Electric Circuits

An Introduction

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Preface

This book is made in reaction to many introductory Electrical Engineering texts, which tend to assume a Sophomore- or even Junior-level understanding of Mathematics. In contrast, we aim our text at Freshmen, who may or may not have completed Calculus I.

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Underlying Fundamentals

1.1 Review of Algebra

1.2 Units

What is electric charge?

- Fundamental property of matter
- Most matter is neutral on a macroscopic level
- Atoms are composed of:
  - protons, which have a positive charge
  - electrons, which have a negative charge
  - neutrons, which do not possess a net charge
- The charge of an electron is constant and the most fundamental unit of charge. However, it is very small
- A more useful unit of charge is the Coulomb, which is equivalent to roughly 6.24×10^{18} electrons.

Current, Voltage, Resistance

The three concepts we will be most interested in throughout this book are current, voltage, and resistance.

Current is:

- The amount of charge moving through a region (usually a wire) in a given amount of time.
- Measured in Amperes – Coulombs per second (1 A = 1 C/s)

Voltage is:

- The amount of energy present in a given amount of charge

- Also called "electric potential"

- Think about a ball on a hill, which has some potential energy. In this analogy, the mass of the ball is like the charge of our particle, the height of the hill is the voltage, and the potential energy is the electric energy of the particle.

- Measured in Volts – Joules per Coulomb (1 V = 1 J/C)

Resistance is:

- A measure of the amount of energy it takes to get some charge through a circuit element in a certain amount of time

- Measured in Ohms, which does not have a useful conversion to more basic units

- Think of a very thin pipe. In order to get water through the pipe, you will need to push harder and harder as the pipe gets thinner (or, alternatively, you will have to push harder in order to get more water through the same pipe). The pipe has some fluid resistance, just as our circuit elements could have some electric resistance.

- We will cover this more when we get to Ohm's Law

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The First Laws

2.1 Ohm's Law on a Single Resistor

Ohm's Law describes the amount of energy it takes to push some amount of charge through a resistor. A reasonable analogy is a ball moving quickly as it approaches a patch of mud. It starts off with some amount of kinetic energy, then as it moves through the mud, it starts to slow down, losing some portion of the kinetic energy it has, until it clears the puddle.

Unfortunately, our analogy breaks down a bit, since our electrons are moving at the same speed the entire time. We are not siphoning off the kinetic energy of the electrons, but rather the energy stored in the electrical potential, or voltage.

Ohm's Law states that the voltage drop *over* a resistor is proportional to both the resistance, R, and the current *through* the resistor:

$$\Delta V = iR \tag{2.1}$$

There is emphasis placed on the words *over* and *through* due to how we measure the voltage and current in a resistor for Ohm's Law. We do not care about the voltage at any one place, but rather only the change between the two ends. Likewise, whatever current is present at any point on the resistor will be present for all of it. Neither occurs at a single point, but rather describes the entire resistor.

Looking at Figure 2.1, you can see that we have labeled the two voltages on either side. When we apply these voltages to ΔV, we should always start on the tail end of our current (that is, the direction our current is coming from) and end on the head.

Looking back to the pipe analogy from section 1.2, ΔV is the loss of energy, i is the flow rate through the thin pipe, and R is how hard it is to get through the pipe. More flow, whether fluid or electric, is more loss of energy, and a smaller pipe or larger resistance also means more loss.

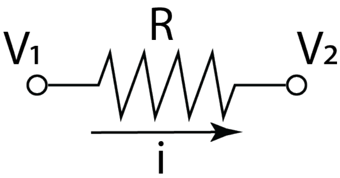

Figure 2.1: $\Delta V = V_1 - V_2$ for the circuit above, since energy is lost as charge passes through the resistor.

2.2 Ohm's Law on a Simple Circuit

2.3 Kirchoff's Current Law

2.4 Watt's Law

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Equivalent Circuits

This chapter introduces the concept of an equivalent circuit. Two equivalent circuits share common values of voltage/current at a point of interest, usually the source. In this chapter, we will simplify complex resistive circuits down to the simplest version (such as those analyzed in Section 2.2). Once simplified, we will expand them again through methods known as voltage division and current division to find the current through each element and the voltage at each node.

3.1 Resistors in Series

First, lets go over in a bit more detail what we mean by **series**. Two elements are in series if and only if:

- Those elements share a common node
- No other elements share the same node

In this case, there is exactly one path for current to follow through both resistors. Because of this, we can write the following:

$$\Delta V_1 = V_A - V_C = iR_1 \tag{3.1}$$

$$\Delta V_2 = V_C - V_B = iR_2 \tag{3.2}$$

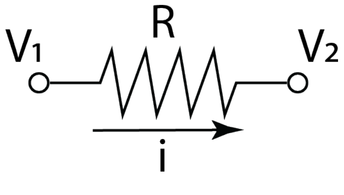

Figure 3.1: The two resistors above are the only circuit elements connected to node C, which means that any current from node A to C will also have to travel from C to B.

Now, our goal is to create simpler circuit which has the same characteristics as seen by nodes A and B. The only way to get simpler is to have a single circuit between nodes A and B with a resistance which we still need to determine.

We want our new circuit to have the following characteristics:

- The current through the new resistor should still be the i that passes through R_1 and R_2
- The voltage difference across the new resistor should be $V_A - V_B$

We can fulfill those two requirements by summing equations 3.1 and 3.2:

$$\Delta V_1 + \Delta V_2 = V_A - V_B = i(R_1 + R_2) \tag{3.3}$$

Equation 3.3 essentially states that we can replace both resistors with a single resistor that has a resistance

$$R_{EQ} = R_1 + R_2. \tag{3.4}$$

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