Four Dudes on a Moped

Part D: Shaft Analysis, Fatigue, and Ball Bearing Selection

By Ethan Schaper, Dara Safe, Joey Schwirtz, Dan O'Brien

College of Engineering

University of Wisconsin - Madison

December 9, 2022

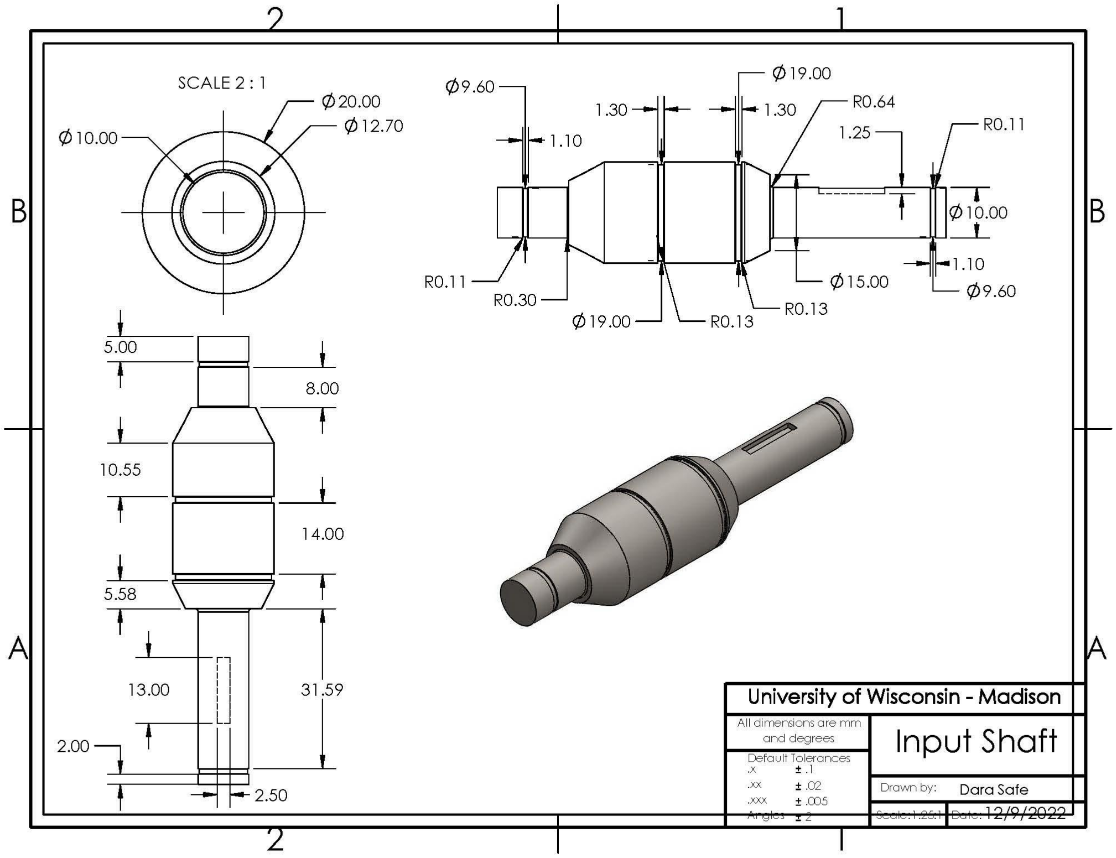

Figure 1: SolidWorks drawing of input shaft

Table 1: Fatigue analysis results at all critical occasions on shaft 1 from left to right as shown in the provided gearbox diagram.

Location*	**SR1 at B1**	**SR2 at B2**	**SR3 at P1**	**SH2 at P1**	**Key 1 at P1**
Safety Factor	21.120	7.686	21.12	12.99	1.603
Kt Torque	1.50	2.30	1.50	1.62	1.30
Kt Bending	N/A	3.80	N/A	2.00	1.30
Diameter (mm)	10	20	10	15	10

*SR- Snap Ring, B- Bearing, SH- Shoulder, P- Pinion, G- Gear

From Table 1 and 2, the safety factors at B1 and B2 are high due to limitations of Table 14.1 in the textbook. According to Table 4, the calculated bore of B1 was determined to be 10mm, the smallest value in the table, hence the team was limited in selecting a smaller bore diameter for a more appropriate safety factor to suit the stresses at that location. The slightly high safety factor at bearing two is also due to the limitation in the tables to sepcify an exact bore diameter, so the team opted to round the bore size to 20 mm. Thus the strength of the snap ring is much greater than the load the ring needs to handle in order to do their job. The constraints of setting the bore size caused the safety factors at P1 to be higher than they should have been for the stresses that they experience. The safety factor of key 1 at pinion one is the lowest amongst the rest of the safety factors on this input shaft which is expected. However, this safety factor is not too low to raise any concerns about the integrity of the shaft so need not there be any justification for this calculated value.

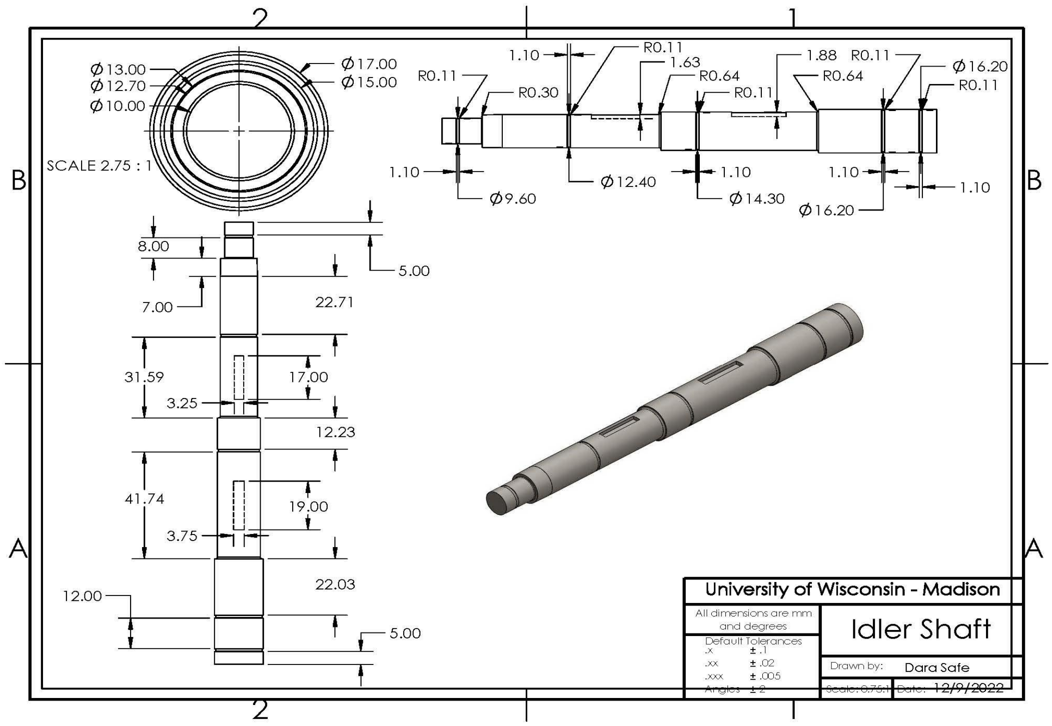

Figure 2: SolidWorks drawing of idler shaft

Table 2: Fatigue analysis results at all critical occasions on shaft 2 from left to right as shown in the provided gearbox diagram.

Location*	**SR5 at G1**	**SH4 at G1**	**Key 2 at G1**	**SR6 at P2**	**SH5 at P2**	**Key 3 at P2**
Safety Factor	2.315	7.286	1.593	1.549	16.280	1.645
Kt Torque	2.00	1.70	1.30	2.00	1.25	1.30
Kt Bending	3.50	2.05	1.30	3.50	1.70	1.30
Diameter (mm)	13	15	13	15	17	15

*SR- Snap Ring, B- Bearing, SH- Shoulder, P- Pinion, G- Gear

It should be noted that the shoulders of the idler shaft contain high safety factors at the shoulders for the P2 and G1 location. Shoulders are holding devices that require a larger diameter size, which leads to a larger safety factor that is needed for the stresses experienced at the mentioned locations. Once again the safety factors at keys 2 and 3 are reasonable values and the lowest amongst the other safety factors besides the snap ring at pinion 2 on this shaft which is expected.

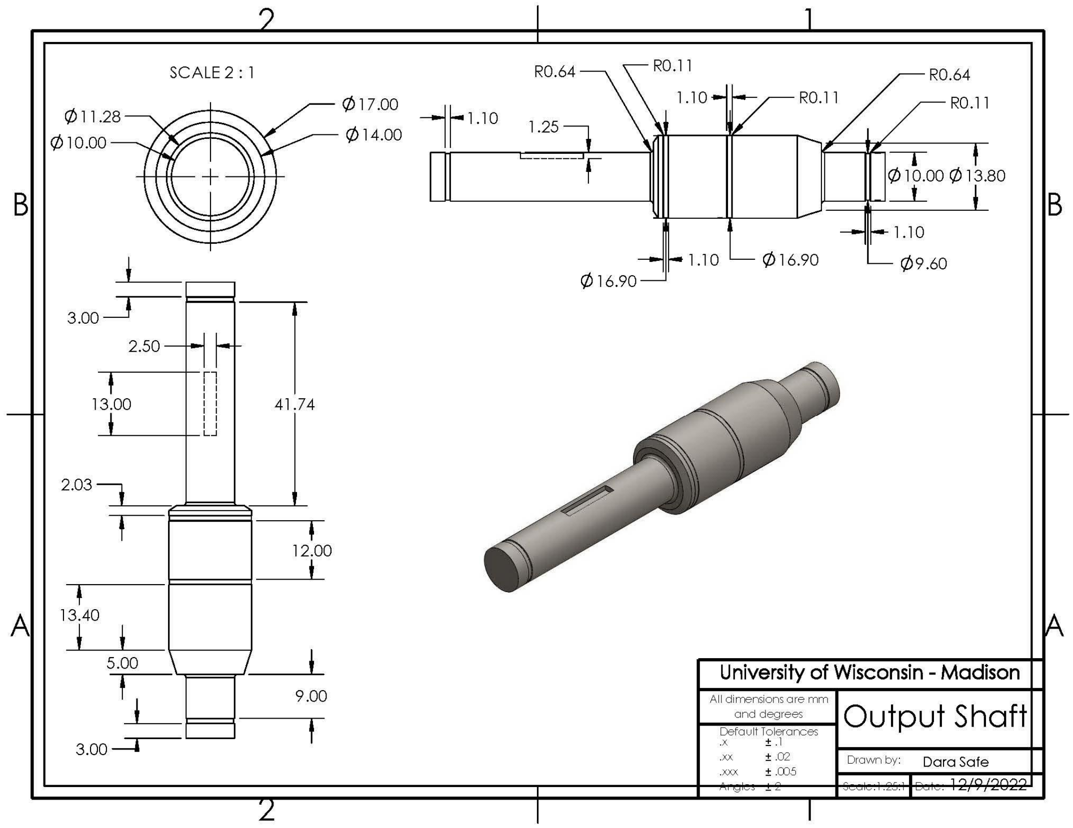

Figure 3: SolidWorks drawing of output shaft

Table 3: Fatigue analysis results at all critical occasions on shaft 3 from left to right as shown in the provided gearbox diagram.

Location*	**SR8 at G2**	**Key 4 at G2**	**SH7 at G2**	**SR9 at B5**	**SR10 at B6**
Safety Factor	2.036	1.603	5.503	4.657	4.657
Kt Torque	2.00	1.30	1.65	1.50	1.50
Kt Bending	3.70	1.30	1.95	N/A	N/A
Diameter (mm)	17	17	14	10	10

*SR- Snap Ring, B- Bearing, SH- Shoulder, P- Pinion, G- Gear

Table 4: Summary of bearing calculations and selections.

Location	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**
F_r (N)	187.7	374.05	85.97	214.89	754.39	377.24
C_{req} (kN)	0.639	1.273	0.297	0.731	2.567	1.284
Series	L00	L00	L00	200	200	200
Bore (mm)	10	20	10	17	17	10
Selected Bearing	L00	L04	L00	203	203	200

Table 4 above contains the results of bearing calculations using the Palmgren's formula. This equation is used to select a bearing for a specific set of design variables such as the application load bearing capacity, impact factor, reliability factor, rated life, and desired life in cycles. The result of this equation gives the team a required rated load bearing capacity, C_{req}, which is used to select a specific bore size and bearing type. The application load bearing capacity, F_r, is the radial force that each bearing experiences and is calculated by shaft force analysis which was completed in part A of the project. Bearings 2 (input shaft), 5 (output shaft), and 6 (output shaft) experience the greatest radial loads. The desired life of the two stage reverted speed reducer in the moped is 14,000 hours because it's critical our machined product may last for a long time and reliability is very important. The deep groove ball bearings with radial contact are selected by gathering the required rated load bearing capacity and the load rating to determine the bore size in table 14.1 in the Juvinall textbook. The load rating of the first three bearings is L for extra light. The load rating for the last three bearings is 2 for light.

Table 5: Updated static analysis at critical locations from Part B with new shaft design.

	Shaft 1	Shaft 2	Shaft 3
Critical Location	B2	P2	B5
Diameter at Critical Location (mm)	20	15	17

Safety Factor	27.21	5.82	5.56
Equivalent Stress (MPa)	8.85	41.35	43.35

According to Table 5, the safety factor of the bearing two is particularly large at the critical location of Shaft 1 because the team was limited to the smallest bearing in textbook table 14.1 as mentioned below Table 1.

Table 6: Material properties of our chosen material 1045 HR steel.

	Yield Strength (MPa)	Ultimate Tensile Strength (MPa)	Hardness (Bhm)
1045 HR Steel	414	638	180

As depicted by Table 6, the material chosen for each shaft is 1045 HR Steel, as opposed to the originally selected 6061 Aluminum, because the values needed for calculation were readily available to the team in the class textbook, Fundamentals of Machine Component Design by Robert C. Juvinall. 1045 hot rolled steel is considered a good material for a gear shaft because of the high strength-to-weight ratio, which implies that part will be strong enough, while keeping them light enough to be practical. Furthermore, the yield strength of 414 MPa and ultimate strength of 638 MPa ensures that the shaft will not fail prematurely based on the team's stress calculations. 1045 HR Steel is also a cost-effective material, which is an important consideration when manufacturing the moped gearbox to minimize costs.

Figure 4 displays the sequence of manufacturing processes the steel will undergo to produce each shaft for the gearbox. The first step in the manufacturing process is to acquire a supply of 1045 hot rolled steel billets with a diameter of atleast twenty millimeters so that all specified dimension are attainable through machining. The billets are then cut into desired lengths using a band saw, and sent to be machined by a computer numeric control (CNC). CNC machines allow for highly precise machining of a complex part like a shaft to make sure that the dimensions are accurate for gear, bearing, and snap ring placement. Next, a milling machine will mill the keyways at the pinion and gear location so they will be held in place by a corresponding key. Afterwards, the shaft is polished to preserve the mechanical properties, as well as reduce friction and wear when in contact with other parts. Lastly, the shaft is inspected for proper strength and quality before it is assembled for the gearbox. Some of the most important things to inspect for would be cracks, imperfections, and stress fractures to avoid unnecessary stress concentration which would lead to failure under operating conditions.

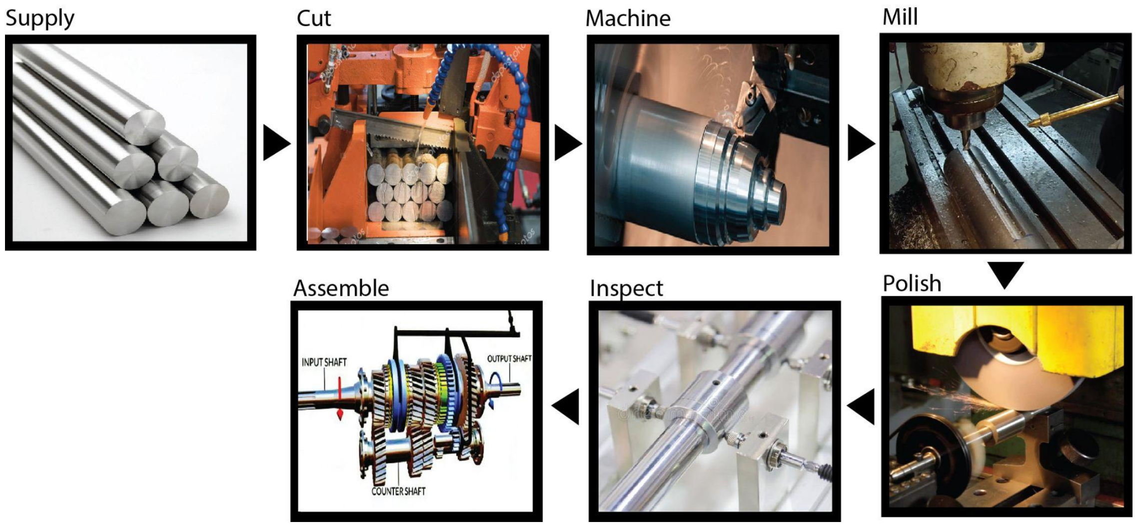

Figure 4: Process flow for shaft manufacturing

Discussion

The minimum shaft diameter varies amongst the different types of analyses and design factors. When first calculating the minimum shaft diameters in the initial static analysis the team had designed the three shafts with Multipurpose 6061 Aluminum because it's cheaper than steel while also being strong enough calculate a desirable safety factor. Since the addition of the fatigue analysis, bearing selections, and key selections the team needed a material with more accessible properties such as 1045 HR Steel. In the initial static analysis where the team was working with designing straight shafts, the minimum diameters required at the critical locations were made the minimum diameters for the entire shafts. This causes the minimum diameters to be larger than the team needs them to be for most of the shafts. The additional calculations conducted for fatigue analysis and bearing selection allowed the team to alter and diversify the diameter sizes along the shafts. The minimum diameter or bore are very similar in size for the fatigue analysis and bearing selections. The range for these calculations is 10mm to 20mm.

Considering the new shaft design and material, the critical loads and strengths have changed. The static shaft analysis critical locations remain the same at bearing 2, bearing 5, and pinion 2. The fatigue strength critical locations are at the keys because that's where the lowest safety factors are found. The bearing capacity at bearing 5 holds much more radial load than the other bearings. The team observes the fatigue strength to be the most critical because of the lowest safety factors. The static strength isn't considered the most critical because the new shaft design and steel increases the safety factors at the critical locations enough to recognize the static strength in the shafts to withstand the stresses imposed on them. The load bearing capacity is also not the most critical because of the correct selection of bearing by the team.

Conclusion

For this part of the project the team's object was to design the shafts with appropriate dimensions of shoulder, keyways, snap rings, to mount the bearings and gears. Additionally the team completed a fatigue strength analysis to determine a sufficient minimum shaft diameter across all locations along the shaft. Next, deep groove ball bearings were chosen for each respective location on all three shafts based on the force analysis and calculated shaft diameters. After the bearing types and new shaft dimensions were acquired, the team updated their original static strength analysis. The formulas and equations used were based from the Fundamentals of Machine Component Design textbook and solved in EES as shown in Appendix A. Also, all mechanical property values and bearing dimensions were based from the textbook as well. With that said, the team was restricted from using the original material, 6061 Aluminum, due to the limited data, hence the next best alternative was chosen, 1045 HR Steel. After these changes were made due to the constraints presented, the team modeled the shaft in SolidWorks and created a detailed drawing of each shaft showing the top, side, and front profile. One the design was set, the team considered the sequence of manufacturing to bring the best quality shaft possible while striving to minimize costs.

Assumptions

1. Neglect radial force
2. Force acts at tip (no load sharing)
3. No sliding force
4. Non-uniform force distribution
5. Stress concentration
6. Non-zero contact velocity
7. 99% reliability
8. Moderate shock on driven machine
9. Light shock on power source
10. Gears are machined and manufactured with form cutters, Hobs
11. The reliability factor has a standard deviation of 8%
12. Lubricant temperature is less than 160 degrees Fahrenheit
13. The gears have a tooth filet radius of 0.35/P
14. Assume the cycle life to be 10e7 for finding Sfe
15. Keyway is a sled runner keyway
16. Assume mechanical properties of 1045 HR Steel

Appendix A

EES Code


```
"PROJECT PART A"

"Given Values"
theta = 20 [deg]
eta = 0.975
omega_in_rpm = 6000 [rev/min]
G_r = 4.5
m = 0.003 [m]
W_dot_in_hp = 4 [hp]
n_P = 18

LB1B2 = 0.03 [m]
LB2P1 = 0.03 [m]
LB3G1 = 0.05 [m]
LG1P2 = 0.05 [m]
LP2B4 = 0.05 [m]
LG2B5 = 0.03 [m]
LB5B6 = 0.03 [m]

"Calculated Values"
W_dot_in = W_dot_in_hp*convert(hp,W) "Power converted from horse power to watts"
d_P = n_P*m "Diameter of P calculated from m and number of teeth"
r_P = d_P/2 "Radius calculated from diameter"
omega_out_rpm = omega_in_rpm/G_r "Omega out calculated from omega in and gear ratio"
(omega_out_rpm/omega_in_rpm) = ((n_P^2)/(n_G^2)) "Number of teeth on G gears calcualted from omega in, omega out, and
 number of teeth on P"

d_G = n_G*m "Diameter of G calculated from the number of teeth on gear G and m"

r_G = d_G/2 "Radius calculated from diameter"

omega_in = 628.52 [rad/sec] "Omega in converted to radians per second"
omega_out = 139.63 [rad/sec] "Omega out converted to radians per second"

"Force calculations on shaft 1"
F_P1_tan*r_P = W_dot_in/omega_in "Sum of torques on shaft 1 equal 0 to find tangential force"
F_P1_rad = F_P1_tan*tan(theta) "Calulated radial force on gear P1 from tangential force"

(-B_2_z*(LB2P1)) - (F_P1_tan*(LB1B2+LB2P1)) = 0 "Sum of the moments on shaft 1 in Y direction to find B_2_z"
B_2_z + B_1_z + F_P1_tan = 0 "Sum of the forces in the Z direction on shaft 1 to find B_1_z"

(B_2_y*(LB2P1)) + (F_P1_rad*(LB1B2+LB2P1)) = 0 "Sum of the moments on shaft 1 in Z direction to find B_2_y"
B_2_y + B_1_y + F_P1_rad = 0                    "Sum of the forces in the Y direction on shaft 1 to find B_1_y"

"Force calculations on shaft 2"
F_G1_rad = eta*F_P1_rad "Radial force on G1 after efficiency loss"
F_G1_tan = eta*F_P1_tan "Tangential force on G1 after effiiciency loss"

F_G1_tan*r_G = F_P2_tan*r_P "Sum of the torques on shaft 2 to find tangential force on P2"
F_P2_rad = F_P2_tan * tan(theta) "Calcualted radial force on gear P2 from tangential force"

(F_G1_tan*LB3G1) - (F_P2_tan*(LB3G1+LG1P2)) - (B_4_z*(LB3G1+LG1P2+LP2B4)) = 0 "Sum of moments about bearing 3 in Y
 directionto find B_4_z"
B_3_z = F_G1_tan - F_P2_tan - B_4_z "Sum of forces in the Z direction to find B_3_z"

(-F_G1_rad*(LB3G1))-(F_P2_rad*(LB3G1+LG1P2))+(B_4_y*(LB3G1+LG1P2+LP2B4)) = 0 "Sum of moments about bearing 3 in Z
 direction to find B_4_y"
```

```
B_3_y = F_G1_rad + F_P2_rad - B_4_y "Sum of the forces in the Y direction to find B_3_y"

"Force calculations on shaft 3"
F_G2_tan = eta*F_P2_tan "Tangential force on G2 after efficiency loss"
F_G2_rad = eta*F_P2_rad "Radial force on G2 after efficiency loss"

B_5_z*LG2B5 = F_G2_tan*(LG2B5+LB5B6) "Sum of the moments about bearing 6 to find B_5_z"
B_6_z = F_G2_tan - B_5_z "Sum of forces in the Z direction to find B_6_z"

B_5_y*LG2B5 = -F_G2_rad*(LG2B5+LB5B6) "Sum of the moments in the z direction on bearing 6"
B_6_y = -B_5_y-F_G2_rad "Sum of the forces in the Y direction to find B_6_y"

"Torque calculations"
T_P1 = F_P1_tan*r_p "Calculated torque on P1"
T_P2 = F_P2_tan*r_p "Calculated torque on P2"

T_G1 = -F_G1_tan*r_g "Calcualted torque on G1"
T_G2 = F_G2_tan*r_g "Clacualted torque on G2"

"Moment Calculations"
M_G1_z = B_3_y*LB3G1 "Moment force on G1 in the Z direction"
M_G1_y = B_3_z*LB3G1 "Moment force on G1 in the Y direction"

M_P1_z = F_P1_tan*LB2P1 "Moment force on P1 in the Z direction"
M_P1_y = F_P1_rad*LB2P1 "Moment force on P1 in the Y direction"

M_G2_z = -F_G2_tan*LG2B5 "Moment force on G2 in the Z direction"
M_G2_y = F_G2_rad*LG2B5 "Moment force on G2 in the Y direction"

M_P2_y = (B_3_z*(LB3G1+LG1P2))-(F_G1_tan*LP2B4) "Moment force on P2 in the Y direction"
M_P2_z = (B_3_y*(LB3G1+LG1P2))-(F_G1_rad*LP2B4) "Moment force on P2 in the Z direction"


"PROJECT PART B"


"Shaft 1 Analysis"

S_y = 241000000 [Pa] "4140 Steel"
d = 0.025 [m] "Assumed diameter"

V_max_1 = sqrt(F_P1_rad^2+F_P1_tan^2)

tau_T_torque1 = (16*T_P1)/(pi*d^3) "Shear stress due to torque"
tau_T_shear1 = (16*V_max_1)/(3*pi*d^2) "Shear stress due to shear force"
tau_T_xz1 = tau_T_torque1 + tau_T_shear1 "Shear stress in the XZ plane"

M_max_1 = sqrt(M_P1_y^2+M_p1_z^2)
sigma_S_x1 = (32*M_max_1)/(pi*d^3) "Sigma X on side"

tau_S1 = (16*T_P1)/(pi*d^3)  "Shear stress in the XY plane"

sigma_ave_1 = (sigma_S_x1 + 0)/2
R1 = sqrt((tau_S1^2)+((sigma_S_x1/2)^2))

sigma_1_shaft1 = sigma_ave_1 + R1
```

```
sigma_2_shaft1 = sigma_ave_1 - R1

sigma_e_1 = sqrt((((sigma_1_shaft1-sigma_2_shaft1)^2) + ((sigma_2_shaft1)^2) + ((sigma_1_shaft1)^2))/2)

SF1 = S_y/sigma_e_1 "Safety factor on shaft 1 using DET"
//SF1 = 2.5

"Shaft 2 Analysis"

V_max_2 = sqrt(B_4_y^2+B_4_z^2)

tau_T_torque2 = (16*T_G1)/(pi*d^3) "Shear stress due to torque"
tau_T_shear2 = (16*(V_max_2))/(3*pi*d^2) "Shear stress due to shear force"
tau_T_xz2 = tau_T_torque2 + tau_T_shear2 "Shear stress in the XZ plane"

M_max_2 = sqrt(M_P2_y^2+M_P2_z^2)

sigma_S_x2 = (32*M_max_2)/(pi*d^3) "Sigma X on side"

tau_S_torque2 = (16*T_P2)/(pi*d^3) "Shear stress due to torque"
tau_S_xy2 = tau_S_torque2 "Shear stress in the XY plane"

sigma_ave_2 = (sigma_S_x2 + 0)/2
R2 = sqrt((tau_s_xy2^2)+((sigma_S_x2/2)^2))

sigma_1_shaft2 = sigma_ave_2 + R2
sigma_2_shaft2 = sigma_ave_2 - R2

sigma_e_2 = sqrt((((sigma_1_shaft2-sigma_2_shaft2)^2) + ((sigma_2_shaft2)^2) + ((sigma_1_shaft2)^2))/2)

SF2 = S_y/sigma_e_2 "Safety factor on shaft 2 using DET"
//SF2 = 2.5

"Shaft 3 Analysis"

V_max_3 = sqrt(B_6_y^2+B_6_z^2)

tau_T_torque3 = (16*T_G2)/(pi*d^3) "Shear stress due to torque"
tau_T_shear3 = (16*V_max_3)/(3*pi*d^2) "Shear stress due to shear force"
tau_T_xz3 = tau_T_torque3 + tau_T_shear3 "Shear stress in the XZ plane"

M_max_3 = sqrt(M_G2_y^2+M_G2_z^2)

sigma_S_x3 = (32*M_max_3)/(pi*d^3) "Sigma X on side"

tau_S_torque3 = (16*T_G2)/(pi*d^3) "Shear stress due to torque"
tau_S_xy3 = tau_S_torque3

sigma_ave_3 = (sigma_S_x3 + 0)/2
R3 = sqrt((tau_s_xy3^2)+((sigma_S_x3/2)^2))

sigma_1_shaft3 = sigma_ave_3 + R3
sigma_2_shaft3 = sigma_ave_3 - R3

sigma_e_3 = sqrt((((sigma_1_shaft3-sigma_2_shaft3)^2) + ((sigma_2_shaft3)^2) + ((sigma_1_shaft3)^2))/2)
```

```
SF3 = S_y/sigma_e_3 "Safety factor on shaft 3 using DET"
//SF3 = 2.5

d_abs = abs(d)

"PROJECT PART C"

"Bending fatigue failure analysis on second gear pairing"

S_u = 1020000000 [Pa] "Su of 4140 steel"
H_B = 302 "Hardness of 4140 steel"

N_p_actual = 18
N_g_actual = 38 "Actual calculated was 38.18, so we chose 38"
b = 0.03175 [m]

F_g2 = F_G2_tan
F_P2 = F_P2_tan

P = N_p_actual/d_p

J_g  = 0.28 "Fig 15.23"
J_p = 0.24 "Fig 15.23"

V_pitchLine2 = omega_out*(d_g/2)

K_v2 = 2.3 "Fig 15.24"
K_o = 1.5 "Table 15.1"
K_m = 1.6 "Table 15.2"
C_L = 1.0 "Bending load"
C_g = 0.85 "P < 0.85"
C_s = 0.70 "Fig 8.13"
K_r = 0.814 "Table 15.3"
K_t = 1.0 "T < 160"
K_ms = 1.4 "One way bending"

S_n` = S_u/2

sigma_G2 = ((F_g2*P)/(b*J_g))*K_v2*K_o*K_m
sigma_P2 = ((F_p2*P)/(b*J_p))*K_v2*K_o*K_m

S_n2 = S_n`*C_L*C_g*C_s*K_r*K_t*K_ms

SF_g2 = S_n2/sigma_g2 "Safety factor for gear 2 using bending analysis"
SF_p2 = S_n2/sigma_p2 "Safety factor for pinion 2 using bending analysis"
//SF_p2 = 1.5
//SF_g2 = 1.5

"Surface fatigue failure analysis on second gear pairing"

R_partc = d_g/d_p
phi = 20 [deg] "Given pressure angle"
I = ((sin(phi)*cos(phi))/2) * (R_partc/(R_partc+1))
C_p = 191000000 [Pa^(1/2)] "Table15.46"
F_t2 = F_P2_tan
C_r = 1 "Table 15.6"
```

```
C_Li = .86 "Fig 15.27"

S_fe = ((2800000[Pa]*H_B) - 69000000[Pa])

SF_surface2 = ((((S_fe)*C_Li*C_r)^2)*b*d_p*I)/(((C_p)^2*1e-6)*F_t2*K_v2*K_o*K_m) "Safety factor for the second gear pair using
 surface fatigue"
//SF_surface2 = 1.5

"Bending fatigue failure analysis on first gear pairing"

V_pitchLine1 = omega_in*(d_p/2)
K_v1 = 3.6

F_g1 = F_G1_tan
F_P1 = F_P1_tan
F_t1 = F_P1_tan

sigma_G1 = ((F_g1*P)/(b*J_g))*K_v1*K_o*K_m
sigma_P1 = ((F_p1*P)/(b*J_p))*K_v1*K_o*K_m

S_n1 = S_n`*C_L*C_g*C_s*K_r*K_t*K_ms

SF_g1 = S_n1/sigma_g1 "Safety factor for gear 1 using bending analysis"
SF_p1 = S_n1/sigma_p1 "Safety factor for gear 2 using bending analysis"
//SF_p1 = 1.5
//SF_g1 = 1.5

"Surface fatigue failure analysis on second gear pairing"

SF_surface1 = ((((S_fe)*C_Li*C_r)^2)*b*d_p*I)/(((C_p)^2*1e-6)*F_t1*K_v1*K_o*K_m) "Safety factor for the first gear pair using
 surface fatigue"
//SF_surface1 = 1.5


"PROJECT PART D"

"Standard values for entire Part D"
Su = 638000000 [Pa] "1045 HR"
Sy = 414000000 [Pa]

rated_life = 14000 [hr]
L = rated_life*(omega_out_rpm)*60[min/hr]
L_r = 9e7 [rev]
K_a = 1.0
K_r_bearing = .21

F_r =377.24 [N] "Radial force that was canged for each bearing to calcualte bearing size"

C_req = F_r*K_a*(L/(K_r_bearing*L_r))^0.3 "Finding correct bearing size and type"

"Constants for Sn"
C_L_4 = 1
C_G_4 = 0.9
C_S_4 = 0.6
C_T_4 = 1
C_R_4 = 0.814
```

```
Sn = 0.5*Su*C_L_4*C_G_4*C_S_4*C_T_4*C_R_4

"Bearing 1 Snap Ring 1"

d_sr1 = 0.01 [m] "Diamter of shaft at snap ring 1"

K_t_sr1t = 1.5 "From book"
q_t_sr1t = 0.5 "From book"
K_f_sr1t = 1 + ((K_t_sr1t-1)*q_t_sr1t)

sigma_em_sr1 = K_f_sr1t*((16*T_P1)/(pi*d_sr1^3)) "Mean stress"

1/SF_sr1 = sigma_em_sr1/Su "SF at snap ring 1"

"Bearing 2 Snap Ring 2"

d_sr2 = 0.02[m] "Diameter of shaft at snap ring 2"

K_t_sr2t = 2.3 "From book"
K_t_sr2b = 3.8 "From book"

q_sr2 = 0.5 "From book"

K_f_sr2t = 1 + ((K_t_sr2t-1)*q_sr2)
K_f_sr2b = 1 + ((K_t_sr2b-1)*q_sr2)

sigma_em_sr2 = K_f_sr2t*((16*T_P1)/(pi*d_sr2^3)) "Mean stress"
sigma_ea_sr2 = K_f_sr2b*((32*M_Max_1)/(pi*d_sr2^3)) "Alternating stress"

1/SF_sr2 = (sigma_em_sr2/Su) + (sigma_ea_sr2/Sn) "SF at snap ring 2"

"Pinion 1 Snap Ring 3"

d_p1 = 0.01 [m] "Diameter of shaft at pinion 1"

q_0 = 0.5 "From book"

K_t_2 = 1.5 "From book"
K_f_2 = 1 + ((K_t_2-1)*q_0) "From book"

sigma_em_p1 = K_f_2*((16*T_P1)/(pi*d_p1^3)) "Mean stress"

(1/SF_P1_4) = sigma_em_p1/Su "SF at pinoin 1 with new shaft design"

"Pinion 1 Shoulder 2"

M_s2 = F_P1_rad*b "Moment at shoulder 2"

K_t_s2t = 1.62 "From book"
K_t_s2b = 2.00 "From book"

q_s2t = 0.65 "From book"
q_s2b = 0.4 "From book"

d_s2 = 0.015[m] "Diameter of shaft at shoulder 2"
```

```
K_f_s2t = 1 + ((K_t_s2t-1)*q_s2t)
K_f_s2b = 1 + ((K_t_s2b-1)*q_s2b)

sigma_em_s2 = K_f_s2t*((16*T_P1)/(pi*d_s2^3)) "Mean stress"
sigma_ea_s2 = K_f_s2b*((32*M_s2)/(pi*d_s2^3)) "Alternating stress"

1/SF_s2 = (sigma_em_s2/Su) + (sigma_ea_s2/Sn) "SF at shoulder 2"

"Key1"

th_key1 = d_p1/4 "Width of the key"
L_key1 = 0.013 [m] "Length of the key"

T_key1_shear = (d_p1/2)*L_key1*th_key1*0.58*Sy "Max shear torque"
T_key1_bearing = ((d_p1/2)+(th_key1/4)) * Sy *L_key1 * (th_key1/2) "Max bearing torque"

sigma_key1_shear = 1.3* (16*T_key1_shear)/(pi*d_p1^3) "Shear stress"
sigma_key1_bearing = 1.3* (16*T_key1_bearing)/(pi*d_p1^3) "Bearing stress"

SF_key1 = Sy/sigma_key1_shear "SF of key 1"

"Shaft 2"

"Gear 1 Snap Ring 5"

d_g1 = 0.013 [m] "Diameter of shaft 2 at gear 1"

K_t_g1b = 3.5 "From book"
K_t_g1t = 2.0 "From book"

K_f_g1b = 1 + ((K_t_g1b-1)*q_g1)
K_f_g1t = 1 + ((K_t_g1t-1)*q_g1)

q_g1 = 0.65 "From book"

M_G1 = sqrt(M_G1_z^2+M_G1_y^2) "Moment at gear 1"

sigma_em_g1 = K_f_g1t*((16*abs(T_G1))/(pi*d_g1^3)) "Mean stress"
sigma_ea_g1 = K_f_g1b*((32*M_G1)/(pi*d_g1^3)) "Alternating stress"

1/SF_g1_4 = (sigma_em_g1/Su) + (sigma_ea_g1/Sn) "SF at gear 1 snap ring 5"

"Gear 1 Shoulder 4"

d_s4 = 0.015[m] "Diameter of shaft 2 at shoulder 4"

K_t_s4t = 1.70 "From book"
K_t_s4b = 2.05 "From book"

q_s4t = 0.90 "From book"
q_s4b = 0.85 "From book"

K_f_s4t = 1 + ((K_t_s4t-1)*q_s4t)
K_f_s4b = 1 + ((K_t_s4b-1)*q_s4b)
```

```
sigma_em_s4 = K_f_s4t*((16*T_G1)/(pi*d_s4^3)) "Mean stress"
sigma_ea_s4 = K_f_s4b*((32*M_G1)/(pi*d_s4^3)) "Alternating stress"

1/SF_s4 = (sigma_em_s4/Su) + (sigma_ea_s4/Sn) "SF at shoulder 4"

"Gear 1 Key 2"

th_key2 = d_g1/4 "Width of key 2"
L_key2 = 0.017 [m] "Length of key 2"

T_key2_shear = (d_g1/2)*L_key2*th_key2*0.58*Sy "Max shear torque"
T_key2_bearing = ((d_g1/2)+(th_key2/4)) * Sy *L_key2 * (th_key2/2) "Max bearing torque"

sigma_key2_shear = 1.3* (16*T_key2_shear)/(pi*d_g1^3) "Shear stress at key"
sigma_key2_bearing = 1.3* (16*T_key2_bearing)/(pi*d_g1^3) "Bearing stress at key"

SF_key2 = Sy/sigma_key2_shear "SF of key 2"


"Pinion 2 Snap Ring 6"

d_p2 = 0.015 [m] "Diameter of shaft 2 at snap ring 6"

K_t_p2b = 3.5 "From book"
K_t_p2t = 2.0 "From book"

q_p2 = 0.65 "From book"

K_f_p2b = 1 + ((K_t_p2b-1)*q_p2)
K_f_p2t = 1 + ((K_t_p2t-1)*q_p2)

sigma_em_p2 = K_f_p2t*((16*abs(T_P2))/(pi*d_p2^3)) "Mean stress"
sigma_ea_p2 = K_f_p2b*((32*M_max_2)/(pi*d_p2^3)) "Alternating stress"

1/SF_p2_4 = (sigma_em_p2/Su) + (sigma_ea_p2/Sn) "SF of snap ring 6"

"Pinion 2 Shoulder 5"

M_s5 = F_P2_rad*(0.03[m]-(b/2)) "Moment at shoulder 5"

K_t_s5t = 1.25 "From book"
K_t_s5b = 1.70 "From book"

q_s5t = 0.93 "From book"
q_s5b = 0.73 "From book"

d_s5 = 0.017[m] "Diameter of shaft 2 at shoulder 5"

K_f_s5t = 1 + ((K_t_s5t-1)*q_s5t)
K_f_s5b = 1 + ((K_t_s5b-1)*q_s5b)

sigma_em_s5 = K_f_s5t*((16*T_P2)/(pi*d_s5^3)) "Mean stress"
sigma_ea_s5 = K_f_s5b*((32*M_s5)/(pi*d_s5^3)) "Alternating stress"

1/SF_s5 = (sigma_em_s5/Su) + (sigma_ea_s5/Sn) "SF of shoulder 5"
```

```
"Pinion 2 Key 3"

th_key3 = d_p2/4 "Width of key 3"
L_key3 = 0.019 [m] "Length of key 3"

T_key3_shear = (d_p2/2)*L_key3*th_key3*0.58*Sy "Max shear torque"
T_key3_bearing = ((d_p2/2)+(th_key3/4)) * Sy *L_key3 * (th_key3/2) "Max bearing torque"

sigma_key3_shear = 1.3* (16*T_key3_shear)/(pi*d_p2^3) "Shear stress"
sigma_key3_bearing = 1.3* (16*T_key3_bearing)/(pi*d_p2^3) "Bearing stress"

SF_key3 = Sy/sigma_key3_shear "SF of key 3"

"Shaft 3"

"Bearing 5 Snap Ring 8"

d_b5 = 0.017 [m] "Diameter of shaft 3 at bearing 5"

K_t_5t = 2.0 "From book"
K_t_5b = 3.7 "From book"

q_5t = 0.65 "From book"
q_5b = 0.60 "From book"

K_f_5t = 1 + ((K_t_5t-1)*q_5b)
K_f_5b = 1 + ((K_t_5b-1)*q_5b)

sigma_em_b5 = K_f_5t*((16*T_G2)/(pi*d_b5^3)) "Mean stress"
sigma_ea_b5 = K_f_5b*((32*M_max_3)/(pi*d_b5^3)) "Alternating stress"

1/SF_b5 = (sigma_em_b5/Su) + (sigma_ea_b5/Sn) "Safety factor of snap ring 8"

"Bearing 6 Snap ring 9"

d_b6 = 0.01 [m] "Diameter of shaft 3 at bearing 9"
K_t_6t = 1.5 "From book"
q_6t = 0.65 "From book"

K_f_6t = 1 + ((K_t_6t-1)*q_6t)

sigma_em_b6 = K_f_6t*((16*T_G2)/(pi*d_b6^3)) "Mean stress"

1/SF_b6 = sigma_em_b6/Su "SF of snap ring 9"

"Gear 2 Shoulder 6"

M_s6 = F_G2_rad*(0.03[m]-(b/2)) "Moment at gear 2 shoulder 6"

K_t_s6t = 1.65 "From book"
K_t_s6b = 1.95 "From book"

q_s6t = 0.90 "From book"
q_s6b = 0.87 "From book"

d_s6 = 0.014[m] "Diameter of shaft 3 at shoulder 6"
```

```
K_f_s6t = 1 + ((K_t_s6t-1)*q_s6t)
K_f_s6b = 1 + ((K_t_s6b-1)*q_s6b)

sigma_em_s6 = K_f_s6t*((16*T_G2)/(pi*d_s6^3)) "Mean stress"
sigma_ea_s6 = K_f_s6b*((32*M_s6)/(pi*d_s6^3)) "Alternating stress"

1/SF_s6 = (sigma_em_s6/Su) + (sigma_ea_s6/Sn) "SF at shoulder 6"

"Gear 2 Snap Ring 10"

d_g2 = 0.01 [m] "Diameter of shaft 3 at snap ring 10"

K_t_sr8t = 1.5 "From book"
q_sr8t = 0.65 "From book"
K_f_sr8t = 1 + ((K_t_sr8t-1)*q_sr8t)

sigma_em_sr8 = K_f_sr8t*((16*abs(T_G2))/(pi*d_g2^3)) "Mean stress"

1/SF_sr8 = sigma_em_sr8/Su "SF at snap ring 10"

"Gear 2 Key 4"

th_key4 = d_g2/4 "Width of key 4"
L_key4 = 0.013 [m] "Length of key 4"

T_key4_shear = (d_g2/2)*L_key4*th_key4*0.58*Sy "Max shear torque"
T_key4_bearing = ((d_g2/2)+(th_key4/4)) * Sy *L_key4 * (th_key4/2) "Max bearing torque"

sigma_key4_shear = 1.3* (16*T_key4_shear)/(pi*d_g2^3) "Shear stress"
sigma_key4_bearing = 1.3* (16*T_key4_bearing)/(pi*d_g2^3) "Bearing stress"

SF_key4 = Sy/sigma_key4_shear "SF at key 4"
```

SOLUTION

Unit Settings: SI C kPa kJ mass deg

b = 0.03175 [m] {1.25 [in]}	$B_{1,y}$ = 63.97 [N]
$B_{1,z}$ = 175.8 [N]	$B_{2,y}$ = -127.9 [N]
$B_{2,z}$ = -351.5 [N]	$B_{3,y}$ = 85.69 [N]
$B_{3,z}$ = -6.93 [N]	$B_{4,y}$ = 109 [N]
$B_{4,z}$ = -185.2 [N]	$B_{5,y}$ = -258 [N]
$B_{5,z}$ = 708.9 [N]	$B_{6,y}$ = 129 [N]
$B_{6,z}$ = -354.5 [N]	C_g = 0.85
$C_{G,4}$ = 0.9	C_L = 1
C_{Li} = 0.86	$C_{L,4}$ = 1
C_p = 1.910E+08 [$Pa^{(1/2)}$]	C_r = 1
C_{req} = 1284 [N] {1.284 [kN]}	$C_{R,4}$ = 0.814
C_s = 0.7	$C_{S,4}$ = 0.6
$C_{T,4}$ = 1	d = 0.025 [m]
d_{abs} = 0.025 [m]	d_{b5} = 0.017 [m]
d_{b6} = 0.01 [m]	d_G = 0.1146 [m]
d_{g1} = 0.013 [m]	d_{g2} = 0.01 [m]
d_P = 0.054 [m]	d_{p1} = 0.01 [m]
d_{p2} = 0.015 [m]	d_{s2} = 0.015 [m]
d_{s4} = 0.015 [m]	d_{s5} = 0.017 [m]
d_{s6} = 0.014 [m]	d_{sr1} = 0.01 [m]

d_{sr2} = 0.02 [m]	η = 0.975
F_{g1} = 171.4 [N]	$F_{G1,rad}$ = 62.38 [N]
$F_{G1,tan}$ = 171.4 [N]	F_{g2} = 354.5 [N]
$F_{G2,rad}$ = 129 [N]	$F_{G2,tan}$ = 354.5 [N]
F_{P1} = 175.8 [N]	$F_{P1,rad}$ = 63.97 [N]
$F_{P1,tan}$ = 175.8 [N]	F_{P2} = 363.5 [N]
$F_{P2,rad}$ = 132.3 [N]	$F_{P2,tan}$ = 363.5 [N]
F_r = 377.2 [N]	F_{t1} = 175.8 [N]
F_{t2} = 363.5 [N]	G_r = 4.5
H_B = 302	I = 0.1092
J_g = 0.28	J_p = 0.24
K_a = 1	$K_{f,2}$ = 1.25
$K_{f,5b}$ = 2.62	$K_{f,5t}$ = 1.6
$K_{f,6t}$ = 1.325	$K_{f,g1b}$ = 2.625
$K_{f,g1t}$ = 1.65	$K_{f,p2b}$ = 2.625
$K_{f,p2t}$ = 1.65	$K_{f,s2b}$ = 1.4
$K_{f,s2t}$ = 1.403	$K_{f,s4b}$ = 1.893
$K_{f,s4t}$ = 1.63	$K_{f,s5b}$ = 1.511
$K_{f,s5t}$ = 1.233	$K_{f,s6b}$ = 1.827
$K_{f,s6t}$ = 1.585	$K_{f,sr1t}$ = 1.25
$K_{f,sr2b}$ = 2.4	$K_{f,sr2t}$ = 1.65
$K_{f,sr8t}$ = 1.325	K_m = 1.6
K_{ms} = 1.4	K_o = 1.5
K_r = 0.814	$K_{r,bearing}$ = 0.21
K_t = 1	$K_{t,2}$ = 1.5
$K_{t,5b}$ = 3.7	$K_{t,5t}$ = 2
$K_{t,6t}$ = 1.5	$K_{t,g1b}$ = 3.5
$K_{t,g1t}$ = 2	$K_{t,p2b}$ = 3.5
$K_{t,p2t}$ = 2	$K_{t,s2b}$ = 2
$K_{t,s2t}$ = 1.62	$K_{t,s4b}$ = 2.05
$K_{t,s4t}$ = 1.7	$K_{t,s5b}$ = 1.7
$K_{t,s5t}$ = 1.25	$K_{t,s6b}$ = 1.95
$K_{t,s6t}$ = 1.65	$K_{t,sr1t}$ = 1.5
$K_{t,sr2b}$ = 3.8	$K_{t,sr2t}$ = 2.3
$K_{t,sr8t}$ = 1.5	K_{v1} = 3.6
K_{v2} = 2.3	L = 1.120E+09 [rev]
LB1B2 = 0.03 [m]	LB2P1 = 0.03 [m]
LB3G1 = 0.05 [m]	LB5B6 = 0.03 [m]
LG1P2 = 0.05 [m]	LG2B5 = 0.03 [m]
LP2B4 = 0.05 [m]	L_{key1} = 0.013 [m]
L_{key2} = 0.017 [m]	L_{key3} = 0.019 [m]
L_{key4} = 0.013 [m]	L_r = 9.000E+07 [rev]
m = 0.003 [m]	M_{G1} = 4.298 [N-m]
$M_{G1,y}$ = -0.3465 [N-m]	$M_{G1,z}$ = 4.284 [N-m]
$M_{G2,y}$ = 3.87 [N-m]	$M_{G2,z}$ = -10.63 [N-m]
$M_{max,1}$ = 5.611 [N-m]	$M_{max,2}$ = 10.75 [N-m]
$M_{max,3}$ = 11.32 [N-m]	$M_{P1,y}$ = 1.919 [N-m]
$M_{P1,z}$ = 5.273 [N-m]	$M_{P2,y}$ = -9.262 [N-m]
$M_{P2,z}$ = 5.45 [N-m]	M_{s2} = 2.031 [N-m]
M_{s5} = 1.869 [N-m]	M_{s6} = 1.822 [N-m]
n_G = 38.18	$N_{g,actual}$ = 38
n_P = 18	$N_{p,actual}$ = 18
ω_{in} = 628.5 [rad/sec]	$\omega_{in,rpm}$ = 6000 [rev/min]
ω_{out} = 139.6 [rad/sec]	$\omega_{out,rpm}$ = 1333 [rev/min]
P = 333.3 [1/m]	ϕ = 20 [deg]
q_0 = 0.5	q_{5b} = 0.6

q_{5t} = 0.65
q_{8t} = 0.65
q_{g1} = 0.65
q_{p2} = 0.65
q_{s2b} = 0.4
q_{s2t} = 0.65
q_{s4b} = 0.85
q_{s4t} = 0.9
q_{s5b} = 0.73
q_{s5t} = 0.93
q_{s6b} = 0.87
q_{s6t} = 0.9
q_{sr2} = 0.5
q_{sr8t} = 0.65
$q_{t,sr1t}$ = 0.5
R1 = 2.395E+06 [Pa]
R2 = 4.744E+06 [Pa]
R3 = 7.576E+06 [Pa]
$rated_{life}$ = 14000 [hr]
r_G = 0.05728 [m]
r_P = 0.027 [m]
R_{partc} = 2.121
SF1 = 53.15
SF2 = 26.98
SF3 = 17.68
SF_{b5} = 2.036
SF_{b6} = 4.657
SF_{g1} = 6.229
$SF_{g1,4}$ = 2.315
SF_{g2} = 4.714
SF_{key1} = 1.603
SF_{key2} = 1.593
SF_{key3} = 1.645
SF_{key4} = 1.603
SF_{p1} = 5.205
$SF_{P1,4}$ = 21.12
SF_{p2} = 3.939
$SF_{p2,4}$ = 1.549
SF_{s2} = 12.99
SF_{s4} = 7.286
SF_{s5} = 16.28
SF_{s6} = 5.503
SF_{sr1} = 21.12
SF_{sr2} = 7.686
SF_{sr8} = 4.657
$SF_{surface1}$ = 1.508
$SF_{surface2}$ = 1.141
$\sigma_{1,shaft1}$ = 4.225E+06 [Pa]
$\sigma_{1,shaft2}$ = 8.247E+06 [Pa]
$\sigma_{1,shaft3}$ = 1.126E+07 [Pa]
$\sigma_{2,shaft1}$ = -566412 [Pa]
$\sigma_{2,shaft2}$ = -1.241E+06 [Pa]
$\sigma_{2,shaft3}$ = -3.887E+06 [Pa]
$\sigma_{ave,1}$ = 1.829E+06 [Pa]
$\sigma_{ave,2}$ = 3.503E+06 [Pa]
$\sigma_{ave,3}$ = 3.688E+06 [Pa]
$\sigma_{ea,b5}$ = 6.147E+07 [Pa]
$\sigma_{ea,g1}$ = 5.231E+07 [Pa]
$\sigma_{ea,p2}$ = 8.514E+07 [Pa]
$\sigma_{ea,s2}$ = 8.582E+06 [Pa]
$\sigma_{ea,s4}$ = 2.455E+07 [PA]
$\sigma_{ea,s5}$ = 5.855E+06 [Pa]
$\sigma_{ea,s6}$ = 1.236E+07 [Pa]
$\sigma_{ea,sr2}$ = 1.715E+07 [Pa]
$\sigma_{em,b5}$ = 3.367E+07 [Pa]
$\sigma_{em,b6}$ = 1.370E+08 [Pa]
$\sigma_{em,g1}$ = 3.754E+07 [Pa]
$\sigma_{em,p1}$ = 3.021E+07 [Pa]
$\sigma_{em,p2}$ = 2.444E+07 [Pa]
$\sigma_{em,s2}$ = 1.005E+07 [Pa]
$\sigma_{em,s4}$ = -2.414E+07 [Pa]
$\sigma_{em,s5}$ = 1.254E+07 [Pa]
$\sigma_{em,s6}$ = 5.972E+07 [Pa]
$\sigma_{em,sr1}$ = 3.021E+07 [Pa]
$\sigma_{em,sr2}$ = 4.985E+06 [Pa]
$\sigma_{em,sr8}$ = 1.370E+08 [Pa]
$\sigma_{e,1}$ = 4.534E+06 [Pa]
$\sigma_{e,2}$ = 8.932E+06 [Pa]
$\sigma_{e,3}$ = 1.363E+07 [Pa]
σ_{G1} = 5.552E+07 [Pa]
σ_{G2} = 7.336E+07 [Pa]
$\sigma_{key1,bearing}$ = 2.505E+08 [Pa]
$\sigma_{key1,shear}$ = 2.583E+08 [Pa]
$\sigma_{key2,bearing}$ = 2.520E+08 [Pa]
$\sigma_{key2,shear}$ = 2.599E+08 [Pa]
$\sigma_{key3,bearing}$ = 2.441E+08 [Pa]
$\sigma_{key3,shear}$ = 2.517E+08 [Pa]
$\sigma_{key4,bearing}$ = 2.505E+08 [Pa]
$\sigma_{key4,shear}$ = 2.583E+08 [Pa]
σ_{P1} = 6.643E+07 [Pa]
σ_{P2} = 8.778E+07 [Pa]
$\sigma_{S,x1}$ = 3.658E+06 [Pa]
$\sigma_{S,x2}$ = 7.006E+06 [Pa]
$\sigma_{S,x3}$ = 7.377E+06 [Pa]
Sn = 1.402E+08 [Pa]
Su = 6.380E+08 [Pa]
Sy = 4.140E+08 [Pa]
S_{fe} = 7.766E+08 [Pa]
S_{n1} = 3.458E+08 [Pa]
S_{n2} = 3.458E+08 [Pa]
$S_{n'}$ = 5.100E+08 [Pa]
S_u = 1.020E+09 [Pa]
S_y = 2.410E+08 [Pa]
τ_{S1} = 1.547E+06 [Pa]
$\tau_{S,torque2}$ = 3.199E+06 [Pa]
$\tau_{S,torque3}$ = 6.617E+06 [Pa]
$\tau_{S,xy2}$ = 3.199E+06 [Pa]
$\tau_{S,xy3}$ = 6.617E+06 [Pa]
$\tau_{T,shear1}$ = 508071 [Pa]
$\tau_{T,shear2}$ = 583796 [Pa]
$\tau_{T,shear3}$ = 1.025E+06 [Pa]
$\tau_{T,torque1}$ = 1.547E+06 [Pa]

$\tau_{T,torque2}$ = -3.199E+06 [Pa]
$\tau_{T,torque3}$ = 6.617E+06 [Pa]
$\tau_{T,xz1}$ = 2.055E+06 [Pa]
$\tau_{T,xz2}$ = -2.616E+06 [Pa]
$\tau_{T,xz3}$ = 7.642E+06 [Pa]
θ = 20 [deg]
th_{key1} = 0.0025 [m]
th_{key2} = 0.00325 [m]
th_{key3} = 0.00375 [m]
th_{key4} = 0.0025 [m]
T_{G1} = -9.816 [N-m]
T_{G2} = 20.3 [N-m]
$T_{key1,bearing}$ = 37.84 [N-m]
$T_{key1,shear}$ = 39.02 [N-m]
$T_{key2,bearing}$ = 83.63 [N-m]
$T_{key2,shear}$ = 86.23 [N-m]
$T_{key3,bearing}$ = 124.4 [N-m]
$T_{key3,shear}$ = 128.3 [N-m]
$T_{key4,bearing}$ = 37.84 [N-m]
$T_{key4,shear}$ = 39.02 [N-m]
T_{P1} = 4.746 [N-m]
T_{P2} = 9.816 [N-m]
$V_{max,1}$ = 187 [N]
$V_{max,2}$ = 214.9 [N]
$V_{max,3}$ = 377.2 [N]
$V_{pitchLine1}$ = 16.97 [m/s] {3341 [ft/min]}
$V_{pitchLine2}$ = 7.997 [m/s] {1574 [ft/min]}
$\dot{W}_{in}$ = 2983 [W]
$\dot{W}_{in,hp}$ = 4 [hp]

No unit problems were detected.

KEY VARIABLES
C_{req} = 1284 [N] {1.284 [kN]}