decide on joint consumption, savings, and labor supply for the female member. Given the behavioral, technology, and policy parameters, and for a given state vector z realized at the beginning of working age $j < J_R$, they go through the following decision-making procedure:

1. ***Female employment status*** (ℓ): Every household considers three possible employment types (or extensive margins of labor supply) for its female member: staying at home ($\ell = 0$), working part-time ($\ell = 1$), or working full-time ($\ell = 2$). The chosen employment status ℓ for a woman then determines her:

   (a) Lower and upper bounds of work hours

$$n = \begin{cases} 0 & \text{if staying at home } (\ell = 0) \\ (0, \bar{n}_1] & \text{if working part-time } (\ell = 1) \\ (\bar{n}_1, 1) & \text{if working full-time } (\ell = 2) \end{cases} \tag{46}$$

   where $\bar{n}_1$ is the normalized work hour ceiling for part-time employment. I assume $l^f > 0$, implying the maximum full-time work hours are strictly less than 1.

   (b) Next-period human capital according to the law of motion

$$log(h^f_{j+1,\theta,\ell}) = log(h^f_{j,\theta,\ell}) + (\xi_{1,\theta,\ell} - \xi_{2,\theta,\ell} \times j)\, \mathbf{1_{\{\ell \neq 0\}}} - \delta_h(1 - \mathbf{1_{\{\ell \neq 0\}}}) \tag{47}$$

where δ_h is the depreciation rate of human capital when not working. A working woman, on the other hand, accumulates human capital at a diminishing rate over age. Her human capital gain rate is governed by the coefficient $\xi_{1,\theta,\ell} - \xi_{2,\theta,\ell} \times j$, a composite of two parameters $\xi_{1,\theta,\ell}$ and $\xi_{2,\theta,\ell}$ that depend on education and employment status.[30]

2. ***ℓ-specific next-period assets*** (a_+) ***and labor supply*** (n): For each employment status $\ell \in \{0, 1, 2\}$, the household then chooses ℓ-specific joint consumption $c(\ell, z)$, next-period asset holdings $a_+(\ell, z)$, and female work hours $n(\ell, z)$ from a choice set $\mathcal{C} \equiv \{(c, n, a_+) \in \mathcal{R}^{++} \times [0, 1) \times \mathcal{R}^+\}$ to maximize its expected lifetime utility. That is,

   (a) The household decides on the ℓ-specific optimal allocation of next-period assets $a^*_+(\ell, z)$ by solving the following value function:

$$V(z,\ell) \quad = \quad \max_{c,\,n,\,a_+} \left\{ u(c, l^m, l^f, \theta, \lambda) + \beta \sum_{\Lambda} \int_{S^2} V(z_+)\, d\Pi(\lambda_+, \eta^m_+, \eta^f_+ \mid \lambda, \eta^m, \eta^f) \right\} \tag{48}$$

s.t.

$$\begin{aligned}
(1+\tau^c)c + (a_+ - a) + \mathbf{1}_{\{\lambda=1,4\}} n \times CE_\theta &= y_\lambda + \mathbf{1}_{\{\lambda=1,4\}} FTB_\theta + beq - T_\lambda \\
l^f &= 1 - n - \mathbf{1}_{\{\lambda=1,2\}} \frac{\chi_{\lambda,\ell}}{2} - \mathbf{1}_{\{\lambda=4\}} \chi_{\lambda,\ell} \\
l^m &= 1 - n^m_\lambda - \frac{\chi_{\lambda,\ell}}{2} \quad \text{if } \lambda \neq 4 \\
c &> 0 \\
a_+ &\geq 0
\end{aligned} \tag{49}$$

   where $y_\lambda = \mathbf{1}_{\{\lambda \neq 4\}} y^m_\lambda + \mathbf{1}_{\{\ell \neq 0\}} y^f + ra$ is the household market income; $CE_\theta = w(1 - sr) \sum_{k=1}^{nc_\theta} \kappa_i$ is the net formal child care expense per work hour; sr is the CCS rate; κ_i is the hourly child care cost for the k^{th} child as a fraction of wages; $FTB_\theta = nc_\theta \times tr^A + tr^B$ is the total FTB transfer comprising tr^A from (39) and tr^B from (40); τ^c is the consumption tax; and $T_\lambda = \mathbf{1}_{\{\lambda \neq 4\}} tax^m + tax^f$ is the total income tax payment where tax^i for $i \in \{m, f\}$ is calculated using the tax function (38). Leisure is strictly positive, such that $l^i \in (0, 1]$. Bequest motives are not operative. Households are born with no wealth ($a_1 = 0$), and each living working-age household aged j receives a uniform lump-sum accidental bequest, beq, from deceased households in the same period.

   (b) For each $a_+(\ell, z)$, the household simultaneously solves (numerically) for the corresponding female work hours $n(\ell, z) = n(a_+|\ell, z)$ that satisfies the intra-temporal trade-off equation:[31]

$$n(a_+|\ell,z) = \frac{a_+(\ell,z) + \frac{\nu}{1-\nu}\left(1 - \mathbf{1}_{\{\lambda=1,2\}}\frac{\chi_{\lambda,\ell}}{2} - \mathbf{1}_{\{\lambda=4\}}\chi_{\lambda,\ell}\right)\left(1 - EMTR_{y^f,\lambda}\right) we^f_{\theta,\ell} - (NLI_\lambda - T_\lambda)}{we^f_{\theta,\ell}\left[1 + \frac{\nu}{1-\nu}\left(1 - EMTR_{y^f,\lambda}\right)\right] - \mathbf{1}_{\{\lambda=1,4\}} CE_\theta} \tag{50}$$

   On the right-hand side (RHS), $EMTR_{n,\lambda}$ represents the Effective Marginal Tax Rate (EMTR) with respect to female labor earnings y^f, and NLI_λ is the total non-labor income. These terms are expressed as:

[30] Human capital gains reflect experience, skill acquisition, and other improvements derived from work that translate into higher future labor returns. Thus, the law of motion employed is grounded in the learning-by-doing framework rather than on-the-job training. The latter would require an agent to actively invest in human capital by splitting her work hours between productive and training times. A part of the complication of such a setup arises from the difficulty in identifying returns to productive time in data, as we do not observe them.

[31] A woman's future human capital $h^f_{j+1,\theta,\ell}$ is conditional only on her current employment status ℓ and education θ. Her work hours n do not affect her future human capital since employment status ℓ is chosen first and confines her labor hour choices within predefined and mutually exclusive ranges. In other words, because work hours cannot influence her current employment status, they have no impact on her human capital accumulation process.

$$
\begin{aligned}
EMTR_{y^f,\lambda} &= \underbrace{\frac{\partial T_\lambda}{\partial y^f}(n)}_{MTR} + \mathbf{1}_{\{\lambda=1,4\}} \left(\underbrace{\overbrace{\frac{CE_\theta}{we^f_{\theta,\ell}}}^{\text{net child care costs}} + \left(\overbrace{wn \times \frac{\partial sr}{\partial y^f}(n)}^{\text{CCS phase-out rate}} - \overbrace{\frac{n}{e^f_{\theta,\ell}} \times \frac{\partial sr}{\partial n}}^{\text{CCS phase-in rate}} \right) \sum_{i=1}^{nc_\theta} \kappa_i}_{CCS} \right) \\
&\quad + \mathbf{1}_{\{\lambda=1,4\}} \left(\overbrace{\underbrace{nc_\theta \times \frac{\partial tr^A}{\partial y^f}(n)}_{FTB\text{-}A} + \underbrace{\frac{\partial tr^B}{\partial y^f}(n)}_{FTB\text{-}B}}^{\text{FTB phase-out rates}} \right) \qquad (51) \\
NLI_\lambda &= (1+r)a + \mathbf{1}_{\{\lambda=1,4\}} \left(nc_\theta \times tr^A(n) + tr^B(n) \right) \qquad (52)
\end{aligned}
$$

Equation (50) shows that the income tax T_λ affects labor supply through two primary channels: (i) it directly reduces non-labor income NLI_λ, creating a negative income effect (IE) that encourages more labor hours, and (ii) the marginal tax rate $\frac{\partial T_\lambda}{\partial y^f}(n)$ that distorts female labor supply. Equation (51) also demonstrates that parents face additional distortions. First, hourly child care expenses make work inherently more costly for mothers. Second, the phase-out rates of the FTB and CCS programs function as implicit marginal tax rates on labor income. Although the CCS subsidy rate (sr) lowers the net child care costs and thus the $EMTR_{y^f,\lambda}$, the CCS phase-out rate due to means-testing (net of the phase-in rate due to work hour test) counteracts this by adding to the $EMTR_{y^f,\lambda}$. Parent households also face increased $EMTR_{y^f,\lambda}$ stemming from the FTB phase-out rates. These results align with the simulated EMTR schedules (Figures 2 and 3) in Section 2, which show the extent to which the FTB and CCS phase-out rates negate the intended work incentive effects of the CCS program. Further details on the derivation of Equation (50) are provided in Subsection E.1 of the Appendix.

3. ***Optimal choice*** (c^*, n^*, a_+^*): Each ℓ-specific optimal value $V(\ell, z)$ is associated with an optimal pair $a_+^*(\ell, z)$ and $n^*(\ell, z)$. The household picks an employment status ℓ^* for its female member such that:

$$
\ell^* = argmax \{MAX(V(0,z), V(1,z), V(2,z)\}
$$

The maximal attainable utility is therefore $V^*(z) = V(\ell^*, z)$. The corresponding optimal next-period assets and female work hours are $a_+^* = a_+^*(\ell^*, z)$ and $n^* = n^*(\ell^*, z)$, respectively.[32] Given a_+^* and n^*, the optimal consumption c^* is obtained via the household budget constraint (49).

Working-age single male households. Single male households do not make labor supply decisions and follow an exogenous labor supply profile over their life cycle. Given the behavioral, technology, and policy parameters, and for a given state vector z realized at the beginning of each period $j < J_R$, they choose an optimal pair $\{a_+^*(z), c^*(z)\}$ to maximize their expected lifetime utility subject to the budget constraint (54). The problem for single male households reduces to a consumption-savings problem:

$$
V(z) = \max_{c,\, a_+} \left\{ u(c,\theta) + \beta \sum_{\Lambda} \int_{S^2} V(z_+)\, d\Pi(\lambda_+, \eta_+^m \mid \lambda, \eta^m) \right\} \qquad (53)
$$

s.t.

$$
\begin{aligned}
(1+\tau^c)c + (a_+ - a) &= y_\lambda + beq - T_\lambda \\
l^m &= 1 - n_\lambda^m \\
c &> 0 \\
a_+ &\geq 0
\end{aligned} \qquad (54)
$$

where $y_\lambda = y_\lambda^m + ra$, and $T_\lambda = tax^m$ based on the tax function (38).

[32] To break a tie in cases where $V(z|\ell_a) = V(z|\ell_b)$ and $\ell_a \neq \ell_b$, I assume the household chooses ℓ_a if $n(\ell_a) < n(\ell_b)$. In other words, households always prefer ℓ with fewer work hours.

4.7.2 Retirees

Retirement at age J_R is mandatory, at which point the education and transitory shock states become absorptive states. Additionally, since retirees do not have dependent children, they are not eligible for child benefits. They may be eligible for the Age Pension, which is means-tested based on their income and assets. The pension payouts are not conditional on earnings history but vary according to family type, λ. A single household receives two-thirds of the pension payment available to a couple. The state vector of a retired household aged $J_R \leq j \leq J$ therefore reduces to $z^R = \{\lambda, a\} \in \{1,2,3,4\} \times R_+$, and their choice set is $\mathcal{C}^{\mathcal{R}} \equiv \{(c, a_+) \in \mathcal{R}^{++} \times \mathcal{R}^{+}\}$. The retired household's optimization problem simplifies to:

$$V(z^R) \quad = \quad \max_{c,\, a_+} \left\{ u(c, \lambda) + \beta \sum_{\Lambda} V(z_+^R)\, d\Pi(\lambda_+|\lambda) \right\} \tag{55}$$

$$\begin{aligned} \text{s.t.} \quad (1+\tau^c)c + (a_+ - a) \quad &= \quad ra + pen \\ c \quad &> \quad 0 \\ a_+ \geq 0 \quad & \mathit{and} \quad a_{J+1} = 0 \end{aligned} \tag{56}$$

where pen is the Age Pension described in Equation (44).

4.8 Competitive equilibrium

The distribution of households. Let $\phi_t(z)$ denote the stationary density and $\Phi_t(z)$ the cumulative distribution of households aged j at time t, unadjusted for population growth.[33] Given that all households enter the economy with identical female human capital set at unity ($h^f_{j=1} = 1$) and no assets ($a_{j=1} = 0$), the initial distribution of newborn households (aged $j = 1$) in every period t is determined by:

$$\begin{aligned} \sum_{\Lambda \times \Theta} \int_{A \times H \times S^2} d\Phi_t(\lambda_1, a_1, h^f_1, \theta, \eta^m_1, \eta^f_1) \quad &= \quad \sum_{\Lambda \times \Theta} \int_{S^2} d\Phi_t(\lambda_1, 0, 1, \theta, \eta^m_1, \eta^f_1) = 1, \quad \text{and} \\ \phi_t(\lambda_1, 0, 1, \theta, \eta^m_1, \eta^f_1) \quad &= \quad \prod_{x \in \{\lambda_1, \theta, \eta^m_1, \eta^f_1\}} \pi(x) \end{aligned}$$

where h^f_j is shorthand for $h^f_{j,\theta,\ell}$, and $\pi(x)$ is the unconditional probability density of a state vector $x \in \{\lambda_1, \theta, \eta^m_1, \eta^f_1\}$ for newborns, with $\lambda_1 \in \Lambda,\ \theta \in \Theta$, and $\eta^m_1, \eta^f_1 \in S$.

From age $j = 2$ onward, the next-period population density $\phi_+(z_+)$ evolves according to the following law of motion:

$$\phi_+(z_+) = \sum_{\Lambda \times \Theta} \int_{A \times H \times S^2} \mathbf{1}_{\{a_+ = a_+(z,\Omega),\, h^f_+ = h^f_+(z,\Omega)\}} \times \pi(\lambda_+|\lambda) \times \pi(\eta^m_+|\eta^m) \times \pi(\eta^f_+|\eta^f)\, d\Phi(z) \tag{57}$$

where we suppress the age and time subscripts for brevity; Ω is a vector of behavioral, technology and policy parameters at time t; $\pi(\eta^i_+|\eta^i)$ is the conditional probability of η^i_+ given η^i, obtained from discretizing the AR(1) stochastic earnings process ϵ^i, as shown in Equations (31) and (32), for $i \in \{m, f\}$; and $\pi(\lambda_+|\lambda)$ is the transition probability of λ_+ given λ taken from Table 1. Assets and human capital are endogenous states that evolve continuously. The share of households on each pair of $\{a_+, h^f_+\}$ is obtained through bilinear interpolation of a_+ and h^f_+ on their respective discretized domains.

Aggregate variables. There are J number of generations living in every time period t. Let $\mu_{j,t}$ denote the share of households belonging to cohort j at time t, such that $\sum_{j=1}^{J} \mu_{j,t} = 1$. Taking into account the optimal allocations $\{c(z_j, \Omega_t),\ n(z_j, \Omega_t),\ a_+(z_j, \Omega_t)\}_{j=1}^{J}$ for a model economy governed by Ω_t in period t, the aggregate consumption C_t, wealth A_t, female labor force participation rate LFP_t, male work hours NM_t, female work hours NF_t, and labor supply in efficiency units for males LM_t and females LF_t are expressed as below, with

[33] Since the population growth rate g_N is constant, it is factored in as a weighting factor when aggregating across cohorts. Mortality, which is age-dependent, is incorporated trough the transition probabilities of family type λ, as described in Table 1. Thus, $\phi_t(z)$ also reflects the share of surviving households aged j at time t.

the subscript t omitted for simplicity.[34]

$$
\begin{aligned}
C &= \sum_{j=1}^{J} \sum_{\Lambda\times\Theta} \int_{A\times H\times S^2} c(z_j,\Omega)\mu_j \, d\Phi(z_j) \\
A &= \sum_{j=1}^{J} \sum_{\Lambda\times\Theta} \int_{A\times H\times S^2} a(z_j,\Omega)\mu_j \, d\Phi(z_j) \\
LFP &= \sum_{j=1}^{J_R-1} \sum_{\Lambda\times\Theta} \int_{A\times H\times S^2} \mathbf{1}_{\{n(z_j,\Omega)>0\}}\mu_j \, d\Phi(z_j) \\
NM &= \sum_{j=1}^{J_R-1} \sum_{\Lambda\times\Theta} \int_{A\times H\times S^2} n^m_{j,\lambda}\mu_j \, d\Phi(z_j) \\
NF &= \sum_{j=1}^{J_R-1} \sum_{\Lambda\times\Theta} \int_{A\times H\times S^2} n(z_j,\Omega)\mu_j \, d\Phi(z_j) \\
LM &= \sum_{j=1}^{J_R-1} \sum_{\Lambda\times\Theta} \int_{A\times H\times S^2} h^m_{j,\lambda} e^{\theta+\eta^m_j} n^m_{j,\lambda}\mu_j \, d\Phi(z_j) \\
LF &= \sum_{j=1}^{J_R-1} \sum_{\Lambda\times\Theta} \int_{A\times H\times S^2} h^f_{j,\theta,\ell} e^{\theta+\eta^f_j} n(z_j,\Omega)\mu_j \, d\Phi(z_j)
\end{aligned}
$$

The aggregate government variables at time t are

$$
\begin{aligned}
T^C &= \tau^c C \\
T^K &= \tau^k (Y - wAL)
\end{aligned}
$$

where in every period t, $tax(z_j,\Omega)$ is calculated using Equation (38); $ftb(z_j,\Omega) = tr^A(z_j,\Omega) \times nc_{j,\theta} + tr^B(z_j,\Omega)$ is the sum of FTB-A of Equation (39) and FTB-B of Equation (40); $ccs(z_j,\Omega)$ is the CCS with a subsidy rate sr_j from Equation (41); and $pen(z^R_j,\Omega)$ is the Age Pension from Equation (44). In the company tax (T^K) equation, L refers to the total labor supply in efficiency units, an aggregate of both LM and LF.

Definition of competitive equilibrium. Given the household, firm, and government policy parameters, the demographic structure, the goods and factor prices, a steady-state equilibrium at time t is characterized by the following conditions:

(a) The collection of individual household decisions $\{c(z_j,\Omega_t),\, n(z_j,\Omega_t),\, a_+(z_j,\Omega_t)\}_{j=1}^{J}$ solves the household problems (48), (53), and (55);

(b) The firm chooses labor and capital inputs to solve its profit maximization problem (35);

(c) The government periodic budget constraint (45) is satisfied;

(d) The factor markets clear, such that $K^s_t = K^d_t = K_t$ and $L^s_t = L^d_t = L_t$, where

$$K^s_t = A_t - B_{F,t} - B_t, \tag{58}$$

$$L^s_t = LM_t + LF_t; \tag{59}$$

where L^s_t an unweighted sum of LM_t and LF_t;

(e) The goods market clears

$$
\begin{aligned}
Y_t &= C_t + I_t + G_t + NX_t \\
NX_t &= (1+n)(1+g)B_{F,t+1} - (1+r)B_{F,t} \\
B_{F,t} &= A_t - K_t - B_t
\end{aligned}
$$

[34]Since the household mass is normalized to one, aggregate variables are equivalent to per-household variables. Per capita variables in each period t can be obtained by normalizing the aggregate values by the total population (i.e., the number of adults).

where $I_t = (1+n)(1+g)K_{t+1} - (1-\delta)K_t$ is investment; NX_t is the trade account, with $NX_t > 0$ denoting a trade account surplus; $B_{F,t}$ represents the foreign capital under the no-arbitrage condition for a small open economy, where $B_{F,t} > 0$ indicates a capital outflow (or a capital account deficit);[35]

(f) The lump-sum bequest is the total untapped private wealth left by deceased agents at the beginning of time t. Given the known survival probabilities, the total amount of bequests available can be accurately predicted. That is,

$$BQ_t = \sum_{j=1}^{J} \sum_{\Lambda \times \Theta} \int_{A \times H \times S^2} (1-\psi_{j,\lambda})(1+r_t) a(z_j, \Omega_t)\, d\Phi_t(z_j).$$

where $\psi_{j,\lambda}$ is the conditional survival probability for household type λ at age j.[36] The bequest to each surviving household is determined by a general formula:

$$beq_{j,t} = \left[\frac{b_{j,t}}{\sum_{j=1}^{J} b_{j,t}\, m_{j,t}}\right] BQ_t,$$

where $b_{j,t}$ is the share of bequests for surviving households aged j at time t, and $m_{j,t}$ is the mass of households.[37] I assume bequests are uniformly distributed among living working-age households. In this case, $b_{j,t} = \frac{1}{JR-1}$ if $j < J_R$ and $b_{j,t} = 0$ otherwise. Thus, the amount of bequest to a household aged j at time t is:

$$beq_{j,t} = \frac{BQ_t}{\sum_{j=1}^{J_R-1} m_{j,t}}$$

4.9 Welfare

Welfare refers to ex-ante welfare, which concerns the long-run well-being of newborn households under the veil of ignorance. This theoretical construct assumes that households, upon entering the economy, possess perfect information about the economic environment, including their own preferences, constraints, technology, and policy parameters. All policy reforms are anticipated and fully incorporated into the households' decision processes over their life cycles. That is, there is no element of surprise.[38]

The normative welfare criterion is utilitarian. No additional assumptions about the societal aversion to inequality are imposed. I assess welfare changes using the Consumption Equivalent Variation (CEV), which measures the consumption changes necessary to make a newborn household in the benchmark economy as well off as its counterpart in the reformed economy. Formally, for a household type z_j, I define its CEV at time $t = T$ as:

$$W(c_T, l_T) = W\left[c_0 \times (1 + CEV(z_j, \Omega_T)),\ l_0\right] \tag{60}$$

[35] Refer to Section J in the Appendix for a detailed explanation of $B_{F,t}$ and NX_t.

[36] For a married household ($\lambda = 1, 2$), $\psi_{j,\lambda} = 1 - (1-\psi_j^m)(1-\psi_j^f)$ is the probability that both spouses survive and the household maintains its marital status.

[37] Uniform accidental bequests esnure that the wealth of deceased households is distributed among the living, thus maintaining aggregate wealth. Alternative methods to handle leftover wealth include introducing an annuity market, where households fully annuitize their savings through contracts with financial intermediaries. However, annuity markets are relatively small worldwide, including in Australia where only 3.5% of assets in pension accounts are held in annuities, with a limited number of providers (see 2023 Treasury's discussion paper). A different method involves incorporating a parent-child linkage in the household's objective function, though this is computationally expensive as it requires an additional continuous state element to track wealth bequeathed to children. This increases the dimensionality of the problem and is not desirable. Therefore, given the small size of aggregate accidental bequests and the focus of this study on child benefits aimed at supporting low-income parents, introducing bequest heterogeneity might lead to unnecessary complexity.

[38] Non-newborn households aged $j = 2, \ldots, J$ living in the reform period t would not have anticipated the reform. These households already committed to their initial decisions under the status quo regime when they entered the economy in period $t - j + 1$. Thus, the reform exerts different welfare effects on these cohorts living through the transition. However, due to computational limitations, I do not study transitional dynamics of reforms.

where $W(c_T, l_T)$ represents the optimal expected lifetime utility, $V(z_j, \Omega_T)$, expressed as a function of the optimal consumption, $c_T := c_T(z_j, \Omega_T)$, and leisure, $l_T := l_T(z_j, \Omega_T)$, in the new steady state T. Given this definition, together with the household preferences from Subsection (4.2), we can derive a closed-form solution for CEV:

$$CEV(z_j, \Omega_T) = \left[\left(\frac{V(z_j, \Omega_T)}{V(z_j, \Omega_0)} \right)^{\frac{1}{\nu\left(1-\frac{1}{\gamma}\right)}} - 1 \right] \times 100 \tag{61}$$

where Ω_0 and Ω_T denote the policy parameters in the status quo at time $t = 0$ and the new regime at time $t = T$, respectively.

The total CEV at time T is obtained by aggregating households' CEVs across z_j, weighted by their population share, $\mu_{j,T}$:

$$CEV_{total} = \sum_{j=1}^{J} \sum_{\Lambda \times \Theta} \int_{A \times H \times S^2} CEV(z_j, \Omega_T) \mu_{j,T} \, d\Phi(z_j)$$

The optimal policy over a policy parameter space $x \in X$ is formally defined as:

$$x^* = arg\ max\ \{CEV_{total}\}$$

Explanation: To illustrate how the CEV method captures welfare changes, consider a simple two-agent economy. Both agents, A and B, have identical CRRA preferences and differ only with respect to their initial levels of consumption. Suppose the status quo (SQ) endowments are $\{A : 25, B : 75\}$, making A relatively worse off. Two alternative regimes are introduced: Regime 1 ($R1$), where a transfer is made from A to B, and Regime 2 ($R2$) where the opposite policy is pursued. The transfer amounts are identical in both cases. Figure 6 shows the utility possibility frontier, the Social Welfare Functions (SWF), and the implied individual CEVs for the status quo and the two reformed economies.

The utilitarian SWF accounts for the concavity of the household utility function. It implies that for the same amount of transfer, a redistribution from the worse-off agent A to the better-off agent B deteriorates the overall welfare, and vice versa. Assuming a unit mass population, the total CEV in Regime 1 is $CEV_{R1} = 0.5(-40\%) + 0.5(13.33\%) = -13.34\%$, representing an average decline in consumption of 13.34% relative to the status quo. Conversely, the same transfer from B to A in Regime 2 results in $CEV_{R2} = 13.34\%$, a welfare improvement equivalent to an average increase of 13.34% in consumption. This simple example helps explain how the welfare changes of single mothers, as a vulnerable group, could significantly influence the overall welfare outcomes throughout most policy experiments conducted in this study.

Notably, if the distribution of agents is not uniform, a policy move such as $R1$ could be welfare-improving at the aggregate level. Consider a case where A makes up only 20% of the population. This leads to $CEV_{R1} = 0.2(-40\%) + 0.8(13.33\%) \approx 2.67\%$, an increase in the overall welfare while the losses are concentrated in A. Because vulnerable groups, such as single mothers, often form a minority of the population, this highlights the need to carefully consider the distribution of welfare changes in policy evaluations.

4.10 Welfare decomposition

Adapting the approach of Bhandari et al. (2021), I decompose welfare changes in consumption and leisure into 3 components: *Efficiency (or Level)*, *Distribution (or Equity)*, and *Insurance*.

Suppose an economy is at its pre-reform (initial) steady state at time $t = 0$, and a post-reform (final) steady state at time $t = T$. Let $c_t := c(z_j, \Omega_t)$ and $l_t := 1 - n(z_j, \Omega_t)$ be the optimal consumption and leisure allocation for a household aged j with state vector z at time t. Suppressing the age subscript j, the decomposition of

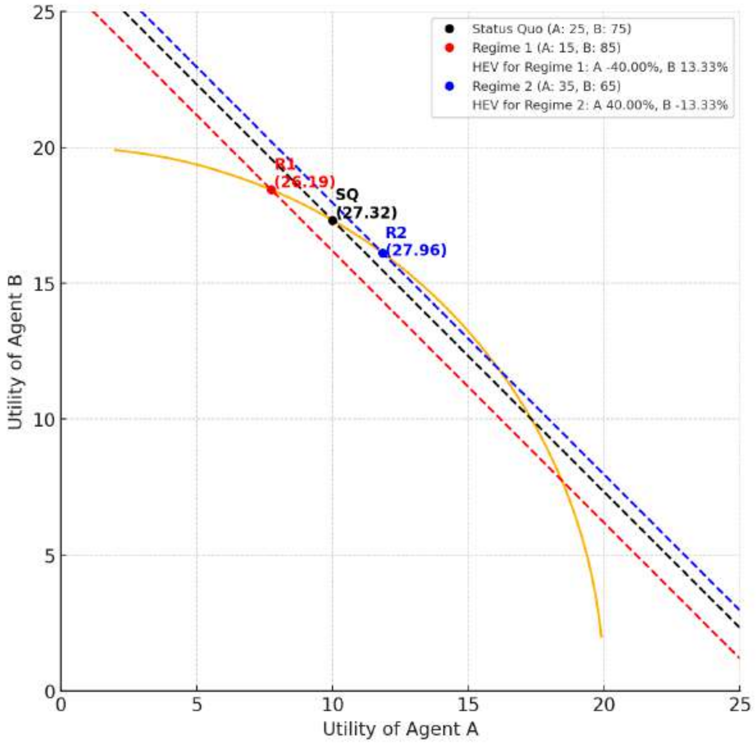

Figure 6: **Example Utilitarian Social Welfare Function and Consumption Equivalent Variation (CEV).**
Notes: Each agent i's preference, for $i \in \{A, B\}$*, is represented by* $u(c_i) = \dfrac{c_i^{1-1/\gamma}}{1-1/\gamma}$*. The initial allocation is* $(c_A, c_B) = (25, 75)$*, making A poorer in consumption. Regime 1 transfers 10 units of consumption from A to B, and vice versa for regime B. Dashed lines represent the Social Welfare Function for each case.*

consumption and leisure in time t is as follows:

$$c_t = E(c_t) \times \frac{E_i(c_t)}{E(c_t)} \times \frac{c_t}{E_i(c_t)} = C_t \times d_t^c \times (1+\epsilon_t^c) \tag{62}$$

$$l_t = E(l_t) \times \frac{E_i(l_t)}{E(l_t)} \times \frac{l_t}{E_i(l_t)} = E(l_t) \times d_t^l \times (1+\epsilon_t^l) \tag{63}$$

For a household of age j and state vector z, the first term $C_t = E(c_t)$ in the consumption equation captures the expected or average consumption level. In the second term, i represents a characteristic that the household shares with a subset of the population, such as family type λ or education θ. Denoting a group by its characteristic i, $d_t^c = \dfrac{E_i(c_t)}{E(c_t)}$ is therefore the household's ex-ante consumption share, which is the average consumption of group i relative to the population average. The last term, $1 + \epsilon_t^c = \dfrac{c_t}{E_i(c_t)}$, is the household's ex-post consumption risk, defined as the realized consumption level relative to its expected consumption as a member of group i. The decomposition for leisure in Equation (63) follows a similar structure, with its components interpreted analogously.

Following the scheme above, the consumption and leisure changes between the two economies can be written as

$$1 + \Delta c = \frac{c_T}{c_0} = \underbrace{\frac{C_T}{C_0}}_{(a)\text{ Efficiency/Level}} \times \underbrace{\frac{d_T^c}{d_0^c}}_{(b)\text{ Distibution/Equity}} \times \underbrace{\frac{1+\epsilon_T^c}{1+\epsilon_0^c}}_{(c)\text{ Insurance}} \tag{64}$$

$$1 + \Delta l = \frac{l_T}{l_0} = \frac{E(l_T)}{E(l_0)} \times \frac{d_T^l}{d_0^l} \times \frac{1+\epsilon_T^l}{1+\epsilon_0^l} \tag{65}$$

Equations (64) and (64) express consumption and leisure changes due to a reform in terms of three components. Consider the case of consumption. Term (a), $\dfrac{C_T}{C_0}$, is the change in expected or average consumption level for a household of age j in the new regime relative to the status quo, reflecting the efficiency or level effect. Aggregating welfare changes from these consumption level changes over the life cycle captures the *allocative efficiency effect*. Term (b), $\dfrac{d_T^c}{d_0^c}$, is the change in the ex-ante share of consumption, representing the *distributional (or equity) effect* of the reform. Finally, term (c), $\dfrac{1+\epsilon_T^c}{1+\epsilon_0^c}$, is the change in the degree to which the household's realized consumption deviates from its expectation (for being in group i), reflecting the difference in its ex-post consumption risks between time 0 and T, and thus the *insurance effect* of the reform.

From Equation (64), post-reform consumption allocations can be decomposed into three terms, reflecting the different stages of changes:

$$\hat{c}_E = \left(\frac{C_T}{C_0}\right) \times c_0 \tag{66}$$

$$\hat{c}_D = \left(\frac{C_T}{C_0} \times \frac{d_T^c}{d_0^c}\right) \times c_0 = \frac{d_T^c}{d_0^c} \times \hat{c}_E \tag{67}$$

$$\hat{c}_I = \left(\frac{C_T}{C_0} \times \frac{d_T^c}{d_0^c} \times \frac{1+\epsilon_T^c}{1+\epsilon_0^c}\right) \times c_0 = \frac{1+\epsilon_T^c}{1+\epsilon_0^c} \times \hat{c}_D = c_T \tag{68}$$

The case of leisure from Equation (64) is analogous:

$$\hat{l}_E = \left(\frac{E(l_T)}{E(l_0)}\right) \times l_0 \tag{69}$$

$$\hat{l}_D = \left(\frac{E(l_T)}{E(l_0)} \times \frac{d_T^l}{d_0^l}\right) \times l_0 = \frac{d_T^l}{d_0^l} \times l_E \tag{70}$$

$$\hat{l}_I = \left(\frac{E(l_T)}{E(l_0)} \times \frac{d_T^l}{d_0^l} \times \frac{1+\epsilon_T^l}{1+\epsilon_0^l}\right) \times l_0 = \frac{1+\epsilon_T^l}{1+\epsilon_0^l} \times \hat{l}_D = l_T \tag{71}$$

I then proceed by decomposing the overall welfare changes into two sets of components. The first set comprises effects stemming from the changes in consumption from c_0 to c_T. Analogously, the second set addresses changes in leisure from l_0 to l_T.

The welfare effect due to consumption changes (Δc) is captured by fixing leisure at its status quo level l_0, and is decomposed into consumption allocative efficiency effect (CEV_{CE}), consumption distributional/equity effect (CEV_{CD}), and consumption insurance effect (CEV_{CI}). Given the CEV definition (60), and the post-reform consumption components in (66), (67), and (68), these effects are formally defined as

$$\text{Allocative efficiency effect of } \Delta c : V_{CE} := W(\hat{c}_E, l_0) = W\left(c_0 \times (1+CEV_{CE}),\ l_0\right) \tag{72}$$

$$\text{Distributive/equity effect of } \Delta c : V_{CD} := W(\hat{c}_D, l_0) = W\left(\hat{c}_E \times (1+CEV_{CD}),\ l_0\right) \tag{73}$$

$$\text{Insurance effect of } \Delta c : V_{CI} := W(c_T, l_0) = W\left(\hat{c}_D \times (1+CEV_{CI}),\ l_0\right) \tag{74}$$

where I suppress notations for the state vector z and policy parameter vector Ω. Once the consumption effects are accounted for, consumption is held constant at the new optimal allocation, c_T. The leisure allocative efficiency effect (CEV_{LE}), leisure distributional/equity effect (CEV_{LD}), and leisure insurance effect (CEV_{LI}) due to changes in leisure (Δl) are then defined as

$$\text{Allocative efficiency effect of } \Delta l : V_{LE} := W(c_T, \hat{l}_E) = W\left(c_T \times (1+CEV_{LE}),\ l_0\right) \tag{75}$$

$$\text{Distributive/equity effect of } \Delta l : V_{LD} := W(c_T, \hat{l}_D) = W\left(c_T \times (1+CEV_{LD}),\ \hat{l}_E\right) \tag{76}$$

$$\text{Insurance effect of } \Delta l : V_{LI} := W(c_T, l_T) = W\left(c_T \times (1+CEV_{LI}),\ \hat{l}_D\right) \tag{77}$$

The solutions to Equations (72)-(77) provide the decomposed welfare effects of consumption and leisure changes in the final steady state at time T. Given the household preferences in Subsection 4.2, their closed-form solutions can be expressed as:

$$CEV_{CE} = \left[\left(\frac{V_{CE}(z_j, \psi_T)}{V_0(z_j, \psi_0)} \right)^{\dfrac{1}{\nu\left(1-\frac{1}{\gamma}\right)}} - 1 \right] \times 100 \quad ; \quad CEV_{CD} = \left[\left(\frac{V_{CD}(z_j, \psi_T)}{V_{CE}(z_j, \psi_T)} \right)^{\dfrac{1}{\nu\left(1-\frac{1}{\gamma}\right)}} - 1 \right] \times 100$$

$$CEV_{CI} = \left[\left(\frac{V_{CI}(z_j, \psi_T)}{V_{CD}(z_j, \psi_T)} \right)^{\dfrac{1}{\nu\left(1-\frac{1}{\gamma}\right)}} - 1 \right] \times 100 \quad ; \quad CEV_{LE} = \left[\left(\frac{V_{LE}(z_j, \psi_T)}{V_{CI}(z_j, \psi_T)} \right)^{\dfrac{1}{\nu\left(1-\frac{1}{\gamma}\right)}} - 1 \right] \times 100$$

$$CEV_{LD} = \left[\left(\frac{V_{LD}(z_j, \psi_T)}{V_{LE}(z_j, \psi_T)} \right)^{\dfrac{1}{\nu\left(1-\frac{1}{\gamma}\right)}} - 1 \right] \times 100 \quad ; \quad CEV_{LI} = \left[\left(\frac{V_{LI}(z_j, \psi_T)}{V_{LD}(z_j, \psi_T)} \right)^{\dfrac{1}{\nu\left(1-\frac{1}{\gamma}\right)}} - 1 \right] \times 100$$

5 Calibration

The economy is modeled on a balanced growth path, where aggregate consumption, investment, and capital grow at a constant rate of $g = g_A + g_N$, while the time endowment for work and leisure is fixed. The parametric functions for preferences and technology are chosen to reflect the observed macroeconomic facts and to ensure comparability with the past research on related issues.

I calibrate the model to match key statistics of the Australian economy from 2012 to 2018, a period of relative stability in macroeconomic indicators, including household consumption and asset growth.[39] Externally calibrated parameters are summarized in Table 2. These parameters are based on estimates from the HILDA survey, widely used estimates in similar studies on Australia, and statistics provided by Australian government bodies, such as the Australian Bureau of Statistics (ABS), and international organizations like the World Bank. The remaining micro and macro parameters are calibrated internally to match key model moments with corresponding data moments. These parameters and their calibration targets are summarized in Table 3.

To evaluate the model's performance, I compare a set of targeted and non-targeted data moments with their model-generated counterparts. Results, as shown in Table 4, indicate that the benchmark model generally demonstrates a good fit with key aggregate empirical characteristics of the Australian economy. Notwithstanding, some discrepancies are notable, particularly in the life cycle profile of labor force participation for mothers. I discuss potential causes for these discrepancies and suggest some potential solutions for future work.

5.1 Demographics

A model period is one year. Households enter the model economy at age 21 ($j = 1$) as workers, retire at age 65 ($j = J_R$ =45), and can live up to a maximum age of 100 ($j = J = 80$).[40] The time-invariant average conditional survival probabilities for males and females ($\psi_{j,m}$ and $\psi_{j,f}$) are calculated using the 2001-2019 ABS Life Tables.

The growth rate of newborn households is kept constant at $g_N = 1.6\%$, which reflects the average annual population growth rate in Australia from 2012-2018 (Profile of Australia's population, AIHW 2024). Newborn household masses by family type, $\pi(\lambda)$, are estimated shares by marital and parental statuses for households aged 50-65 from HILDA data. Married households comprise 59% of the newborns, with 88% being parents, leading to $\pi(1) = 0.52$ and $\pi(2) = 0.06$. Single households, 60% of whom are women, make up the remaining 41%, thus resulting in $\pi(3) = 0.17$ and $\pi(4) = 0.25$.

[39] For further details, see the RBA report on wealth and consumption indicators. Additionally, this period is suitable because it allows for the use of 2018 policy parameters for the FTB and CCS, following significant reforms to these programs (e.g., changes to the FTB-A payment rates, income-test thresholds, FTB-B primary earner thresholds, and other adjustments to tax offsets to streamline the system), thus providing a closer approximation to the present tax and child benefit systems.

[40] I set productivity to zero from age J_R onward, making retirement mandatory.

Parameter	Value	Target
Demographics		
Maximum lifespan	$J = 80$	Age 21-100
Retirement age	$J_R = 45$	Age Pension age 65
Population growth	$g_N = 1.6\%$	Average (ABS 2012-2018)
Survival probabilities	ψ_m, ψ_f	Life Tables (ABS 2010-2019)
Measure of newborns	$\{\pi(\lambda_1),\, \pi(\lambda_2),\, \pi(\lambda_3),\, \pi(\lambda_4)\} = \{0.52,\, 0.06,\, 0.17,\, 0.25\}$	Marital and parental status at age 50-65 (HILDA 2012-2018)
Technology		
Labor aug. tech. growth	$g_A = 1.3\%$	Prod. growth per hour (World Bank 2012-2018)
Output share of capital	$\alpha = 0.4$	Treasury 2019
Real interest rate	$r = 4\%$	World Bank 2012-2018
Households		
Relative risk aversion	$\sigma = \frac{1}{\gamma} = 3$	Standard values 2.5-3.5
Exogenous male labor hours	n_λ^m	Age-profiles of average work hours for male workers (HILDA)
Male human capital profile	h_λ^m	Age-profiles of median male hourly wages (HILDA)*
Education		
Measure of $\{\theta_L, \theta_H\}$ type households	$\{\pi(\theta_L),\, \pi(\theta_H)\} = \{0.7,\, 0.3\}$	College-to-HS ratio (ABS 2018)
Fiscal policy		
Income tax progressivity	$\tau = 0.2$	Tran and Zakariyya 2021a
Consumption tax	$\tau^c = 8\%$	$\tau_c \frac{C}{Y} = 4.5\%$
Company profit tax	$\tau^k = 10.625\%$	$\tau^k \left(\frac{Y - wL}{Y}\right) = 4.25\%$
Government debt to GDP	$\frac{B}{Y} = 20\%$	Average (CEIC 2012-2018)
Government general purchase	$\frac{G}{Y} = 21\%$	Net of FTB, CCS and Age Pension (APH)
FTB, CCS, and Pension parameters		HILDA tax-benefit model

Table 2: Externally calibrated parameters

Notes: (*) The age-profile of median male hourly wages is estimated by regressing log(wage) on quadratic age terms and four dummy variables for gender, marital status, employment type, and time. All hourly wage estimates are then normalized by the average hourly wages of 21-year-old, low-education, married men working full-time.

Parameter	Value	Target
Households		
Discount factor	$\beta = 0.99$	Savings rate 5%-8% (ABS 2013-2018)
Taste for consumption	$\nu = 0.55$	Female work hours = 28.2 per week (HILDA 2012-2018)
Fixed cost function		
Maximum fixed cost	$\{\chi^{max}_{\lambda=\{1,4\},\ell},\ \chi^{max}_{\lambda=2,\ell}\}$	
Full-time ($\ell = 2$)	$\{0.645,\ 0.650\}$	LFP of mothers (71.1%) and non-mothers (73.4%)
Part-time ($\ell = 1$)	$\{0.543,\ 0.645\}$	FT share of mothers (53.6%) and non-mothers (68.9%)*
Female human capital		
Depreciation rate	$\delta_h = 0.074$	Male-female wage gap at age 50**
Accumulation rate for:	$(\xi_{1,\theta,\ell},\ \xi_{2,\theta,\ell})$	
Low-Ed working part-time	$(0.01,\ 0.00045)$	FT wage profile of low-ed male***
Low-Ed working full-time	$(0.0275,\ 0.001125)$	PT wage profile of low-ed male
High-Ed working part-time	$(0.04,\ 0.0015)$	FT wage profile of high-ed male
High-Ed working full-time	$(0.065,\ 0.0025)$	PT wage profile of high-ed male
Technology		
Capital depreciation rate	$\delta = 0.07172$	$\frac{K}{Y} = 3.2$ (ABS 2012-2018)
Transitory shocks		
Persistence parameter	$\rho = 0.98$	Literature
Variance of shocks	$\sigma^2_v = 0.01425$	Gini coefficient of male wages at age 21, $GINI_{j=1,m} = 0.35$
Fiscal policy		
Maximum pension payment	$pen^{max} = 30\% \times Y$	Pension share of GDP, $\frac{\mathcal{P}_t}{Y_t} = 2.4\%$ (Treasury 2021)

Table 3: Internally calibrated parameters

Notes: (*) See Subsection for details on the calibration of the slope parameter $\chi^s_{\lambda,\ell}$ and the inflection point $\bar{j}_\lambda$ of the age profiles of fixed costs. (**) Age 50 is chosen to allow sufficient time for δ_h to take effect on female labor supply decisions. (***) I calibrate the female human capital accumulation and depreciation rates for a type $\{\theta, \ell\}$ woman so that her age-profile of wages aligns with that of her male counterpart if she works continuously without time off. The target male moments (i.e., male age-profiles of wages) are HILDA estimates for each $\{\theta, \ell\}$ pair. Some adjustments (e.g., excluding data near retirement age) were made to better fit the male profiles, particularly for groups with noisier data, such as single men.

5.2 Preferences

The subjective discount factor is calibrated to $\beta = 0.99$ to ensure that the household savings rate stays between 5% and 8%, as reported by ABS National Accounts statistics. The elasticity of intertemporal substitution is set at $\gamma = \frac{1}{3}$, within the standard range of values in the literature.[41]

The taste-for-consumption parameter $\nu = 0.55$ is calibrated to align the model's implied average female weekly work hours with the observed average of 28 hours. The fixed time cost parameters from Equation (34) are calibrated to match labor force participation rates and the full-time employment shares for both mothers and non-mothers with observed data.

Let λ_m denote all households with mothers ($\lambda = \{1,4\}$), and λ_{nm} denote those without mothers ($\lambda = \{2,3\}$). The calibration of fixed cost parameters involves two steps. First, the maximum fixed cost parameters for part-time ($\chi^{max}_{\lambda_i,\ell=1}$) and full-time work ($\chi^{max}_{\lambda_i,\ell=2}$) for each $i \in \{m, nm\}$ are jointly calibrated to match the model's labor force participation rates for mothers (LFP_m) and non-mothers (LFP_{nm}) to observed data. Then, the full-time-to-part-time fixed cost ratios for mothers $\left(\frac{\chi^{max}_{\lambda_m,2}}{\chi^{max}_{\lambda_m,1}}\right)$ and non-mothers $\left(\frac{\chi^{max}_{\lambda_{nm},2}}{\chi^{max}_{\lambda_{nm},1}}\right)$ are calibrated so that their respective full-time employment shares (FT_m and FT_{nm}) in the model align with their data counterparts. Specifically, in the first step, the calibration procedure sets $\chi_{\lambda_i,1} = \chi_{\lambda_i,2}$, and in the second step, $\chi_{\lambda_i,1}$ is adjusted while holding $\chi_{\lambda_i,2}$ constant at the values obtained in the first step. This process results in $\left\{\chi^{max}_{\lambda_m,1}, \chi^{max}_{\lambda_m,2}\right\} = \{0.645, 0.650\}$ for mothers and $\left\{\chi^{max}_{\lambda_m,1}, \chi^{max}_{\lambda_m,2}\right\} = \{0.543, 0.645\}$ for non-mothers. Furthermore, I assume that married households ($\lambda = \{1,2\}$) are perfectly altruistic, meaning couples share fixed time costs $\chi_{\lambda,\ell}$ equally.

Parameters associated with the steepness ($\chi^s_{\lambda,\ell}$) and inflection point ($\bar{j}_\lambda$) of the fixed cost function are then adjusted to capture the declining rates and peaks, respectively, of life cycle profiles of full-time employment shares for both mothers and non-mothers.[42] For mothers, I set $\left\{\chi^s_{\lambda_m,1}, \chi^s_{\lambda_m,2}\right\} = \{0.002, 0\}$ and $\bar{j}_{\lambda_m} = 10$, while for non-mothers, $\left\{\chi^s_{\lambda_{nm},1}, \chi^s_{\lambda_{nm},2}\right\} = \{0.001, 0\}$ and $\bar{j}_{\lambda_{nm}} = 50$.

5.3 Endowments

Labor productivity. Every adult household member is subject to idiosyncratic transitory earnings shocks, η^i for $i \in \{m, f\}$. These shocks follow an identical AR(1) process with persistence ρ and variance of innovation σ^2_υ. I set $\rho = 0.98$ to stay within the bounds of common values in the literature, and $\sigma_\upsilon = 0.01425$ to achieve a Gini index of 0.35 for the efficiency wage distribution of newborn male workers aged $j = 1$ in the model. This configuration results in a Gini coefficient of 0.3766 (non-targeted) for the working-age male population.[43]

The Rouwenhorst method is employed to discretize the shock values into three grid points $\{0.4281,\ 1,\ 2.3358\}$ with the following Markov transition probabilities[44]

$$\begin{bmatrix} 0.9801 & 0.0198 & 0.0001 \\ 0.0099 & 0.9802 & 0.0099 \\ 0.0001 & 0.0198 & 0.9801 \end{bmatrix}$$

I assume two education types—low (θ_L) and high (θ_H)—realized at birth, representing individuals with at most a high school degree and those with a bachelor's degree or higher qualifications, respectively. Education θ influences the parameters $\{\xi_{1,\theta,\ell}, \xi_{2,\theta,\ell}\}$ that govern human capital trajectories, thereby determining effective

[41] $\beta = 0.99$ yields a growth-adjusted discount factor $\tilde{\beta} = \beta(1+g)^{\nu\left(1-\frac{1}{\gamma}\right)} = 0.9807$ for the balanced-growth path steady-state economy.

[42] The model-generated life cycle profiles of full-time employment shares relative to the data are reported in Table 4.

[43] More precisely, σ_υ is calibrated to match the Gini index of the model's male efficiency wage distribution with that of the observed male earnings distribution, which includes variations in work hours. The rationale is that the exogenous male work hour profiles employed in the model are normalized average values. Since the model lacks an endogenous source of hour variation for men, I use the transitory shock process that drives the male efficiency wages to also capture the exogenous work hour fluctuations.

[44] The Rouwenhorst method matches exactly the first and second moments of the continuous process but cannot capture higher-order moments of shocks (e.g., skewness and kurtosis), which are important for understanding the magnitude and probability of extreme earnings shocks.

wages. The proportions of low- and high-education households are $\pi(\theta_L) = 0.7$ and $\pi(\theta_H) = 0.3$, based on the college-high school ratio in the 2018 ABS data.

I abstract from men's labor supply decisions and assume they always work full-time. Their age-profiles of normalized average work hours (n^m_λ) are externally estimated by family type.[45]

I estimate hourly wage age-profiles from HILDA data for single and married males. They serve as proxies for male age profiles of human capital h^m_λ in the model. Female human capital $h^f_{\theta,\ell}$ evolves endogenously over the life cycle, governed by education θ and employment status ℓ. Human capital gain parameters for women, $\{\xi_{1,\theta,\ell}, \xi_{2,\theta,\ell}\}$, are calibrated so that the life cycle paths of human capital for single and married women mirror those of their male counterparts should they choose to work continuously without time off. The parameter values for each $\{\theta, \ell\}$ pair are presented in Table 3.

Children. Children are deterministic and exogenous. Based on HILDA survey data, which indicates that a plurality of parents (42%) have two children, the model households are assumed to have two children over their lifetimes.[46] Heterogeneity in the timing of childbirth is linked to the household's education level θ. The longitudinal study of Australian children (LSAC) annual statistics report in 2017 shows that the largest share of first-time mothers aged 15-19 concentrates within the low-education group (67.7%), while only around 10% of first-time mothers aged 25-37 have low education. In contrast, nearly half of first-time mothers in the older age group hold a bachelor's degree or higher. Reflecting this fact, I assign the first child's birth to low-education (θ_L) parents at age 21 ($j = 1$, the youngest in the model), and to high-education (θ_H) parents at age 28 ($j = 8$). In both groups, the second child arrives three years after the firstborn, at age 24 and 31, respectively.[47] Moreover, for tractability, and based on the observation that women constitute the majority of lone parents (87.21%) in the sample, I assume that all single women have children, whereas single men are childless.

Child care cost. I assume that there is no informal child care and that formal care services operate in a perfectly competitive market environment with uniform quality and pricing, thus abstracting from variations in regional costs and types of child care providers. Using a conservative estimate of \$12.5 per hour, the cost of child care amounts to 52% of the average hourly wage of a 21-year-old male in the model. The total formal child care costs for a household aged j is the sum of costs for all dependent children. I further assume that child care costs (κ) decline once a child reaches six years of age (school age). More precisely, working mothers pay the full cost of formal child care for children aged 0-5 years, and one-third of the cost thereafter. This reduction reflects the assumption that public schools are free, and that parents only incur expenses for out-of-school-hours (OOSH) care and extracurricular activities.[48]

5.4 Technology

The production function is $Y = K^\alpha (AL)^{1-\alpha}$, where the capital output share is set at $\alpha = 0.4$ for Australia. The labor-augmenting technology A is normalized to 1 in the benchmark economy. Given Australia's average

[45] Estimates from HILDA show that male labor supply is stable across parental and marital statuses. Empirical exercises using logistic regressions of workforce participation on lagged FTB benefits and demographic controls also suggest minimal work disincentives from family benefits for men. For example, a \$10,000 increase in the FTB transfer is associated with only a 1 percentage point (*pp*) decline in participation for fathers (p-value = 0.18), compared to a statistically significant 4.3*pp* drop in participation for mothers. Similarly, Doiron and Kalb (2004) find that increases in child care costs have a negligible effect on male labor supply in Australia. Empirical evidence thus far points to a highly inelastic male labor supply. Hence, for computational feasibility and given the model's focus on women, male labor supply is treated as exogenous.

[46] The proportion of parents with two children is based on a restricted sample of older households (aged 50 and above). This ensures that the statistics reflect the number of children households have over their life cycles. The data shows that 12% of parents have one child, 42% have two, 28% have three, and the remainder have four or more.

[47] According to the Australian Institute of Health and Welfare (AIHW) report, child spacing remains approximately three years, although the average age of mothers at the birth of their first and second children rose from 27.9 and 31 years in 2009 to 29.4 and 31.9 years in 2019.

[48] OOSH services operate before school (6:30am-9am), after school (3pm-6pm), and during vacation periods (7am-7pm). I reduce the cost to one-third of the original to account for the fact that school-age children spend less time in child care on average (only 40% of children aged 6-8 participate in any form of child care, and the rate declines to 20% by age 12). For further information on child care usage, see the AIFS report on child care and early child hood education in Australia, and for information on the average cost of care for a child, refer to the 2005 DSS report on costs of children. I use recent information for the hourly child care costs and assume the cost ratio for school-age children relative to preschool-age children has remained stable since 2005.

annual GDP growth per hour worked of 1.3%, the labor-augmenting technology growth rate g_A is set at 0.013. Using the firm's first-order conditions (36) and targeting a capital-to-GDP ratio of $K/Y = 3.2$, the capital depreciation rate δ is derived to be 0.07172.

5.5 Fiscal policy

Taxes. The progressivity parameter is set at $\tau = 0.2$, following Tran and Zakariyya 2021a. The tax scale parameter ζ, which controls the overall size of the tax system (or tax burden), is used as an endogenous variable to balance the budget in all policy experiments. The consumption tax rate $\tau^c = 8\%$ targets a consumption tax share of GDP $\frac{\tau^c C}{Y}$ of 4.5%, based on an average consumption-to-GDP ratio $\frac{C}{Y} = 56.3\%$ according to the 2012-2018 ABS data. The company profit tax rate τ^k is calculated to be 10.625% such that the company tax share of GDP, $\tau^k \left(\frac{Y - wL}{Y}\right) = 4.25\%$, where $\frac{wL}{Y} = 1 - \alpha = 0.6$.

Family Tax Benefit and Child Care Subsidy. The policy parameters—including base and maximum payment rates, income-test thresholds, and phase-out rates—for the Family Tax Benefit (FTB) Parts A and B and the Child Care Subsidy (CCS) programs are based on the actual 2018 Australian government policy settings. See Subsection H.2 in the Appendix for detailed information.

Means-tested Age Pension. The Age Pension's income and assets test thresholds, along with their respective phase-out rates, are based on 2018 values. In the benchmark economy, the maximum pension payout p^{max} is calibrated to be 30% of per capita income to achieve a total pension share of GDP of 2.4%, in line with the Treasury 2021 Retirement Income Review.

General government expenditure and debt. General government expenditure G is defined as all government spending other than the two child benefit programs (FTB and CCS) and the Age Pension, which are explicitly modeled and respond endogenously to counterfactual reforms. According to the Budget Review 2020-21, total government expenditure is 25% of GDP. After accounting for the estimated expenditures on the FTB and CCS (1.4%) and the Age Pension (2.4%), the exogenous general expenditure is 21.2% of GDP. Public debt B is set at 20% of GDP, reflecting the average public debt share prior to the COVID-19 pandemic.

5.6 Benchmark economy

The model performance is assessed by comparing key aggregate and life cycle moments generated by the model with their corresponding data counterparts.

Aggregate macro variables. I examine targeted and non-targeted aggregate macroeconomic moments in the benchmark economy. Table 4 demonstrates that the benchmark model generally aligns well with the observed data at the aggregate level.

Life-cycle profiles. Figure 7 reports the age-profiles of labor force participation rates (non-targeted) and full-time employment shares (targeted) for mothers and non-mothers, comparing the model-generated moments with the observed data.

The benchmark model is able to capture the general patterns of full-time employment share profiles for both mothers and non-mothers. However, it misses the dip in mothers' profiles between age 30 and 40. The implied participation profile for non-mothers closely tracks the data, but slightly understates participation by 2 to 5 percentage points (*pp*) before age 35 and overstates it by similar margins after age 45.

The model's implied participation profile for mothers, however, presents challenges. While the aggregate labor force participation rates align well with the data, the benchmark model predicts significantly higher labor force participation among mothers during child-bearing and rearing years, followed by a sharp decline after their children become independent.

Several assumptions in the model may contribute to this discrepancy: (i) credit constraints, (ii) absence of non-partner family insurance, (iii) lack of informal care, (iv) perfectly flexible work hour arrangements, (v) lack of job search frictions and switching costs between part-time and full-time jobs, and (vi) exogenous and

Moments	Model	Data	Source
Targeted			
Capital, K/Y	3.2	3-3.3	ABS (2012-2018)
Savings, S/Y	8.5%	5-8%	ABS (2013-2018)
Female work hours	23.6	28.2	HILDA (2012-2018)
LFP of mothers	73.3%	71.1%	HILDA (2012-2018)
LFP of non-mothers	74.2%	73.4%	HILDA (2012-2018)
FT share for working mothers	54.6%	53.6%	HILDA (2012-2018)
FT share for working non-mothers	71%	68.9%	HILDA (2012-2018)
Consumption tax, T^C/Y	3.6%	4.50%	APH Budget Review
Corporate profit tax, T^K/Y	4.25%	4.25%	APH Budget Review
Age Pension, P/Y	2.3%	2.4%	ABS (2012-2018)
Gini coefficient (male aged 21)	0.35	0.35	HILDA (2012-2018)
Non-targeted			
Consumption, C/Y	45.5%	54-58%	ABS (2012-2018)
Investment, I/Y	32.3%	24-28%	ABS (2013-2018)
Female LFP	70.7%	71.5%	HILDA (2012-2018)
Scale parameter, ζ	0.8978	0.7237	Tran and Zakariyya 2021b
Income tax, T^I/Y	4.9%	11%	APH Budget Review
Child-related transfers (FTB + CCS)	1%	1.45%	ABS (2012-2018)

Table 4: **Key macroeconomic variables: Model vs. Data moments**

Notes: (*) Multiple sources, including my estimates using HILDA survey data, confirm these ranges of participation rates for mothers. (**) I target a Gini coefficient of 0.35 for the male earnings distribution at birth (age 21 or $j = 1$). This results in a Gini coefficient of 0.3766 for the male earnings distribution over the entire working age.

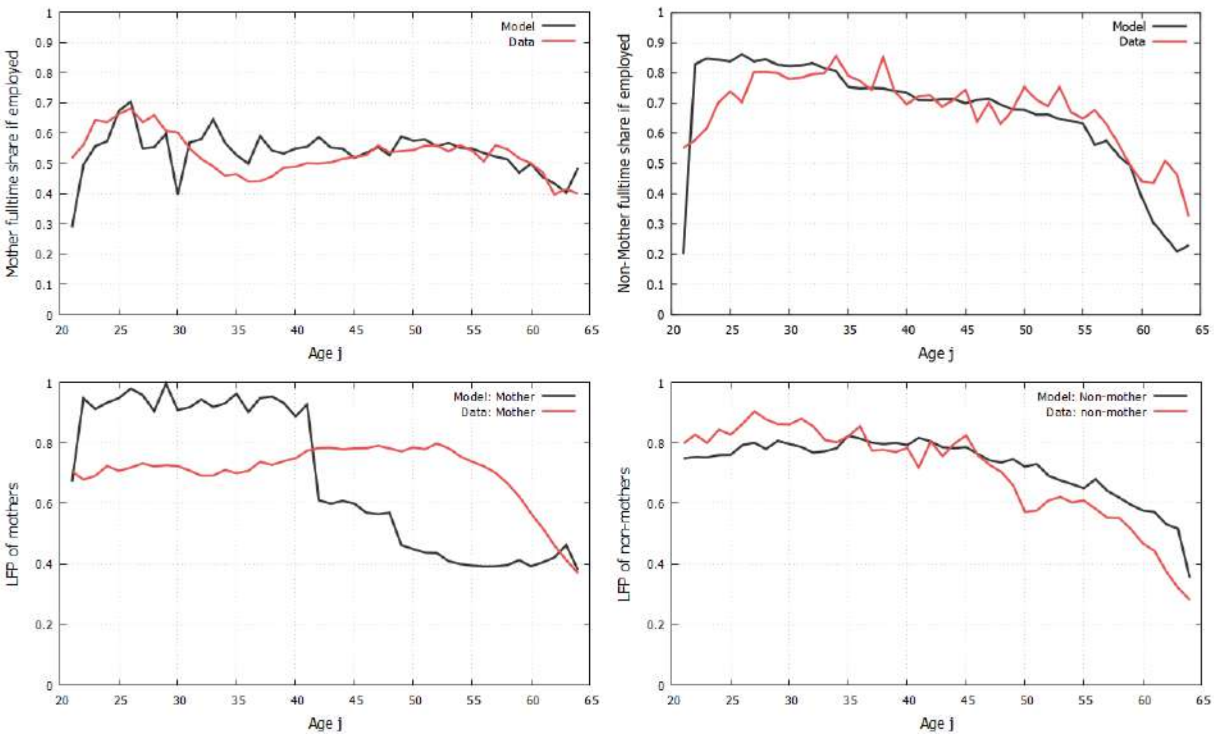

Figure 7: **Model vs Data:** Life-cycle profiles of labor supply of women.

Top-left: Full-time share of mothers (targeted); **Top-right:** Full-time share of non-mothers (targeted); **Bottom-left**: Labor force participation of mothers (non-targeted); **Bottom-right**: Labor force participation of non-mothers (non-targeted).

deterministic children.

Assumptions (i) through (v) may lead more mothers in the model, especially single mothers, to opt for minimal work hours (thus being counted as part of the labor force) to increase consumption, as children can considerably reduce per capita consumption. This may explain why the model captures the full-time shares of employment but displays discrepancies in overall participation. Assumption (vi), which restricts childbirth to the first 10 years of working life, excludes older mothers from the model. This reduces the need for self-insurance via labor supply after age 40, which could account for the sharp decline in labor force participation among mothers.

To address these limitations, future improvements to the model should consider relaxing these assumptions. In addition, incorporating richer features of the income process for individuals and couples could enhance the model's ability to match beyond the first-order moments.

6 Quantitative analysis

In this section, I investigate the interaction between tax progressivity and means-tested child benefits, and propose an optimal joint system. Optimality is defined in terms of the overall welfare of newborns under the veil of ignorance (or ex-ante welfare). The analysis also considers how each counterfactual reform impacts key macroeconomic variables—such as female labor supply, consumption, and output—and the distribution of welfare changes across demographic groups.[49]

In all policy experiments, discrepancies between the government's consolidated tax revenues and expenditures are resolved by adjusting the overall size (burden) of the tax system through the tax scale parameter ζ_t, thus ensuring the government budget equation (45) is balanced at time t according to the following rule:[50]

$$\zeta_t = \frac{w_t L_t + (B_{t+1} - B_t) + T_t^C + T_t^K - (G_t + Tr_t + \mathcal{P}_t + r_t B_t)}{\sum_j \sum_{\Lambda \times \Theta} \int_{A \times H \times S^2} \left(\widetilde{y}^{m\ 1-\tau}_{j,\lambda \neq 4} + \widetilde{y}^{f\ 1-\tau}_{j,\lambda \neq 3} \right) \mu_{j,t} \, d\Phi_t(z_j)} \tag{78}$$

where $\widetilde{y}^i_{j,\lambda}$ is the taxable income for $i \in \{m, f\}$, as defined in Subsection 4.5.1.

Table 5 summarizes overall welfare outcomes across different counterfactual experiments. It shows that most policy re-configurations bring about overall welfare losses relative to the status quo, but there are three promising reforms. The first reform (a) retains the benchmark means-tested child benefits and optimizes tax progressivity. The second (b) and third (c) reforms involve a partial universalization of the means-tested child benefit system. In particular, both reforms maintain the status quo Child Care Subsidy (CCS) program while replacing the means-tested lump-sum benefits (FTB) with an optimal universal lump-sum benefit per child, referred to as '*Universal Lump-Sum Child Benefits*' or '*Universal FTB*'. The two reforms differ only in their treatment of tax progressivity: the former keeps tax progressivity at its benchmark value $\tau = 0.2$, whereas the latter searches for an optimal pair of tax progressivity and universal lump-sum child benefit rate, $\{\tau^*, \bar{tr}^*\}$. The following subsections examine the aggregate and distributional impacts of these counterfactual policies

6.1 Optimal tax progressivity under the benchmark child benefits

The previous study by Tin and Tran (2024) finds that the benchmark means-tested child benefits, combined with the status quo tax progressivity of $\tau = 0.2$, improve overall welfare but reduce aggregate labor supply and output. From an equity standpoint, the study suggests that the scheme could be desirable, as it redistributes welfare from high-education married parents and single men to more vulnerable groups, namely, low-education married parents and single mothers. Building on this work, I extend the analysis to explore a scenario in

[49] Following Subsection 4.10, welfare changes are decomposed into six components: consumption allocative efficiency effect (CEV_{CE}), consumption distributional effect (CEV_{CD}), consumption insurance effect (CEV_{CI}), leisure allocative efficiency effect (CEV_{LE}), leisure distributional effect (CEV_{LD}), and leisure insurance effect (CEV_{LI}).

[50] ζ_t affects the overall tax burden across all income levels, while holding constant the tax progressivity τ. See Subsection 4.5.1 for further explanation.

	τ	No CCS	No FTB	No FTB and CCS	No means-testing	Universal FTB
τ	$+^{(a)}$					
No CCS	$-$	$-^{(e)}$				
No FTB	$-$	$-$	$-^{(f)}$			
No FTB and CCS	$-$	NA	NA	$-$		
No means-testing	$-$	$-$	$-$	NA	$-^{(d)}$	
Universal FTB	$+^{(c)}$	$-$	$-$	NA	NA	$+^{(b)}$

Table 5: Summary of overall welfare outcomes across selected reforms.

Notes: Experiment (a) involves testing different levels of tax progressivity under the existing means-tested child benefits. Experiment (d), referred to as the '*Baseline universal child benefits*' reform, eliminates means-testing from both the FTB and CCS while retaining demographic eligibility criteria. Experiment (c) focuses on the joint optimization of tax progressivity and universal lump-sum child benefit (FTB) levels, while keeping the CCS structure at the status quo.

which the government can adjust tax progressivity while maintaining the existing means-tested child benefit system. This approach deepens the understanding of the interaction between tax and child benefit systems by assessing whether optimizing the tax system can complement the main objective of child benefits in improving the welfare of vulnerable parents.

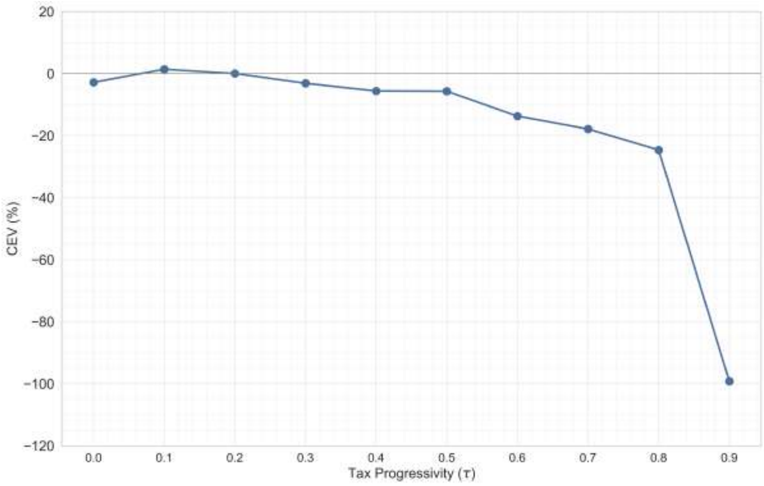

Figure 8: **Overall welfare changes over tax progressivity under the benchmark means-tested child benefits.**

Figure 8 shows the overall welfare changes, relative to the status quo, across different levels of tax progressivity. The results indicate that the optimal tax policy requires lowering progressivity to $\tau^* = 0.1$ compared to the current level of $\tau = 0.2$.

As detailed in Table 6, this reform leads to a 1.38% improvement in overall welfare, measured in consumption equivalent terms. The regime also produces several notable aggregate changes. First, it creates mixed effects on female labor supply: women work 5.71% longer hours on average but reduce their participation by 2.77 percentage points (pp). Second, the overall tax burden (determined by the tax scale parameter ζ) remains nearly unchanged, which implies that the increase in average tax rate by 4.85pp is likely due to two factors: (i) the shrinking zero-tax income zone as progressivity decreases, resulting in more low-income households paying taxes, and (ii) an increase in work hours among women, pushing more of them into higher tax brackets. Third, despite a 0.5% fall in output, aggregate consumption increases modestly by 0.5%.

These results suggest two crucial points: (i) there is a trade-off between intensive and extensive labor supply due to the adjustment in tax progressivity, and (ii) the increased consumption may be a key driver of the overall welfare improvement. The following discussion delves deeper into these outcomes.

Intensive-extensive labor supply trade-off. Figure 9 illustrates that the trade-off between intensive and extensive margins of labor supply occurs across all demographics. Given the minimal change to the tax scale parameter ζ (+0.007), these behavioral responses cannot be attributed to changes in the overall tax burden.

Optimal tax progressivity under the benchmark means-tested child benefits			
CCS size, %	+7.14	Fe. Hours, %	+5.71
FTB size, %	0	Fe. Human cap. (H), %	+0.77
Average tax rate, *pp*	+4.85	Consumption (C), %	+0.50
Tax scale (ζ)	+0.007	Output (Y), %	−0.50
Fe. Lab. Force Part. (LFP), *pp*	−2.77	Welfare (CEV), %	+1.38

Table 6: **Aggregate effects of the optimal tax progressivity reform under the benchmark means-tested child benefits.**

Notes: Results are reported as changes relative to the levels in the benchmark economy.

Instead, the primary mechanism affecting female labor supply decisions in the new regime is the lower tax progressivity. Labor efficiency changes (bottom panel of Figure 9), which combines intensive and extensive labor supply margins, indicate that the reform primarily encourages labor supply among high-education mothers. However, while the more proportional system reduces tax liabilities and distortions (lower MTR) for higher income brackets, it raises them for lower-income earners. In other words, this tax structure generally incentivizes longer work hours but discourages participation among women in low-education households, who tend to remain in lower income brackets even with increased hours worked.[51]

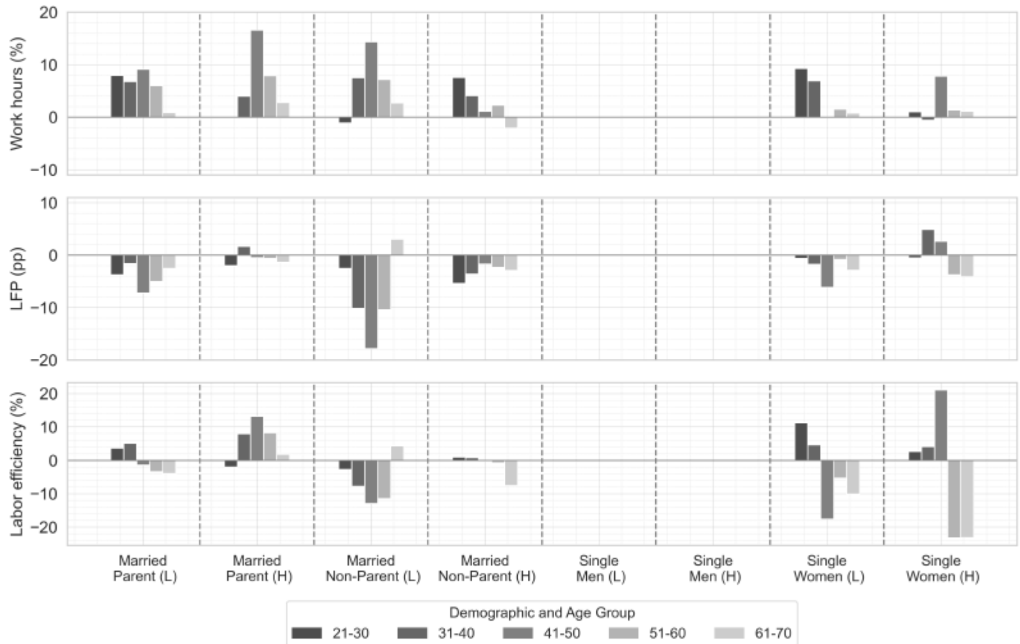

Figure 9: **Female labor supply responses to the optimal tax progressivity reform under the benchmark child benefits by age and demographic.** (**Top:** Work hours, **Middle:** Labor force participation, **Bottom:** Labor in efficiency units).

Sources and distribution of welfare changes under the optimal tax progressivity. Consistent with the aggregate results in Table 6, the decomposition of welfare changes in Figure 10 suggests that the main channel through which welfare improvements manifest is the increase in consumption allocative efficiency (CEV_{CE}).[52] The welfare effects related to leisure are relatively trivial, partly due to the trade-off between work hours and participation.[53]

[51] That is, low-education women's weaker earning potential makes it less likely that they benefit from the more favorable tax treatment at higher income levels.

[52] See Subsection 4.10 for formal definitions of CEV and its components.

[53] Notably, lowering tax progressivity also has minimal effects on the distribution and insurance components of consumption. In Subsection 6.1.1, I demonstrate that this result is not unique to the case of tax progressivity reform. Motivated by these findings, the analysis in Subsection F.2 reveals that access to the CCS, which enhances parents' self-insurance capacity, helps cushion the impacts of other policy reforms on the distributional and insurance components of welfare.

The optimal tax regime redistributes welfare to a subset of parents, namely, low-education single mothers and high-education married parents, at the expense of the majority, including vulnerable groups such as low-education married parents (Figure 11). The overall welfare improvement is made possible only by the significant positive welfare effect experienced by single mothers, outweighing the smaller combined losses for the rest of the population (excluding high-education married parents).

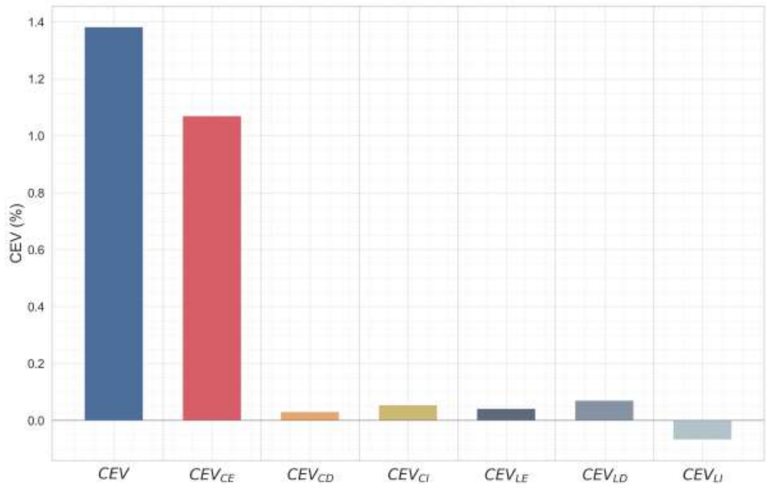

Figure 10: **Decomposition of welfare changes under the optimal tax progressivity ($\tau_y^* = 0.1$) and benchmark means-tested child benefits.**

Low-education single mothers see substantial gains in consumption efficiency and welfare. As shown in Figure 9, this group significantly increases their work hours during the first 20 years of life, with minimal change in labor force participation. This additional labor effort allows them to boost their consumption by almost 10% during younger years (Figure 12), despite a subsequent decline between ages 40 and 70. The net positive welfare effect suggests that, in the initial steady state, young low-education single mothers had high marginal utilities of consumption, implying a high proportion of severely hand-to-mouth households, likely due to factors such as early childbearing and credit constraints.[54]

For the losers of this reform, their losses also stem from changes in consumption allocative efficiency. Figure 12 depicts a common pattern within this group: they reduce consumption in their younger years in favor of accumulating more wealth to subsidize later-life consumption. Although the more proportional tax schedule may help offset means-testing distortions at higher income levels, it makes consumption smoothing more costly for low-income households due to their increased tax liabilities. With savings and female labor supply being the only active self-insurance mechanisms, some households shift toward savings. This also reduces consumption among young households, especially those with low education, as all households enter the economy with zero wealth and need time to build human capital. These consumption and wealth patterns are especially pronounced for high-education single mothers, who lack family insurance, and for low-education married households, whose limited earning potential restricts them to the low-income bracket. Ultimately, their reduced consumption and increased wealth accumulation lead to less allocatively efficient consumption profiles, resulting in welfare losses.

In general, this policy experiment reveals two key insights. First, the interaction between taxes and child benefits matters for enhancing overall welfare. Given the existing means-tested child benefits, the optimal tax system features lower progressivity to alleviate the EMTR for higher earnings, which enables low-education single mothers to increase their work hours, earnings, and consumption during the critical early years. Therefore, if reforming child benefits is not feasible, adjusting the tax system may serve as an indirect route to improve overall welfare and support some of the most vulnerable groups in the population.

[54]The model suggests 62.5 of single mothers are hand-to-mouth at age 21; however, low-education single mothers have, on average, 10% lower consumption than their high-education peers and less than one-third the consumption of married households.

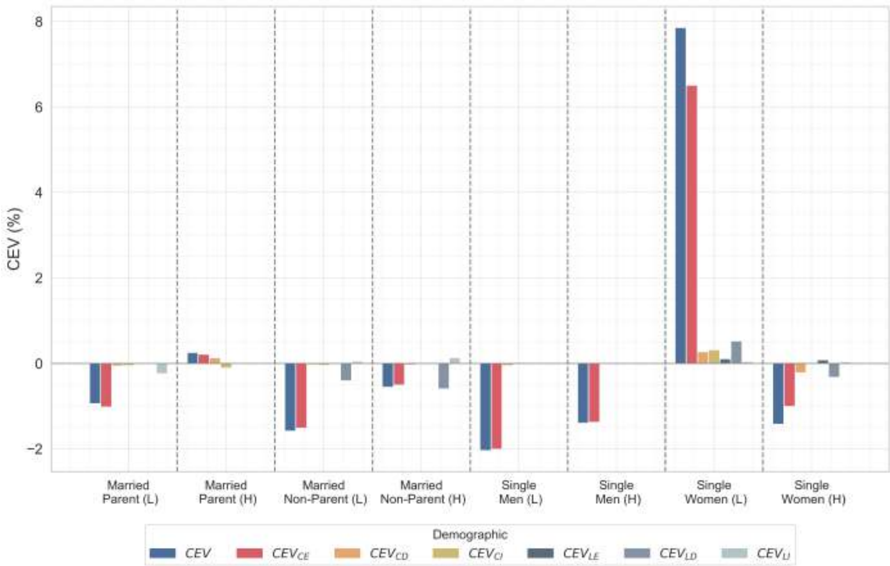

Figure 11: **Distribution of welfare changes under the optimal tax progressivity ($\tau_y^* = 0.1$) and benchmark means-tested child benefits.**

Second, this experiment emphasizes the adverse distributional effects of an isolated tax reform. Even with optimal progressivity, the majority of the population, including low-education married parents, are made worse off. Reduced tax progressivity shifts tax distortions and liabilities toward low-income brackets, making low-income employment more costly. For most women, this leads to an intensive-extensive labor supply trade-off (increasing work hours while reducing participation) and a switch to savings as a self-insurance vehicle. Consequently, their allocative efficiency in consumption declines, causing up to 2% welfare losses. Since the losers constitute a larger share of the population, this reform would not enjoy majority support.

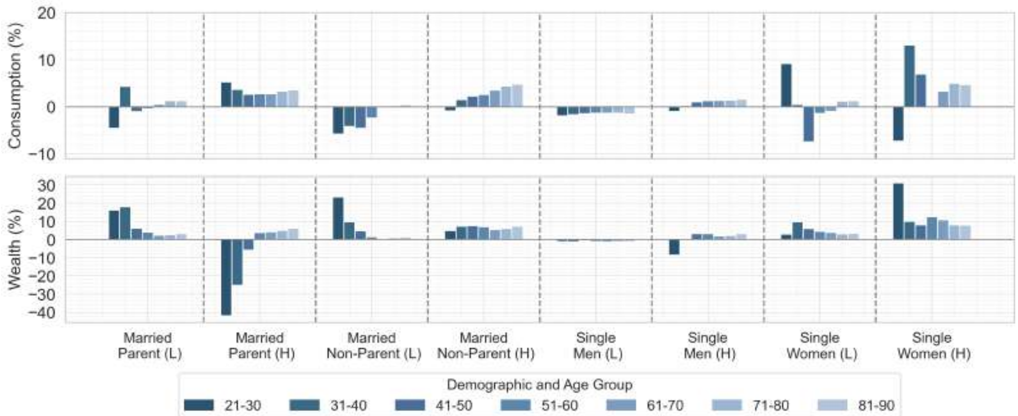

Figure 12: **Household consumption and wealth responses to the optimal tax progressivity reform under the benchmark child benefits by age and demographic** (**Top:** Consumption, **Bottom:** Wealth).

6.1.1 Deviations from optimal progressivity

To better understand the interaction between tax and child benefit systems, I extend the analysis to examine the aggregate and distributional implications of two tax progressivity levels that deviate from the optimal value $\tau^* = 0.2$. The aggregate results in Table 7 indicate that both proportional ($\tau = 0$) and highly progressive ($\tau = 0.6$) tax systems lead to overall welfare losses.

Under the proportional system, although the tax scale ζ remains virtually unchanged, the larger shift of tax liabilities from high- to low-income brackets (relative to the optimal tax system) magnifies the intensive-extensive labor supply trade-off. Female work hours rise by approximately 8%, while participation falls by 4.62*pp*. However, despite the longer hours and higher human capital (+0.97%), consumption and output decrease by 0.39% and 0.46%, respectively, and overall welfare declines by 2.86%.

Deviations from optimal tax progressivity under the benchmark means-tested child benefits			
	$\tau = 0$	$\tau^* = 0.1$	$\tau = 0.6$
CCS size, %	+7.14	+7.14	+14.29
FTB size, %	−5.55	0	−11.11
Average tax rate, *pp*	+3.66	+4.85	+12.37
Tax scale (ζ)	+0.014	+0.007	−0.14
Fe. LFP, *pp*	−4.62	−2.77	+9.80
Fe. Hour, %	+7.99	+5.71	−7.29
Fe. H. cap, %	+0.97	+0.77	+0.20
Cons (C), %	−0.39	+0.50	−2.21
Output (Y), %	−0.46	−0.50	−2.16
Welfare (CEV), %	−2.86	+1.38	−13.72

Table 7: **Aggregate implications of deviations from the optimal tax progressivity under the benchmark means-tested child benefits:** $\tau = 0$, $\tau^* = 0.1$, and $\tau = 0.6$.
Notes: Results are reported as changes relative to the levels in the benchmark economy.

Conversely, a highly progressive system at $\tau = 0.6$ produces the opposite effects. Female work hours decline by 7.29%, while participation increases by 9.8*pp*. This regime introduces a new adverse force on households and the overall economy. It not only heightens tax distortions and liabilities for higher earners but also reduces the tax scale parameter ζ by 0.14 points, implying a significant increase of the overall tax burden on the working population. The combination of higher tax progressivity and lower average work hours brings about greater fiscal pressure, even with an expanded female workforce. Specifically, the economy in this regime consists of more low-hour, low-income workers paying lower taxes, leading to a greater overall tax burden (i.e., an upward shift in the tax schedule) to balance the budget. Consequently, the average tax rate rises by 12.37%, while consumption and output shrink by 2.21% and 2.16%, respectively, causing a substantial welfare loss of 13.72%.[55]

As depicted in Appendix Figure 27, the magnitude of welfare loss increases exponentially as tax progressivity deviates further from the optimal level, driven almost exclusively by declines in consumption efficiency. Moreover, Figure 28 in the Appendix reveals consistent outcomes in aggregate and distributional terms. When $\tau = 0.6$, the higher MTR for higher earners and the increased overall tax burden significantly worsen consumption allocative efficiency and welfare for all households, parents included. Their losses suggest that the reduced labor earnings under a highly progressive tax regime are not adequately compensated by the child benefits they receive.[56]

On the contrary, under a proportional tax regime (Appendix Figure 29), welfare gains are observed only among high-education married households, mainly due to favorable consumption efficiency and leisure distributional effects over their life cycle.[57] Nevertheless, these gains are insufficient to offset the losses incurred by other demographics, resulting in a 2.86% welfare loss for the average newborn household.

Compared to scenarios with increasing tax progressivity (Appendix Figure 27), the overall welfare impact in the proportional regime is less severe. Additionally, the loss under this new regime stems from diverse factors, including negative consumption distributional (CEV_{CD}) and leisure efficiency (CEV_{LE}) effects, although consumption allocative efficiency remains the primary driver. For most demographics, including low-education parents, the increased cost of low-income work under the proportional system prompts some to exit the labor

[55]Other factors, such as the expansion of the CCS program and the contracted consumption tax revenue (as aggregate consumption falls), also contribute to the increased demand for income tax revenue.

[56]Based on Figure 40, most households experience sustained declines in consumption over the life cycle, with the exception of working-age single mothers whose consumption falls only in later stages of life.

[57]A higher leisure distributional effect (CEV_{LD}) implies that these households can expect above-average leisure relative to the population under the new tax policy. These households typically work longer hours and save more under a proportional scheme. The accelerated wealth accumulation, in turn, allows them to increase their expected leisure.

force while others extend their work hours. The net result is a deterioration of the allocative efficiency in both consumption and leisure, leading to welfare losses. Low-education single mothers, in particular, while seeing some improvements in consumption allocative efficiency (for reasons discussed in Subsection 6.1), face larger reductions in leisure allocative efficiency (CEV_{LE}) and consumption distributional (CEV_{CD}) effects. Hence, excessively low tax progressivity can erode allocative efficiency for most households and result in inequitable consumption redistribution for vulnerable single mothers, reducing their expected consumption relative to the population average.

In summary, these findings offer two key policy lessons. First, the impacts of tax progressivity reforms are asymmetric. Lowering tax progressivity reduces welfare, but it does so without raising the overall tax burden. In contrast, increasing progressivity creates fiscal stresses, elevating the overall tax burden that amplifies welfare losses. Second, the interaction between tax policy and means-tested child benefits is complex. Optimizing the tax system solely for overall welfare can inadvertently undermine the objectives of child benefit programs. A proportional regime primarily benefits high-education married parents, whereas a highly progressive one benefits nobody. As demonstrated, despite the provision of child benefits, labor supply remains an essential self-insurance mechanism for parents. The effects of tax reforms on the labor earnings of various parental groups should therefore be carefully considered.

6.2 Optimal child benefits under the benchmark tax progressivity

The optimal tax system ($\tau^* = 0.1$), while moderately increasing overall welfare, proves detrimental to most low-education households by placing greater tax liabilities and distortions on low-income brackets, ultimately reducing their consumption allocative efficiency and welfare.

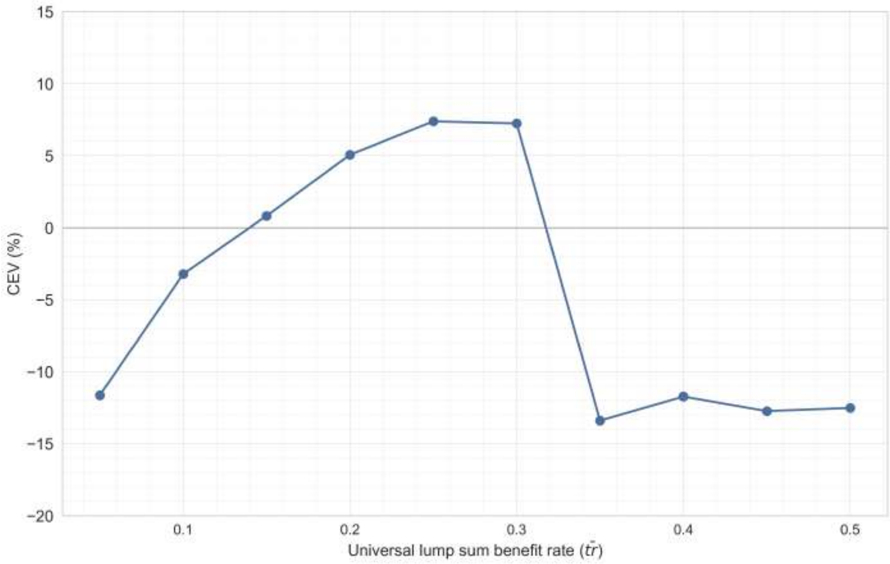

Figure 13: **Overall welfare changes over different levels of universal lump-sum child benefits per child** ($\bar{tr}^* =$ 25%×**median income in 2018) under the benchmark tax progressivity** ($\tau = 0.2$).

In light of these findings, I explore an alternative welfare-improving reform based on Table 5, specifically a policy referred to as *Universal Lump-Sum Child Benefits* or *Universal FTB*. This reform partially universalizes the child benefit system by removing means-testing and demographic criteria from the FTB program to provide uniform lump-sum transfers (per child) to all parents, while keep the status quo means-tested CCS structure unchanged. This setup allows subsidies adjust endogenously in response to changes in female labor supply, thus having fiscal implications. Within this new policy environment, I then examine whether an optimal universal child benefit rate can deliver better overall and parental welfare outcomes compared to the optimal tax reform in Subsection 6.1, while also accounting for broader distributional effects to highlight trade-offs between the welfare of parents and non-parents.

Figure 13 indicates that the optimal universal lump-sum benefit per child, denoted by $\bar{tr}^*$, is 25% of median income in 2018 or around AUD 15,000 per annum. As shown in Table 8, this optimized child benefit plan results in a significant 7.39% increase in overall welfare. Macroeconomic outcomes also display a modest 0.94% increase in consumption, but the reform comes at a cost, with declines in female labor force participation, work hours, and output by 1.83*pp*, 3.48%, and 1.26%, respectively.

Aggregate implications of optimal child benefits under the benchmark tax progressivity			
	$\bar{tr} = 15\%$	$\bar{tr}^* = 25\%$	$\bar{tr} = 35\%$
CCS size, %	0	0	+14.29
FTB size, %	+166.67	+341.67	+519.44
Average tax rate, *pp*	+5.53	+6.85	+20.59
Tax scale (ζ)	−0.025	−0.050	−0.22
Fe. LFP, *pp*	−0.09	−1.834	+1.59
Fe. Hour, %	−0.63	−3.48	−3.84
Fe. H. cap, %	−0.32	+0.27	+0.09
Cons (C), %	+0.75	+0.94	−12.29
Output (Y), %	+0.01	−1.26	−15.81
Welfare (CEV), %	+0.82	+7.39	−13.40

Table 8: **Aggregate implications of optimal universal lump-sum child benefits per child at three levels of $\bar{tr}$:** 15% (first column), 25% (second column) and 35% (third column) of median income in 2018.
Notes: Results are reported as changes relative to the levels in the benchmark economy. Median income is approximately AUD 60,000 in 2018 dollars.

Female labor supply and consumption responses. Unlike the tax reform discussed in Subsection 6.1, which leads to intensive-extensive labor supply trade-off, the optimal child benefit scheme results in declines in both labor force participation and work hours, stemming from the drop in labor supply of mothers (Figure 14).

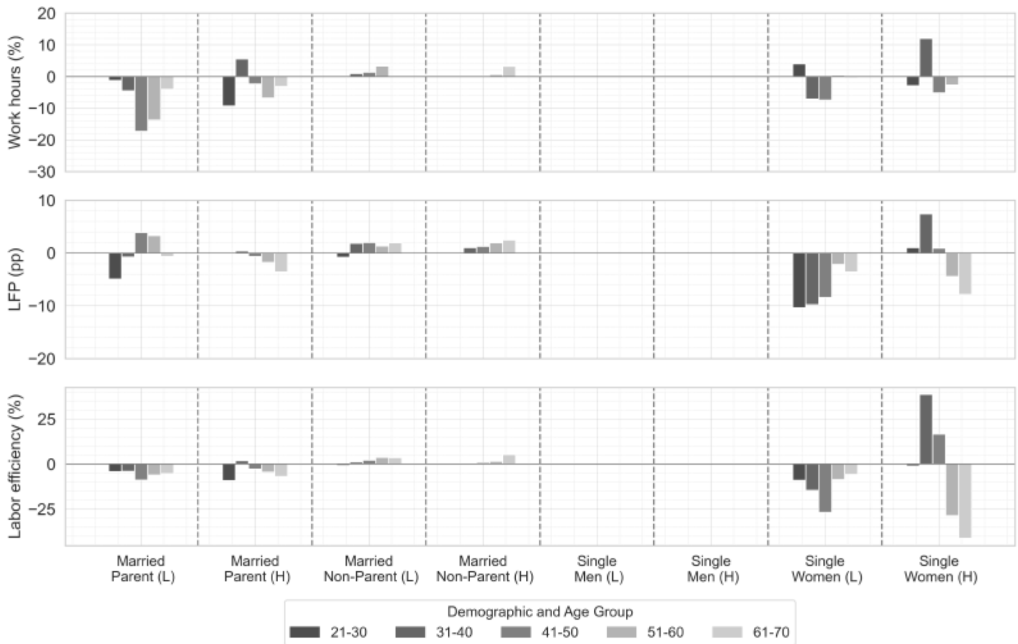

Figure 14: **Female labor supply responses to the optimal child benefit system** ($\bar{tr}^* = 25\%$) **under the benchmark tax progressivity** ($\tau = 0.2$) **by age and demographic.** (**Top:** Work hours, **Middle:** Labor force participation, **Bottom:** Labor in efficiency unit).

For non-parents, the negative wealth effect due to the increased tax burden to fund the 341.67% larger FTB spending, as reflected by a 6.85*pp* jump in the average tax rate in Table 8, appears to make them exert stronger work effort. Nonetheless, their overall labor supply response remains modest, with increases hovering around 2% (Figure 14). At the same time, this group experiences a sustained consumption decline. Married non-parents' consumption, for instance, falls by approximately 4% throughout their life cycle. These childless couples also save more under the new regime, with up to 10% increase in wealth near retirement (Figure 15), thus demonstrating more reliance on savings as a means of self-insurance. Single male households show similar changes in their consumption and wealth profiles, though to a greater extent since my model forbids this group from adjusting their labor supply.

For parent households, responses vary but generally reflect significant reductions in labor supply. This suggests the dominance of the positive wealth effect from the new universal child benefits, especially among those with low education. Case in point, young low-education single mothers reduce their participation by about 10*pp* during most of their prime working years, which results in as much as 25% loss in labor efficiency. For low-education parents overall, Figure 15 indicates that the lost labor earnings are offset by substantial increases in wealth as a source of self-insurance. Ultimately, low-education parents increase their average consumption levels considerably during younger years, although their reduced work effort causes their consumption to fall later in life once they exit the programs.

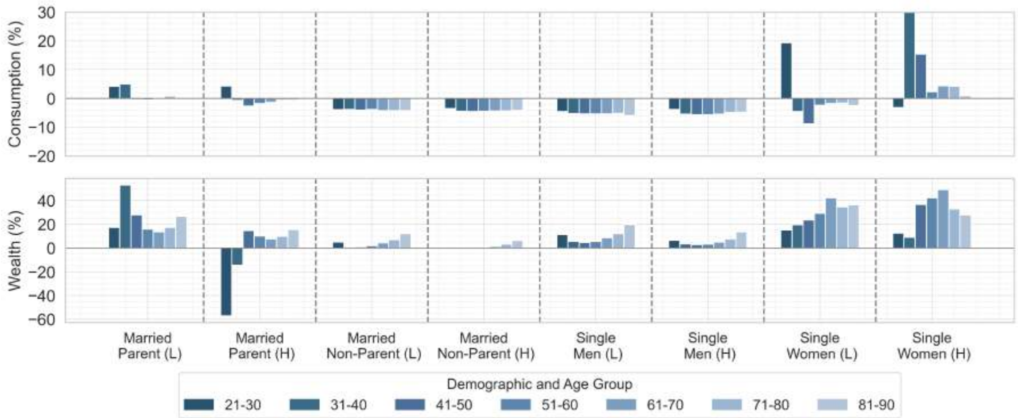

Figure 15: **Household consumption and wealth responses to the optimal child benefit system** ($\bar{tr}^* = 25\%$) **under the benchmark tax progressivity** ($\tau = 0.2$) **by age and demographic** (**Top:** Consumption, **Bottom:** Wealth).

An exception is high-education single mothers, whose responses deviate from the norm. Figure 14 shows a substantial increase in their labor supply and consumption between the ages of 31 and 40, corresponding to the arrival of their second child. The labor response suggests that the work incentives from the negative wealth effect of higher taxes, coupled with the absence of means-testing, outweigh the disincentive effects of the transfers. This group also experiences significant increases in wealth by as much as 45% between ages 51 and 60 (Figure 15), which helps explain their increased leisure after age 50.

Distribution of welfare changes. Figure 15 suggests that the overall welfare gains under the optimal child benefit system are driven exclusively by welfare improvements of parents, with the biggest beneficiaries, low-education single mothers, experiencing up to 23% increase in welfare. The changes in their labor supply and consumption suggest that these gains can be attributed to the significantly increased consumption and leisure during the critical child-rearing period. Additionally, these results imply that, for parent households, the benefits they receive under this reform more than compensates for the adverse impacts due to the rising tax burden. Conversely, the increased tax burden to fund the universal FTB imposes large costs on non-parents, resulting in a welfare decline of approximately 4% post-reform.

As opposed to the optimal tax reform, which fails to benefit all parents, the optimal child benefit regime accomplishes this goal while also delivering greater welfare improvements overall and for vulnerable families. However, this achievement also entails a higher welfare loss for non-parent households, raising serious equity concerns.

Deviations from the optimal benefit payment. Figure 17 indicates that while a smaller transfer payment may impose a lighter tax burden and smaller welfare losses for non-parents, it may also be insufficient

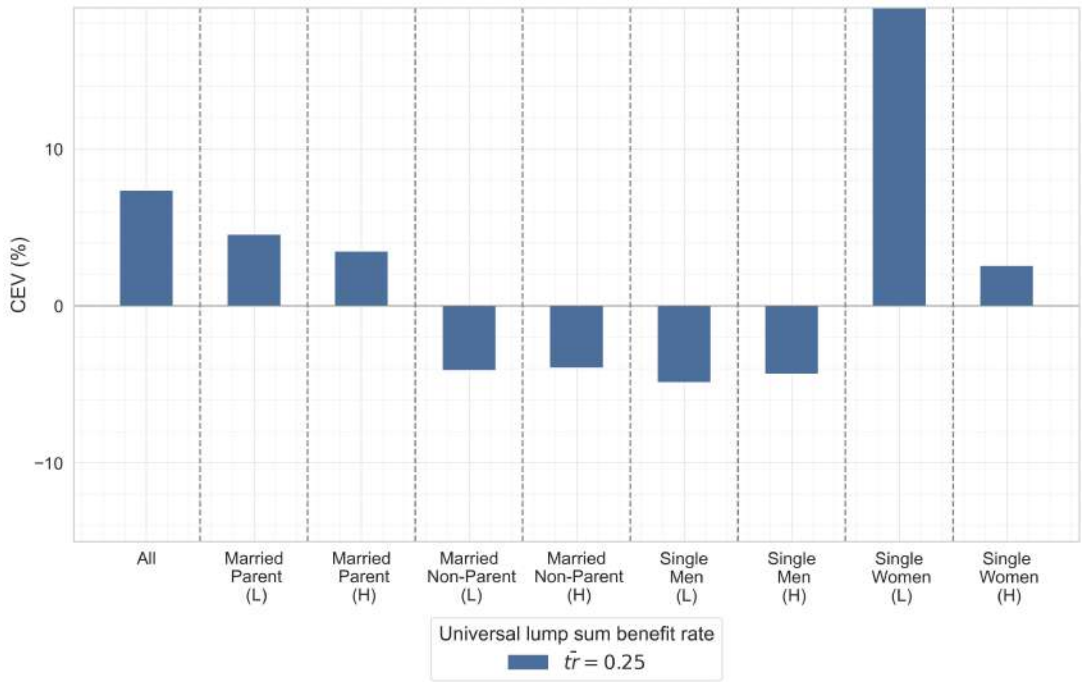

Figure 16: **Distribution of welfare changes under the optimal child benefit system** ($\bar{tr}^* = 25\%$) **and benchmark tax progressivity** ($\tau = 0.2$)**.**

to offset reduced earnings of recipients due to high taxes, resulting in welfare losses for the intended beneficiaries. As illustrated, a universal program with $\bar{tr} = 15\%$ leads to approximately a 1% welfare loss for low-education single mothers.

In contrast, expanding universal child benefits negatively impacts all households, including vulnerable parents. As shown in Table 8, a more generous payment at 35% of median income (or AUD 21,000 in 2018) causes the average tax rate to rise by 20.59pp. The tax scale parameter ζ falls by 0.22 points—over four times the change under the optimal child benefit scheme. This implies a significant tax burden, causing the welfare of non-parents to fall by as much as 30%. Although non-parents bear the largest losses, this regime also negates the benefits of short-term transfers for parents, resulting in an approximate 12% welfare loss for this group.

Composition of welfare changes. As illustrated in Appendix Figure 30, overall welfare changes as the child benefit rate $\bar{tr}$ varies follow an almost hump-shaped profile, with positive welfare outcomes for payments between 15% and 30%. However, increasing the transfer beyond 30% leads to a sharp welfare decline of around 12%. Additionally, the figure shows that these changes in welfare are almost exclusively driven by changes in consumption allocative efficiency consumption, while the leisure insurance effect (CEV_{LI}) is present but modest. In general, overall welfare and consumption efficiency change by similar magnitudes as the payment level varies.

These findings warrant a closer examination of the factors driving welfare changes under the optimal child benefit system. At the aggregate level, Table 9 reveals that the average newborn household gains 7.39% in welfare, driven by a 5.47% increase in consumption allocative efficiency and a 2.44% rise in leisure insurance. While there are losses from reduced consumption insurance (CEV_{CI}) and leisure allocative efficiency (CEV_{LE}), these effects are relatively small.

The dominant role of consumption allocative efficiency aligns with the consumption patterns in Figure 15. The significant post-reform consumption increases among parent households during child-bearing and rearing years—particularly for vulnerable groups such as low-education single mothers who face self-insurance constraints—enhance allocative efficiency despite some decreases in consumption as they age. Moreover, the positive leisure insurance effect indicates lower ex-post leisure risk. This likely stems from increased public support and savings under the optimal child benefit regime, allowing parents (who form the majority) to achieve

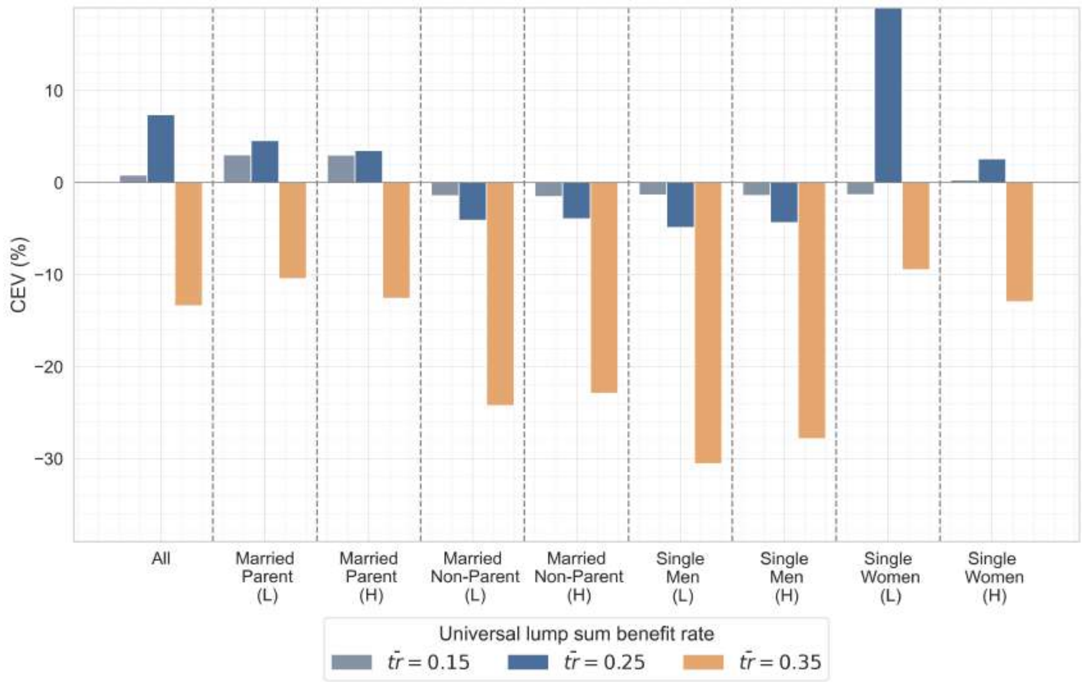

Figure 17: **Distribution of welfare changes over different levels of universal child benefits per child** ($\bar{tr}$) **under the benchmark tax progressivity** ($\tau = 0.2$).

better leisure outcomes under adverse shocks.

Across demographic groups, Appendix Figure 31 reveals similar mechanisms underlying their welfare changes. For non-parents, welfare losses are driven almost entirely by declines in consumption allocative efficiency, with minimal contributions from other factors. The dominance of consumption allocative efficiency in explaining welfare improvements is also evident among parents, especially low-education single mothers whose welfare increases by nearly 14% from this factor alone. Since parents experience significant consumption improvements during their younger years, followed by lower consumption in later life, this implies that young parent households have higher marginal utility of consumption than their older selves, reflecting binding credit constraints on their decisions in the initial steady state.

Low-education single mothers are the only group to see pronounced increases in consumption allocative efficiency and leisure insurance. Their improved ex-post leisure outcomes can be attributed to the new universal child support payments and a significant increase in wealth over their life cycle (Figure 15). Indeed, their enhanced leisure insurance is largely responsible for the overall rise leisure insurance observed in Table 9.

Welfare (%)	CEV	CEV_{CE}	CEV_{CD}	CEV_{CI}	CEV_{LE}	CEV_{LD}	CEV_{LI}
$\bar{tr}^* = 25\%$	$+7.39$	$+5.47$	$+0.046$	-0.32	-0.76	$+0.13$	$+2.44$

Table 9: **Decomposition of overall welfare changes under the optimal universal lump-sum child benefits** ($\bar{tr}^* = 25\%$) **and the benchmark tax progressivity** ($\tau = 0.2$).

These results offer three key lessons. First, the child benefit reform generates significantly stronger welfare effects—nearly an order of magnitude larger—compared to the optimal tax regime in Subsection 6.1.

Second, the optimal child benefit reform is highly advantageous for vulnerable parents, particularly low-education single mothers, by allowing them to more efficiently allocate consumption and improve their ex-post leisure outcomes. However, the reform also results in substantial reductions in labor supply and, consequently, lower human capital for this group.

Third, the optimal child benefit system is inequitable, transferring welfare from non-parents to parents. Similar to the findings in Tin and Tran (2024), a more generous transfer leads to welfare losses for all households, including parents. In other words, excessive short-term child benefits fail to offset the adverse effects of the associated tax burden on the intended beneficiaries. In contrast, a less generous system, while alleviating the tax burden on non-parents, may provide inadequate support for parents.

6.3 Optimal taxes and child benefits

The tax and child benefit reforms exhibit distinct quantitative and qualitative impacts. Next, I explore their joint optimal design to assess whether combining these two systems can yield further aggregate or distributional improvements. As in the previous sections, I simplify child-related policy instruments to the benefit rate ($\bar{tr}$) and tax progressivity (τ), deferring a broader set of means-testing parameters—such as phase-out rates and income-test thresholds—for future work. Specifically, the current analysis focuses on a counterfactual reform that jointly optimizes tax progressivity and benefit rates of the *Universal Lump-Sum Child Benefit program* (or *Universal FTB*), while retaining the existing structure of the means-tested CCS program. Based on the simulation results, I then propose an optimal policy mix—a pair $\{\tau^*, \bar{tr}^*\}$—that maximizes ex-ante welfare. The outcomes related to equity and key macroeconomic variables, such as female labor supply, output, and parental welfare, are also examined.

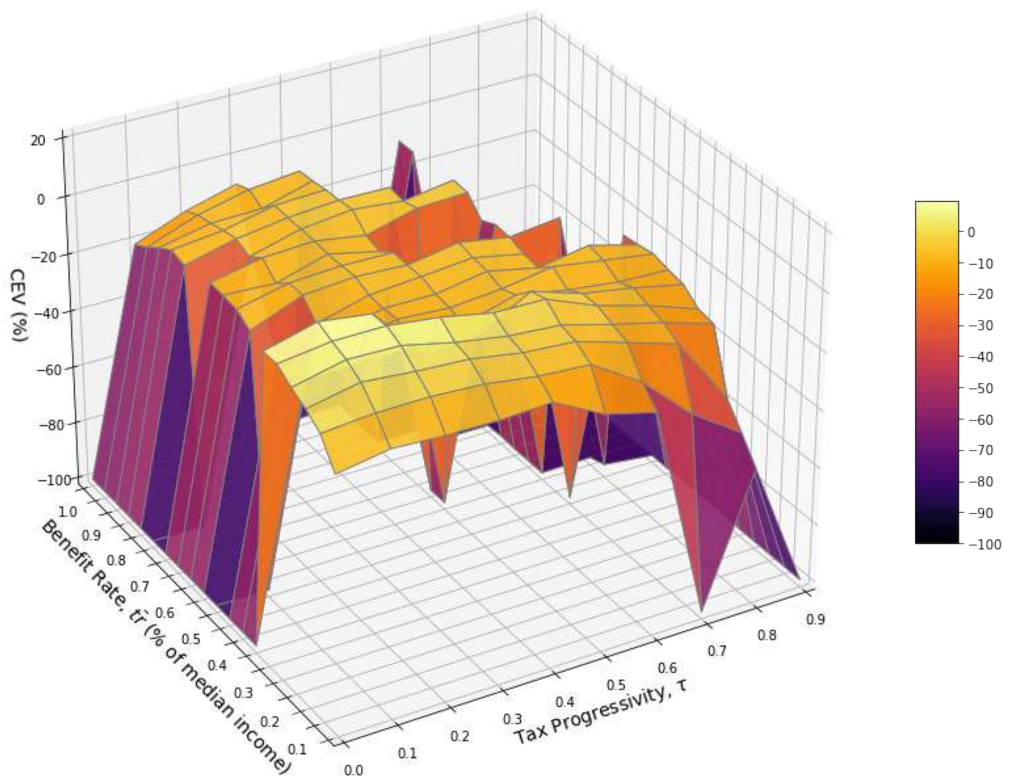

Figure 18: **Overall welfare changes over different combinations of tax progressivity (τ) and universal lump-sum child benefits per child ($\bar{tr}$).**

Notes: For a cross-sectional view at $\tau^* = 0.1$, refer to Appendix Figure 32.

6.3.1 Optimal tax progressivity and universal lump-sum child benefits per child

As shown in Figure 18 and Appendix Figure 32, within the confines of parameter values explored, the joint optimal design of taxes and child benefits integrate features from individual reforms. Specifically, it calls for a lower tax progressivity of $\tau^* = 0.1$ and a generous universal lump-sum benefit per child at $\bar{tr}^* = 30\%$ of median income, or approximately AUD $18,000$ in 2018. This transfer is 5 percentage points (*pp*) higher than the optimal rate of the standalone child benefit reform in Subsection 6.2. This means a household with two children—regardless of income, marital status, or children's ages—would receive \$$36,000$ annually.

At the aggregate level, Table 10 indicates that the jointly optimized tax and child benefit system results in a 1.37% higher aggregate consumption and a 9.64% rise in overall welfare. However, it also leads to significant declines in female labor force participation and work hours, by 5.04*pp* and 5.23%, respectively, contributing to a 1.06% reduction in output.

Household responses under the optimal tax and child benefit system. Figure 19 provides insights into labor supply and consumption responses. For non-parent couples, who do not receive child benefits, their decisions are influenced solely by the tax consequences of the joint system reform. The optimal policy's less progressive tax structure shifts tax liabilities from higher- to lower-income brackets, encouraging longer work hours but reducing participation, particularly for women with limited earning potential, such as those with low education.

Aggregate implications of optimal tax and universal child benefits			
	$\bar{tr} = 20\%$	$\bar{tr}^* = 30\%$	$\bar{tr} = 40\%$
CCS size, %	−3.06	−28.43	−8.49
FTB size, %	+252.78	+430.56	+608.33
Average tax rate, *pp*	+4.94	+6.57	+22.36
Tax scale (ζ)	−0.004	−0.029	−0.212
Fe. LFP, *pp*	−4.87	−5.04	−2.35
Fe. Hour, %	+0.92	−5.23	−5.43
Fe. H. cap, %	−0.45	−0.35	−0.86
Cons (C), %	+1.39	+1.37	−12.57
Output (Y), %	+0.44	−1.06	−16.3796
Welfare (CEV), %	+5.57	+9.64	−14.89

Table 10: **Aggregate implications of optimal tax progressivity and universal lump-sum child benefits per child at three levels of payment:** 20% (first column), 30% (second column) and 40% (third column) of median income in 2018.
Notes: Results are reported as changes relative to the levels in the benchmark economy. Median income is approximately AUD 60,000 in 2018 dollars.

Additionally, Table 10 reports a dramatic 430.56% increase in FTB spending under the optimal joint system. While this is partially offset by a rise in consumption tax revenue (due to the 1.37% increase in consumption) and a 28.43% decrease in CCS spending, the universal FTB still exerts significant pressure on the tax system and causes the tax scale parameter ζ to decrease by 0.029. This pushes the tax schedule upward, shrinks the zero-income tax zone, and raises the average tax rate by 6.57%. The resulting tax burden constitutes a negative wealth effect and further increases the marginal tax rates (MTR) for all workers, especially those in the lower income brackets.

Compared to an alternative regime with optimal progressivity τ^* but lower benefit payment ($\bar{tr} = 20\%$), where the tax scale remains virtually unchanged (Table 10), the additional tax burden under the joint optimal system does not alter the general pattern of responses. The intensive-extensive margin trade-off of labor supply due to tax progressivity remains evident. However, the increased tax burden does moderately increase work hours and participation among childless women due to its negative wealth effect (Figure 50 in the Appendix). Furthermore, as depicted in Figures 20 and 51, the tax burden has a noticeable adverse impact on non-parents' consumption, particularly during their younger years, with consumption falling by as much as 8%.

For parent households, their labor supply profiles in Figure 19 reveal that the joint optimal design leads to a significant drop in female labor supply. While parents and non-parents face similar tax incentives, the universal lump-sum scheme effectively pays mothers to reduce their work hours and participation. This positive wealth effect is especially pronounced among low-education single mothers, whose work hours and participation decline by up to 25% and 15*pp*, respectively, during their prime working years. Figure 20 suggests that the decreased labor supply is also partly due to the improved ability to accumulate wealth from the universal benefits, allowing parents to self-insure against future risks without needing to work as much. Consequently, parents, particularly young single mothers, see substantial improvements in their consumption and leisure

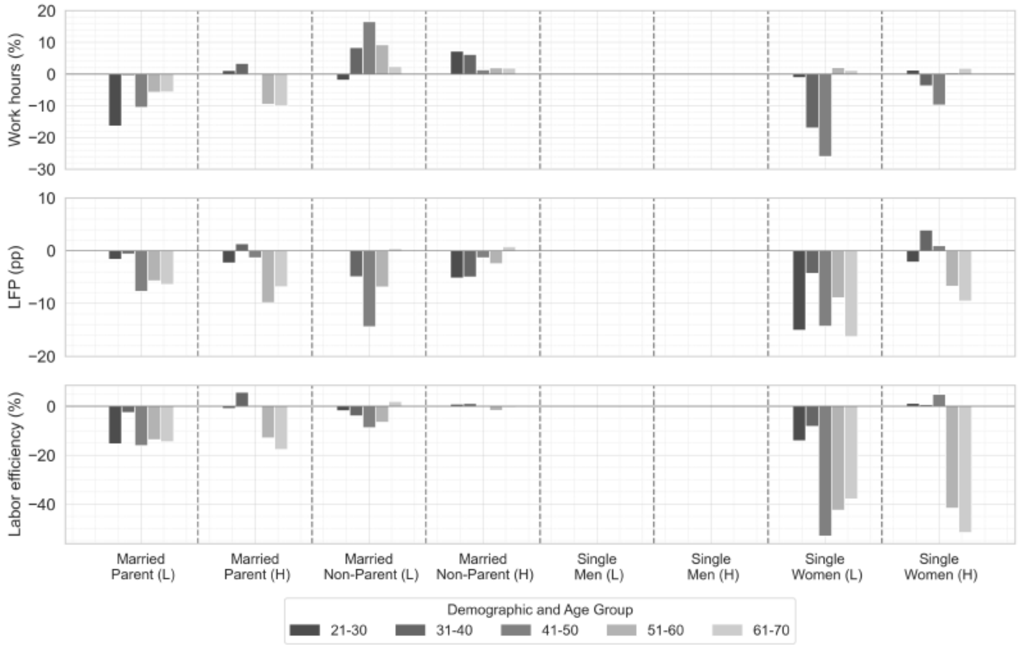

Figure 19: **Female labor supply responses to the optimal tax and child benefit system by age and demographic.** (**Top:** Work hours, **Middle:** Labor force participation, **Bottom:** Labor in efficiency unit).

profiles under the jointly optimized tax and child benefit reform.

Distribution of welfare changes. Gains in average consumption and leisure over the life cycle for parents, contrasted with declines for non-parents, help explain the distribution of welfare changes under the new regime. As depicted in Figure 21, all parent households enjoy substantial welfare improvements, with low-education single mothers seeing the largest welfare increase of up to 27%. High-education married parents, while receiving the smallest gains, still benefit from a 5% welfare boost. Because parents constitute 77% of the model population, the reform would likely garner majority support. However, these gains come at a cost of 4-7% welfare losses for non-parent households. Therefore, despite the 9.64% increase in overall welfare, as in Subsection 6.2, the joint optimal system redistributes welfare from non-parents to parents.

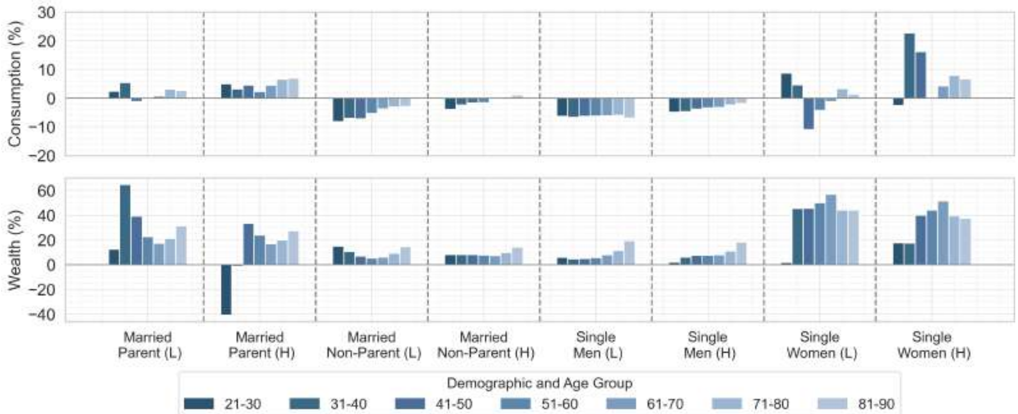

Figure 20: **Household consumption and wealth responses to the optimal tax and child benefit system by age and demographic.** (**Top:** Consumption, **Bottom:** Wealth).

6.3.2 Welfare effects across the three major reforms

Does the joint optimal tax and child benefit regime provide better welfare outcomes compared to the individual system reforms? To what extent are the welfare outcomes driven by the child benefit reform rather than by the

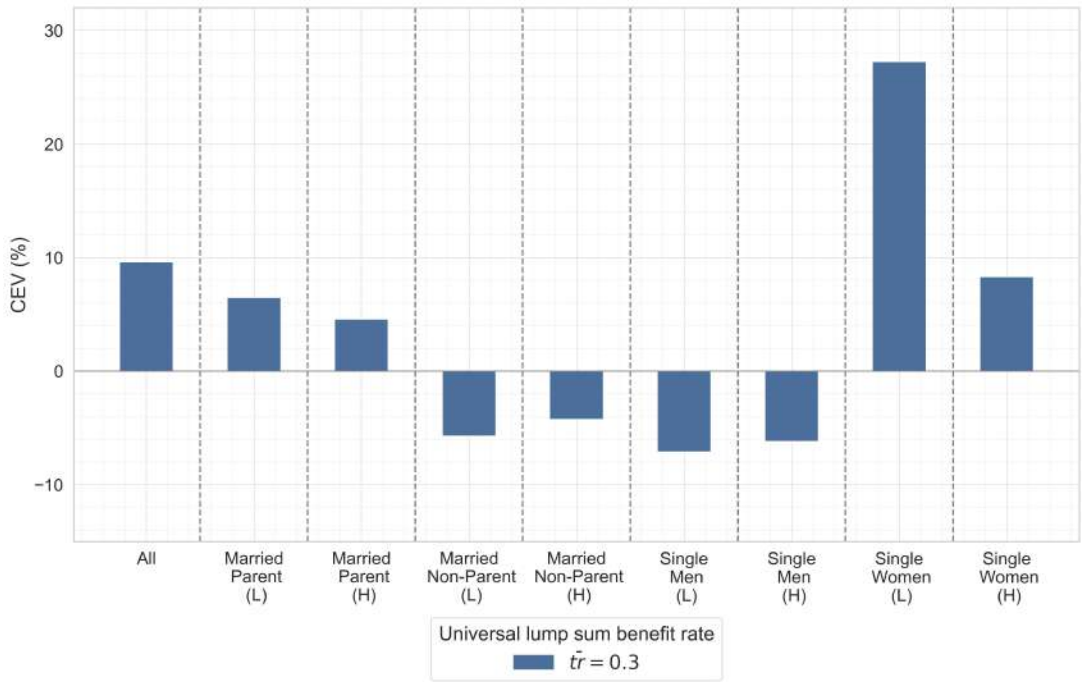

Figure 21: **Distribution of welfare changes under the optimal tax and child benefit system.**

tax progressivity? Why is the transfer higher under the combined system optimization relative to the standalone child benefit reform? To address these questions, I compare welfare outcomes across the three key reforms: (i) the optimal tax system ($\tau^* = 0.1$) from Subsection 6.1, (ii) the optimal child benefit system ($\bar{tr}^* = 25\%$) from Subsection 6.2., and (iii) the jointly optimized system ($\tau^* = 0.1$ and $\bar{tr}^* = 30\%$). Additionally, I introduce welfare outcomes by demographic group for a regime that combines individual optimal reforms, $\tau = 0.1$ and $\bar{tr} = 0.25$, without adjusting the transfer amount to mach the jointly optimized reform system.

Welfare (%)	All	Married Parent (L)	Married Parent (H)	Married Non-parent (L)	Married Non-parent (H)	Single Men (L)	Single Men (H)	Single Women (L)	Single Women (H)
$\tau^* = 0.1$	+1.38	−0.94	+0.25	−1.59	−0.56	−2.04	−1.40	+7.86	−1.42
$\bar{tr}^* = 25\%$	+7.39	+4.59	+3.50	−4.10	−3.95	−4.87	−4.35	+22.80	+2.59
τ=0.1, $\bar{tr}$=25%	+7.21	+5.95	+4.35	−4.40	−3.20	−5.15	−4.55	+16.09	+8.50
τ^*=0.1,$\bar{tr}^*$=30%	**+9.64**	**+6.49**	**+4.59**	**−5.73**	**−4.26**	**−7.12**	**−6.21**	**+27.27**	**+8.32**

Table 11: **Distribution of welfare changes by demographic under the three key reforms: First row**: Optimal tax system with benchmark child benefits; **Second row**: Optimal child benefit system with benchmark tax progressivity; **Third row**: Combined individual optimal reforms; **Fourth row**: Optimal tax and child benefit system.
Notes: Results are reported as changes relative to the levels in the benchmark economy.

Comparing welfare outcomes. In terms of overall and parental welfare impacts, results in Table 11 indicate that the joint optimal design of taxes and child benefits yields significantly larger gains than the individual reforms of either system. The overall welfare improvement under the joint system is 9.64%—1.3 times greater than the 7.39% achieved under the optimal child benefit system and 7 times higher than the 1.38% increase under the optimal tax system. Similar patterns are observed across parent groups, thereby underscoring the importance of a holistic approach in designing tax and child benefit systems to effectively meet policy objectives.

The key reform behind welfare changes. The largest welfare gains overall and for parents come from the universal lump-sum child benefit system (or universal FTB). Even when implemented in isolation, the optimal child benefit system significantly enhances welfare for all parents, with the greatest gains seen among low-education single mothers, whose welfare increases by almost 15*pp* more than under the tax reform. Thus, the universal reform more than compensates parents for the adverse effects of the tax burden, allowing these households to attain substantial welfare improvements.

The reverse holds true for non-parent households. They do not receive child-related transfers but bear the fiscal pressure of funding the system. Due to the heavier tax load under the child benefit reform (compared to the initial status quo and the optimal tax regime), non-parent households experience substantial welfare losses when the partial universalization of the child benefit system is introduced. Their welfare declines further under the jointly optimized system due to the larger child benefits and therefore fiscal stress. For vulnerable non-parent households, such as low-education childless couples, welfare losses reach -5.73%, more than triple those under the optimal tax system (-1.59%) and 1.4 times higher than under the optimal child benefit reform (-4.1%). The joint optimal taxes and child benefits therefore worsen the inequitable redistribution problem, magnifying welfare gains for the winners and losses for the losers.

Larger transfers under the joint optimal system. The joint optimal tax and child benefit system provides a 5*pp* higher lump-sum transfer to parents compared to the standalone optimal child benefit system. As demonstrated in Table 11, this additional benefit is directed primarily toward low-education parents, especially single mothers.

A closer comparison between the second and third rows of Table 11 reveals that simply incorporating lower tax progressivity ($\tau = 0.1$), matching the optimal tax system, into the optimal child benefit regime ($\bar{tr} = 0.25\%$) yields notable welfare gains for parents with high education. In particular, highly-educated single mothers experience a significant welfare increase, from 2.59% to 8.5% (relative to the benchmark level).

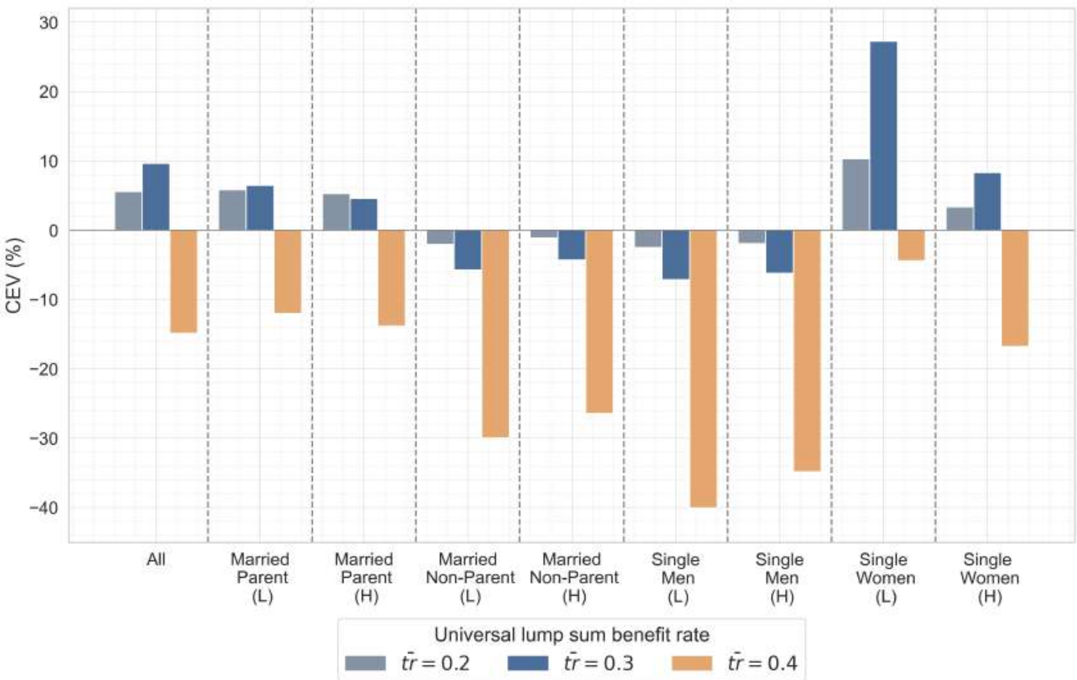

Figure 22: **Distribution of welfare changes under the optimal tax progressivity** ($\tau^* = 0.1$) **over different levels of universal child benefits per child** ($\bar{tr}$)**.**

However, the decreased tax progressivity adversely affects single mother households who, due to the universal child benefits, reduce their labor supply and thus fall into lower income brackets. As a result, the smaller progressivity exposes them to higher tax liabilities, reducing their welfare gain from 22.8% relative to the benchmark to 16.09%, representing nearly a one-third decline.

On net, these changes lead to a slight reduction in overall welfare, from 7.39% to 7.21%. Given the important role of low-education single mothers in driving overall welfare outcomes, the joint optimal system seeks to compensate this group for the losses incurred by the tax reform. This compensation explains the additional universal transfer under the joint optimal tax and transfer system, which increases single mothers' welfare to 27.27%. Notably, this extra transfer offers minimal benefits to high-education parents while substantially increasing welfare losses for non-parents due to the higher tax burden.

Furthermore, Figure 22 indicates that as the universal lump-sum child benefit system expands, welfare declines across all demographics. The increased tax burden needed to fund a more generous universal payment not only deteriorates welfare of non-parents but also negates the short-term benefits for parents. An overly generous system ultimately fails to deliver positive welfare outcomes for the intended beneficiaries. Conversely, a smaller system with a benefit rate of $\bar{tr} = 20\%$, while not maximizing overall welfare, provides moderate gains for all parents at significantly smaller welfare costs for non-parents—about one-third of the losses they would experience under the optimal scenario.

Composition of welfare changes. Figure 23 shows that as the generosity of universal child benefits varies, welfare changes are primarily driven by changes in consumption allocative efficiency (CEV_{CE}). Once the benefit rate exceeds the optimal level of $\bar{tr}^* = 30\%$, rising fiscal pressures take hold and result in a decline in consumption efficiency and overall welfare. For example, Table 10 demonstrates that increasing the benefit rate to 40% significantly raises the overall tax burden, reflected by a steep 0.212 decrease in ζ, and causes the average tax rate to surge by 22.36%. This, in turn, depresses household consumption over the life cycle compared to the optimal reform (as illustrated in the bottom panel of Figure 51 in the Appendix). Consequently, both consumption allocative efficiency and overall welfare decrease by approximately 15%.

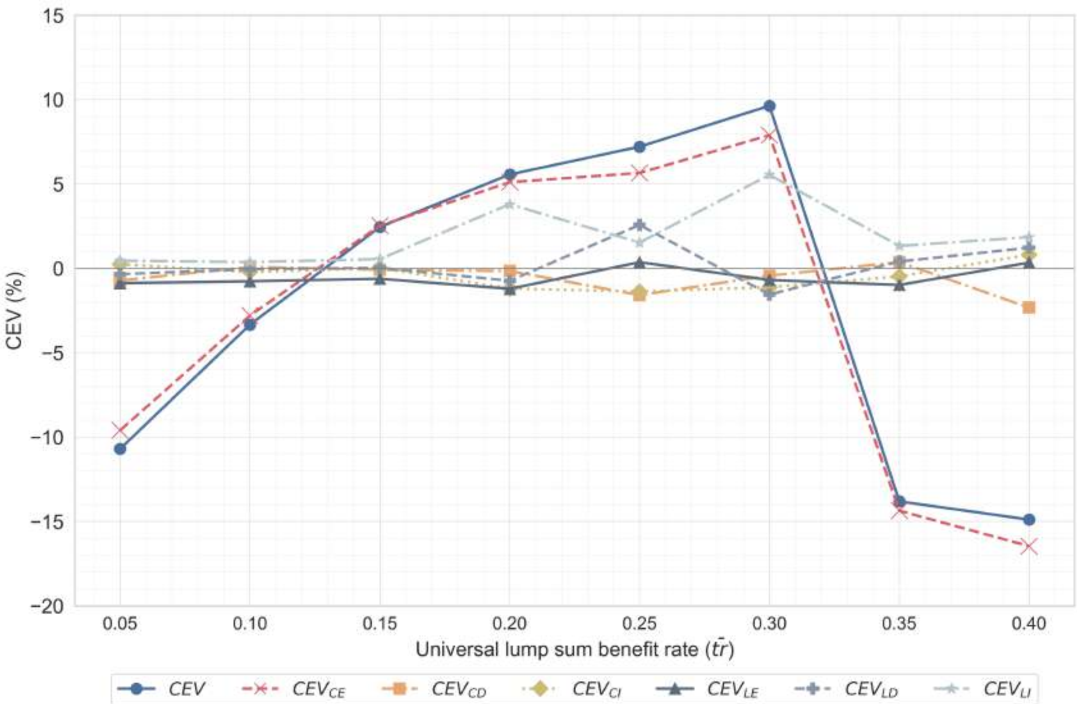

Figure 23: **Decomposition of overall welfare changes under the optimal tax progressivity** ($\tau^* = 0.1$) **over different payment rates of universal lump-sum child benefits** ($\bar{tr}$)**.**

While CEV_{CE} remains the dominant factor in most scenarios, moderate child benefit payments corresponding to $\bar{tr} \in \{20\%, 25\%, 30\%\}$—including the optimal level—also yield gains in leisure insurance (CEV_{LI}). This suggests that newborn households entering the post-reform economy experience better ex-post leisure outcomes relative to the status quo.

As shown in Table 12, under the jointly optimized system with $\tau^* = 0.1$ and $\bar{tr}^* = 30\%$, welfare improvements are driven primarily by a 7.9% increase in consumption allocative efficiency (CEV_{CE}) and a 5.55% rise in leisure insurance (CEV_{LI}). There are also modest negative effects on consumption distribution (CEV_{CD}) and insurance (CEV_{CI}), as well as adverse impacts on leisure allocative efficiency (CEV_{LE}) and distribution

(CEV_{LD}). Regardless, these losses are relatively small and are outweighed by the more substantial gains in CEV_{CE} and CEV_{LI}.

A comparison of the three key reforms in Table 12 suggests that much of the overall and composition of welfare gains under the joint optimal system can be explained by the universalization of lump-sum child benefits. Optimizing tax progressivity contributes a modest 1.07% increase in CEV_{CE} with minimal impacts on other components of welfare. It is only through the incorporation of the child benefit reform that significant improvements in CEV_{CE} and CEV_{LI}, along with moderate losses in the other welfare components, materialize.

Welfare (%)	CEV	CEV_{CE}	CEV_{CD}	CEV_{CI}	CEV_{LE}	CEV_{LD}	CEV_{LI}
$\tau^* = 0.1$	+1.38	+1.07	+0.03	+0.06	+0.04	+0.07	−0.07
$\bar{tr}^* = 25\%$	+7.39	+5.47	+0.05	−0.32	−0.76	+0.13	+2.44
$\tau^* = 0.1, \bar{tr}^* = 30\%$	**+9.64**	**+7.90**	**−0.43**	**−1.12**	**−0.69**	**−1.55**	**+5.55**

Table 12: **Decomposition of overall welfare changes under the three key reforms: Top row**: Optimal tax system with benchmark child benefits; **Middle row**: Optimal child benefit system with benchmark tax progressivity; **Bottom row**: Optimal tax and child benefit system.

In short, this comparison reveals that the child benefit reform is largely responsible for the welfare changes under the joint system optimization. It allows households to better smooth consumption over their life cycle (CEV_{CE}), and although leisure allocation becomes less efficient (CEV_{LE}), households enjoy more favorable ex-post leisure outcomes in adverse circumstances (CEV_{LI}) than in the initial steady state.[58] However, as average consumption rises post-reform, some demographic groups do find themselves with reduced ex-ante shares of consumption (CEV_{CD}) and leisure (CEV_{LD}). Additionally, the decline in labor supply and human capital under the joint optimal system reduces households' earnings potential and weakens their ability to self-insure against negative shocks, contributing to higher ex-post consumption risk (or lower consumption insurance, CEV_{CI}).

Composition of welfare changes by demographic. Consistent with the overall welfare composition, Figure 24 demonstrates that the primary driver of welfare changes for most demographic groups under the jointly optimized tax and child benefit schemes is the consumption allocative efficiency (CEV_{CE}) effect.

Case in point, parent households experience significant consumption allocative efficiency gains. This group sees substantially increased consumption during their younger years when child care responsibilities are most prominent (Figure 20). While some face reduced consumption at certain stages, the overall reallocation of consumption throughout their life cycle generates a positive efficiency effect, leading to a welfare boost of at least 5%.

Single mothers, especially those with low education, benefit most from the optimal system. Unlike other demographic groups, their welfare improvements stem from multiple factors. The largest contributor remains the 20% increase in their consumption allocative efficiency, but their welfare is further enhanced by positive leisure insurance effects. These improvements arise partly from the universal child benefits received when their children are dependent and the increased wealth buffer accumulated under the reform (Figure 20). However, single mothers also experience negative effects on consumption insurance (CEV_{CI}) and adverse distributional outcomes in both consumption (CEV_{CD}) and leisure (CEV_{LD}). The decline in consumption insurance likely reflects their increased reliance on child benefits and savings, which diminishes their labor efficiency units (Figure 19) and consequently heightens their ex-post consumption risk. On the other hand, the negative consumption and leisure distributional effects suggest that, despite their consumption and leisure gains, single mothers can expect to consume less and work more relative to the new population averages.

These findings offer three insights. First, the proposed optimal design of taxes and universal lump-sum child benefits—featuring lower tax progressivity at $\tau^* = 0.1$ and a universal transfer of 30% of median income—outperforms the status quo system and previously considered individual reforms in terms of both overall and

58These adverse circumstances mean realizations of bad states, such as low asset holdings and negative earnings shocks.

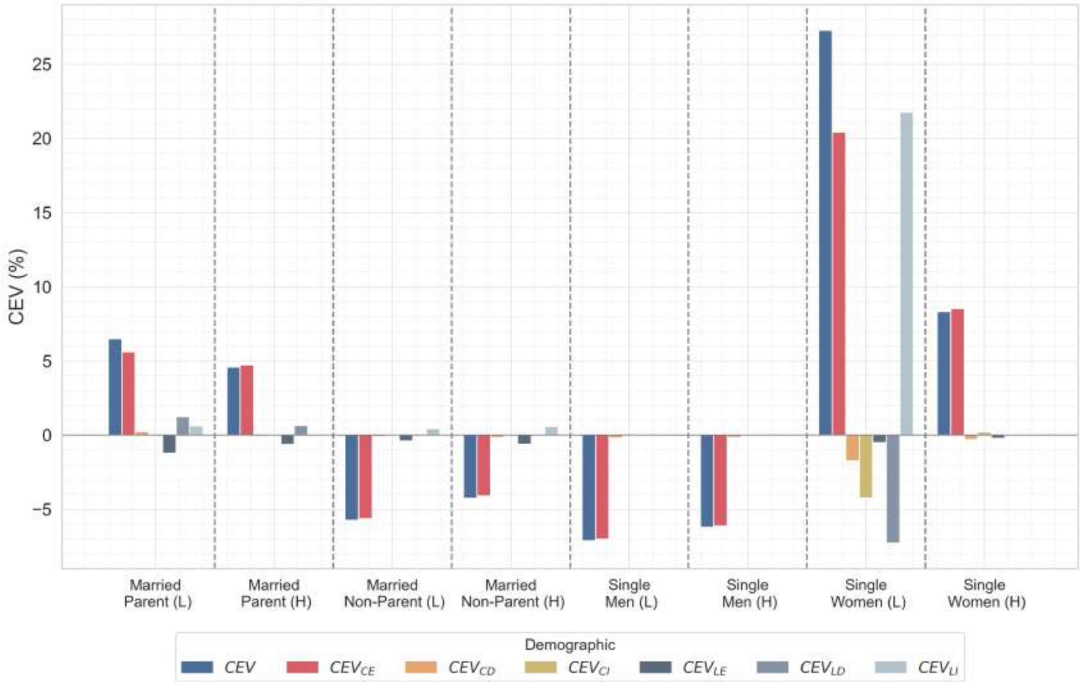

Figure 24: **Decomposed welfare changes by demographic under the optimal tax and child benefit system.**

parental welfare improvements. This attests to the importance of joint policy design in achieving policy targets. In this study, I demonstrate that the joint optimal system adjusts tax progressivity to benefit high-education households and offset the resulting losses for vulnerable parents by increasing transfers.

Second, the optimal system raises parental and overall welfare but presents equity challenges. The regime transfers welfare from non-parents to parents, thus raising distributional concerns. Consistent with Tin and Tran (2024), I show that adjusting the generosity of the universal system does not resolve this redistributional issue. In fact, a more generous system may harm all households, including parents, as the increasing tax burden eventually outweighs the positive effects of child support. In contrast, a less generous system offers smaller overall and parental welfare gains but imposes lower costs on non-parents, making it potentially more viable in certain policy contexts.

Third, as discussed, designing tax and child benefit systems to improve overall and parental welfare requires carefully balancing the short-term benefits of transfers against the adverse effects of rising tax burdens over the life cycle, which grow exponentially with the size of transfers. Striking this balance is crucial not only for society at large but also for the recipients of child benefits. Therefore, a life cycle perspective is integral to assessing the effectiveness of child benefit policies.

7 Conclusion

This paper studies the joint design of taxes and child benefits, evaluating the aggregate and distributional implications of the proposed optimal reform.

First, the findings reveal the close interconnection between tax and child benefit systems. An optimal tax reform, which does not account for parenthood, suggests a lower tax progressivity that promotes labor supply, primarily among highly educated women, and yields moderate welfare improvement. However, this shifts tax liabilities towards lower income brackets, causing welfare losses for some vulnerable parents. As such, an isolated tax reform, even if optimized, may undermine the objectives of child benefit programs. Evaluating distributional effects of tax reforms on welfare program beneficiaries is therefore crucial.

Second, assuming the existing Child Care Subsidy (CCS) program remains intact, the study finds that a joint optimal tax and child benefit system offers larger parental and overall welfare improvements than separate tax or child benefit reforms. This approach reduces tax progressivity ($\tau^* = 0.1$), benefiting high-education parents, while compensating low-education parents with a generous universal lump-sum benefit (per child) at 30% of median income in 2018 (approximately AUD $18,000$). However, non-parents bear the brunt of the additional tax burden from reduced progressivity and financing transfers to parents. A less generous system, while yielding smaller gains for parents, would impose considerably lower costs on non-parents, potentially making it more viable. Conversely, excessive transfers may fail to offset the adverse impacts of the tax burden, leading to welfare losses for all households, including parents.

Lastly, the distributions of welfare changes across reforms in this study underscore the unique vulnerabilities of low-education parents, particularly single mothers. In general, since parents constitute the majority and face distinct child-related constraints, optimizing for overall welfare inherently favors policies that benefit parents, often to the detriment of non-parents. Among parents, low-education single mothers are most affected by these reforms. Their welfare changes have a significant impact on overall welfare outcomes. This suggests that research on child-related welfare programs should explicitly account for the well-being of low-education single mothers to more accurately assess policy impacts and achieve targeted welfare improvements.

There are several caveats due to the assumptions made for tractability and computational ease. First, the model abstracts from male labor supply, fertility, marriage, and child quality decisions. Second, assumptions regarding children and child care costs could be refined to improve realism and account for behavioral responses among older mothers. Third, transitory shocks are modeled using a normally distributed innovation term, though empirical evidence (see Tin and Tran (2023) for Australia) suggests that shocks may follow a non-linear and non-Gaussian distribution. Fourth, the welfare of households living along the transition path is not accounted for. Finally, the current joint system design focuses on tax progressivity and the lump-sum child benefit rate. Expanding the policy space to include phase-out rates and income-test thresholds for means-tested child benefits and subsidies could yield greater overall and distributional improvements.

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Appendix

A Data: Additional empirical results

A.1 Taxes, child benefits, and EATR

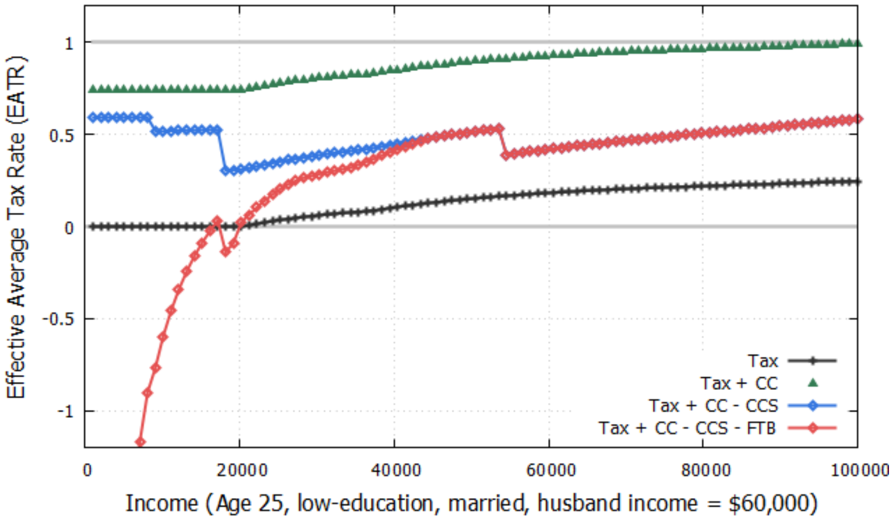

Figure 25: **Effective Average Tax Rate (EATR) schedule of a representative young low-education (at most high school) married mother with two children, whose husband earns $60,000$ in 2018.**
Notes: The black line is the average income tax rate (ATR), including Low Income Tax Offset (LITO). The dotted green line is the EATR when the average gross child care cost is added to the ATR. The blue line is the EATR that incorporates the average net child care cost (accounting for the Child Care Subsidy (CCS)). The red line is the total EATR schedule when the average net child care cost and Family Tax Benefit (FTB) is accounted for.

Figure 25 depicts a simulated effective average tax rate (EATR) schedule for a young mother of two children whose husband earns approximately the median income (around $60,000 in 2018). The figure shows that child benefits, particularly the FTB and CCS, significantly increase progressivity in the EATR—the average rate of tax and child care costs net of child benefits—for the mother than what could be achieved under the income tax system alone. This enhanced progressivity primarily stems from the generosity and non-mutually exclusive nature of the two benefit schemes. For example, for families situated below the median pre-government earnings, the FTB can account for up to 40% of their gross income. Moreover, through the CCS, low-income working parents could also receive up to 85% subsidy on their child care fees. Consequently, the child benefit programs in conjunction with the moderately progressive tax regime, where zero-tax zone extends until $18,200 followed by a low tax rate of 19% for earnings below $37,000, result in a strong redistributive effect.

A.2 Taxes, child benefits, and EMTR over the life cycle

EMTRs for a mother also vary over her life cycle since the child-related costs and transfers are conditional on the number and age of dependent children. Figure 26 shows two simulated life cycle EMTR profiles for a married mother of the same socioeconomic and demographic attributes as in Figure 2, comparing scenarios where she (i) stays at home (left panel) and (ii) works part-time (right panel).

In the stay-at-home scenario, the mother's EMTR is initially high, driven by child care fees and the phasing-out of child benefits. The MTR is nil since her first dollar earned is in the zero-tax bracket. However, with her first child born at age 21 and second child three years later, the EMTR peaks early in her economic life due to the high hourly formal care fees but decreases by age 30 as her children age and child care costs decline. The CCS partially offsets the child care costs, thus reducing her EMTR. Conversely, via its phase-out rate,

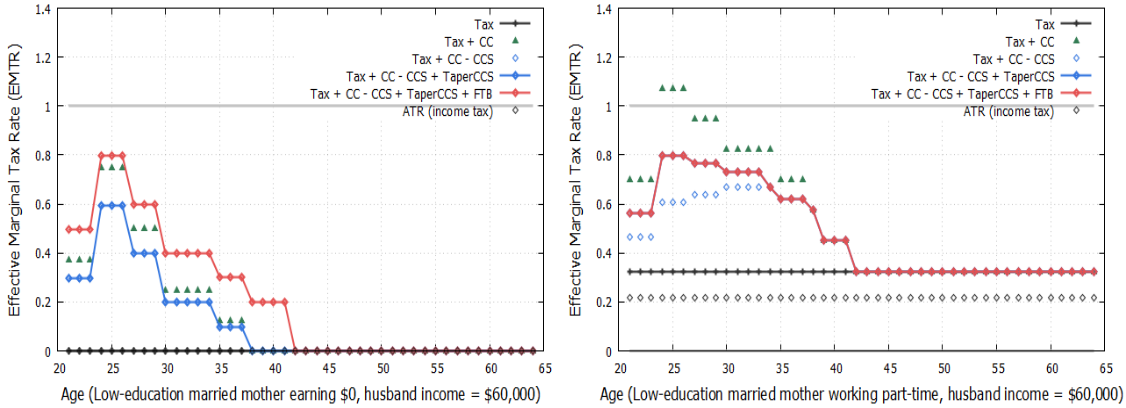

Figure 26: Life cycle profiles of Effective Marginal Tax Rate (EMTR) of a representative low-education married mother (with two children) whose husband earns $60,000 in 2018 if she stays at home (**left panel**) or works part-time (**right panel**). In the right panel, note how the red line (total EMTR) overlaps the blue line (EMTR without FTB). This suggests the FTB phase-out rate has no effect on the EMTR.

Notes: These lines show the cumulative effects, stacked successively. The black dotted line is the average income tax rate (ATR). The black solid line is the marginal tax rate (MTR), including Low Income Tax Offset (LITO). The dotted green line is the EMTR when the marginal rate of the gross child care cost (CC) is added on top of the MTR. The light dotted blue line is the EMTR that also incorporates the CCS. The heavy solid blue line accounts for both the CCS and its phase-out rate. The solid red line is the total EMTR schedule when the FTB's phase-out rate is included.

the FTB adds approximately 20 cents to the EMTR over her life cycle, raising the profile beyond what arises naturally from the child care expense.

In the part-time scenario, the MTR rises to around 35%, but the joint effect of the FTB and CCS appears more favorable. Because the CCS rate scales with work hours, it more than halves her EMTR from child care costs, despite her family income being in the subsidy's phase-out zone. The FTB, on the other hand, has no influence on her work decision at the margin. The mother's family income positions her beyond the FTB's cutout point, and consequently, the FTB does not alter her EMTR profile.

These observations reveal that the joint effect of the tax and child benefits is heterogeneous and non-linear over the life cycle. In most cases, the net effect of child benefits weakens the work incentive effect of the tax-free zone.

For the part-time mother, the means-tested child benefits lower her total EMTR profile, thus negating the work disincentives from taxes (35% MTR) and child care costs. This net favorable effect can be attributed to the fact that: (i) the FTB has completely phased out, and (ii) the CCS provides substantial support for part-time workers.

B Deviations from optimal progressivity: Supplementary results

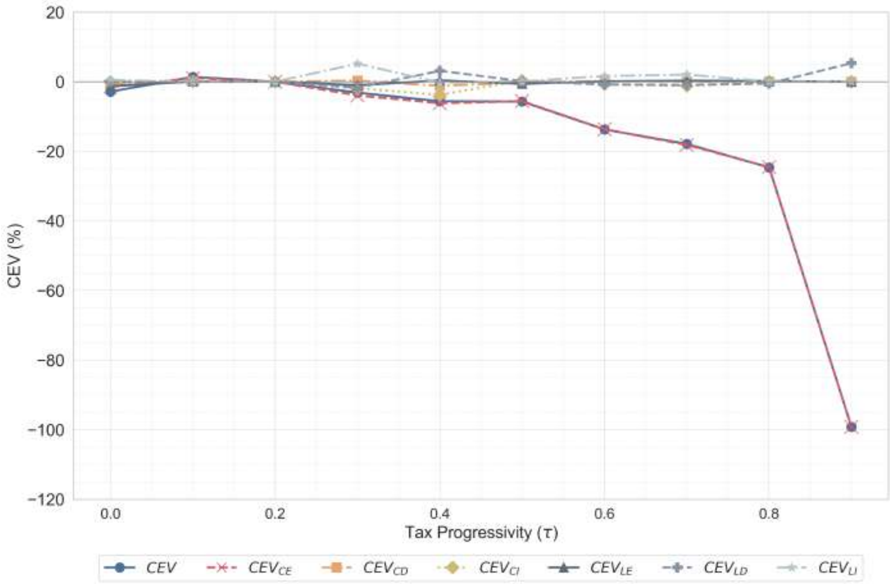

Figure 27: **Decomposition of welfare changes over tax progressivity under the benchmark means-tested child benefits.**

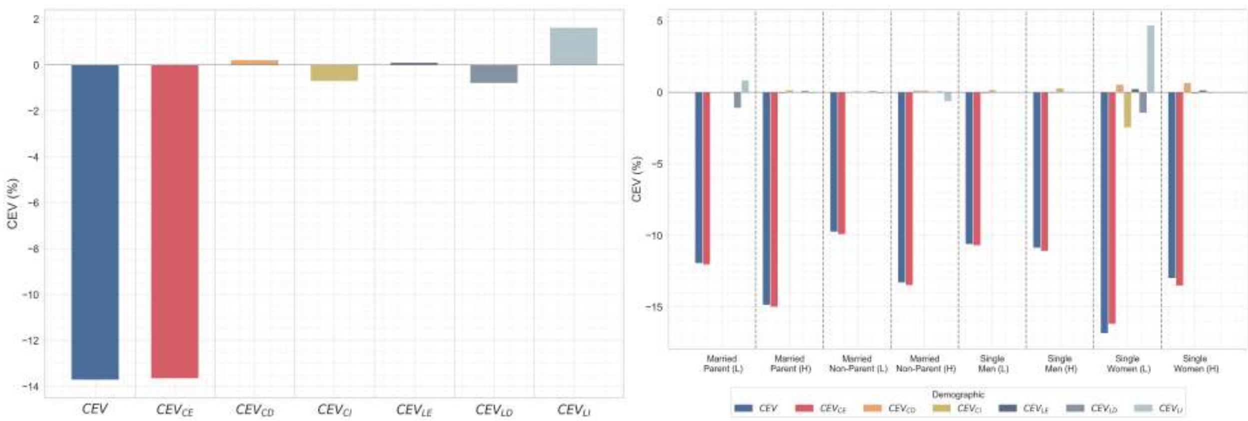

Figure 28: **Decomposition of welfare changes under the benchmark means-tested child benefits (FTB and CCS) and a highly progressive tax regime ($\tau = 0.6$). Left panel**: Overall; **Right panel**: By demographic.

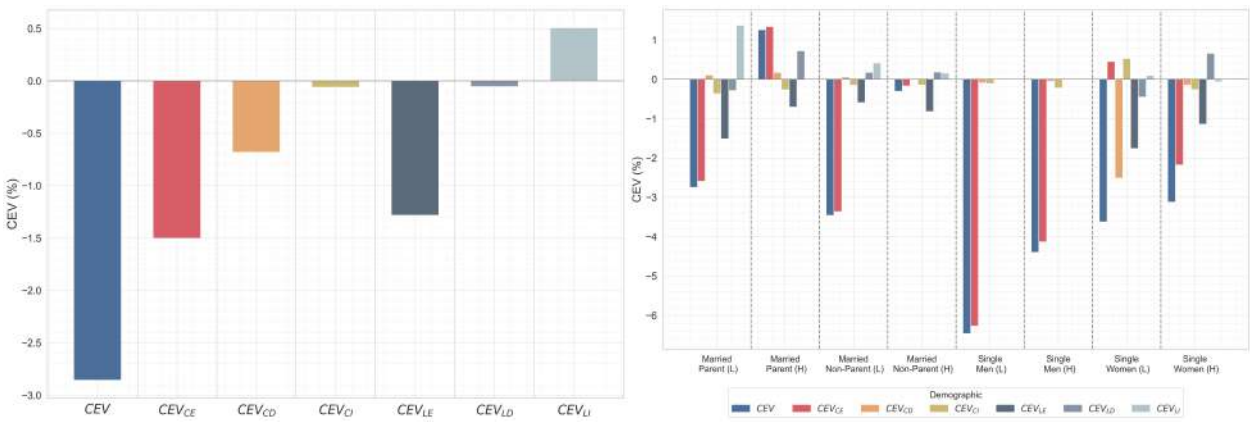

Figure 29: **Decomposition of welfare changes under the benchmark means-tested child benefits (FTB and CCS) and a proportional tax regime ($\tau = 0$). Left panel**: Overall; **Right panel**: By demographic.

C Optimal child benefits under the benchmark tax progressivity: Supplementary results

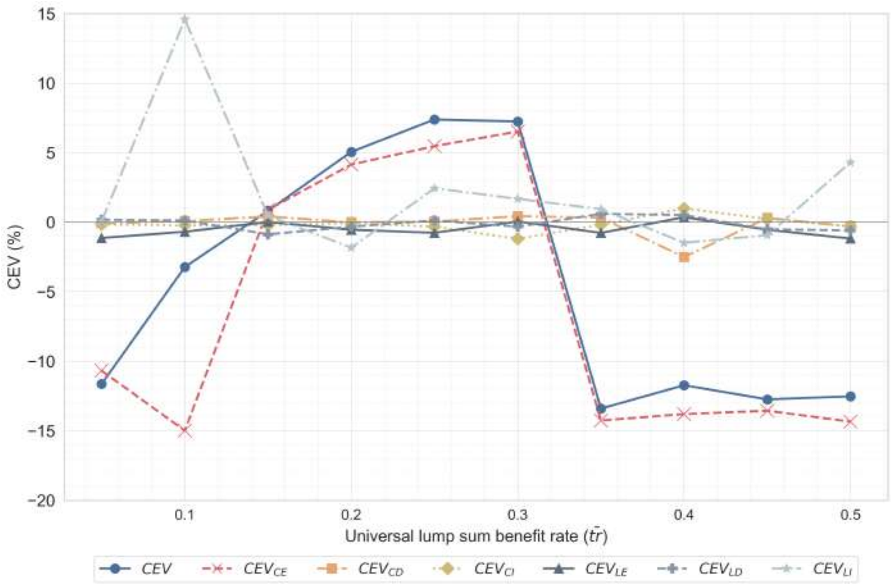

Figure 30: **Decomposition of overall welfare changes over different payment rates of universal lump-sum child benefits ($\bar{tr}$) under the benchmark tax progressivity ($\tau = 0.2$).**

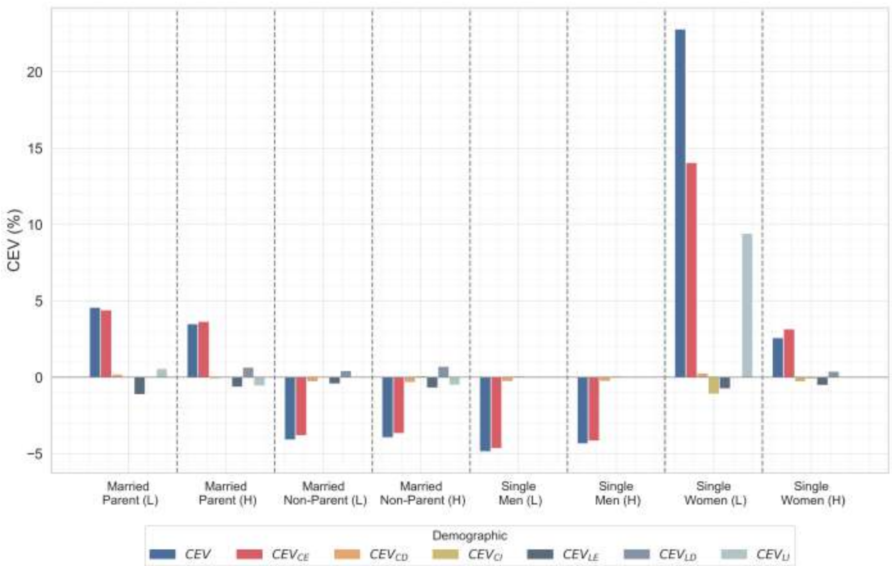

Figure 31: **Decomposed welfare changes under the optimal child benefit system ($\bar{tr}^* = 25\%$) and benchmark tax progressivity ($\tau = 0.2$) by demographic.**

D Optimal taxes and child benefits: Supplementary results

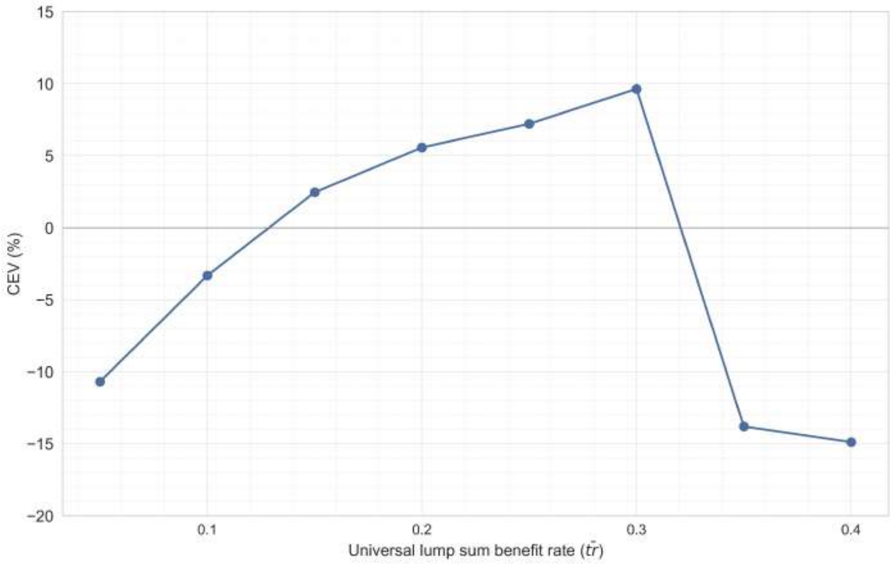

Figure 32: **Overall welfare changes over different payment levels of universal lump-sum child benefits** ($\bar{tr}$) **under the optimal tax progressivity** ($\tau^* = 0.1$)**.**

E Simple model: Derivations

E.1 Working-age households' intra-temporal trade-off equation (50)

The First-Order Conditions for working-age households are:

$$u'_{\tilde{c}} = m \times p\iota_{\lambda,\theta} \tag{79}$$

$$u'_{1-n} = m \times we^{f}_{\theta,\ell}\left(1 - \mathbf{1}_{\{\lambda=1,2\}}\frac{\chi_{\lambda,\ell}}{2} - \mathbf{1}_{\{\lambda=4\}}\chi_{\lambda,\ell}\right)(1 - EMTR_{y^f,\lambda}) \tag{80}$$

$$\beta E\left[(1 + r + EMTR_{a_+,\lambda}) \times u'_{\tilde{c}_+} \mid \lambda, \eta^m, \eta^f\right] = \frac{p_+ \times \iota_{\lambda,\theta+}}{p \times \iota_{\lambda,\theta}} \times u'_{\tilde{c}} \tag{81}$$

where $\tilde{c} = \dfrac{c}{\iota_{\lambda,\theta}}$ is the scaled household consumption; u'_i denotes the marginal utility with respect to a decision variable $i \in \{\tilde{c}, 1-n\}$; $p = 1 + \tau_c$ is the price of consumption goods; m is the Lagrange multiplier; and $EMTR_{y^f,\lambda}$and $EMTR_{a_+,\lambda}$ are the effective marginal tax rates on labor and capital earnings, respectively. Because male labor supply is exogenous, Equation (80) does not apply to single-male households ($\lambda = 3$).

Note that, $EMTR_{y^f,\lambda}$, $EMTR_{a_+,\lambda}$ and NLI_λ differ by family type. Furthermore, the progressive income tax scheme $T_\lambda(n)$, the means-tested child benefits (FTB and CCS), and the Age Pension program result in non-linear $EMTR_{y^f,\lambda}$ and NLI_λ with respect to labor, and non-linear $EMTR_{a_+,\lambda}$ with respect to future asset holdings. They are expressed as

$$\begin{aligned} EMTR_{y^f,\lambda} &= \frac{\partial T_\lambda}{\partial y^f}(n) + \mathbf{1}_{\{\boldsymbol{\lambda}=\mathbf{1,4}\}}\left[\frac{CE_\theta}{we^f_{\theta,\ell}} + \left(wn \times \frac{\partial sr}{\partial y^f}(n) - \frac{n}{e^f_{\theta,\ell}} \times \frac{\partial sr}{\partial n}\right)\sum_{i=1}^{nc_\theta}\kappa_i\right] \\ &\quad + \mathbf{1}_{\{\boldsymbol{\lambda}=\mathbf{1,4}\}}\left(nc_\theta \times \frac{\partial tr^A}{\partial y^f}(n) + \frac{\partial tr^B}{\partial y^f}(n)\right) \end{aligned} \tag{82}$$

$$\begin{aligned} EMTR_{a_+,\lambda} &= r \times \frac{\partial T_\lambda}{\partial (ra_+)}(a_+) \\ &\quad + \mathbf{1}_{\{\boldsymbol{\lambda}=\mathbf{1,4}\}}\left(r \times \frac{\partial tr^A}{\partial (ra_+)}(a_+) + wn_+ r \times \frac{\partial sr}{\partial (ra_+)}(a_+)\sum_{i=1}^{nc_{\theta+}}\kappa_{i+}\right) \end{aligned} \tag{83}$$

$$NLI_\lambda(n) = (1+r)a + \mathbf{1}_{\{\boldsymbol{\lambda}=\mathbf{1,4}\}}\left(nc_\theta \times tr^A(n) + tr^B(n)\right) \tag{84}$$

Equation (79) and (80) give us the optimal intra-temporal trade-off condition between consumption and leisure:

$$\frac{u'_{1-n}}{u'_{\tilde{c}}} = \frac{we^f_{\theta,\ell}}{p\iota_{\lambda,\theta}}\left(1 - \chi_{\lambda,\ell}\right)\left(1 - EMTR_{y^f,\lambda}\right) \tag{85}$$

Solving (85) with the utility functions from Subsection 4.2 yields the household total consumption as a function of female labor supply

$$c(n) = \frac{\nu}{1-\nu}\frac{we^f_{\theta,\ell}}{p}\left(1 - \chi_{\lambda,\ell}\right)(1 - EMTR_{y^f,\lambda})(1-n) \tag{86}$$

Equation (50) from the working-age household problem in Subsection 4.7 can then be derived by solving a system of two equations: (i) the consumption function (86), and (ii) the household budget constraint (49).

F Extensions

F.1 Optimal progressivity with baseline universal child benefits

I consider a child benefit reform termed *baseline universal child benefits,* proposed by Tin and Tran (2024), where means-testing from both the FTB and CCS is eliminated but demographic eligibility criteria and the baseline payment rates of the two programs are retained.[59] As summarized in Table 5, unlike in Tin and Tran (2024), this reform is welfare deteriorating even when implemented together with an optimal tax progressivity. This segment discusses the similarities and differences between findings of the two papers, including potential causes behind the divergence.

Aggregate implications of the baseline universal child benefits			
	$\tau = 0$	$\tau = 0.2$	$\tau = 0.5$
CCS size, %	+90.09	+133.30	+56.98
FTB size, %	+122.22	+122.22	+122.22
Average tax rate, *pp*	+4.84	+5.21	+10.04
Tax scale (ζ)	−0.003	−0.031	−0.093
Fe. LFP, *pp*	−4.35	+0.21	+5.36
Fe. Hour, %	+7.19	+0.95	−8.42
Fe. H. cap, %	+0.99	+1.13	+0.03
Cons (C), %	+0.50	+0.92	−0.42
Output (Y), %	+0.24	+0.41	−0.28
Welfare (CEV), %	−1.96	−1.38	−6.50

Table 13: **Aggregate implications of the baseline universal child benefits at three tax progressivity levels:** $\tau = 0$, $\tau = 0.2$, and $\tau = 0.5$.
Notes: Results are reported as changes relative to the levels in the benchmark economy.

Macroeconomic outcomes, welfare effects and their composition at the aggregate level. At the benchmark progressivity of $\tau = 0.2$, the baseline universal child benefits improve female labor supply, human capital, consumption, and output (Table 13). However, removing means-testing causes a significant expansion in both the FTB and the CCS programs—by 133.3% and 122.22%, respectively—despite the demographic criteria being retained to curb benefit spending. Funding this expansion necessitates increased tax revenue, resulting in a higher overall tax burden on all workers, as reflected by a 0.031 point decrease in the tax scale parameter ζ.[60] Consequently, the average tax rate rises by 5.21%, dampening the intended work incentives from removing means-testing. In essence, while the reform eliminates wage distortions caused by means-testing, it simultaneously introduces larger tax liabilities and distortions. These counteracting forces help explain the relatively modest increases in female labor supply, output, and consumption.

Despite some aggregate improvements, the policy results in an overall welfare loss of 1.38%. Figure 33 shows further that only by maintaining the status quo tax progressivity of $\tau = 0.2$ can the welfare losses under the new child benefit system be minimized. Any deviation from the current progressivity leads to greater losses, stemming from declines in consumption allocative efficiency (CEV_{CE}), especially as the tax system becomes more progressive and tax burden increases (see Subsection 6.1.1).

Distribution of welfare changes and their composition by demographic. Aligned with the composition of the overall welfare changes, Figure 34 shows that, at the benchmark tax progressivity of $\tau = 0.2$, both winners and losers under this new regime experience welfare changes driven mainly by consumption allocative efficiency. Moreover, except for single men who do not make labor decisions in the model, all demographic groups also experience moderate losses from reduced leisure allocative efficiency.

For non-parents, who are not eligible for child benefits, these losses are attributable solely to the increased overall tax burden, which deteriorates their allocative efficiency in both consumption and leisure. Among

[59] Details related to demographic criteria and their effects on child benefit payments are provided in Subsection H.3 in the Appendix.
[60] See explanation in Subsection 4.5.1.

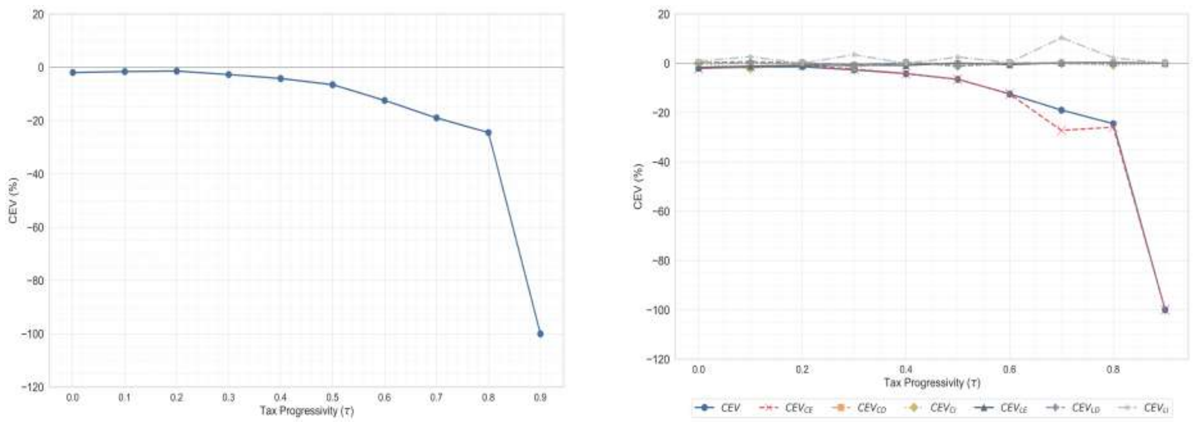

Figure 33: **Overall (left panel) and decomposed welfare changes (right panel) over tax progressivity under the baseline universal child benefits.**

parents, welfare outcomes vary, with couples benefiting while singles face losses. For brevity and comparability with Tin and Tran (2024), the following discussion focuses on single mothers—the primary target of child benefit programs—who fare worse than other groups.

To understand this outcome, recall that single mothers lack family insurance and have limited self-insurance capacity through work and savings due to child-related costs and early parenthood, which penalize their household consumption. Furthermore, since the pre-reform child benefits are means-tested based on family income, single mothers' earnings often fall below the income-test threshold, implying they faced no wage distortions and likely already received full child benefits under the status quo. As a result, the baseline universal reform does little to enhance their benefits or reduce wage distortions, thus offering few advantages to offset the higher tax burden under the new regime. These factors contribute to the significant welfare losses for single mothers.

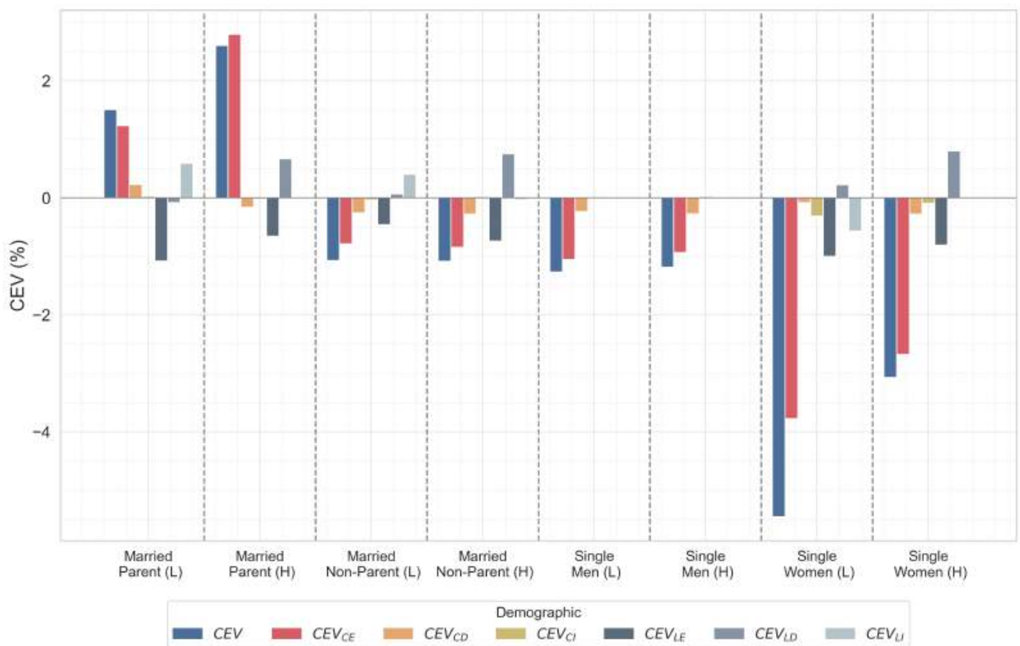

Figure 34: **Decomposition of welfare changes by demographic under the baseline universal child benefits at the benchmark tax progressivity level** ($\tau = 0.2$).

Ultimately, the reform redistributes welfare from single and non-parent households to married parent households, creating an inequitable distributional outcome. As illustrated in Figure 35, adjusting tax progressivity does not address the problem. Increasing progressivity to $\tau = 0.5$ only exacerbates the already unfavorable welfare outcome, leading to economy-wide welfare losses of at least 5% for all household types. In contrast, a proportional tax regime merely shifts the loss from low-education single mothers to low-education married parents.

Similarities and differences to Tin and Tran (2024). These findings, particularly in terms of equity outcomes, resonate with Tin and Tran (2024), who also find that the baseline universal child benefit scheme disadvantages single mothers due to increased tax pressure. Their analysis reveals that adjusting the universal payment rate does not resolve the inequity, and this study shows that altering tax progressivity similarly fails to mitigate the issue.

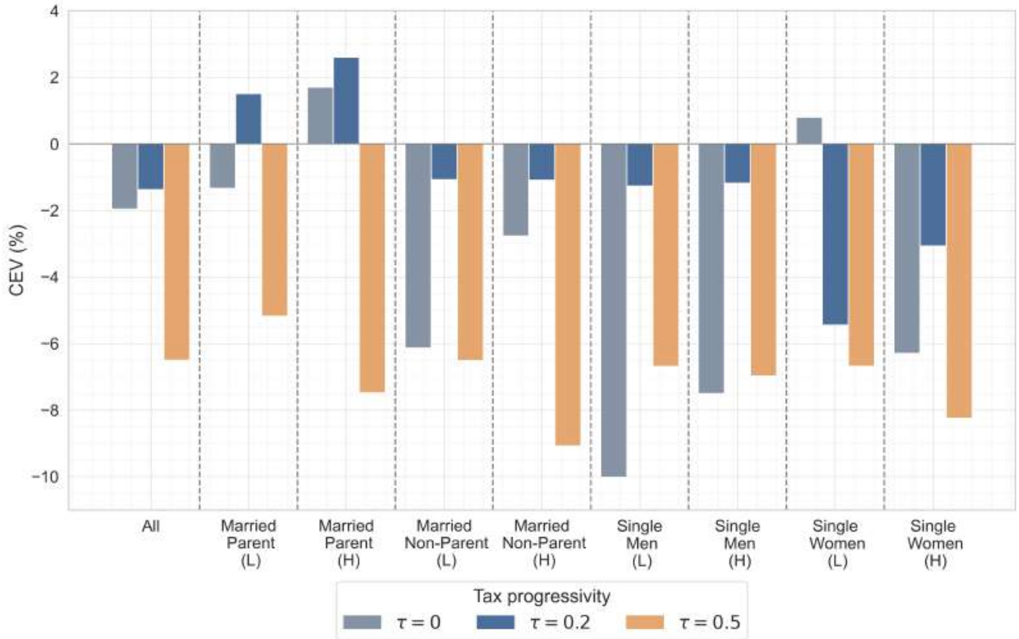

Figure 35: **Distributions of welfare changes by demographic under the baseline universal child benefits for three tax progressivity levels. Blue bars**: Proportional ($\tau = 0$); **Gray bars**: Benchmark, moderate progressivity ($\tau = 0.2$); **Orange bars**: High progressivity ($\tau = 0.5$).

However, the overall welfare outcomes differ between the two studies. Tin and Tran (2024) report a small positive welfare effect, whereas the results here indicate a welfare decline. This divergence can be partially attributed to the differences in female labor supply modeling. In Tin and Tran (2024), where labor decisions are limited to part-time and full-time employment, eliminating means-testing significantly increases female labor supply, expanding the tax base and easing the fiscal strain of the universal regime. The average tax rate in their setting increases by 4.2*pp*.

In contrast, in the current study's configuration, where both the intensive and extensive margins of female labor supply are enabled, their trade-off results in a weaker overall labor supply response compared to Tin and Tran (2024). For instance, Figure 36 shows that married parents tend to increase participation but reduce work hours, while low-education single mothers work longer hours but with fewer of them participate in the workforce. The weaker tax base expansion helps explain the larger increase in the average tax rate (5.2*pp*) in the current setting. The higher tax burden disproportionately affects single mothers, contributing to larger losses in their welfare and overall welfare. These results underscore the importance of modeling the intensive margin of female labor supply decisions to capture the policy effects on low-education single mothers responses and welfare outcomes. Furthermore, despite some variations in the aggregate results, the qualitatively consistent distributional outcomes across both studies provide confidence that the findings concerning the redistributive effects of universal child benefits on vulnerable households are robust.

In conclusion, means-testing plays a pivotal role in alleviating tax burden and enhancing overall welfare.

The tax savings due to means-testing is not only beneficial for non-parents but results in significant welfare improvements for vulnerable parents, particularly low-education single mothers. In this paper, the removal of means testing renders the baseline universal child benefits a lose-lose reform, irrespective of tax progressivity. This also demonstrates that the baseline child benefits for parents are insufficient to justify the increased overall tax burden from universalizing both the FTB and CCS. Subsection 6.3.1 explores an alternative environment, where the CCS is kept unchanged, while tax progressivity and lump-sum child benefit (FTB) payment rates are jointly optimized.

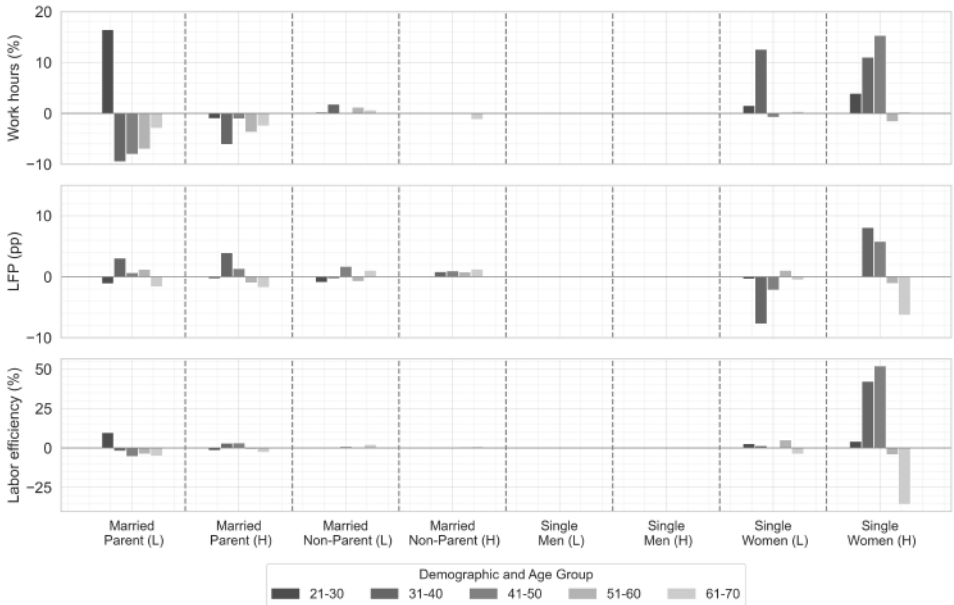

Figure 36: **Female labor supply responses by age and demographic under the baseline universal child benefits and benchmark tax progressivity** ($\tau = 0.2$): (**Top**: work hours, **Middle** : labor force participation, **Bottom** : labor efficiency)

F.2 The role of lump-sum child transfers (FTB) and child care subsidies (CCS)

Previous experiments show the dominance of consumption allocative efficiency (CEV_{CE}) in driving welfare changes, both at the aggregate level and across demographics. Most reforms considered have minimal impact on the distributional and insurance components of welfare, in terms of both consumption (CEV_{CD} and CEV_{CI}) and leisure (CEV_{LD} and CEV_{LI}).

A plausible explanation could be households' ability to adjust their labor supply and savings in response to policy changes, allowing them to maintain relatively stable ex-ante shares and ex-post risks in consumption and leisure under different reform scenarios. For vulnerable parent groups, child benefit programs that relax constraints on their capacity to self-insure through work and savings—such as child-related costs—may help them achieve similar stability. The consistent presence of the FTB and CCS in the counterfactual experiments likely contributes to the relatively muted effects of reforms on equity and insurance.

Thus, to better understand how each program may have influenced welfare outcomes across the three reforms, I extend the analysis by examining the composition and distribution of welfare changes in two policy experiments: one where means-tested lump-sum child benefits (FTB) are removed, and another where child care subsidies (CCS) are removed from the status quo system.

I find that eliminating either the FTB or the CCS brings about significant overall welfare losses in the model economy, with potential reductions of up to 100% in consumption equivalent terms. However, the magnitude and mechanisms behind these losses differ greatly between the two reforms.

The left panel of Figure 37 reveals that removing the FTB causes an approximately 20% loss in overall welfare, primarily driven by a decline in consumption allocative efficiency (CEV_{CE}). Contributions from other

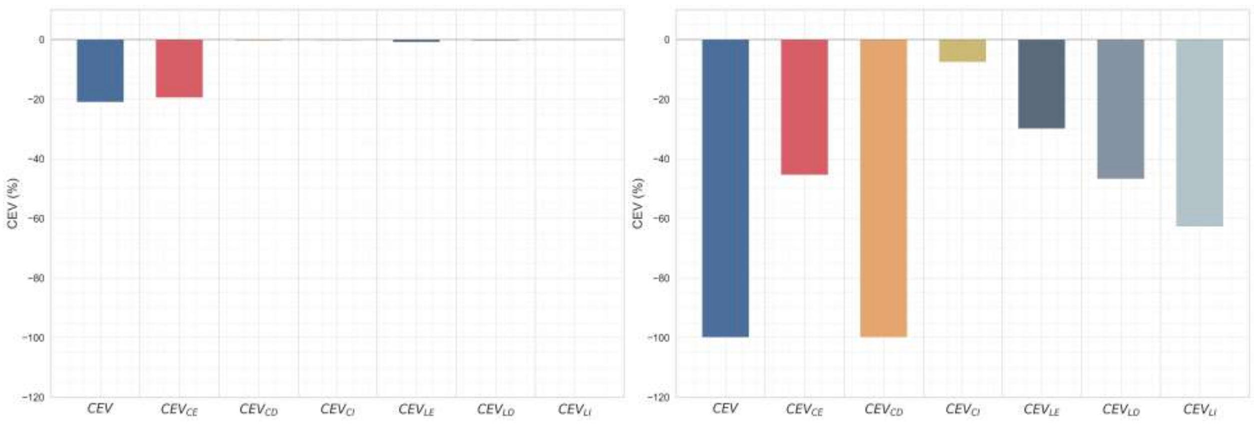

Figure 37: **Decomposition of welfare changes by demographic under the benchmark tax progressivity** ($\tau = 0.2$) **with the removal of one child benefit program: Left panel**: FTB removal; **Right panel**: CCS removal.

components of welfare are minimal. In contrast, the absence of the CCS, as evident in the right panel of Figure 37, produces a significantly greater welfare loss, reaching up to 100%, due to a mixture of factors beyond just consumption efficiency.

Aggregate implications of removing the FTB or the CCS		
	Remove FTB	*Remove CCS*
CCS size, %	+92.86	−100
FTB size, %	−100	+2.78
Average tax rate, *pp*	+4.91	+4.01
Tax scale (ζ)	−0.020	+0.003
Fe. LFP, *pp*	+1.57	−2.69
Fe. Hour, %	+8.58	+5.69
Fe. H. cap, %	+1.34	+0.07
Cons (C), %	−0.37	−0.17
Output (Y), %	+0.18	−0.30
Welfare (CEV), %	−21.06	−100

Table 14: **Aggregate implications of the removing either the FTB or the CCS under the benchmark tax progressivity** ($\tau = 0.2$)**.**
Notes: Results are reported as changes relative to the levels in the benchmark economy.

The relatively stable tax scale parameter ζ in Table 14 suggests that these welfare losses must arise directly from the removal of the child benefit programs themselves, rather than indirectly through changes in the overall tax burden. Additionally, aggregate consumption levels decline only modestly—by 0.37% with the removal of the FTB and 0.17% with the removal of the CCS, prompting the question of why such drastic welfare losses occur.

Figure 38 identifies the welfare declines among low-education married and single mother households as the primary drivers of the overall welfare reduction under these reforms. Removing the FTB results in a 40% welfare loss for low-education single mothers and a 20% loss for their high-education counterparts, mainly due to a reduction in consumption allocative efficiency. The impact of removing the CCS is even more severe, resulting in welfare losses equivalent to 100% in consumption terms for low-education parents. Furthermore, the absence of the CCS significantly increases the importance of distributional and insurance effects in explaining the welfare losses among parents.

Several factors may explain the heavy dependence of these households on subsidies, including (i) limited or nonexistent family insurance, (ii) constrained self-insurance through female labor supply and savings due to early arrive of children and associated costs, and (iii) the inability to borrow in younger years due to credit constraints. These constraints make low-education parents particularly vulnerable compared to the rest of the population. The substantial welfare losses they face—through drastically declined ex-ante shares and height-

Figure 38: **Decomposition of welfare changes by demographic in the absence of one of the child benefit programs under the benchmark tax progressivity** ($\tau = 0.2$). **Top panel**: FTB removal; **Bottom panel**: CCS removal.

ened ex-post risks in consumption and leisure—underscore the critical role the CCS plays in promoting equity and insurance. By reducing child care costs, the CCS alleviates labor supply constraints for single mothers, facilitating their workforce participation and human capital accumulation. The enhanced self-insurance capacity raises their ex-ante shares of consumption and leisure. Moreover, by improving their labor earnings and savings capacity, the subsidies help mitigate these households' ex-post risks, leading to better consumption and leisure outcomes in the face of adverse shocks.

Why does the FTB not offer the same support? There are two plausible reasons. First, lump-sum child benefits are only available while children are dependent, limiting their ability to provide long-term consumption and leisure insurance. Second, the program's means-testing and benefit structure create work disincentives during early phases of life, thus diminishing human capital potentials. Because low-education families must rely on labor earnings once they exit the FTB program, these factors likely contribute to the program's ineffectiveness in boosting their ex-ante consumption and leisure shares or enhancing their ability to self-insure against shocks.

In addition to demonstrating the importance of child benefits for vulnerable parents, this analysis highlights the distinct roles of lump-sum child benefits (FTB) and child care subsidies (CCS). While the FTB primarily enhances consumption allocative efficiency, the CCS is vital for equity and insurance. These differences suggest that policies could be more effectively tailored to specific economic contexts. For example, in economies with weaker private insurance mechanisms, child care subsidies may be more effective in improving long-term parental and overall welfare.

G Quantitative analysis: Supplementary results

G.1 Optimal tax progressivity with benchmark child benefits

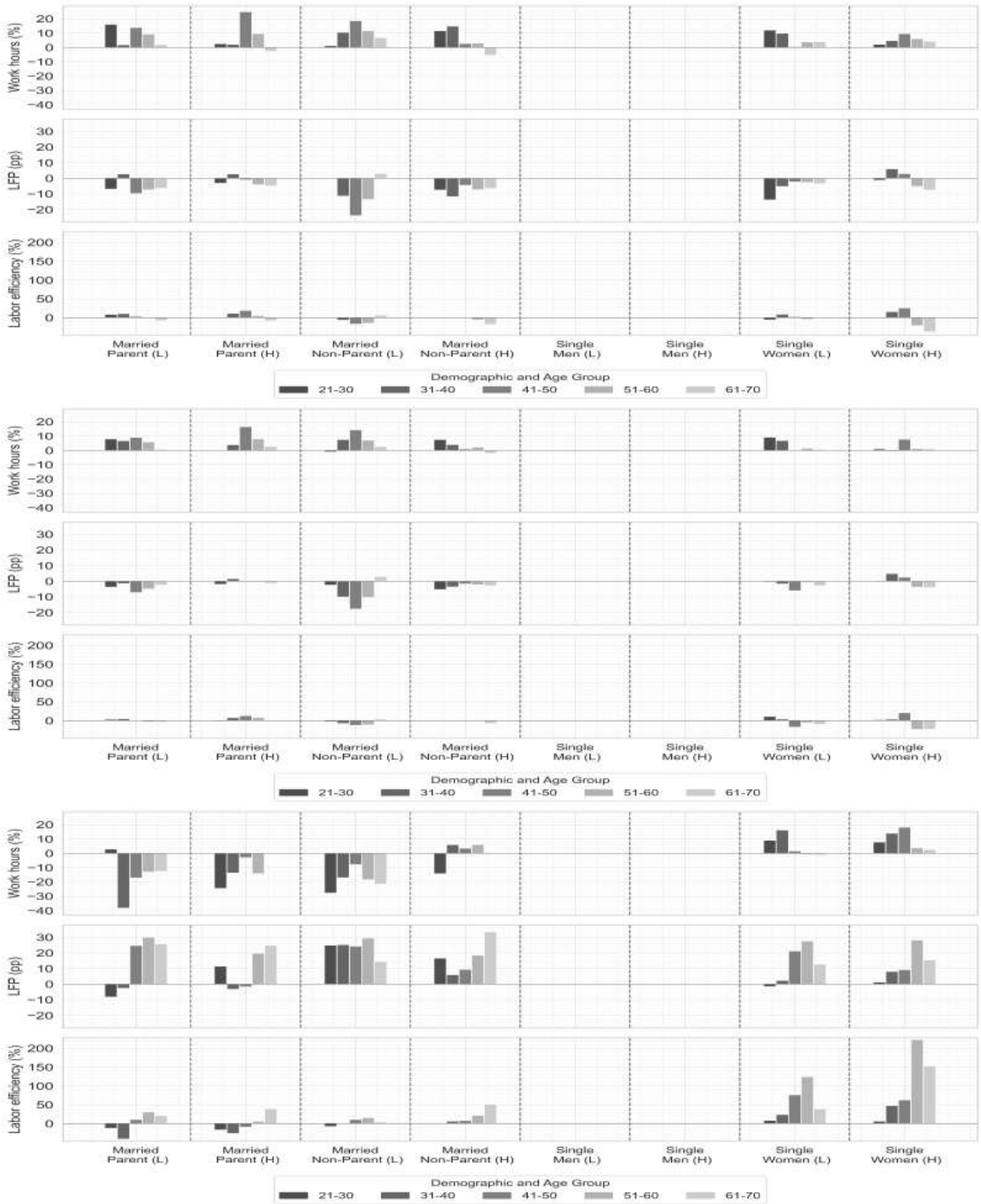

Figure 39: Changes in labor supply (**top row**: work hours, **middle row**: labor force participation, **bottom row**: labor efficiency) by age and demographic under the benchmark FTB and CCS for three tax progressivity levels. **Top panel**: Proportional ($\tau = 0$); **Middle panel**: Optimal progressivity ($\tau = 0.1$); **Bottom panel**: High progressivity ($\tau = 0.6$).

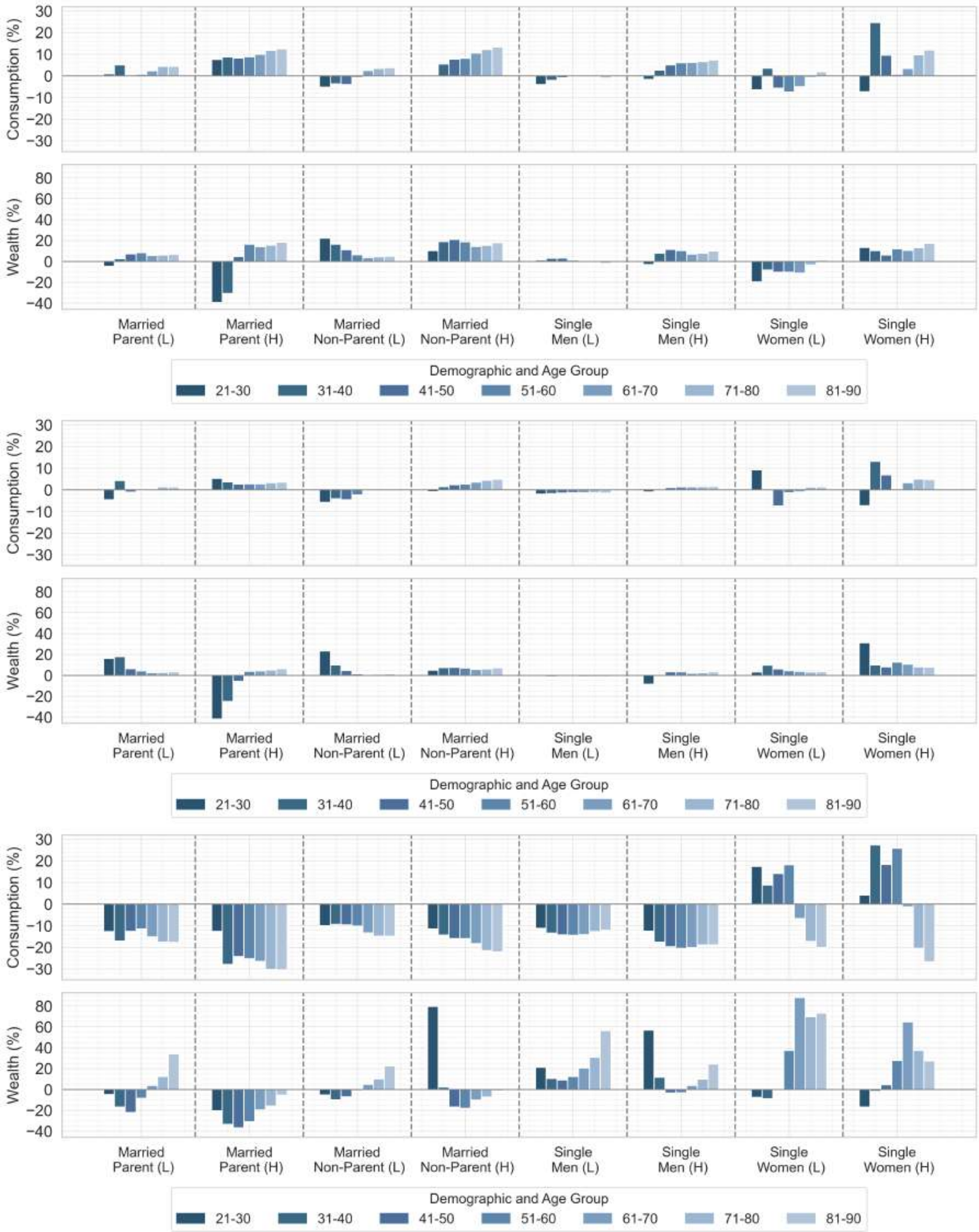

Figure 40: Changes in consumption and wealth (**top row**: consumption, **bottom row**: wealth) by age and demographic under the benchmark FTB and CCS for three tax progressivity levels. **Top panel**: Proportional ($\tau = 0$); **Middle panel**: Optimal progressivity ($\tau = 0.1$); **Bottom panel**: High progressivity ($\tau = 0.6$).

G.2 Optimal child benefits with benchmark tax progressivity

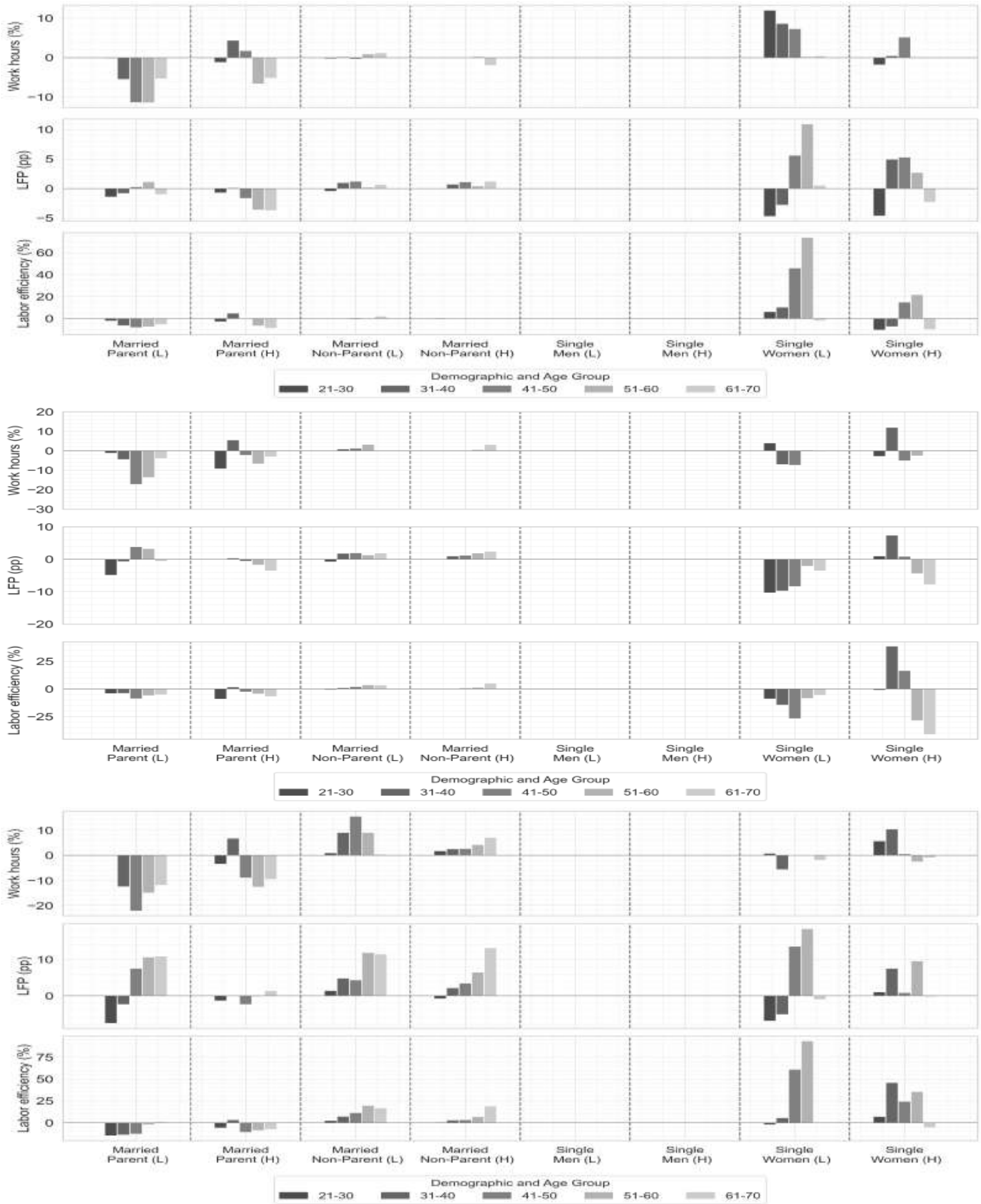

Figure 41: Changes in labor supply (**top row**: work hours, **middle row**: labor force participation, **bottom row**: labor efficiency) by age and demographic under the benchmark tax progressivity and universal child benefits at three payment rates. **Top panel**: $\bar{tr} = 15\%$; **Middle panel**: $\bar{tr}^* = 25\%$; **Bottom panel**: $\bar{tr} = 35\%$.

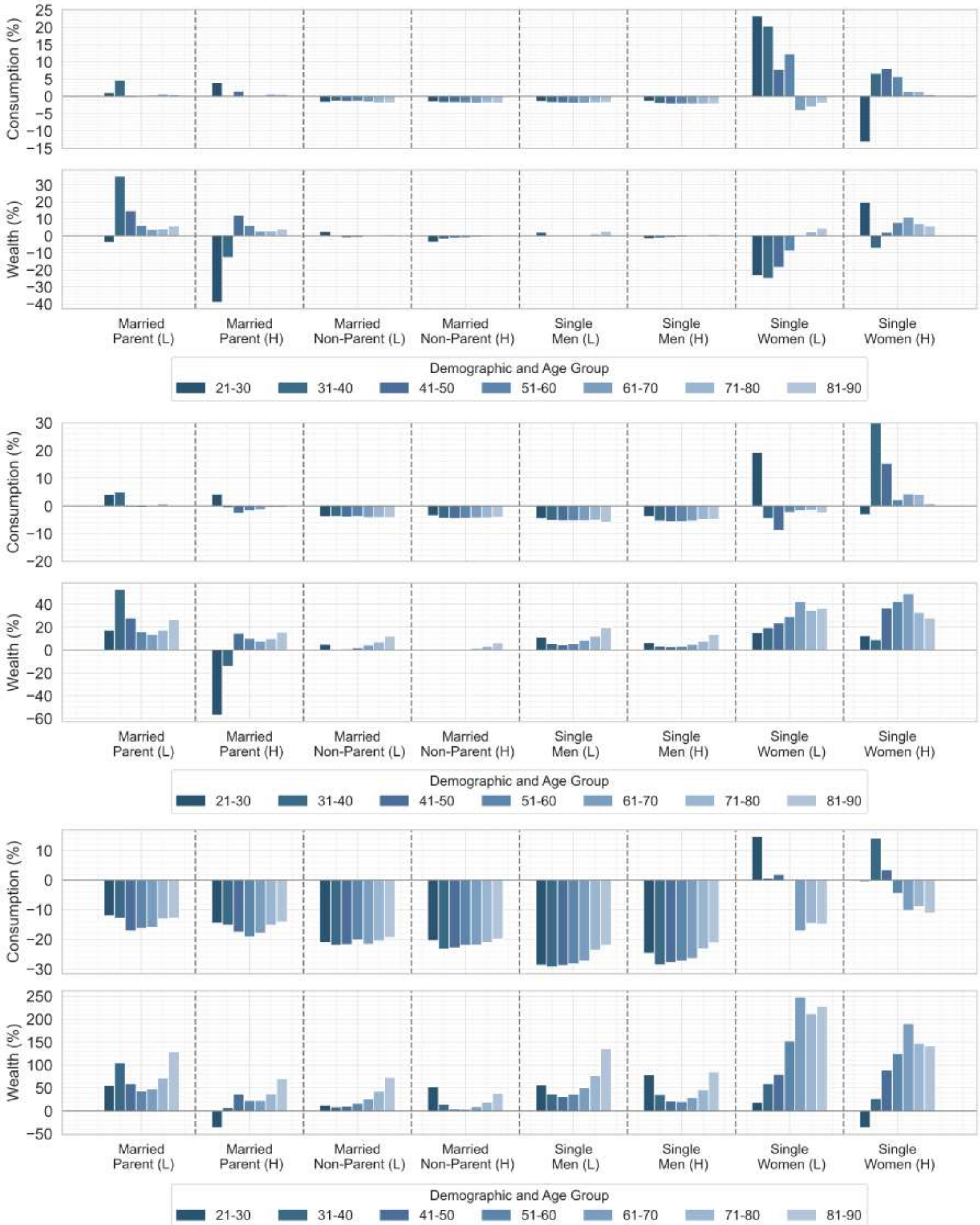

Figure 42: Changes in consumption and wealth (**top row**: consumption, **bottom row**: wealth) by age and demographic under the benchmark tax progressivity and universal child benefits at three payment rates. **Top panel**: $\bar{tr} = 15\%$; **Middle panel**: $\bar{tr}^* = 25\%$; **Bottom panel**: $\bar{tr} = 35\%$.

G.3 Removing CCS

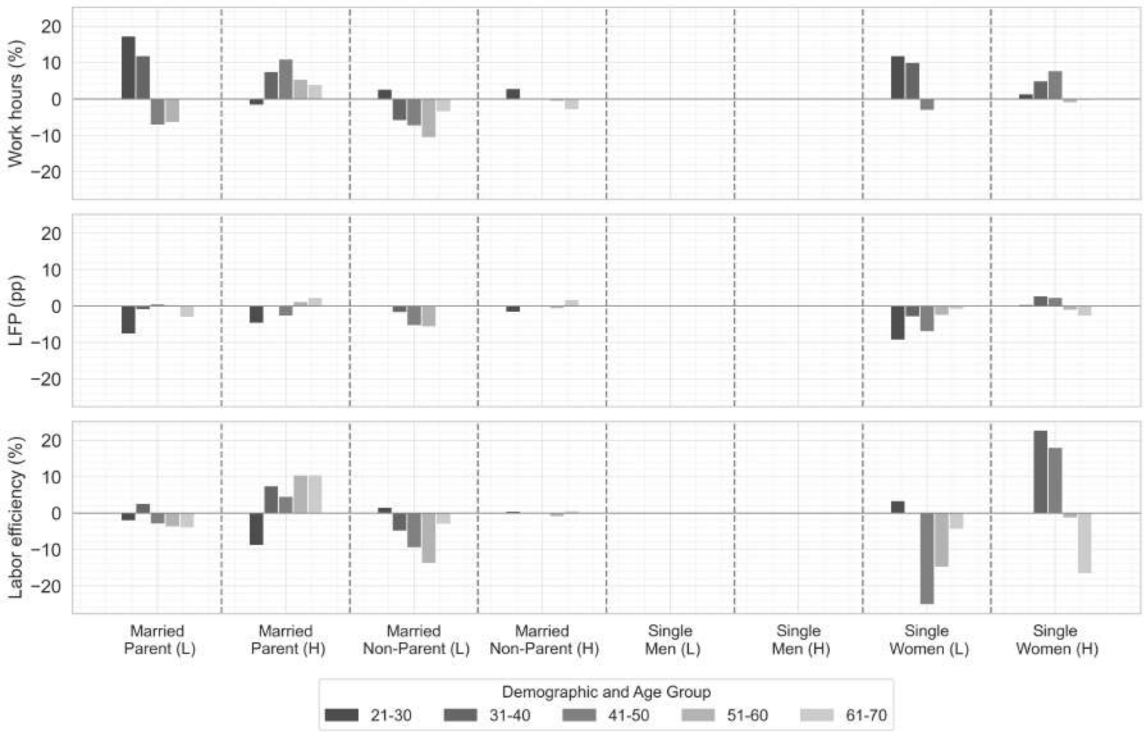

Figure 43: Changes in labor supply (**top**: work hours, **middle**: labor force participation, **bottom**: labor efficiency) by age and demographic in the absence of the child care subsidy (CCS).

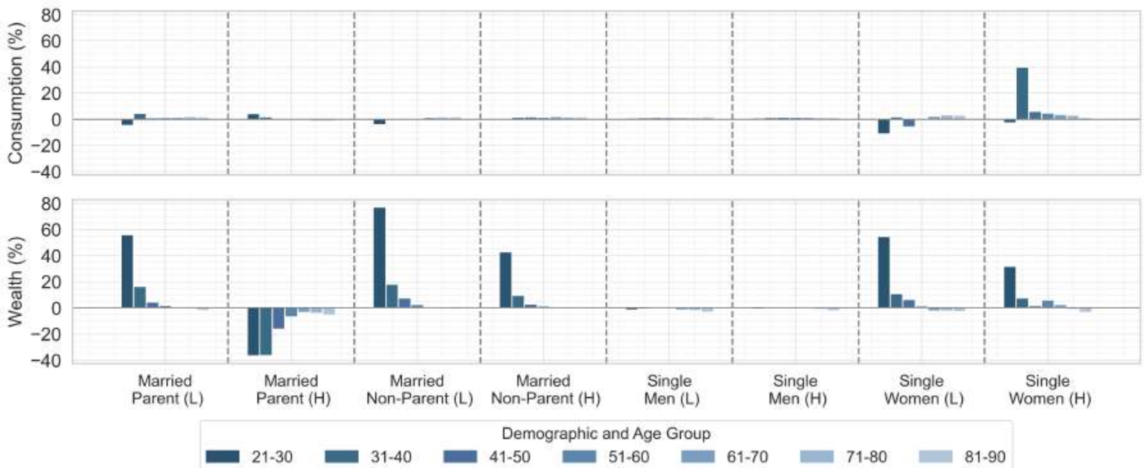

Figure 44: Changes in consumption and wealth (**top**: consumption, **bottom**: wealth) by age and demographic in the absence of the child care subsidy (CCS).

G.4 Removing FTB

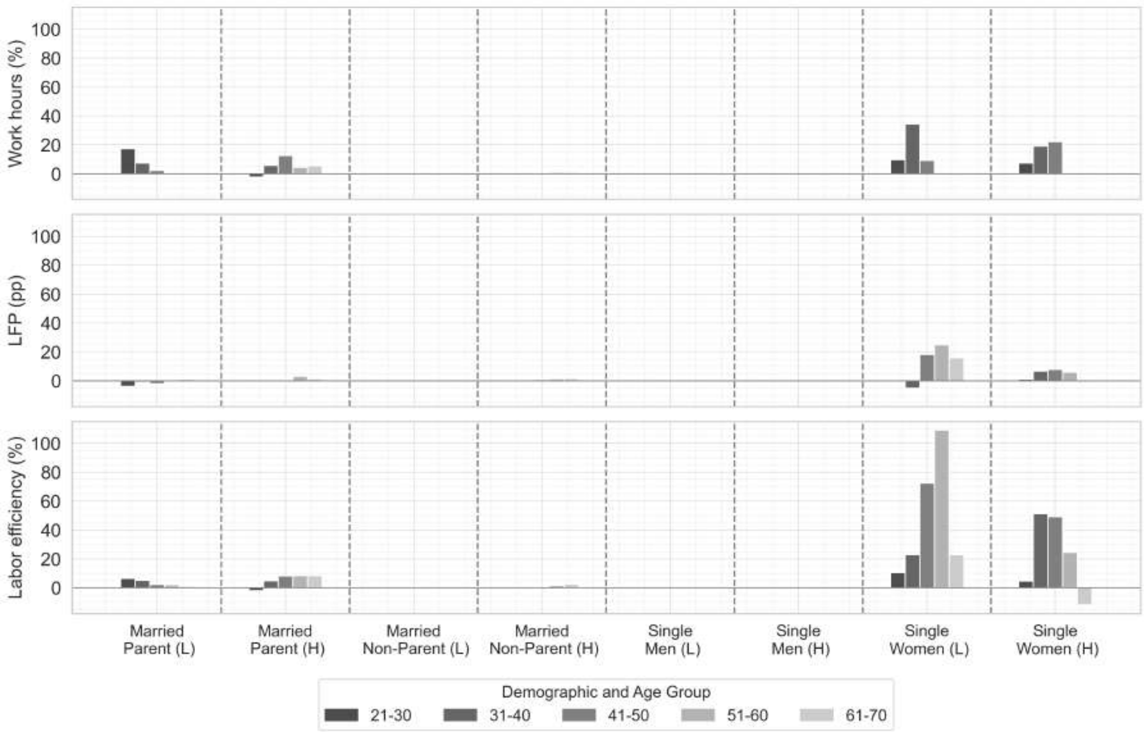

Figure 45: Changes in labor supply (**top**: work hours, **middle**: labor force participation, **bottom**: labor efficiency) by age and demographic in the absence of the Family Tax Benefit (FTB).

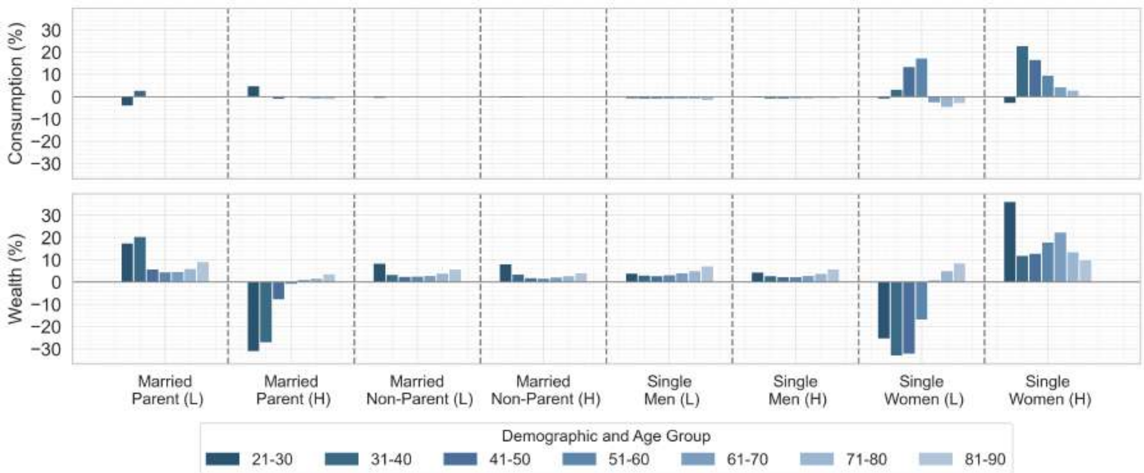

Figure 46: Changes in consumption and wealth (**top**: consumption, **bottom**: wealth) by age and demographic in the absence of the Family Tax Benefit (FTB).

G.5 Baseline universal child benefits

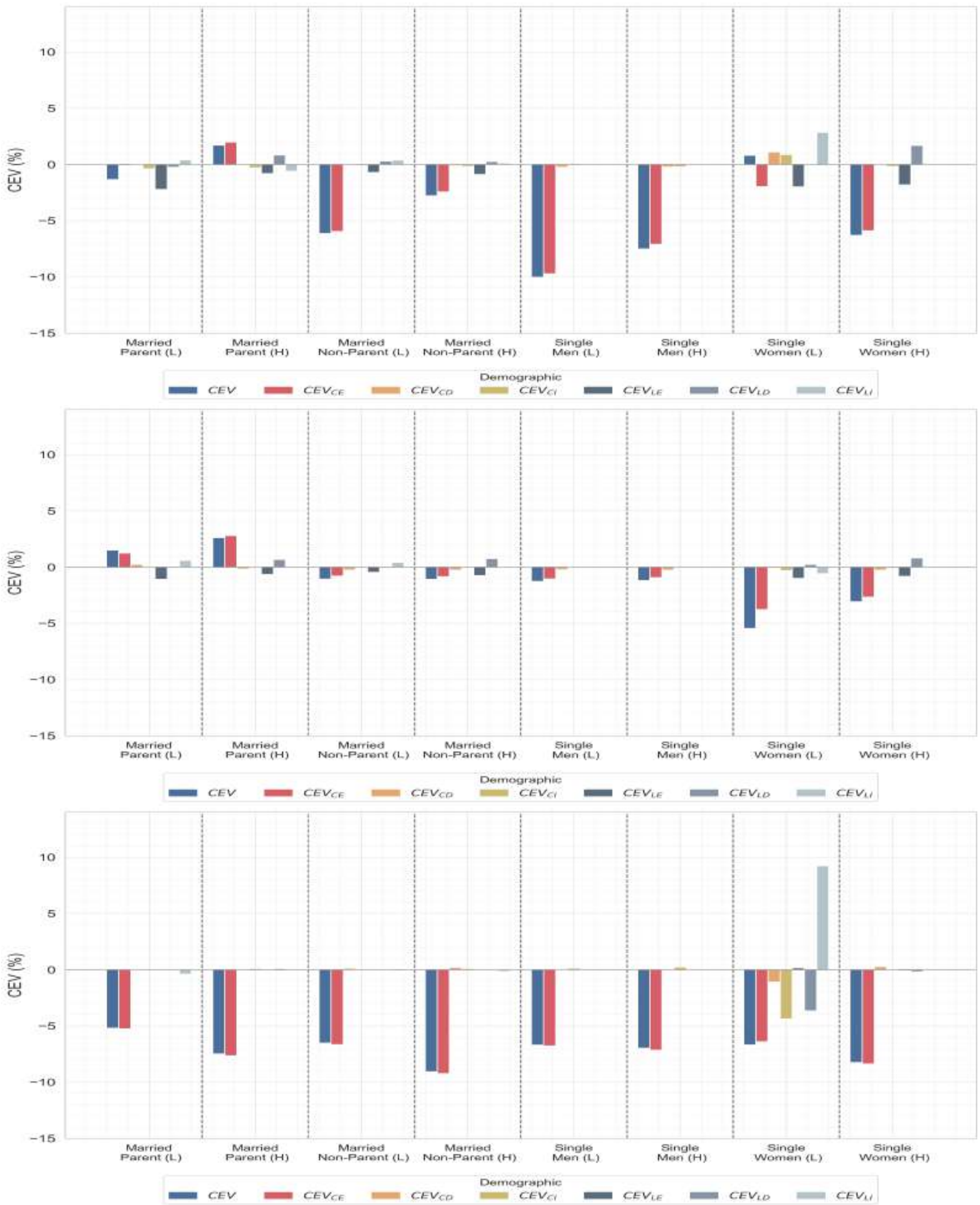

Figure 47: Decomposition of welfare changes by demographic under the baseline universal child benefits for three different tax progressivity levels. **Top panel**: Proportional ($\tau = 0$); **Middle panel**: Moderate progressivity, benchmark ($\tau = 0.2$); **Bottom panel**: High progressivity ($\tau = 0.5$) .

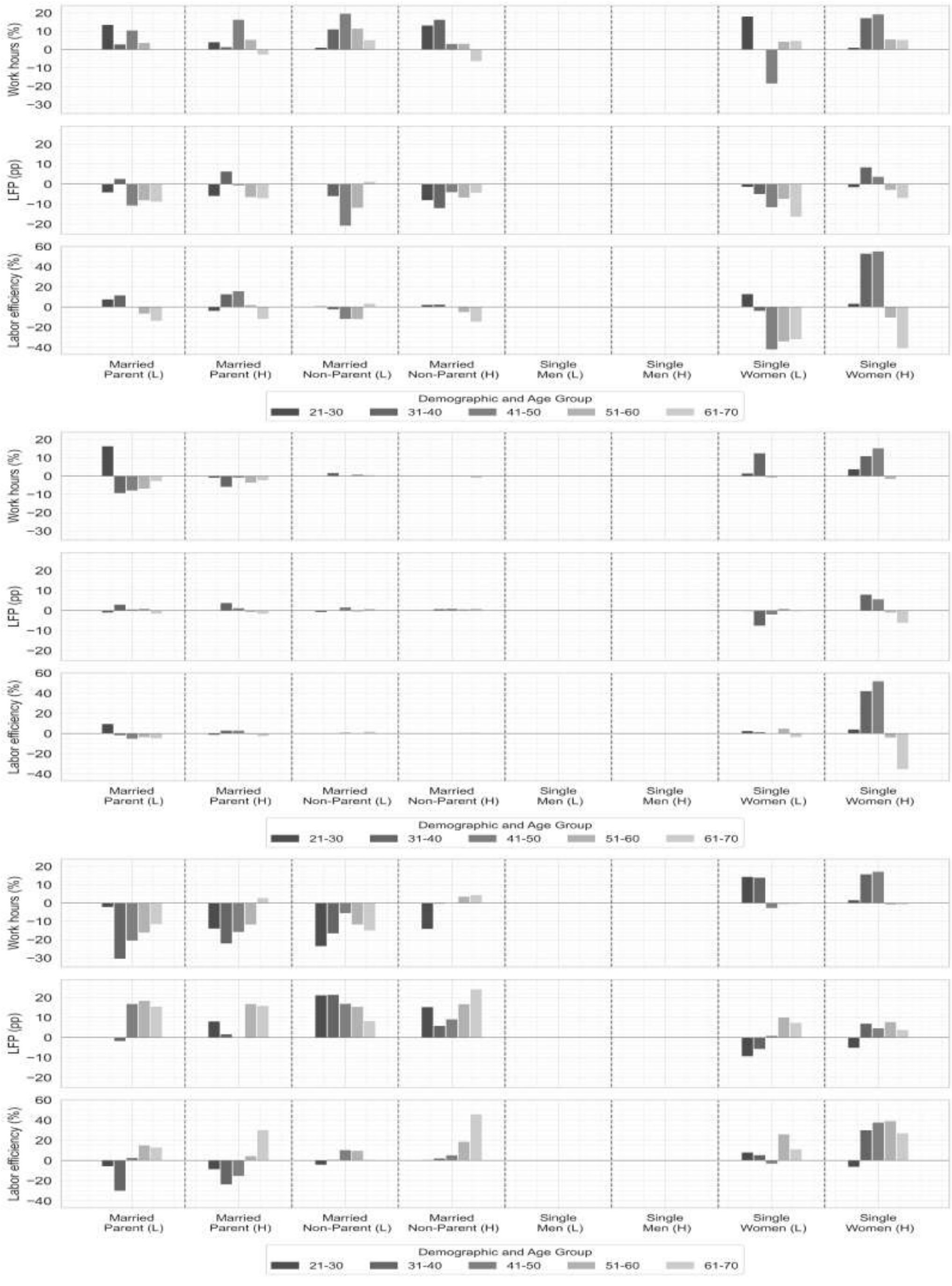

Figure 48: Changes in labor supply (**top row**: work hours, **middle row**: labor force participation, **bottom row**: labor efficiency) by age and demographic under the baseline universal child benefits for three tax progressivity levels. **Top panel**: Proportional ($\tau = 0$); **Middle panel**: Moderate progressivity, benchmark ($\tau = 0.2$); **Bottom panel**: High progressivity ($\tau = 0.5$).

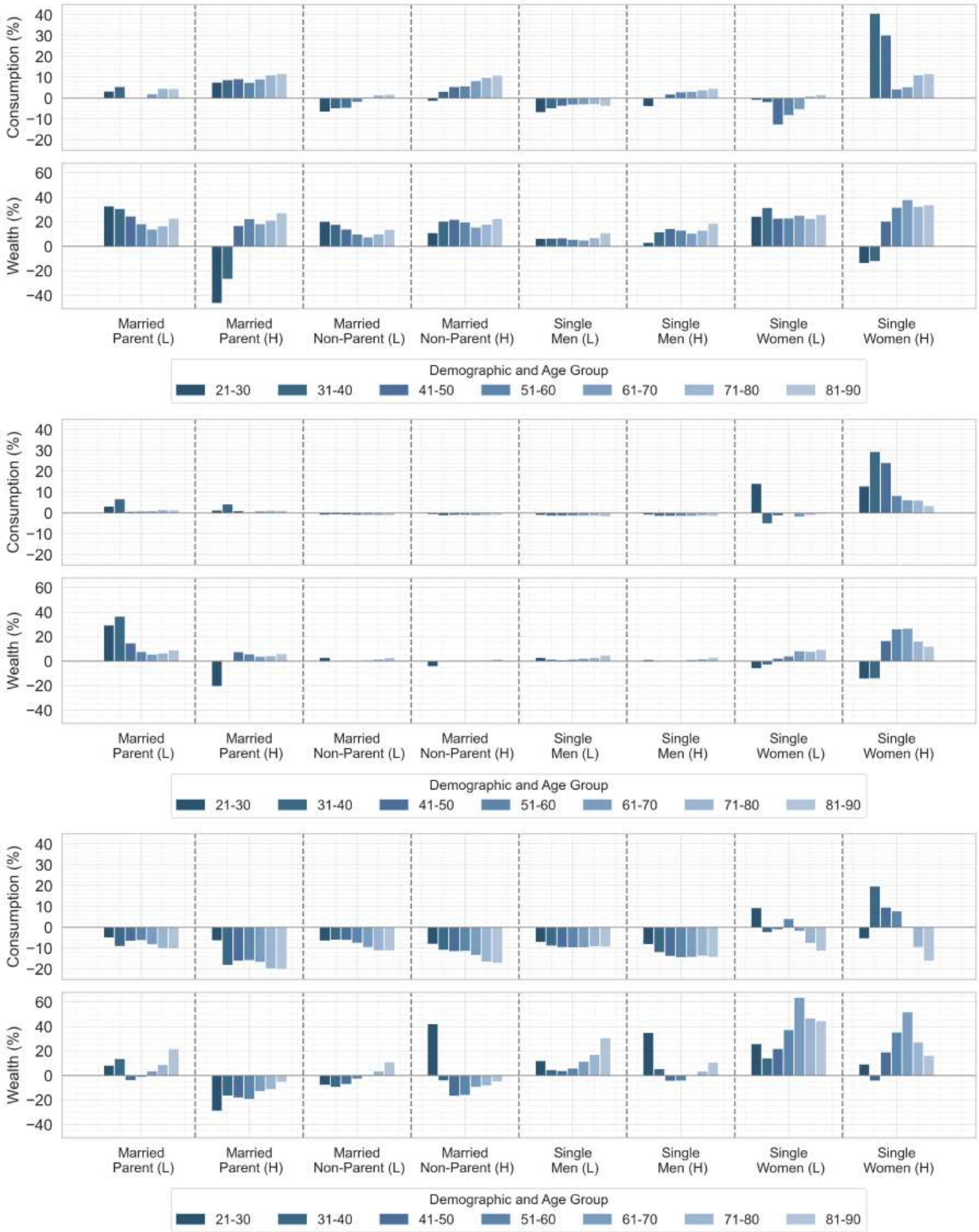

Figure 49: Changes in consumption and wealth (**top row**: consumption, **bottom row**: wealth) by age and demographic under the baseline universal child benefits for three different tax progressivity levels. **Top panel**: Proportional ($\tau = 0$); **Middle panel**: Moderate progressivity, benchmark ($\tau = 0.2$); **Bottom panel**: High progressivity ($\tau = 0.5$).

G.6 Additional results: Optimal tax progressivity and universal lump-sum child benefits per child

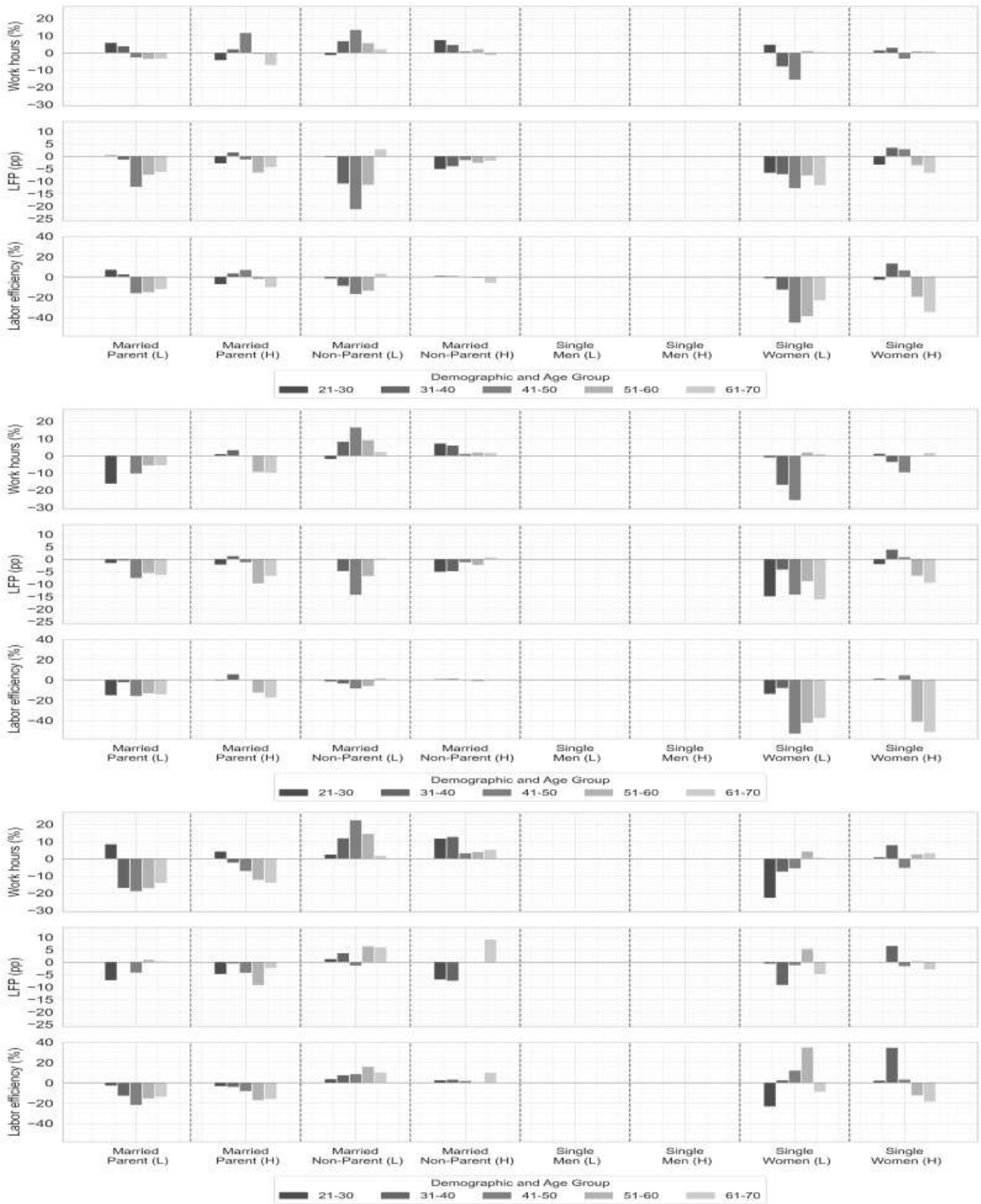

Figure 50: Changes in labor supply (**top row**: work hours, **middle row**: labor force participation, **bottom row**: labor efficiency) by age and demographic under the optimal tax progressivity ($\tau^* = 0.1$) and three different levels of universal lump-sum child benefits per child. **Top panel**: $\bar{tr} = 20\%\times$median income; **Middle panel**: optimal $\bar{tr}^* = 30\%\times$median income; **Bottom panel**: $\bar{tr} = 40\%\times$median income.

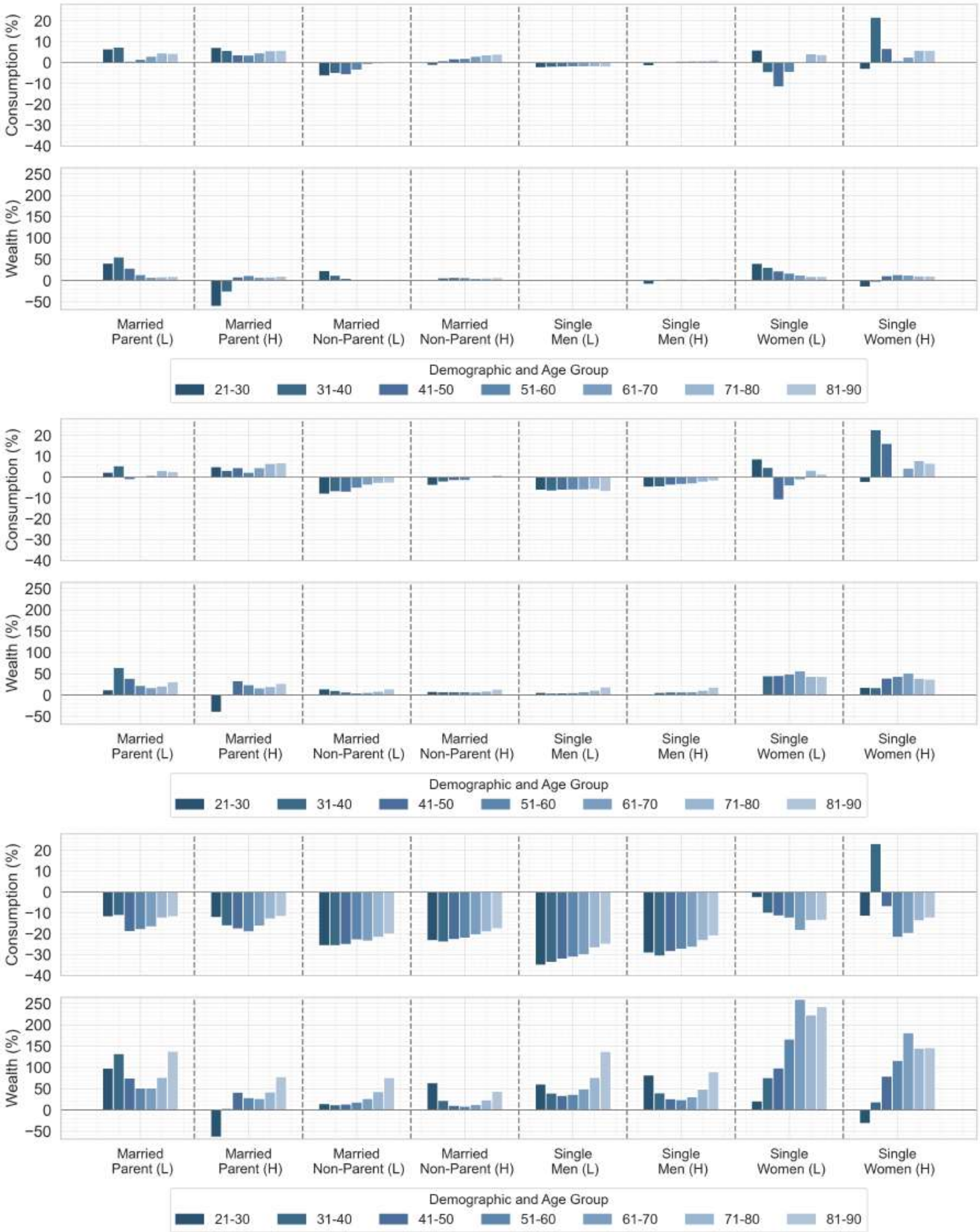

Figure 51: Changes in consumption and wealth (**top row**: consumption, **bottom row**: wealth) by age and demographic under the optimal tax progressivity ($\tau^* = 0.1$) and three different levels of universal lump-sum child benefits per child. **Top panel**: $\bar{tr} = 20\% \times$median income; **Middle panel**: optimal $\bar{tr}^* = 30\% \times$median income; **Bottom panel**: $\bar{tr} = 40\% \times$median income.

G.7 Supplementary figures: Female labor supply profiles

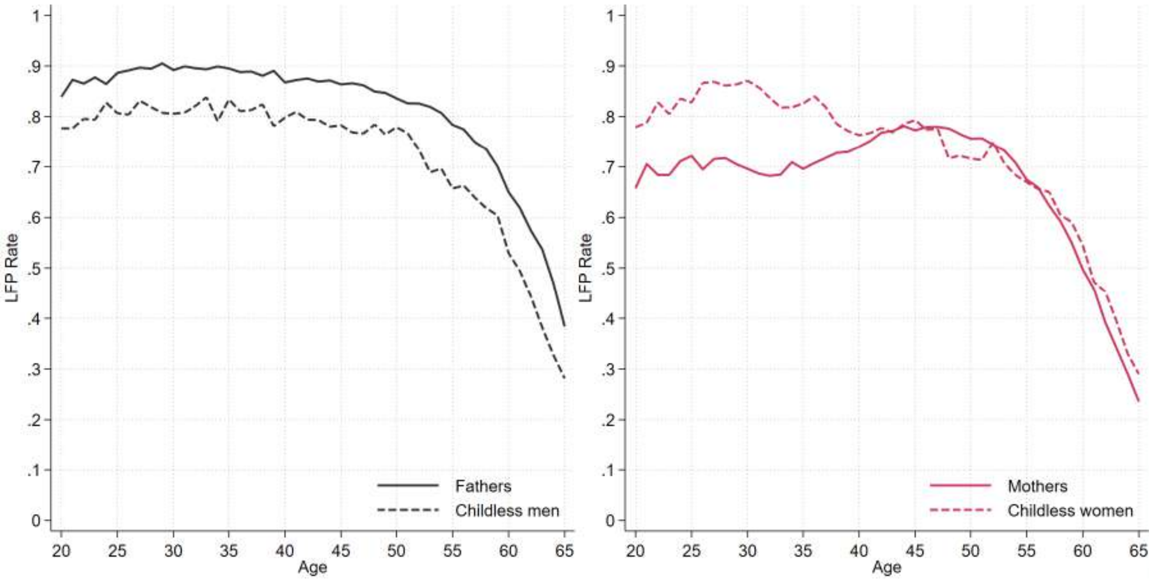

Figure 52: **Age profiles of labor force participation. Left:** fathers (solid) and childless men (dashed). **Right:** mothers (solid) and childless women (dashed).

Notes: The age profiles stitch together 20-year snapshots of life cycle for selected cohorts. The youngest cohort is cohort 12 aged 20-39 in the data, and the oldest cohort is cohort 12 aged 75-94.

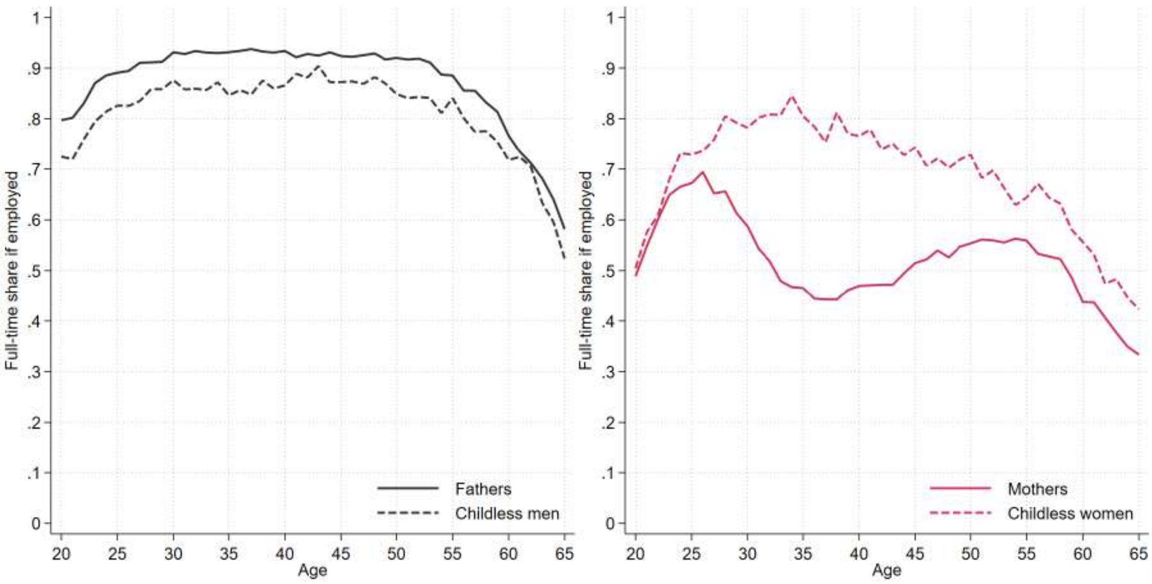

Figure 53: **Age profiles of full-time share of employment. Left:** fathers (solid) and childless men (dashed). **Right:** mothers (solid) and childless women (dashed).

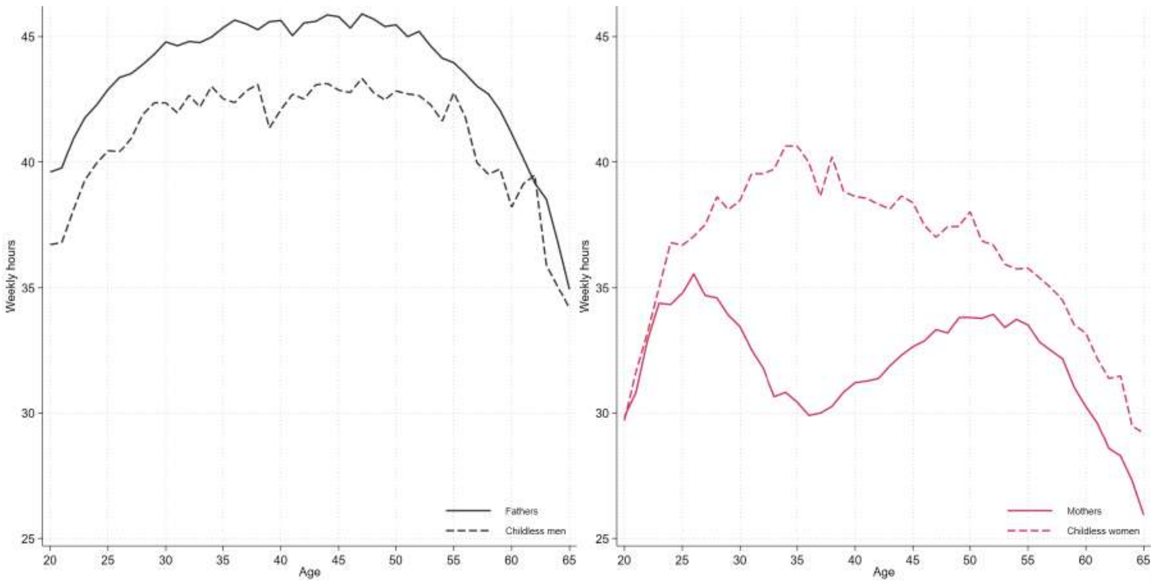

Figure 54: Intensive margin: Age profiles of work hours (if employed) by key demographics (gender and parenthood). Left: fathers (solid) and childless men (dashed). Right: mothers (solid) and childless women (dashed).

Notes: The age profiles stitch together 20-year snapshots of life-cycle for selected cohorts. The youngest cohort is cohort 12 aged 20-39 in the data. The oldest cohort is cohort 4 (aged 60-79) on the left panel and cohort 5 (aged 55-74) on the right panel. We omit the very old cohorts due to data limitation.

H Welfare programs in Australia

H.1 Trends in welfare expenditure

Financial year	Welfare ($b)	Welfare-GDP (%)	Welfare-Revenue (%)
2010-11	140.19	8.43	34.04
2011-12	149.66	8.70	34.20
2012-13	153.24	8.89	33.62
2013-14	155.68	8.88	33.47
2014-15	165.13	9.41	35.15
2015-16	167.68	9.47	34.59
2016-17	165.76	8.95	33.02
2017-18	171.62	8.99	32
2018-19	174.24	8.80	31.18
2019-20	195.71	9.86	36.05

Figure 55: **Welfare expenditure in Australia**

Notes: $ value is expressed in 2019–20 AUD.
Source: Welfare expenditure report by the Australian Institute of Health and Welfare.

Financial year	Families & Children	Old people	Disabled	Unemployed	Others
2009-10	2.51	3.33	1.87	0.48	0.40
2010-11	2.39	3.33	1.94	0.44	0.34
2011-12	2.33	3.43	1.98	0.44	0.52
2012-13	2.31	3.57	2.00	0.49	0.52
2013-14	2.26	3.47	2.02	0.55	0.57
2014-15	2.33	3.79	2.09	0.59	0.61
2015-16	2.32	3.86	2.08	0.60	0.62
2016-17	2.02	3.72	2.01	0.57	0.63
2017-18	1.94	3.67	2.18	0.56	0.65
2018-19	1.81	3.63	2.22	0.49	0.64
2019-20	1.92	3.85	2.53	0.93	0.62

Figure 56: **Welfare expenditure to GDP (%) by target groups**

Source: Welfare expenditure report by the Australian Institute of Health and Welfare.

H.2 Child-related transfer programs in Australia

		2001-05	2006-10	2011-15	2016-20*	Total
Income support	Pensions	51.74%	51.35%	57.67%	60.80%	55.79%
	Parenting payments	9.52%	6.58%	5.61%	4.63%	6.39%
	Allowances	14.80%	9.94%	10.62%	11.54%	11.59%
	Total	76.06%	67.87%	73.90%	76.98%	73.77%
Non-income support	Family payments	23.09%	24.96%	22.18%	18.02%	21.87%
	Bonus payments	0.00%	5.55%	1.31%	1.38%	2.07%
	Other non-income supports	0.59%	1.40%	2.51%	3.45%	2.10%
	Total	23.68%	31.91%	26.00%	22.85%	26.05%
Other public benefits		0.26%	0.22%	0.10%	0.18%	0.18%

Table H.1: **Components of Australian public transfers over time**

*Notes: *The welfare and social security transfers account for roughly 30% of government revenue in the 2016-20 period.*

H.3 Family Tax Benefit part A (FTB-A)

The FTB-A program is a non-taxable transfer paid per child and the amount claimable depends on family's circumstances. In short, it is a function of combined household adjusted taxable income, annual private rent, and age and number of dependent children. Important parameters that determine the levels, kinks and slopes of the FTB-A benefit schedule are:

1. Statutory base and maximum payment rates per qualifying dependent child (i.e., FTB child),
2. Income test thresholds for the base and maximum payments,
3. Withdrawal or taper rates for the base and maximum payments, and
4. Supplements such as the Large Family Supplement (LFS), the Newborn Supplement (NBS), the Multiple Birth Allowance (MBA), the Rent Assistance (RA), and the Clean Energy Supplement (CES) that are added to the statutory base and maximum payment rates per child to derive the total base and maximum payments..

These parameters constitute the main structure of the FTB-A program. Their values may vary from year to year. For our purpose, we adopt the 2018 FTB-A parameters in the initial steady state equilibrium of the model economy.

We first calculate the per child total base payment, b_A, and the per child total maximum payment, m_A, of the FTB-A benefit.

$$\begin{aligned} b_{A,j} = {} & LFS_j + NBS_j + MBA_j + CES_{A,base,j} \\ & + ndep_{[0,17],j} \times FTBA_{base_1} \\ & + ndep_{[18,24],j} \times FTBA_{base_2} \\ & + \mathbf{1}_{\{school=1\}} ndep_{[18,19],j} \times FTBA_{base_3} \\ & + \mathbf{1}_{\{school=0\}} ndep_{[18,21],j} \times FTBA_{base_4} \end{aligned} \tag{H.1}$$

$$\begin{aligned} m_{A,j} = {} & LFS_j + NBS_j + MBA_j + RA_j + CES_{A,max,j} \\ & + ndep_{[0,12],j} \times FTBA_{max_1} \\ & + ndep_{[13,15],j} \times FTBA_{max_2} \\ & + ndep_{[16,17],j} \times FTBA_{max_3} \\ & + ndep_{[18,24],j} \times FTBA_{max_4} \\ & + \mathbf{1}_{\{school=1\}} ndep_{[16,19],j} \times FTBA_{max_5} \\ & + \mathbf{1}_{\{school=0\}} ndep_{[16,17],j} \times FTBA_{max_6} \\ & + ndep_{[18,21],j} \times FTBA_{max_7} \end{aligned} \tag{H.2}$$

where *school* is a binary variable for school attendance and $ndep_{[a,b],j}$ denotes the number of children in the age range $[a, b]$ of parents aged j. $FTBA_{base}$ and $FTBA_{max}$ are parameters corresponding to the statutory base and maximum per dependent child payment rates which vary over age of a child. In 2018, $FTBA_{base} = \{2,266.65;\ 0;\ 2,266.65;\ 0\}$ and $FTBA_{max} = \{5504.20;\ 6938.65;\ 0;\ 0;\ 6938.65;\ 0;\ 0\}$ stated in 2018 AUD.

The income test thresholds for base and maximum payments, $TH_{base}$and TH_{max}, are

$$\begin{cases} TH_{max} & = FTBA_{T_1} \\ TH_{base} & = FTBA_{T_2} + (ndep_{[0,24],j} - 1) \times FTBA_{T_2 A} \end{cases} \tag{H.3}$$

The maximum threshold is fixed while the base threshold expands at the rate of $FTBA_{T_2 A}$ for every addition of a dependent child.

In 2018, the starting income test thresholds $FTBA_T = \{52,706;\ 94,316\}$, and the base payment income test threshold adjustment factor per additional qualifying child $FTBA_{T_2 A} = 0$, stated in 2018 AUD.

We can then calculate the FTB-A benefit.

$$FTBA^0_j(y_h) = \begin{cases} m_{A,j} & \text{if } y_h \le TH_{max} \\ MAX\{b_{A,j},\ m_{A,j} - FTBA_{w_1}(y_h - TH_{max})\} & \text{if } TH_{max} < y_h \le TH_{base} \\ MAX\{0, & \text{if } y_h > TH_{base} \\ \qquad m_{A,j} - FTBA_{w_1}(y_h - TH_{max}), & \\ \qquad b_{A,j} - FTBA_{w_2}(y_h - TH_{base})\} & \end{cases} \tag{H.4}$$

where the total household taxable income $y_h = y_m + y_f + ra$ and $FTBA_w$ is the withdrawal rate. In 2018, $FTBA_w = \{0.20,\ 0.30\}$.

The statutory rates include extra supplement for low income households. In our calculation, this supplement is later deducted from the total benefit payment if a household does not meet the supplement's income test cutoff. The income test is conducted separately once the full benefit has been computed

$$FTBA_j(y_h) = \begin{cases} MAX\{0,\ FTBA^0_j(y_h) - FTBA_{AS} \times (ndep_{[0,12],j} & \text{if } y_h{>}FTBA_{FT1} \\ \qquad\qquad + ndep_{[13,15],j} + \mathbf{1}_{\{school=1\}} ndep_{[1619],j})\} & \\ FTBA^0_j(y_h) & otherwise \end{cases} \tag{H.5}$$

where in 2018, the annual FTB-A supplement adjustment $FTBA_{AS} = 737.30$ and the supplement's income test threshold $FTBA_{FT1} = 80,000$ stated in 2018 AUD.

Below are the formulae used to calculate the LFS, NBS, MBA, CES (for part A and part B), and RA in the model.

Large Family Supplement (LFS):

$$LFS_j = min\{FTBA_{S_1} \times (ndep_{[0,24],j} - FTBA_{C_1} + 1),\ 0\} \tag{H.6}$$

where $ndep_{[a,b],j}$ denotes the number of children in the age range $[a,b]$ of parents aged j, $FTBA_{S_1}$ is the LFS amount per child, and $FTBA_{C_1}$ is the number of dependent children a family must have to be eligible for the LFS for the first child to satisfy the cutoff $FTBA_{C_1}$ and every additional child onward. In 2018, $FTBA_{C_1} = 1$ and $FTBA_{S_1} = 0$.

Newborn Supplement (NBS):

$$NBS_j = \begin{cases} \mathbf{1}_{\{nb_j \geq 1,\ fc_j=1\}} FTBA_{NS_1} \times nb_j + \mathbf{1}_{\{nb_j \geq 1,\ fc_j=0\}} FTBA_{NS_2} \times nb_j & \text{if } ppl = 0 \\ \mathbf{1}_{\{nb_j \geq 2,\ fc_j=1\}} FTBA_{NS_1} \times (nb_j - 1) + \mathbf{1}_{\{nb_j \geq 2,\ fc_j=0\}} FTBA_{NS_2} \times (nb_j - 1) & \text{if } ppl = 1 \end{cases} \tag{H.7}$$

where nb_j denotes the number of newborns to parents aged j, fc_j is a binary variable for first child, ppl is a binary variable for Paid Parental Leave (by default, we set $ppl = 0$), and $FTBA_{NS}$ is the amount of NBS per qualified child. In 2018, $FTBA_{NS} = \{2,158.89;\ 1,080.54\}$ stated in 2018 AUD.

Multiple Birth Allowance (MBA):

$$MBA_j = \begin{cases} \mathbf{1}_{\{sa=3,\ j_c \leq FTBA_{MAGES}\}} FTBA_{MBA_1} + \mathbf{1}_{\{sa \geq 4,\ j_c \leq FTBA_{MAGES}\}} FTBA_{MBA_2} & \text{if } school = 1 \\ \mathbf{1}_{\{sa=3,\ j_c \leq FTBA_{MAGE}\}} FTBA_{MBA_1} + \mathbf{1}_{\{sa \geq 4,\ j_c \leq FTBA_{MAGE}\}} FTBA_{MBA_2} & \text{if } school = 0 \end{cases} \tag{H.8}$$

where sa is the number of dependent children with the same age, $school$ is a binary variable for school attendance, j_c is the age of children sharing birth date, and $FTBA_{MAGE}$ and $FTBA_{MAGES}$ are a child's age cutoffs to be eligible for the MBA if they attend and do not attend school, respectively. $FTBA_{MBA}$ is the MBA payment. For simplicity, we assume there can only be one instance of multiple births for each household. In 2018, $FTBA_{MAGE} = 16$, $FTBA_{MAGES} = 18$, and $FTBA_{MBA} = \{4,044.20;\ 5,387.40\}$ stated in 2018 AUD.

Clean Energy Supplement for the FTB part A ($\mathrm{CES_A}$):

The Clean Energy Supplement for the FTB part A ($\mathrm{CES_A}$) is separated into base and maximum payments. We add the former to the base level and the latter to the maximum level of the FTB-A benefit.

$$\begin{aligned} CES_{A,base,j} &= ndep_{[0,17],j} \times FTBA_{CE_1} + ndep_{[18,19]_{AS},j} \times FTBA_{CE_1} \\ CES_{A,max,j} &= ndep_{[0,12],j} \times FTBA_{CE_2} + ndep_{[13,15],j} \times FTBA_{CE_3} + ndep_{[16,19]_{AS},j} \times FTBA_{CE_3} \end{aligned} \tag{H.9}$$

where $ndep_{[a,b],j}$ denotes the number of children in the age range $[a,b]$ of parents aged j, $school$ is a binary variable for school attendance, $ndep_{[a,b]_{AS},j} = \mathbf{1}_{\{school=1\}} \times ndep_{[a,b],j}$, $FTBA_{CE}$ is the per child amount of the CES_A. In 2018, $FTBA_{CE} = \{36.50;\ 91.25;\ 116.80\}$ in 2018 AUD.

Note that from 2018 onward, only households who had received the CES_A in the previous year were eligible for the supplement. In the baseline model, we assume this is true for all households.

Rent Assistance (RA):

Rent assistance adds to the per child maximum payment of the FTB-A and is available only to FTB-A recipients who rent privately which we assume to hold true for all households in the benchmark model.

$$RA_j(rent) = \begin{cases} MAX\{MIN\{0.75\,(rent - rent_{min}),\ RA_{max}\},\ 0\} & \text{if } FTBA_1 \geq FTBA_{min} \\ 0 & \text{otherwise} \end{cases} \tag{H.10}$$

where $rent$ is the annual rent, $rent_{min}$ is the minimum rent to qualify for the RA, RA_{max} is the cap on the RA benefit, $FTBA_1$ is the FTB-A benefit excluding the RA, $FTBA_{min}$ is the minimum size of the FTB-A for which a household must be qualified to be deemed eligible for the RA. In 2018, expressed in 2018 AUD

$$RA_{max} = \mathbf{1}_{\{ndep_{[0,24],j} \leq 2\}} 4,116.84 + \mathbf{1}_{\{ndep_{[0,24],j} \geq 3\}} 4,648.28\}$$

$$rent_{min} = \mathbf{1}_{\{single=1\}} 4,102.28 + \mathbf{1}_{\{couple=1\}} 6,071.52$$

Before 2013, $FTBA_{min}$ is set to the base FTB-A payment and $FTBA_{min} = 0$ thereafter.

H.4 Family Tax Benefit part B (FTB-B)

The FTB-B program is paid per family. Its objective is to give additional support to single parents and single-earner partnered parents with limited means. Similar to the FTB-A, the FTB-B is a function of age and number of dependent children, but differently, the eligibility and amount claimable are determined by separate tests on spouses' (i.e., primary earner's and secondary earner's) individual taxable income and marital status of the potential recipients. Important parameters that determine the levels, kinks and slopes of the FTB-B benefit schedule are: (i) Maximum payment rate; (ii) Separate income test thresholds on primary and secondary earners; and (iii) Withdrawal or taper rates based on secondary earner's taxable income.

Let $y_{pe} = MAX(y_m, y_f)$ and $y_{se} = MIN(y_m, y_f)$ denote the primary earner's and secondary earner's taxable income, respectively, and let $m_{B_i,j} = FTBB_{max_i} + CES_{B,j}$ be the maximum payment per family. Note that the structure of the FTB-B changed in 2017. The FTB-B formula prior to 2017 is thus different to that from 2017 onwards.

Before 2017:

$FTBB_j(y_m, y_f) =$

$$\begin{cases} cond_1 \times m_{B_1,j} + cond_2 \times m_{B_2,j} & \text{if } y_{pe} \leq FTBB_{T_1} \text{ and } y_{se} \leq FTBB_{T_2} \\ \\ cond_1 \times MAX\{0,\ m_{B_1,j} - FTBB_w(y_{se} - FTBB_{T_2})\} & \text{if } y_{pe} \leq FTBB_{T_1} \text{ and } y_{se} > FTBB_{T_2} \\ +cond_2 \times MAX\{0,\ m_{B_2,j} - FTBB_w(y_{se} - FTBB_{T_2})\} & \end{cases} \tag{H.11}$$

From 2017:

$FTBB_j(y_m, y_f) =$

$$\begin{cases} cond_1 \times m_{B_1,j} + cond_3 \times m_{B_2,j} & \text{if } y_{pe} \leq FTBB_{T_1} \text{ and } y_{se} \leq FTBB_{T_2} \\ \\ cond_1 \times MAX\{0,\ m_{B_1,j} - FTBB_w(y_{se} - FTBB_{T_2})\} & \text{if } y_{pe} \leq FTBB_{T_1} \text{ and } y_{se} > FTBB_{T_2} \\ +cond_3 \times MAX\{0,\ m_{B_2,j} - FTBB_w(y_{se} - FTBB_{T_2})\} & \end{cases} \tag{H.12}$$

where $cond_1 = 1_{\{ndep_{[0,4],j} \geq 1\}}$, $cond_2 = 1_{\{ndep_{[0,4],j}=0,\ (ndep_{[5,15],j} \geq 1 \text{ or } ndep_{[16,18]_{AS},j} \geq 1)\}}$ and $cond_3 = 1_{\{ndep_{[0,4],j}=0,\ ndep_{[5,12],j} \geq 1\}} + 1_{\{ndep_{[0,4],j}=0,\ ndep_{[5,12],j}=0,\ (ndep_{[13,15],j} \geq 1 \text{ or } ndep_{[16,18]_{AS},j} \geq 1),\ single=1\}}$

In 2018, the statutory maximum FTB-B payment $FTBB_{max} = \{4,412.85;\ 3,190.10\}$, the income test thresholds $FTBB_T = \{100,000;\ 5,548\}$ in 2018 AUD, and the withdrawal rate $FTBB_w = 0.20$.

Clean Energy Supplement for the FTB part B ($\mathrm{CES_B}$):

The Clean Energy Supplement for FTB part B ($\mathrm{CES_B}$) adds to the statutory per family payment of the FTB-B benefit.

$$CES_{B,j} = \begin{cases} FTBB_{CE_1} & \text{if } ndep_{[0,4],j} \geq 1 \\ FTBB_{CE_2} & \text{if } ndep_{[0,4],j} = 0 \text{ and } (ndep_{[5,15],j} \geq 1 \text{ or } ndep_{[16,18]_{AS},j} \geq 1) \\ 0 & \text{if } ndep_{[0,4],j} = 0 \text{ and } ndep_{[5,15],j} = 0 \text{ and } ndep_{[16,18]_{AS},j} = 0) \end{cases} \tag{H.13}$$

where $ndep_{[a,b],j}$ denotes the number of children in the age range $[a, b]$ of parents aged j, *school* is a binary variable for school attendance, $ndep_{[a,b]_{AS},j} = \mathbf{1}_{\{school=1\}} \times ndep_{[a,b],j}$, $FTBB_{CE}$ is the per family amount of CES_B. In 2018, $FTBB_{CE} = \{73;\ 51.10\}$ in 2018 AUD.

Note that from 2018 onward, only households who had received the CES_B in the previous year were eligible for the supplement. In the baseline model, we assume this is true for all households.

H.5 Child Care Subsidy (CCS)

The Child Care Subsidy program aims at assisting households with the cost of caring for children aged 13 or younger who are not attending secondary school and is paid directly to approved child care service providers. Eligibility criteria include (i) a test on the combined family income (y_h), (ii) the type of child care service, (iii) age of the dependent child, and (iv) hours of recognized activities (e.g., working, volunteering and job seeking) by parents (n_j^m, n_j^f). The rate of subsidy is also determined by parameters such as income thresholds, work hours, and taper unit (the size of income increment by which the subsidy rate falls by 1 percentage point). Given that the current model is silent on the type of child care and therefore child care fees, we assume the followings:

1. Identical child care service operating within a perfectly competitive framework,
2. No annual cap on hourly fee and on subsidy per child,
3. Households exhaust all the available hours of subsidized care.

The child care subsidy function is

$$CCS(y_h, n_j^m, n_j^f) = \Psi(y_h, n_j^m, n_j^f) \times \begin{cases} CCS_{R_1} & \text{if } y_h \leq TH_1 \\ MAX\{CCS_{R_2},\ CCS_{R_1} - \omega_1\} & \text{if } TH_1 < y_h < TH_2 \\ CCS_{R_2} & \text{if } TH_2 \leq y_h < TH_3 \\ MAX\{CCS_{R_3},\ CCS_{R_2} - \omega_3\} & \text{if } TH_3 \leq y_h < TH_4 \\ CCS_{R_3} & \text{if } TH_4 \leq y_h < TH_5 \\ CCS_{R_4} & \text{if } y_h \geq TH_5 \end{cases} \tag{H.14}$$

where $y_h = y_m + y_f + ra$ and $\omega_i = \dfrac{y_h - TH_i}{taper\ unit}$.
In 2018,

- Taper unit = AUD 3,000;
- Statutory subsidy rates, $CCS_R = \{0.85,\ 0.5,\ 0.2,\ 0\}$;
- Income test thresholds, $TH = \{70,015;\ 175,015;\ 254,305;\ 344,305;\ 354,305\}$ in 2018 AUD;
- Let $n_j^{min} = min\{n_j^m, n_j^f\}$. The adjustment factor is

$$\Psi(y_h, n_j^m, n_j^f) = 0.24_{\{y_h \leq AU\$70{,}015,\ n_j^{min} \leq 8\}} + 0.36_{\{8 < n_j^{min} \leq 16\}} + 0.72_{\{16 < n_j^{min} \leq 48\}} + 1_{\{n_j^{min} > 48\}}$$

  Otherwise, $\Psi(y_h, n_j^m, n_j^f) = 0$.

I Supplementary facts on child benefit programs

I.1 Child care benefit: Intensive and extensive margins

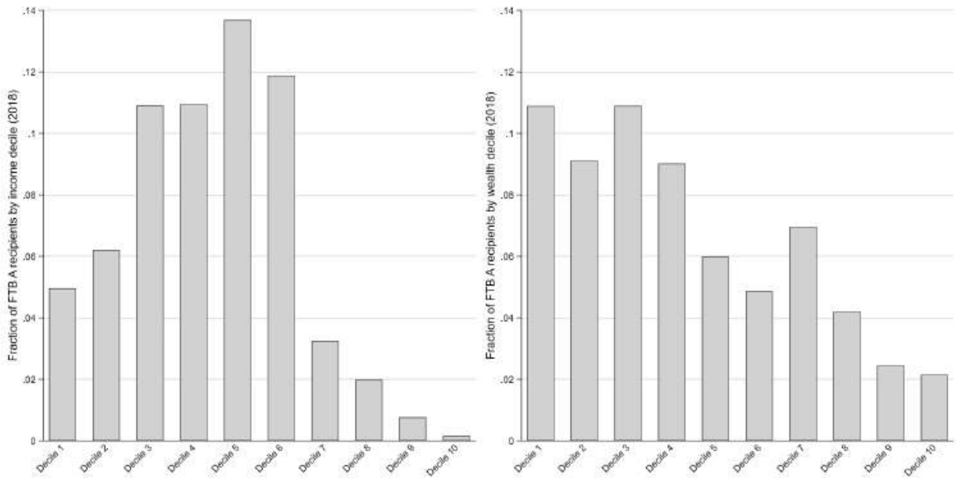

Figure I.1: **FTB-A recipients in 2018. Left**: By income decile, **Right**: By wealth decile

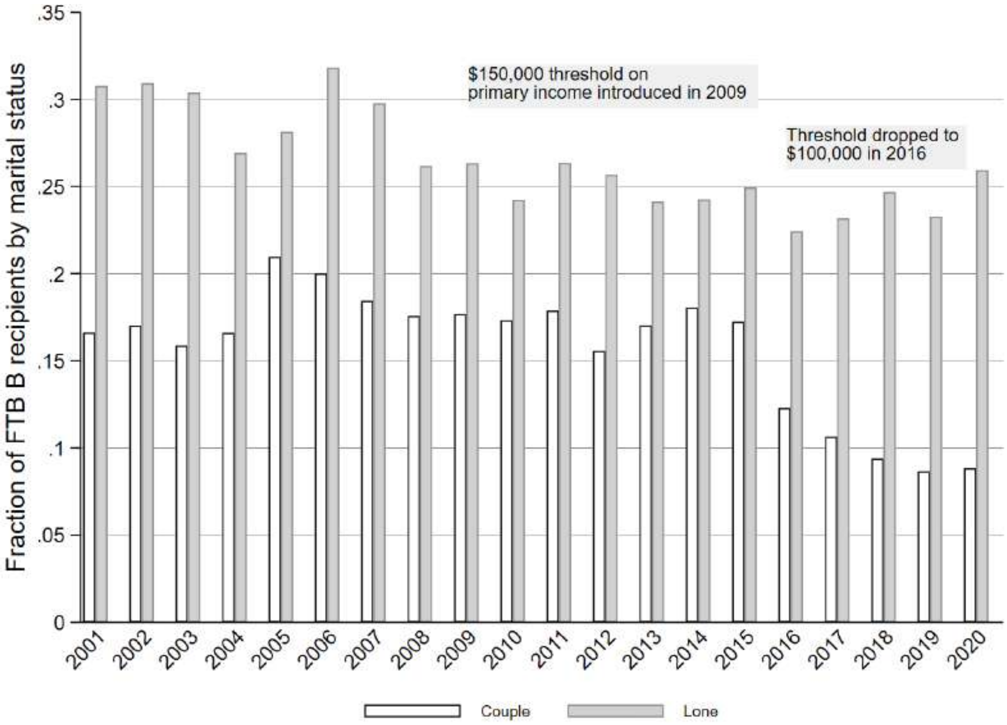

Figure I.2: **Proportion of FTB-B recipients by marital status.**

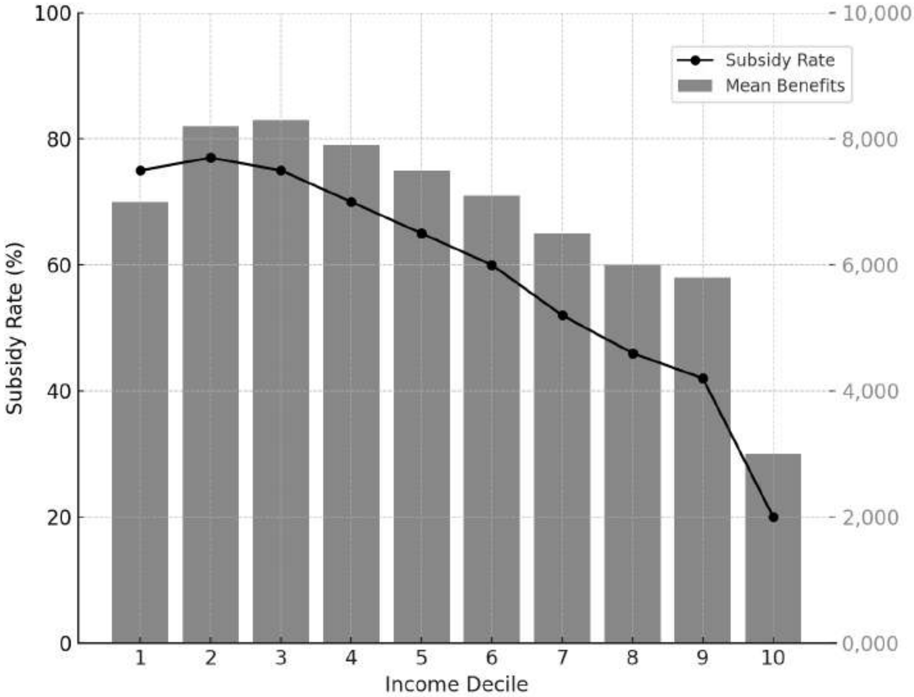

Figure I.3: **Child Care Subsidy rates and Mean Benefits (Subsidies) by income decile.**
Notes: This figure uses data from Table 61 in the 2021 report by the AIFS. The lowest decile earned at most \$31,399. *The top decile earned* \$240,818 *or more.*

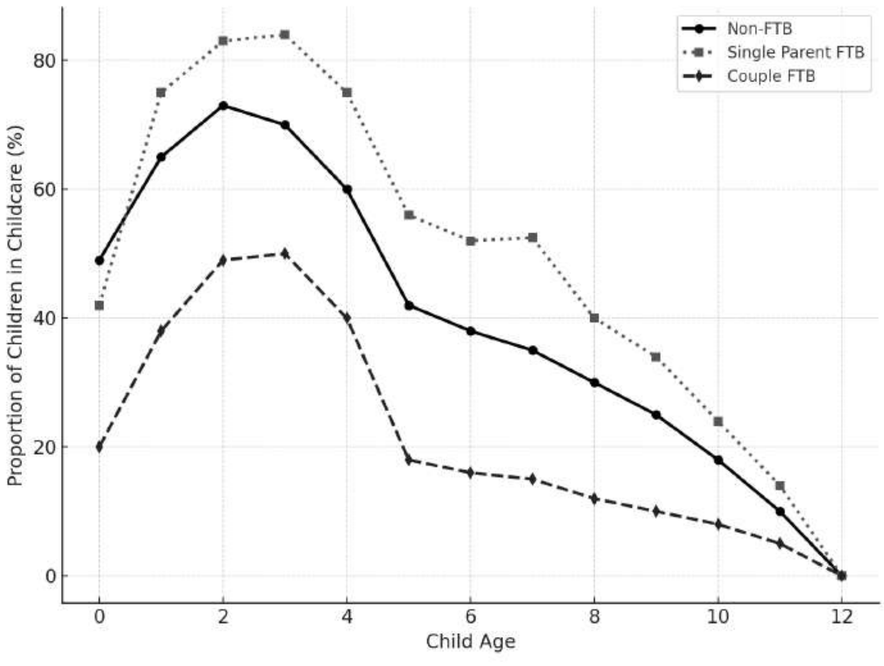

Figure I.4: **Proportion of children in child care by child age and FTB receipt.**
Notes: This figure uses data from Figure 95 in the 2021 report by the AIFS.

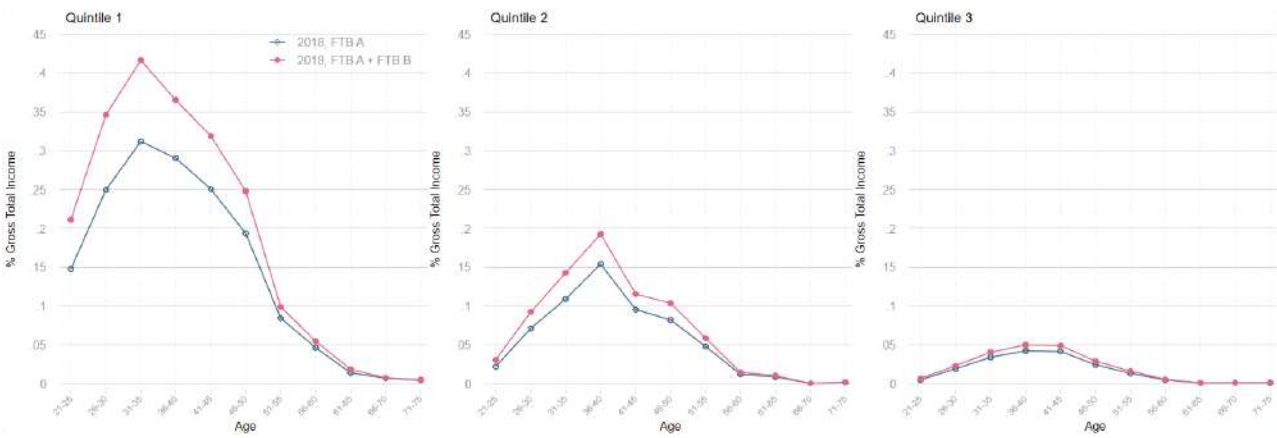

Figure I.5: **Age profiles of FTB share of gross household income for the first three quintiles by family market income in 2018.**

I.2 Supplementary figures: FTB-A parameters and related statistics

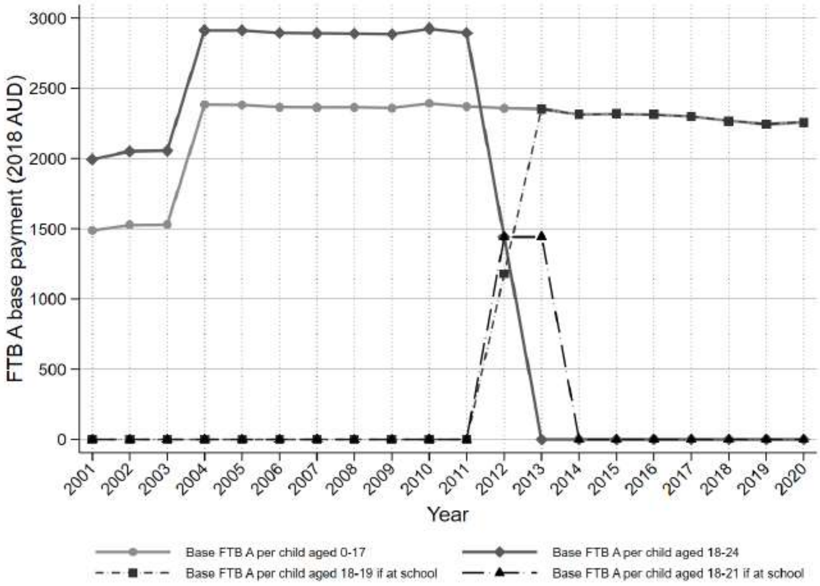

Figure I.6: FTB-A base payment rates per child

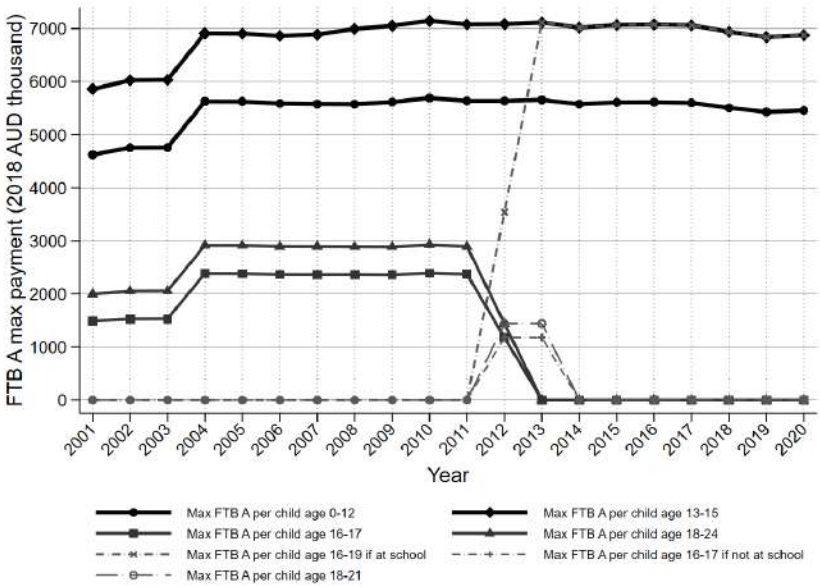

Figure I.7: FTB-A maximum payment rates per child

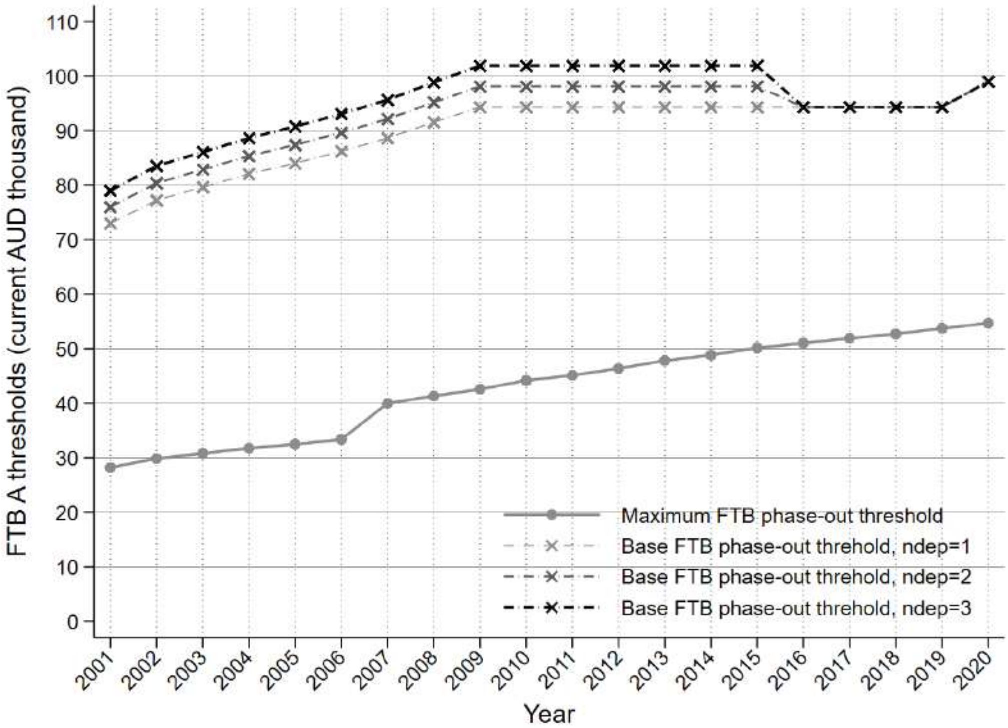

Figure I.8: FTB-A income test thresholds for maximum and base payment rates

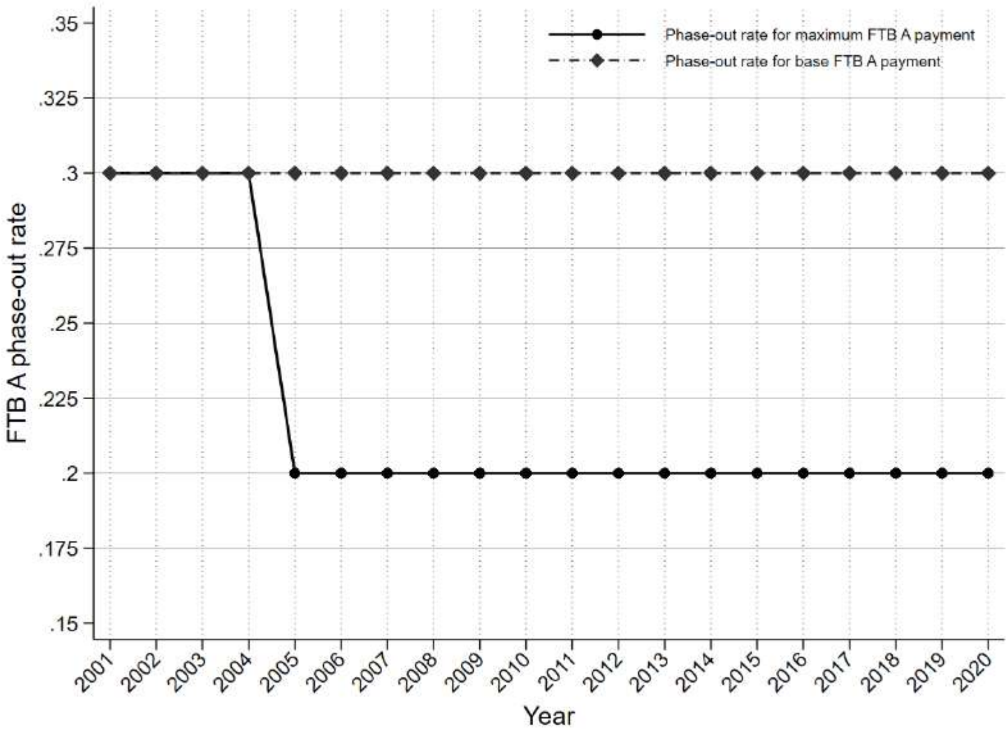

Figure I.9: FTB-A phase-out rates for maximum and base payments

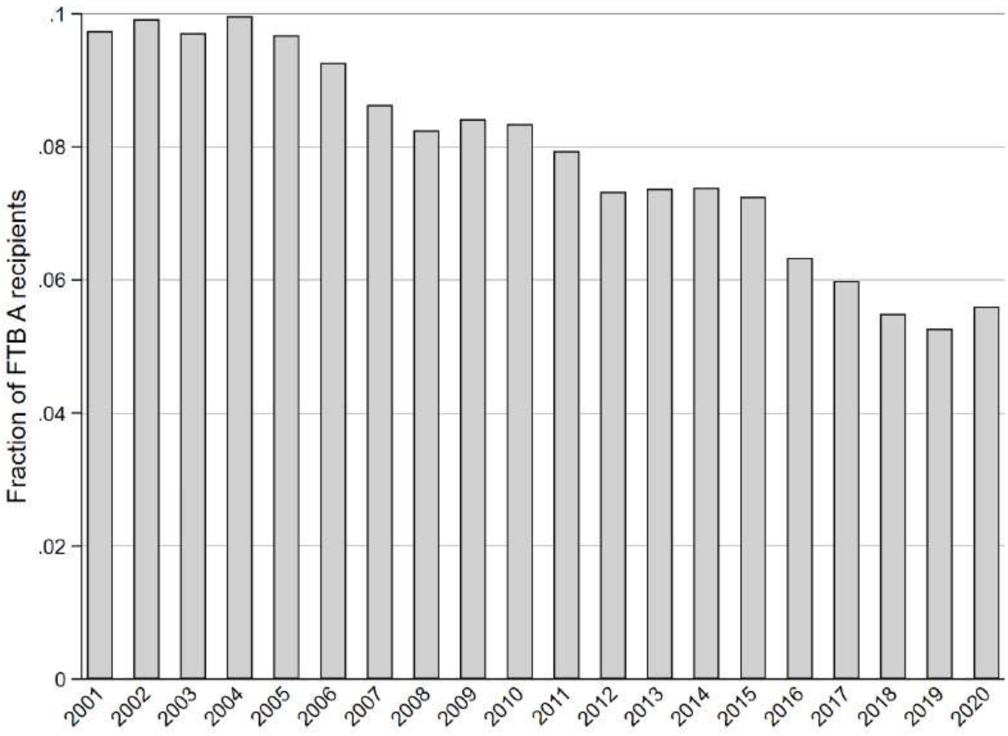

Figure I.10: Proportion of FTB-A recipients over time.

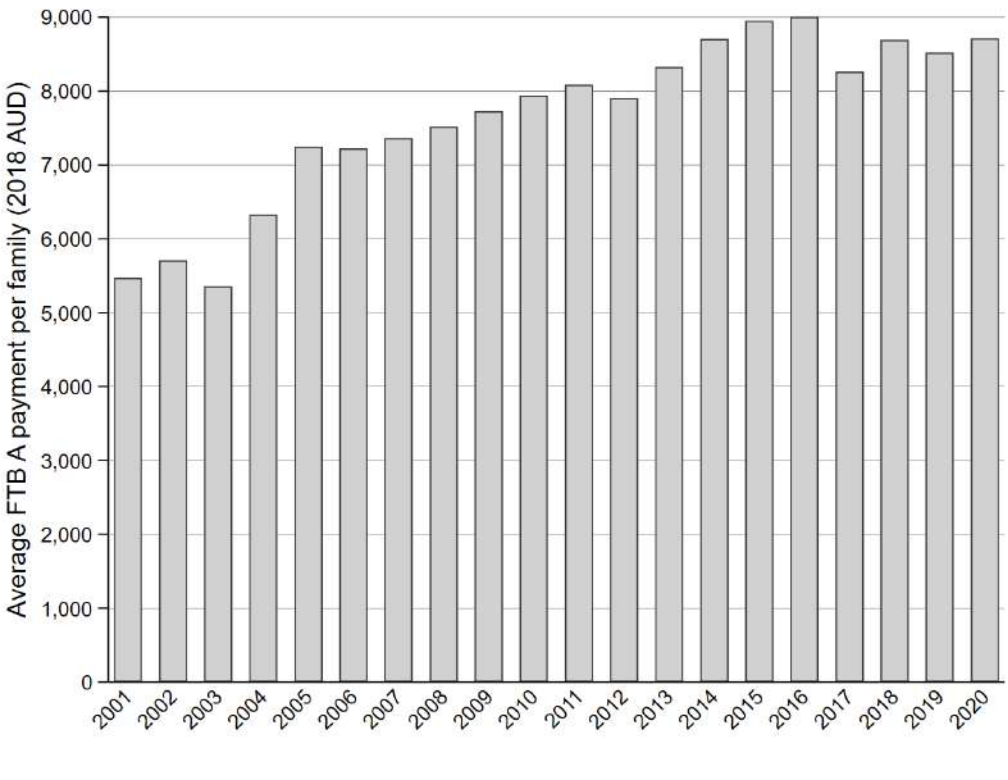

Figure I.11: Average FTB-A payment per family (2018 AUD) over time.

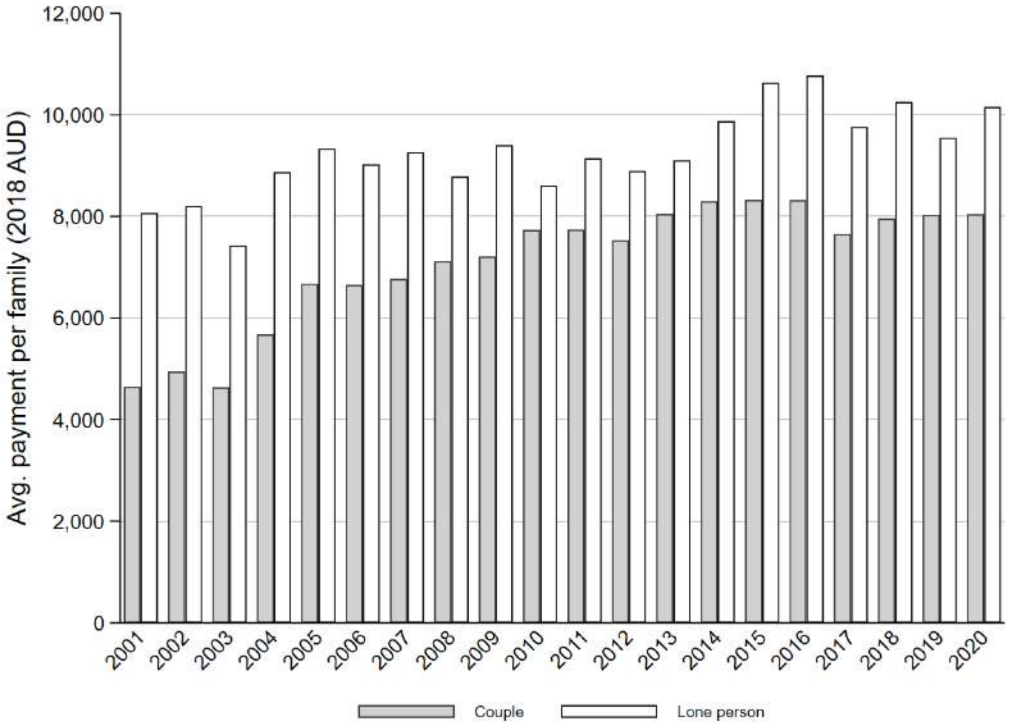

Figure I.12: Average FTB-A payment per family by marital status

The proportion of households receiving the FTB-A (out of all households observed in the survey data) has fallen from 10% in 2001 to slightly over 5% in 2020, (see Figure I.10). This can be attributed, in part, to threshold-creep (inflation pushing incomes above the income-test threshold) and the falling birth rate. Despite the overall decline, the benefit remains concentrated among low-income families.

At the intensive margin, the FTB-A alone represents a significant sum of inflation-indexed transfers. Figures I.6 and I.7 illustrate that there have been minimal changes to the base and maximum statutory payment rates for children under 18 since 2004. Qualified families with a child aged 13-15 could receive up to $7,000. The maximum rate per dependent child aged 12 or younger is slightly lower, but still exceeds $5,500. Given that payments are allocated per child, a two-children family could receive up to $14,000. Moreover, Figure I.11 shows that the benefits delivered to eligible families have been rising. The average FTB-A payout increased from $8,000 to $8,500 over the past decade. Moreover, because the scheme predominantly targets single-earner families, especially single parents, single parent households claimed higher benefits on average compared to couple parent households, as seen in Figure I.12.

I.3 Supplementary figures: FTB-B parameters and related statistics

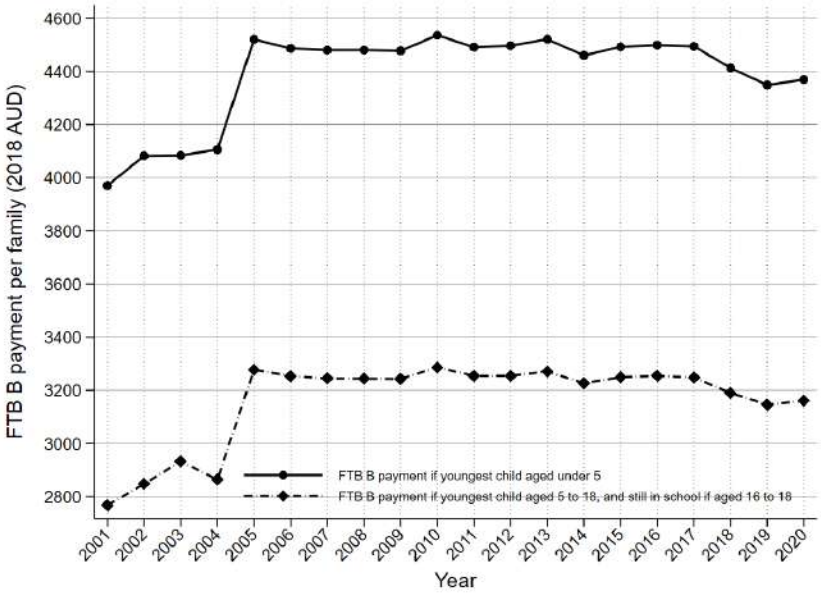

Figure I.13: FTB-B payment rates per family by age of the youngest child

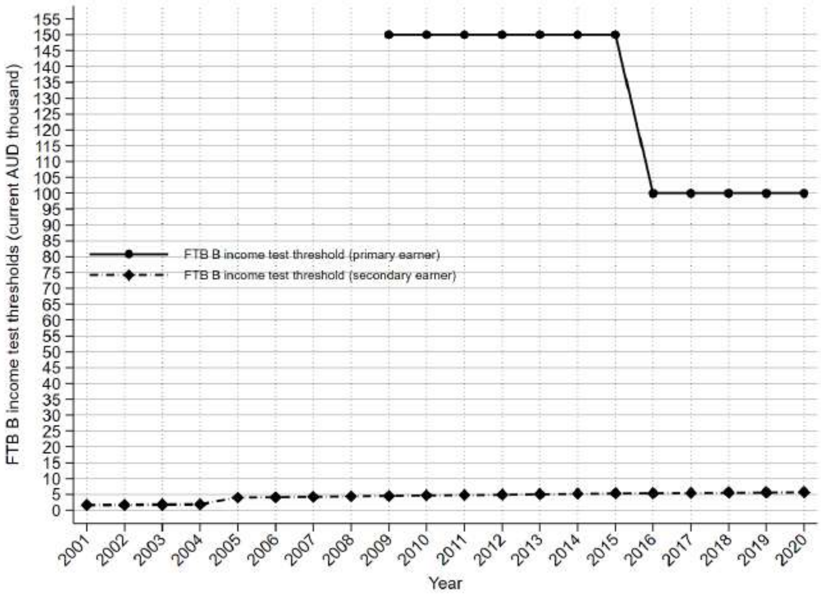

Figure I.14: FTB-B thresholds over time on primary and secondary earners over time eneral equilibrium

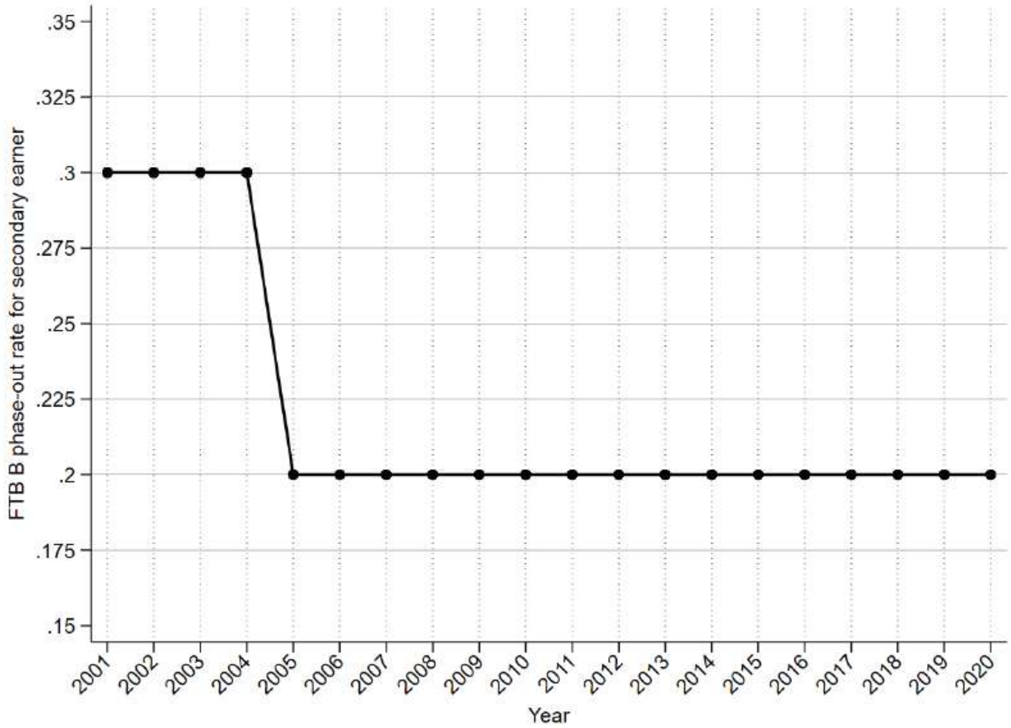

Figure I.15: FTB-B taper rates over time

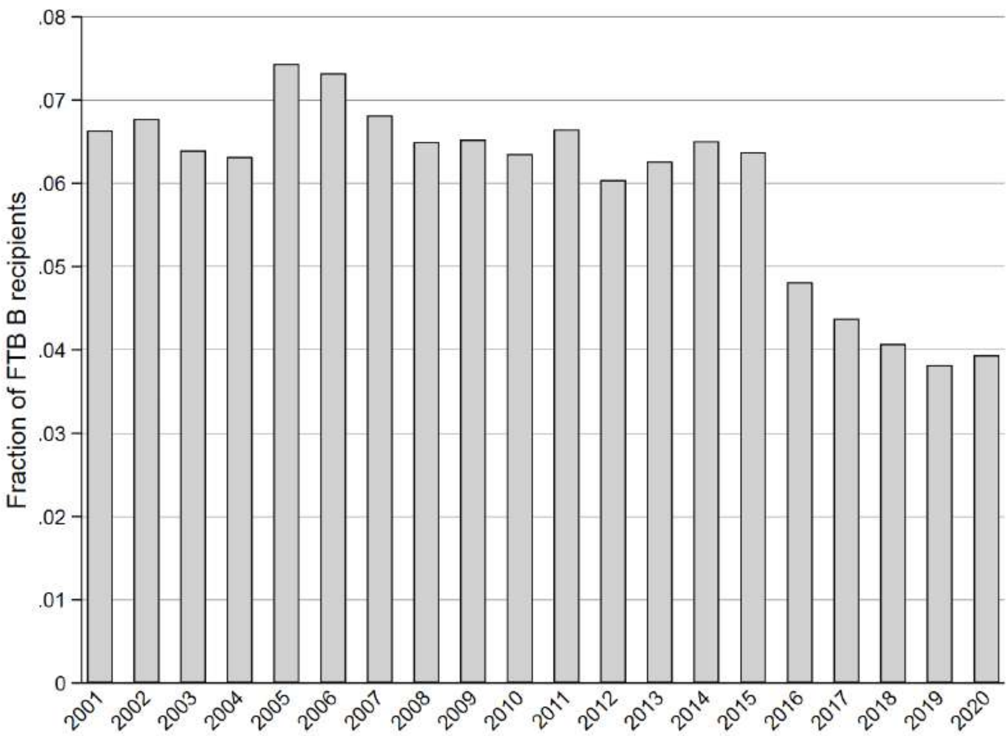

Figure I.16: Proportion of FTB-B recipients over time

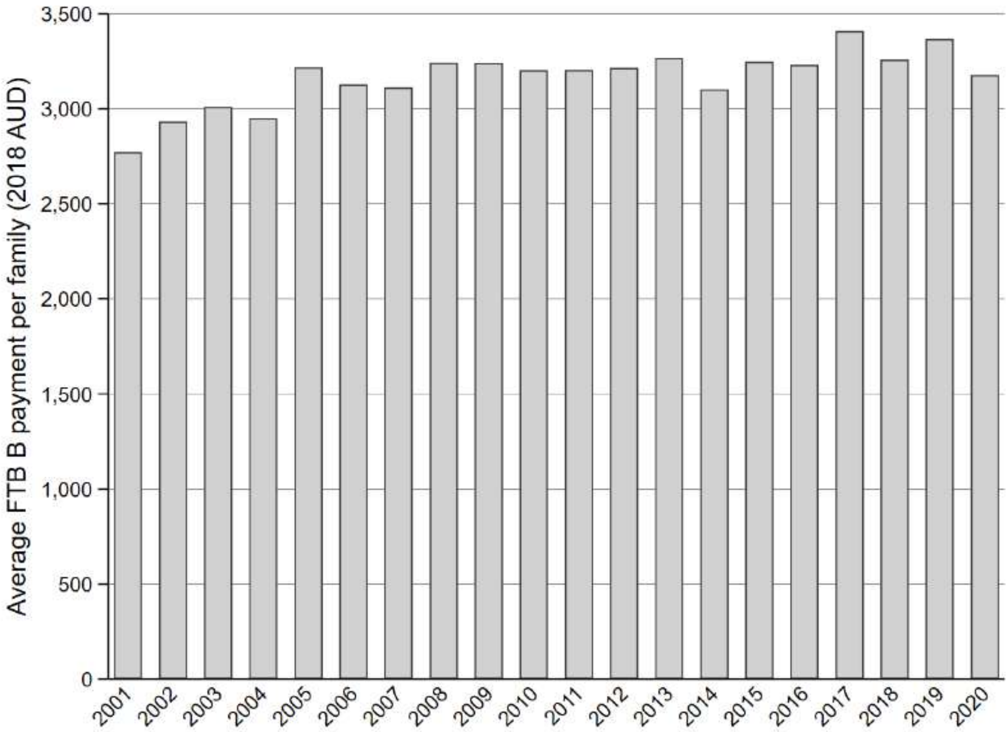

Figure I.17: Average FTB-B payment (2018 AUD) over time

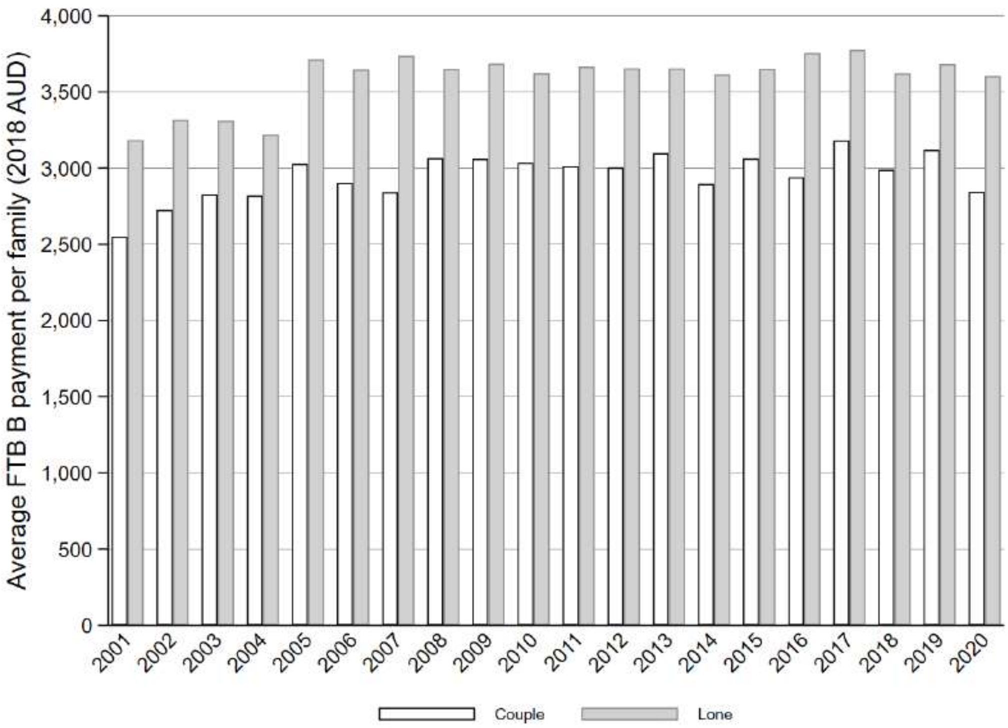

Figure I.18: **Average FTB-B payment by marital status.**

Because FTB-A recipient status is necessary for a household to access the FTB-B benefits, we can infer from Figure I.10 and Figure I.16 that the majority of FTB-A households also claimed the FTB-B. Although the FTB-A is the larger of the two benefits, the FTB-B offers a non-trivial amount. As shown in Figure I.13, the FTB-B payment remained steady at approximately $4,500 for eligible families whose youngest child is under 5 years of age, and $3,200 if their youngest child is between 5 and 18 years old.

At the extensive margins, the proportion of claimants fell over time. Compared to the 2000s and the first half of 2010s, the fraction of partnered FTB-B households dropped by nearly 50% by 2018 (Figure I.16). This could be partially explained by factors similar to those affecting the FTB-A, such as fertility trends and threshold creep. For the FTB-B in particular, the recent drop in couple recipients can also be attributed to the $150,000 (current AUD) income-test threshold for primary earners introduced in 2009, and the subsequent tightening in 2016 as the threshold decreased further to $100,000 (current AUD). These stricter measures, which complemented the existing test on secondary earners, significantly reduced the claimant pool. However, because the primary earner's income test exclusively determines eligibility (controlling the extensive margin), it had no discernible effect on the average benefit rate for recipients. The right panel of Figure I.18 demonstrates that in 2020, eligible single parents could still expect to receive over $3,500, while couple parents could expect just under $3,000 — similar to the amount they would receive in 2005.

I.4 Supplementary figures: CCS-related statistics

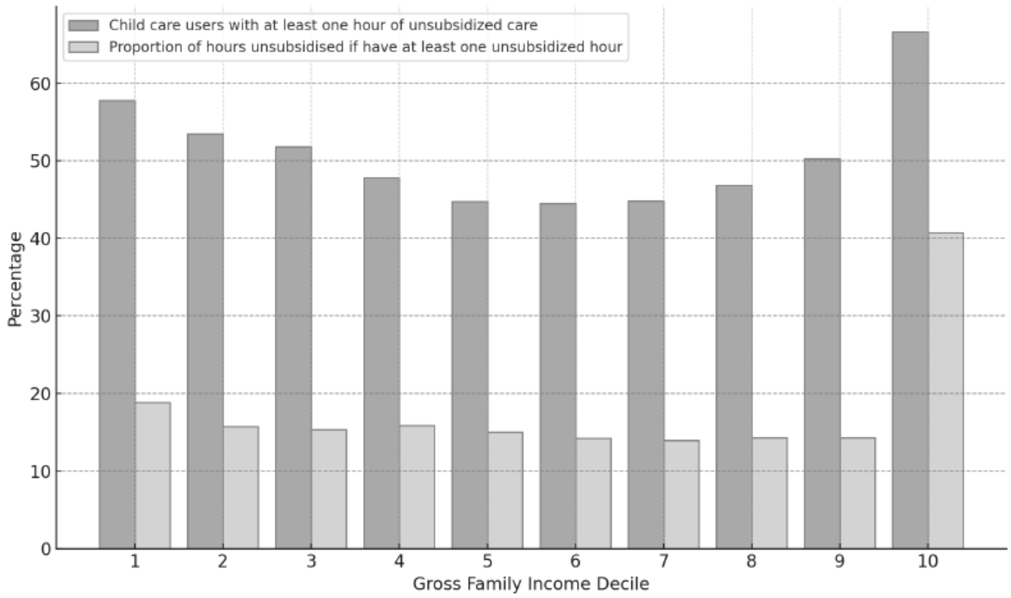

Figure I.19: **Proportion of hours paid for that are unsubsidized by gross family income decile in 2018-19 financial year.**

Notes: This figure uses data from Table 31 in the 2021 Child Care Package Evaluation report by the AIFS. The lowest decile earned at most $31,399$. *The top decile earned* $240,818$ *or more.*

Figure I.19 illustrates the proportion of unsubsidized child care hours, highlighting the program's expansive coverage. Excluding the top decile, the majority of families received fully subsidized child care. Case in point, between 50-55% of families situated around the median income received full subsidies. The prevalence of families with at least one hour of unsubsidized child care increases among the lower deciles, likely due to the work activity requirement. Yet, approximately 40% of families in the bottom decile still received full subsidies. Additionally, even among families with at least one unsubsidized child care hour, provided that they were not in the top income bracket (with annual earnings above AUD 240,818), the average unsubsidized hours did not exceed 20% of their total child care hours.

J Numerical solution method and algorithm

The quantitative model is solved numerically in FORTRAN. I solve the model (a small economy with open capital market) for household optimal allocations, their distributions, and aggregate variables along the initial balanced-growth path steady state equilibrium. The model economy is calibrated to the Australian economy's key micro and macro economic moments during 2012-2018, a relatively stable period. The algorithm is as follows:

1. Parameterize the model and discretize the asset space $a \in [a_{min}, a_{max}]$. The choice of grid points is such that
   - Number of grid points, $N_A = 70$;
   - $a_{min} = 0$ (No-borrowing constraint);
   - The grid nodes on $[a_{min}, a_{max}]$ are fairly dense on the left tail so households are not restricted by an all-or-nothing decision (i.e., unable to save early in the life cycle due to the lack of choices on the grid nodes for small asset levels);
   - a_{max} is sufficiently large so that: (i) household wealth accumulation is not artificially bounded by a_{max}, and (ii) there is enough margin for upward adjustment induced by new policy regimes;
2. In a similar manner, discretize the human capital space $h^f_{\theta,\ell} \in [h^f_{min,\theta,\ell}, h^f_{max,\theta,\ell}]$ for each θ and ℓ types such that
   - Number of grid nodes, $N_H = 25$;
   - $h^f_{min,\theta,\ell} = 1$ for all θ and ℓ;
   - $h^f_{max,\theta,\ell} = h^m_{max,\theta,\ell}$ for every θ and ℓ;
3. Guess the initial values of the endogenous aggregate macro variable L_0, endogenous government policy variable ζ_0, taking $r = r^w$ where r^w is a given world interest rate;
4. Solve the representative firm problem's first-order conditions for market clearing wages, w;
5. Given the vector of the benchmark macro and micro parameters (Ω_0), such as the parameters governing the stochastic processes of lifespan (ψ) and income (η_m, η_f), factor prices (w, r), and the government policy parameters, I jointly solve the household problems for optimal decision rules on future asset holdings (a^+), joint consumption (c), female labor supply (n) and the value function of households via backward induction (from $j = J$ to $j = 1$) using the value function iteration method. The numerical optimization and root finding algorithms are from a toolbox constructed by Hans Fehr and Fabian Kindermann. For a pair of state vector and employment status (z, ℓ), I solve jointly for $a^*_+(\ell, z)$, $c^*_+(\ell, z)$, and $n^*(\ell, z)$ via backward induction using the value function iteration method. Suppressing ℓ and z to ease notations, the household solution algorithm is detailed below:
   (a) First, I assume no left-over assets (bequest) at terminal age. Thus, $a^*_+ = 0$ for households aged $j = J$. Since $n = 0$ by mandatory retirement for all $j \geq J_R$, I solve for the optimal consumption, c^*, by maximizing the household utility.
   (b) For $j = 1, \ldots, J-1$, an initial guess $a_+ \in [a_{min}, a_{max,j})$ is provided, where $a_{max,j}$ is the total income a household has at age j. For every guess of a_+, the corresponding labor supply $n = n(a_+|\ell, z)$ is such that the optimal intra-temporal trade-off equation (50) is satisfied. Because $EMTR_{n,\lambda}$ and NLI_λ in (50) are labor-dependent and non-linear, I solve numerically for n using a root-finding algorithm, $fzero$;
   (c) c is obtained via the household budget constraint (49);
   (d) then solve for the optimal allocations (a^*_+, c^*, n^*) that jointly maximize a household's value using a non-linear solver $fminsearch$ from Fehr and Kindermann's toolbox
6. Starting from a known distribution of newborns ($j = 1$), and given the households' optimal solutions, compute the measure of households across states and over the life cycle by forward induction, using
   - the computed decision rules $\{a^+_j, c_j, \ell_j\}^J_{j=1}$;
   - the time-invariant survival probabilities $\{\psi\}^J_{j=1}$;
   - the Markov transition probabilities of the transitory earnings shocks η;
   - the law of motion of female human capital from Equation (47);

   For determining the next period measure of households on the asset (a) and female human capital (h^f) grids, employ a bi-linear interpolation method;
7. Accounting for the share of agents who are alive, sum over all state elements to arrive at the aggregate levels of assets (A), consumption (C), female labor force participation (LFP), tax revenue, transfers, and others. L, K, C, I and Y are updated via a convex updating process to ensure a stable convergence;
8. Given the aggregate macro variables, solve for endogenous government policy variable, ζ, using the government budget balance equation (78);

9. The goods market convergence criterion for a small open economy at time t is

$$\left|\frac{Y-(C+I+G+NX)}{Y}\right|<\varepsilon$$

where

- the trade balance NX is the difference between current and future government foreign debts. That is, $NX = (1+n)(1+g)B_{F+} - (1+r)B_F$ and $B_F = A - K - B$ is the required foreign capital to clear the domestic capital market;
- $NX < 0$ implies a capital account surplus or current account deficit (net inflow of foreign capital and thus an increase in the foreign indebtedness);
- $\varepsilon = 0.001$.

10. If the goods market convergence criterion is not satisfied, return to step 3 with the initial guesses L_0 and ζ_0 being updated with L and ζ from step 7 and 8, respectively.

The steady-state analyses compare the benchmark economy in the initial steady state with a reformed economy in a new steady state. I capture aggregate macroeconomic changes, ex-ante welfare effect (i.e., effect on future newborns), and the redistributive outcomes of a regime shift in the new steady state. The experimental results, therefore, are concerned with the long-run implications of a policy reform.

However, quantifying the full impact of a policy change also requires investigating the macroeconomic, welfare, and redistributive effects on current generations (non-newborn) living along the transition path. Accounting for the transitional dynamics is crucial for grasping the short-run implications when households do not anticipate the policy reform. This necessitates solving for the transition path of the model economy as it moves from the initial steady state under the status quo to the final steady state equilibrium under the new regime. For the current model, with high dimensionality of state space and non-linearities brought about by child benefits, this is a computationally monumental task. One might need to impose simplifying parametric forms on the social security schemes of interest, and/or shrink the state space by re-formulating certain aspects of the problem. I leave this to future endeavors.