Illustrating Bayes theorem with Gaussian mixtures

NCEO Intensive Course on Data Assimilation

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1 Objective

This activity will allow the student to visualise the application of Bayes theorem in a uni-dimensional setting. In particular, the pdf's (probability density functions) of the prior and likelihood are Gaussian mixtures.

2 Description of the code

2.1 Files

These are the python files used in this activity:

- *ControlBayes.py*. This is the control file. You will run and modify this file.
- *functionsBayes.py*. This file has the code to compute Gaussian mixture pdf's, as well as the point-wise operations in Bayes theorem.

You will run different sections of the file *ControlBayes.py*. These are enumerated as comments of the file (recall that in python # is used for comments). To run **only** a section of a file you can highlight the desired instructions with the mouse, and the press F9.

2.2 Instructions

- The first lines of the file import the different packages that the file uses: numpy, matplotlib, and the functions we have created for this activity.
- *Section 0*. The statistical support of the random variable is defined.
- *Section 1*. In this section you can edit the components of the prior. There is one array for the coefficients (α's), one for the means and one for the standard deviations for each component of the Gaussian mixture. Try changing the values and the number of components. Recall that the size of the three arrays has to be the same.
- *Section 2*. In this section you have to define the value of the actual observation y, as well as the components of the Gaussian mixture. This pdf is overall centred in the observation.
- *Section 3*. This small section contains the commands to compute Bayes theorem.
- *Section 4*. This section contains the instructions to plot the prior, likelihood and posterior pdf's in the same graph.

3 Exercises

1. Let us generate different prior probability density functions. We use Gaussian Mixtures, i.e. densities formed by the sum of Gaussians. This construction is quite versatile. We will simulate the assimilation of an observation $y = 2$ with standard deviation $\sigma_y = 1$ and the following options for priors:

coefficients	μ	σ	properties of the prior
[1]	[-2]	[1]	Gaussian
[1]	[-2]	[2]	Gaussian
[1]	[-2]	[0.5]	Gaussian
[0.5,0.5]	[-1.5,1.5]	[0.5,0.5]	bimodal
[0.25,0.75]	[-1.5,1.5]	[0.5,0.5]	bimodal
[0.8,0.2]	[-1.5,0.0]	[1.5,1.0]	unimodal skewed

   How do the different priors compare to each other? What features can you identify?

2. Repeat these experiments with a different observation value of your choice.