

Let

```

fish :: Int → [Permutation Int]
fish n = [r1 n, r2 n]
r1 :: Int → Permutation Int
r1 n = p [[1..n]]
r2 :: Int → Permutation Int
r2 n = p ([2, (2 * n - 2), 4] ++ [(2 * n - 3), (2 * n - 4) .. (n + 1)])

```

Note that this is a subset of S_{2*n-2} .

Let

```

m n k | k < 0 = let pk = -k in ((r1 n) ^ - pk) * ((r2 n) ^ - pk) * (r1 n) ↑ pk * (r2 n) ↑ pk
      | k ≥ 0 = (r1 n) ↑ k * (r2 n) ↑ k * ((r1 n) ^ - k) * ((r2 n) ^ - k)

```

Then we find that for any $n \geq 5$, we find that $m\ n\ (\pm 2)$ permutes 4 elements and has order 2. Further, for any $n \geq 6$, $m\ n\ (\pm 1, 3)$ permutes 6 elements and has order 3. It appears that for any $n \geq 4$, we have that $m\ n\ k$ has order 3 and permutes 6 elements if $k \not\equiv \pm 2 \pmod n$.

We find that the order of $fish\ 4 = 24$ and $\text{ord}(fish\ 5) = 20160 = 4 * 7!$.

For Fish 5, we find that n_7 is gotten by

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a 'dv' b = (b 'mod' a) ≡ 0
n7 = [(a, b, c, d) | a ← [0..1], b ← [0..1], c ← [0..2], d ← [0..6]
      , let np = 7 ↑ a * 5 ↑ b * 3 ↑ c * 2 ↑ d
      , np 'dv' (5 * 3 ↑ 2 * 2 ↑ 6)
      , np 'mod' 7 ≡ 1]
n5 = [(a, b, c, d) | a ← [0..1], b ← [0..1], c ← [0..2], d ← [0..6]
      , let np = 7 ↑ a * 5 ↑ b * 3 ↑ c * 2 ↑ d, ((7 * 3 ↑ 2 * 2 ↑ 6) 'mod' np) ≡ 0, np 'mod' 5 ≡ 1]
n3 = [(a, b, c, d) | a ← [0..1], b ← [0..1], c ← [0..2], d ← [0..6]
      , let np = 7 ↑ a * 5 ↑ b * 3 ↑ c * 2 ↑ d, ((7 * 5 * 2 ↑ 6) 'mod' np) ≡ 0, np 'mod' 3 ≡ 1]
n2 = [(a, b, c, d) | a ← [0..1], b ← [0..1], c ← [0..2], d ← [0..6]
      , let np = 7 ↑ a * 5 ↑ b * 3 ↑ c * 2 ↑ d, ((7 * 5 * 3 ↑ 2) 'mod' np) ≡ 0, np 'mod' 2 ≡ 1]

```

which gives n_7 to be $[1, 8, 64, 36, 288, 15, 120, 960]$, $n_5 = [1, 16, 6, 96, 36, 576, 56, 21, 336, 126, 2016]$, $n_3 = [1, 4, 16, 64, 10, 40, 160, 7, 28, 112, 448, 70, 280, 1120]$, $n_2 = [1, 3, 9, 5, 15, 45, 7, 21, 63, 35, 105, 315]$