

Review of the ensemble Kalman filter

NCEO Intensive Course on Data Assimilation

University of Reading, 2022

1 Review of Theory

Ensemble Kalman filters are Monte-Carlo implementations of the Kalman filter, or the extended Kalman filter in the case of non-linear evolution models. In the Kalman filter, the analysis equations are:

$$\begin{aligned}\bar{\mathbf{x}}^a &= (\mathbf{I} - \mathbf{KH}) \bar{\mathbf{x}}^b + \mathbf{Ky} \\ \mathbf{A} &= (\mathbf{I} - \mathbf{KH}) \mathbf{B} \\ \mathbf{K} &= \mathbf{BH}^T (\mathbf{HBH}^T + \mathbf{R})^{-1}\end{aligned}\tag{1}$$

where $\bar{\mathbf{x}}^b \in \mathcal{R}^{N_x}$ is the background mean, $\bar{\mathbf{x}}^a \in \mathcal{R}^{N_x}$ is the analysis mean $\mathbf{y} \in \mathcal{R}^{N_y}$ is the observation, $\mathbf{H} \in \mathcal{R}^{N_y \times N_x}$ is the observation operator, $\mathbf{B} \in \mathcal{R}^{N_x \times N_x}$ is the background error covariance, $\mathbf{R} \in \mathcal{R}^{N_y \times N_y}$ is the observation error covariance, and $\mathbf{A} \in \mathcal{R}^{N_x \times N_x}$ is the analysis error covariance.

The ensemble Kalman filter uses a sample estimator for B . Moreover, the equations in (1) are difficult to compute. In this practical we use two formulations:

- The Stochastic Ensemble Kalman filter of Burgers et al (1999) and its localised version.
- The Ensemble Transform Kalman filter of Wang et al. (2004) and Hunt et al (2007). This deterministic square root filter acts on ensemble space.