# OGL

## 1 setup

Let $U = \{[x] : x \in \mathbb{Z},\, x \leq n\}$, $n \in \mathbb{N}$

All functions unless other ways noted maps $U^2 \to U$

## 2 gates

$$OP(a,b) = \begin{array}{c|ccccccc} & 0 & 1 & 2 & \dots & \text{b} & \dots & \text{n} \\ \hline \\ 0 & \text{n} & 0 & 0 & \dots & 0 & \dots & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 & \dots & 0 \\ 2 & 0 & 0 & 1 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \text{a} & 0 & 0 & 0 & \dots & \text{a-1} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \text{n} & 0 & 0 & 0 & \dots & 0 & \dots & \text{n-1} \end{array}$$

$$= \begin{cases} a-1 & a = b \\ 0 & a \neq b \end{cases}$$

$ROT : U \to U$
$\overline{ROT}(a) = OP(a,a)$
$\overline{A_0}(a,b) = a$
$\overline{A_m}(a,b) = ROT(\overline{A_{m-1}})$
$\overline{B_0}(a,b) = b$
$\overline{B_m}(a,b) = ROT(\overline{B_{m-1}})$

$$\overline{F_{ab}}(x,y) = \begin{cases} x-1-a-b & x = y \\ -b & x \neq y \end{cases}$$

$\overline{F_{a0}}(x,y) = OP(\overline{A_a}, \overline{B_a})$
$\overline{F_{ab}}(x,y) = ROT(\overline{F_{a(b-a)}})$
$\overline{0} = OP(\overline{A_0}, \overline{A_1})$
$\overline{0}(x,y) = 0$
$\overline{m} = ROT(\overline{m+1})$
$\overline{m}(x,y) = m$

$$\overline{(a,b,c)}(x,y) = \begin{cases} c & x = a \wedge y = b \\ 0 & x \neq a \vee y \neq b \end{cases}$$

$$\overline{(a,b,c)} = \begin{cases} \overline{0} & c = 0 \\ OP(\overline{F_{(-2-c)0}}(\overline{A_a}, \overline{B_b}), \overline{C+1}) & c \neq n-1 \\ OP(\overline{F_{(c-1)1}}(\overline{A_a}, \overline{B_b}), \overline{0}) & c = n-1 \end{cases}$$

$$\overline{S_a}(x,y) = \begin{cases} a & x = a \wedge y = 0 \\ a & x = 0 \wedge y = a \\ 0 & \end{cases}$$

$\overline{S_a} = \overline{F_{(a-1)(-a)}}(\overline{(0,a,a)}, \overline{(a,0,a)})$
$\overline{AS_{ab}(x,y)} =$
$if(x = 0 \vee y = 0)\{$
$if(x = a \vee y = a)\{$
return b
$\}if(x = a+1 \vee y = a+1)\{$
return b

}
$\}else\{return 0\}$
$\overline{AS_{ab}} = \overline{F_{(b-1)(-b)}}(\overline{S_a}, \overline{S_{a+1}})$

$\overline{PCO_a}(x,y) =$

| | 0 | 1 | 2 | ... | a | a+1 | a+2... | n |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | ... | a | 0 | 0... | 0 |
| 1 | 1 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| 2 | 2 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| a | a | 0 | 0 | 0 | ... | 0 | ... | 0 |
| a+1 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| a+2 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| n | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 |

$\overline{PCO_0} = \overline{0}$
$\overline{PCO_1} = \overline{S_1}$
$\overline{PCO_2} = \overline{F_{0(-1)}}(\overline{S_2}, \overline{AS_{12}})$
$\overline{PCO_a} = \overline{F_{(-2)(0)}}$(the two following tables (needs $\overline{PCO_{(a-1)(0)}}$))

| | 0 | 1 | 2 | ... | a | a+1 | a+2... | n |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | ... | a-1 | 0 | 0... | 0 |
| 1 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| 2 | 1 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| a | a-1 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| a+1 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| a+2 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| n | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 |

| | 0 | 1 | 2 | ... | a | a+1 | a+2... | n |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | ... | a-1 | 1 | 1... | 1 |
| 1 | 1 | 1 | 1 | 1 | ... | 1 | ... | 1 |
| 2 | 1 | 1 | 1 | 1 | ... | 1 | ... | 1 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| a | a-1 | 1 | 1 | 1 | ... | 1 | ... | 1 |
| a+1 | 1 | 1 | 1 | 1 | ... | 1 | ... | 1 |
| a+2 | 1 | 1 | 1 | 1 | ... | 1 | ... | 1 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| n | 1 | 1 | 1 | 1 | ... | 1 | ... | 1 |