half derivative $M\frac{3}{3}$ p1

$$\frac{d^r x}{dx} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0) \cdot x^{(n-r)}}{\overset{!}{\sim}(n-r)} \quad r<1 \qquad \overset{?}{\sim}(x) = \frac{1}{\overset{!}{\sim}(x)}$$

$$e^x = \frac{d^r e^x}{dx} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n-r}}{\overset{!}{\sim}(n-r)} \; ; r<1$$

$\overset{!}{\sim}(x) =$ 

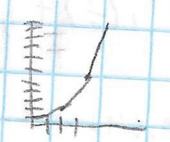
$$\mathrm{stdev}_{(a,b)}(f(x), g(x)) = \sqrt{\int_a^b (f(x)-g(x))^2\, dx}$$

$$\frac{d^r \sin(x)}{dx} @ x=0 \overset{?}{=} \sin(2\pi r) \text{ or } \cos(2\pi r)$$

$\overset{!}{\sim}(x) = (\lceil x \rceil)!$ $\quad \mathrm{stdev}\left(e^x, \frac{d^{\frac{1}{2}} e^x}{dx}, 0, 8\right) = 1337.58$

$\overset{!}{\sim}(x) = (\lfloor x \rfloor)!$ $\quad$ " $\quad = 3665.913$

$\overset{!}{\sim}(x) = \Gamma(x+1)$ $\quad$ " $\quad = .963057$

$\overset{!}{\sim}(x) = (\lceil x \rceil ! - \lfloor x \rfloor !)(x - \lfloor x \rfloor) + \lfloor x \rfloor ! =$  $= 773.5103$

straight lines

$\overset{!}{\sim}(x) = (\lceil x \rfloor)!$ $\quad$ " $\quad = 1164.317$

$\overset{!}{\sim}(x) =$ curves (x^2)

$\mathrm{lines}(x) = (\lceil x \rceil ! - \lfloor x \rfloor !)(x - \lfloor x \rfloor) + \lfloor x \rfloor !$

$$\int_0^x \left((D(\mathrm{lines}(x+1), x) - D(\mathrm{lines}(x-1), x)) \right. ($$

connect
$.2x \; @ x=1$
with $6x \; @ x=2$
with parabola

$$\int \left(\frac{D(2x) - D(6x)}{1 - 2} (x_{-1}) + 2D(1) \right) dx$$

must start with slope one at ~~and~~ and end with slope 2.

$2x \; @ x=1$
$6x \; @ x=2$

$(2ax + b) D(1) = 2 D(2x)$
$(2ax + b) D(2) = D(6x)$

$2a + b = 2$
$4a + b = 6$

$2 - 2a = b$
$4a + 2 - 2a = 6$
$2a = 6 - 2$
$2a = 4$
$a = 2$

$4 + b = 2$
$b = -2$

$P_1 = ~~(D(\mathrm{lines}(x-1)),~~$ $(x-1, D(\mathrm{lines}(x-1)))$ $\quad ~~D(ax^2 + bx + c)(a)~~ = L_1$

$P_2 = (x+1, D(\mathrm{lines}(x+1)))$

line through 2 points: $y = \left(\frac{P_{2,y} - P_{1,y}}{P_{2,x} - P_{1,x}}\right)(x - P_{1,x}) + P_{1,y}$

half derivative part 2

$f(x) = x^2$

$c = 1$

$x = 0$

$$f(x) = \frac{1^2 (x-1)}{0!} + 2(x-1) + \cancel{2} \cdot (x-1)^2$$

$$= (x-1) \cdot \frac{1}{2} + \frac{f^{\frac{1}{2}}(1)(x-1)^{\frac{1}{2}}}{\frac{1}{2}! \cdot 2} + 2(x-1) + (x-1)^2$$

$$f(0) =$$

$$0 = -\frac{1}{2} + \frac{f^{\frac{1}{2}}(1)(-1)^{\frac{1}{2}}}{\frac{1}{2}! \cdot 2} + \frac{-2}{-1} + 1$$

$$0 = -\frac{1}{2} + 3$$

$$-\frac{3}{2} \quad \cancel{\frac{1}{2} = \frac{f^{\frac{1}{2}}(1)(-1)^{\frac{1}{2}}}{\frac{1}{2}! \cdot 2}}$$

$$\frac{1}{1} = \frac{\frac{1}{2}}{1} = \frac{1}{-1}$$

$$\cancel{\frac{1}{2}!(1) = f^{\frac{1}{2}}(0) - \frac{1}{2}! \cdot -1}$$

$$\cancel{-1 = \frac{f^{\frac{1}{2}}(0)(-1)^{\frac{1}{2}}}{\frac{1}{2}! \cdot 2} + \frac{1}{2}}$$

$$\frac{3}{2} = \frac{f^{\frac{1}{2}}(1)(-1)^{\frac{1}{2}}}{\frac{1}{2}! \cdot 2}$$

$$\frac{3 \cdot \frac{1}{2}!}{i} = f^{\frac{1}{2}}(0) \qquad f^{\frac{1}{2}}(1) = -3i \cdot \frac{1}{2}!$$

$$f_1^{(1/2)}(0) = -3i \cdot \frac{1}{2}!$$

Multi Variable Taylors theorem 11/11

Thm: let f to be a function, and V be a vector, furthermore, f take V-length arguments. ~~let f be~~

$$T_f(V) = \sum_{n=0}^{\infty} \frac{\nabla^n \nabla f \underset{@\vec{0}}{\cdot} (V)^n}{n!}$$

Ex: $f(x,y) = x^2 + y^2$

$$T_f(x,y) = \frac{(x^2+y^2) @ (x=0, y=0) \cdot (\langle x, y \rangle)^0}{0!} + \frac{\langle 2x, 2y \rangle @(0,0) \cdot (\langle x, y \rangle)^1}{1!}$$

$$= 0 \qquad\qquad \frac{\langle 0,0 \rangle \cdot \langle x, y \rangle}{1!} = 0$$

$$+ \frac{\langle \langle 2,0 \rangle, \langle 0,2 \rangle \rangle @(0,0) \cdot (\langle x, y \rangle)^2}{2!}$$

$$= \frac{\langle 2,0 \rangle \cdot \langle x^2, y^2 \rangle + \langle 0,2 \rangle \cdot \langle x^2, y^2 \rangle}{2}$$

$$= \frac{2x^2 + 0y^2 + 0x^2 + 2y^2}{2}$$

$$= x^2 + y^2 = f(x,y)$$

$$f(x_0+x, y_0+y) \approx f(x_0, y_0) + \nabla f(x_0, y_0) \cdot (x,y) + \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} Hf(x_0, y_0) \begin{bmatrix} x \\ y \end{bmatrix} + \cdots$$

$$\lim_{x \to R} \frac{\left[\prod\!\!\!\!\!\!\int_{-\infty}^{\infty} (x-r)@r\right]}{\prod\!\!\!\!\!\!\int_{-\infty}^{\infty} (x-r)@r}$$

$$\lim \prod_{i=1}^{\wedge} (x-r_i)\,\Delta r_i$$

log

$$\log \prod\!\!\!\!\!\!\int_{-\infty}^{\infty} f = \int \log f$$

$$\prod\!\!\!\!\!\!\int f = e^{\int \log f}$$

$$\int$$

half derivative M4 p1

$$\frac{d^r x}{dx} = \sum_{n=\lceil r\rceil}^{\infty} \frac{f^{(n)}(0)\cdot x^{n-r}}{\tilde{\Gamma}(n-r)} \qquad ?<r<? \quad \text{maby } -\infty<r<\infty \qquad \tilde{\Gamma}(x)=\frac{1}{\Gamma(x)}$$

$$e^x = \frac{d^r e^x}{dx} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=\lceil r\rceil}^{\infty} \frac{x^{n-r}}{\tilde{\Gamma}(n-r)}$$

$$\text{or } \frac{d^r f(x)}{dx} = \sum_{n=\lfloor r\rfloor}^{\infty} \frac{f^{(n)}(0)\cdot x^{n-r}}{\tilde{\Gamma}(n-r)}$$

$$\frac{d^r \cancel{\alpha} f(x)}{dx} = \frac{f(0)}{\Gamma(r)} + \sum_{n=1}^{\infty} \frac{f^{(n)}(0)\cdot x^{n-r}}{\tilde{\Gamma}(n-r)} \ ; \ r<1$$

$$\text{or } \frac{d^r f(x)}{dx} = f(0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(0)\cdot x^{n-r}}{\tilde{\Gamma}(n-r)} \ ; \ r<1$$

$$e^x = \sum \cancel{f(0)} + \sum_{n=1}^{\infty} \frac{f^{(n)}(0)\cdot x^{n-r}}{\cancel{\ }} \cdot \Gamma(n-r)$$ cont from M2

ans matrix

$$A = \begin{bmatrix} e^1-1 \\ e^2-1 \\ \vdots \\ e^j-1 \end{bmatrix} = [e^j-1]$$

V matrix

$$V = \begin{bmatrix} \Gamma(1-r) \\ \Gamma(2-r) \\ \vdots \\ \Gamma(j-r) \end{bmatrix} = [\Gamma(j-r)]$$

Coeff matrix

$$C = \begin{bmatrix} 1^{(1-r)} & 1^{(2-r)} & \cdots & 1^{(k-r)} \\ 2^{(1-r)} & 2^{(2-r)} & \cdots & 2^{(k-r)} \\ \vdots & \vdots & \ddots & \vdots \\ j^{(1-r)} & j^{(2-r)} & \cdots & j^{(k-r)} \end{bmatrix} = \left[j^{(k-r)} \right]$$

tends toward 0

half derivative part 3

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = \frac{f^{0}(a)}{0!}\cdot\frac{1}{2} + \frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2}\overset{*(x-a)^{\frac{1}{2}}}{} + \sum_{n=1}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) - \frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2}\overset{*(x-a)^{\frac{1}{2}}}{} = \frac{f^{0}(a)}{0!}\cdot\frac{1}{2} + \sum_{n=1}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$-\frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2}\overset{*(x-a)^{\frac{1}{2}}}{} = \frac{f^{0}(a)}{0!}\cdot\frac{1}{2} - f(x) + \sum_{n=1}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f^{\frac{1}{2}}(a) = \frac{-2\cdot\frac{1}{2}!}{(x-a)^{\frac{1}{2}}}\left(-f(x) + \frac{f(a)}{2} + \sum_{n=1}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n\right)$$

$f(x) = x^2$

$a = 0$

$x = 1$

$f'(x) = 2x$

$f''(x) = 2$

$$f_1^{\frac{1}{2}}(0) = \frac{-2\cdot\frac{1}{2}!}{(1-0)^{\frac{1}{2}}}\left(-1 + 0 + \underbrace{\frac{2\cdot 0}{1!}(1-0)}_{0} + \underbrace{\frac{2}{2!}(1-0)^2}_{+1}\right)$$

$$f_1^{\frac{1}{2}}(0) = 0$$

$a=0, x=2$

$$f_2^{\frac{1}{2}}(0) = \frac{-2\cdot\frac{1}{2}!}{\sqrt{2-0}}\left(-4 + 0 + 0 + \underbrace{\frac{2}{2!}(2-0)^2}_{4}\right)$$

$$= 0$$

$a=1, x=2$

$$f_2^{\frac{1}{2}}(1) = \frac{-2\cdot\frac{1}{2}!}{\sqrt{2-1}}\left(-4 + \underbrace{\frac{1^2}{2}}_{\frac{1}{2}} + \underbrace{\frac{2\cdot 1}{1!}(2-1)}_{2} + \underbrace{\frac{2}{2!}(2-1)^2}_{1}\right)$$

$3\frac{1}{2}$

$$= \frac{-2\cdot\frac{1}{2}!}{\sqrt{2-1}}\left(-\frac{1}{2}\right)$$

$$= \frac{1}{2}!$$

$a=1, x=3$

$$f_3^{\frac{1}{2}}(1) = \frac{-2\cdot\frac{1}{2}!}{\sqrt{3-1}}\left(-9 + \underbrace{\frac{1^2}{2}}_{\frac{1}{2}} + \underbrace{\frac{2\cdot 1}{1!}(3-1)}_{4} + \underbrace{\frac{2}{2!}(3-1)^2}_{4}\right)$$

$-9 + 8\frac{1}{2}$

$$= \frac{-2\cdot\frac{1}{2}!}{\sqrt{3-1}}\left(-\frac{1}{2}\right)$$

$$f_3^{\frac{1}{2}}(1) = \frac{\frac{1}{2}!}{\sqrt{2}}$$

half derivative part 1

$$f(x) = \sum_{n=0}^{\infty, ss=1} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) \approx \frac{f^{0}(a)}{0!}\cdot(x-a)^0\cdot\frac{1}{2} + \frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot(x-a)^{\frac{1}{2}}\cdot\frac{1}{2} + \sum_{n=1}^{\infty, ss=1} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$(x-a)^{\frac{1}{2}}\frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2} \approx -f(x) + f^{0}(a)\cdot\frac{1}{2} + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f^{(\frac{1}{2})}(a) \approx \frac{\frac{1}{2}!\cdot 2}{(x-a)^{\frac{1}{2}}}\left(-f(x) + \frac{f(a)}{2} + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n\right)$$

$$\underset{c=1}{f(x)} = x^2 = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)\cdot(x-1)^n}{n!} = \frac{1^2\cdot(x-1)}{0!} + \frac{2\cdot 1\cdot(x-1)}{1!} + \frac{2\cdot(x-1)^2}{2!}$$

$x=2$ $\qquad x^2 = \frac{1^2\cdot(x-1)}{0!}\cdot\frac{1}{2} + \frac{f^{\frac{1}{2}}(1)}{\frac{1}{2}!}(x-1)^{\frac{1}{2}}\cdot\frac{1}{2} + 2(x-1)$

$f^{(\frac{1}{2})}(x) =$

$$4 = 2\cdot\frac{1}{2} + \frac{f^{\frac{1}{2}}(1)\,(2-1)^{\frac{1}{2}}}{\frac{1}{2}!}\cdot\frac{1}{2} + \frac{(x-1)^2}{2}$$

$$-\frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2} + f(x) = \frac{f^{0}(a)}{0!}\cdot\frac{1}{2} + \sum_{n=1} f^{n}(a)\ldots$$

$$4 = \frac{1}{2} + \frac{f^{\frac{1}{2}}(1)}{\frac{1}{2}!\cdot 2} + 3 \qquad 2+1$$

$$4 = 3\tfrac{1}{2} + \frac{f^{\frac{1}{2}}(1)}{\frac{1}{2}!\cdot 2} \qquad f^{(\frac{1}{2})}_{c=1}(2)$$

$$2\cdot\frac{1}{\frac{1}{2}!}\left(\frac{f^{0}(a)}{0!}\cdot\frac{1}{2} - f(x) + \sum f^{n}\ldots\right)$$

$$\frac{1}{2} = \frac{f^{\frac{1}{2}}(1)}{\frac{1}{2}!\cdot 2} \qquad \frac{1}{2}! = f^{\frac{1}{2}}(1)$$

$$\frac{1}{2}! = \frac{d^{\frac{1}{2}}(x^2)}{dx^{\frac{1}{2}}}(1)$$

$$\boxed{f^{(1/2)}_{1}(2) = \frac{1}{2}!}$$

plug in $x=2$

Eln P3

$$(x!) = \left(\sum_{n=0}^{\infty} \frac{\left(\sum_{i=1}^{x} (\ln(i))\right)^n}{n!} \right) = e^{\sum_{i=1}^{x} (\ln(i))}$$

$$2! = \sum_{n=0}^{\infty} \frac{\left(\sum_{i=1}^{2} (\ln(i))\right)^n}{n!}$$

$$= \left(\sum_{i=1}^{2} (\ln(i))\right)^0 + \sum_{i=1}^{2} (\ln(i)) + \frac{\left(\sum_{i=1}^{2} (\ln(i))\right)^2}{2!} + \cdots$$

$$= 1 + \frac{\ln(1)+\ln(2)}{\ln(2)} + \frac{\ln(2)^2}{2!} + \frac{\ln(2)^3}{3!} + \cdots$$

$$= 1.9897 + \cdots$$

$$= e^{\ln(2)}$$

$$\prod_{n=0}^{\infty} f(x) = e^{\sum_{n=0}^{\infty} \ln(f(x))}$$

$$\sum_{n=0}^{\infty} f(x) = \ln\left(\prod_{n=0}^{\infty} e^{f(x)}\right).$$

$$2! \approx \sum_{n=0}^{\infty} \frac{\left(\int_1^2 (\ln(i))\,di\right)^n}{n!}$$

$$x! \approx \sum_{n=0}^{\infty} \frac{\left(\int_1^x \ln(i)\,di\right)^n}{n!}$$

$$2! \approx \sum_{n=0}^{\infty} \frac{(2\ln(2)-1)^n}{n!}$$

Maybe not

$$2! \approx e^{(2\ln(2)-1)}$$

$$x! \approx e^{\int_1^x (\ln(i))\,di} \; ; x \geq 0$$

$$2! \approx 1.4715$$

$$e^{\int_1^x (\ln(i))\,di} \neq \Gamma(x+1) \quad x \notin \mathbb{I}$$

E ln P4

$$f(x) = e^{\sum_{i=1}^{x} \log(i)}$$

$$h(x) = e^{\int_1^x \log(i)\, di}$$

$$0 \overset{?}{<} \lim_{x\to\infty} \left(\frac{f(x)}{h(x)} \right) \overset{?}{<} \infty$$

$$\lim_{x\to\infty} \frac{e^{\sum_{i=1}^{x} \log(i)}}{e^{\int_1^x \log(i)}} = \lim_{x\to\infty}$$

$HCOOH + NaOH \rightarrow NaCOOH + H_2O$

M: ~~2~~ ? -1105

L: 2.02 .03543

Q: ~~.004~~ .00391,5

$H_3PO_4 + 3NaOH \rightarrow 3H_2O(l) + Na_3PO_4$

L 2.026 .02463

.120

Q .0029556

~~$Na\,C_2O_7$~~ HC

$NaHCO_3 + HCl \rightarrow NaCl + H_2CO_3$

$H_2CO_3 \rightarrow H_2O + CO_2$

9 2.5 ~~2.7~~ ?

Eln rl

$$e^{\sum_{n=0}^{\infty} \ln\left(\frac{f^{(n)}(a)}{n!}(x-a)^n\right)} \stackrel{?}{=} \prod_{n=0}^{\infty}\left(\frac{f^{(n)}(a)}{n!}(x-a)^n\right)$$

$$e^{(\ln(f(a)) + \ln(f'(a)(x-a)) + \dots}$$

$$e^{(\ln(f(a)\cdot f'(a)(x-a)\cdot \dots)}$$

$$f(a)\cdot f'(a)(x-a)\cdot \frac{f''(a)}{2!}(x-a)^2 \cdot \dots$$

Alternate Principal Normal unit vector M1P1

$$\vec{N}(t) \overset{?}{=} \lim_{h \to 0} \frac{\vec{r}(t) \times (\vec{r}(t-h) \times \vec{r}(t))}{|\vec{r}(t) \times (\vec{r}(t-h) \times \vec{r}(t))|}$$

$$\vec{N}(t) = \frac{\frac{T'(t)}{dt}}{\left|\frac{T'(t)}{dt}\right|} = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} \qquad \vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{r}(t) \times (\vec{r}(t-h) \times \vec{r}(t)) \underset{h \to 0}{\overset{?}{\propto}} \vec{T}'(t)$$

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = \langle \underset{-1}{8 - 9}, \underset{2}{(4-6)}, \underset{-1}{3-4} \rangle \qquad \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\frac{d\, f(x)}{dx} = \lim_{h \to 0} \left(\frac{f(x) - f(x-h)}{h} \right)$$

$$\vec{r}(t) \times (\vec{r}(t-h) \times \vec{r}(t)) = CR(t) = [a(t), b(t), c(t)]$$

$$a(t) = -x(t)z(t)z(t-h) - x(t)y(t)y(t-h) + (z(t)^2 + y(t)^2)x(t-h)$$

$$b(t) = -y(t)z(t)z(t-h) + (z(t)^2 + x(t)^2)y(t-h) - x(t)y(t)x(t-h)$$

$$c(t) = (y(t)^2 + z(t)^2)z(t-h) - y(t)z(t)y(t-h) - x(t)z(t)x(t-h)$$

$$|CR(t)| = \sqrt{z(t)^2 + y(t)^2 + x(t)^2} \cdot \mathrm{sqrt}\big((y(t)^2 + x(t)^2)z(t-h)^2 + (-2y(t)z(t)y(t-h)$$
$$- 2x(t)z(t)x(t-h))z(t-h) + (z(t)^2 + x(t)^2)y(t-h)^2 - 2x(t)y(t)x(t-h)y(t-h)$$
$$+ (z(t)^2 + y(t)^2)x(t-h)^2\big)$$

$\dfrac{\vec{U}(t)}{|\vec{U}(t)|}$ is always of unit length $\qquad \left|\dfrac{\vec{U}(t)}{|\vec{U}(t)|}\right| = 1$

half derivative M2 r ~~0~~6

$$\frac{d^r x}{d x^r} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0) \cdot x^{(n-r)}}{\tilde{|}\,(n-r)} \qquad r<1$$

$$\text{b/c} \quad \frac{d^1 (x^0)}{dx^1} = 0 \neq x^{-1}$$

$$\tilde{|}(x) = \frac{1}{\tilde{|}(x)}$$

Tilde pipe

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n-r}}{\tilde{|}(n-r)} \; ; r<1$$

$$r<1$$

$$e^x = x^{(0-r)} \cdot \tilde{|}(0-r) + x^{(1-r)} \cdot \tilde{|}(1-r) + \dots$$

$$e^1 = 1^{(0-r)} \cdot \tilde{|}(0-r) + 1^{(1-r)} \tilde{|}(1-r) + \dots$$

$$e^2 = 2^{(0-r)} \cdot \tilde{|}(0-r) + 2^{(1-r)} \tilde{|}(1-r) + \dots$$

answer matrix

$$A = \begin{array}{c} \scriptstyle k\to \\ \begin{array}{c} j=1 \\ j=2 \\ j=3 \\ \\ j=\infty \end{array} \begin{bmatrix} e^1 \\ e^2 \\ \vdots \\ e^j \end{bmatrix} \end{array} = \left[e^j \downarrow j \right]$$

Variable matrix

this what were solving for

$$V = \begin{array}{c} \scriptstyle k\to \\ \begin{array}{c} j=1 \\ j=2 \\ \\ j=\infty \end{array} \begin{bmatrix} \tilde{|}(0-r) \\ \tilde{|}(1-r) \\ \vdots \\ \tilde{|}(j-1-r) \end{bmatrix} \end{array} = \left[\tilde{|}(j-1-r) \downarrow j \right]$$

$\left[f(j) \; \cancel{exx} \downarrow j \right]$ means that ~~each elem~~ it is column vector with values of $f(0), f(1), \dots f(j)$

Coefficient matrix

$$r<1$$

$$C = \begin{array}{c|cccc} & k=1 & k=2 & & k=\infty \\ \hline j=1 & 1^{(0-r)} & 1^{(1-r)} & \cdots & 1^{(k-1-r)} \\ j=2 & 2^{(0-r)} & 2^{(1-r)} & \cdots & 2^{(k-1-r)} \\ & \vdots & \vdots & \ddots & \vdots \\ & j^{(0-r)} & j^{(1-r)} & \cdots & j^{(k-1-r)} \end{array} = \left[j^{(k-1-r)} \; \underset{\downarrow j}{\overset{k}{\to}} \right]$$

maxima: ans : genmatrix(lambda([j,k], %e^j), size, 1)

coef: genmatrix(lambda([j,k], j^(k-1-r)), size, size)

Vars: ~~for~~ linsolve_by_lu(coef, ans)

can't go over twelve with my machine

GenPipeTilde(r, size) := 1/float(linsolve_by_lu(genmatrix(lambda([j,k], j^(k-1-r)), size, size), genmatrix(lambda([j,k], %e^j), size, 1)))

$(a+1)^6$ $\quad (a+b)^0 = 1 \quad (a+b)^1 = a+b \quad (a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 = (a^2+2ab+b^2)(a+b)$

$a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$a^3 + 3a^2b + 3ab^2 + b^3$

coefficients

1 1

1 2 1

1 3 3 1

1 4 6 4 1

powers

0

0,0

1,0 0,1

2,0 1,1 0,2

3,0 2,1 1,2 0,3

$Z_n = aZ_{n-1} + b \qquad Z_0 = c$

$\sum_{z=0}^{n-1} a^z + b$

$b\sum_{z=0}^{n-1} a^z$

$aZ_{n-1} + c$

$c \cdot a^n + b \cdot a^n + b$

$+ b\sum_{z=0}^{n-1} a^z$

$a_n = 2a_{n-1} + 2 \qquad a_0 = 2$

$0: 2$ $\nearrow 4$

$1: 2 \cdot 2 + 2 = 6 \searrow 8 \qquad \cancel{2 \cdot 2^n + 2n}$

$2: 2 \cdot 6 + 2 = 14 = 2(2 \cdot 2 + 2) + 2$ $\nearrow 16$

$3: 2 \cdot 14 + 2 = 30 = 2(2 \cdot (2 \cdot 2 + 2) + 2) + 2$ $\nearrow 32$

$4: 2 \cdot 30 + 2 = 62$

$a_{n-1} = b$

$2^{n+2} - 2 \qquad 2b + 2$

$\ln(2)\ln(b) + \ln(2)$

$a_0 = 2$

$2b + 2 \rightarrow 2^{n+2} - 2 \qquad \ln(b^2 \cdot 2)$ or $(\ln(2^b \cdot 2))$

$\leftarrow \qquad \ln(2^{b+1}$

b c D

$a_n = 2a_{n-1} + 1 \qquad a_0 = 3 \qquad a_0 = 0$

$0: 3$ $\nearrow 2$

$1: 2 \cdot 3 - 1 = 5$ $\nearrow 4$

$2: 2 \cdot 5 - 1 = 9$ $\nearrow 8$

$3: 2 \cdot 9 - 1 = 17$ $\nearrow 16$

$4: 2 \cdot 17 - 1 = 33$

$0: 0 \quad \nearrow -1$

$1: 2 \cdot 0 - 1 = -1 \nearrow -2$

$2: 2 \cdot -1 - 1 = -3 \nearrow -4$

$3: 2 \cdot -3 - 1 = -7 \nearrow -8$

$4: 2 \cdot -7 - 1 = -15$

$0: 1$
$1: 2$ $\nearrow 1$
$2: 4$ $\nearrow 2$

$2^{n+1} + 1 \qquad -2^n + 1$

$\ln(a + b) = \ln(a) \geq \ln(b) \qquad \ln(a) \overset{+}{\downarrow} \ln(b)$

$ELN(x^2 + 3x^4 - 2 + e^x) = \ln$

$ELN(0) = e^{\ln(0)} = 0$

$ELN(2x) = e^{\ln(2) \cdot \ln(x)} = e^{\ln(x^{\ln(2)})}$

$= e^{\ln(2^{\ln(x)})} = x^{\ln(2)}$

$= 2^{\ln(x)}$

$ELN(1) = e^{\ln(1)} = 1$

$ELN(x) = e^{\ln(x)} = x$

$ELN(x+1) = e^{\ln(x) + \ln(1)} = e^{\ln(1 \cdot x)} = x$

$ELN(x+2) = e^{\ln(x) + \ln(2)} = e^{\ln(2x)} = 2x$

Left-associative $f(n)$

$$\bigwedge_{n=1}^{\infty}(2$$

$$f(x)=\bigwedge_{n=1}^{\infty}\left(1+\frac{-1}{n}^{n+1}\right)=\sqrt{2}$$

$$\left(1+\frac{1}{1}\right)^{\left(1-\frac{1}{2}\right)^{\left(1+\frac{1}{3}\right)^{\cdots}}}$$

$\sqrt{2}$, 1.5874, $\sqrt{2}$, 1.5157

$$f(n+1)=f(n)^{\left(1+\frac{-1}{n}^{n+1}\right)}$$

$$f(0)=\left(1+\frac{+1}{1}^{7+1}\right)$$

$n+(-1)^n$

$$\bigwedge_{n=1}^{\infty}\left(1+\frac{-1}{xn}^{(n+1)}\right)$$, Converges to

2, $2\wedge\frac{1}{2}$, $2\wedge\frac{1}{2}\wedge\frac{4}{3}$, $2\wedge\frac{1}{2}\wedge\frac{4}{3}\wedge\frac{3}{4}$, $2\wedge\frac{1}{2}\wedge\frac{4}{3}\wedge\frac{3}{4}\wedge\frac{6}{5}$

$2\wedge\frac{4}{6}$

$2\wedge\frac{2}{3}$, $2\wedge\frac{2}{3}\wedge\frac{3}{4}$

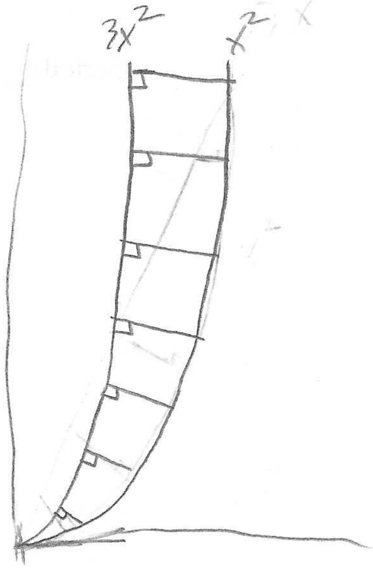
$$e^{f(\ln(x))}$$

$f(x)=5x+3$

$5\ln x+3$

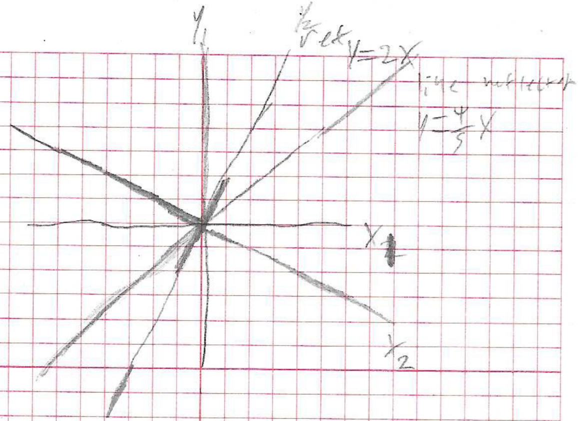
$\ln x^5 \cdot 3$

$x^5 \cdot 3$



find cross sectional area

cross sections are squares, perpendicular to x^2



$\frac{a}{b}$ = a written respect to b / a measured with b

$$1x_2 = \sin(\operatorname{atan}(\tfrac{dx_2}{x_1}))y_1 + \cos(\operatorname{atan}(\tfrac{dx_2}{x_1}))x_1$$

$$1y_2 = \sin(\operatorname{atan}(\tfrac{dy_2}{x_1}))y_1 + \cos(\operatorname{atan}(\tfrac{dy_2}{x_1}))x_1$$

$$1x_1 = \sin(\operatorname{atan}(\tfrac{dx_1}{x_2}))y_2 + \cos(\operatorname{atan}(\tfrac{dy_1}{x_2}))x_2$$

testing flipping over given line

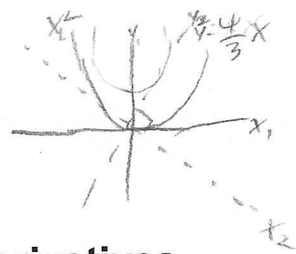
$y_2 = 2x$ $\qquad\qquad$ $f(x) = x^2$

$$1\,y_2 = x_1 \cdot \overset{\frac{1}{\sqrt{5}}}{\cos(\text{atan}(2))} + y_1 \cdot \overset{\frac{2}{\sqrt{5}}}{\sin(\text{atan}(2))}$$

$$1\,x_2 = 1x_1 \cdot \sin(\text{atan}(2)) - 1y_1 \cdot \cos(\text{atan}(2))$$

$y = x^2$

$G(y) = f(x)$ ~~f(x~~ $G(y) = f(x)$



$x_1 = \frac{\sqrt{3}}{2} x_2 + \frac{1}{2} y_2$

$y_1 = -\frac{1}{2} x_2 + \frac{\sqrt{3}}{2} y_2$

$(-\frac{1}{2} x_2 + \frac{\sqrt{3}}{2} y_2) = (\frac{\sqrt{3}}{2} x + \frac{1}{2} y)^2$

Applications of Derivatives – Summary for interpreting graphs

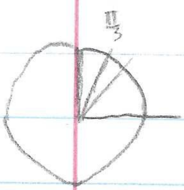
Review: What do the derivatives tell us?

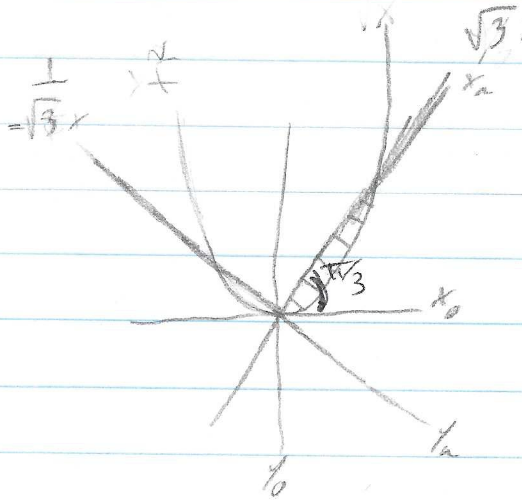
Derivative	What does it tell us?
1st	The rate of change of the function: instantaneous slope of the tangent to the curve
2nd	The rate of change of the slope of the tangent to the curve (1st derivative): concavity of the curve

On a graph of:	(What to look for) If:	(What it means) then:
f(x) (original function)	f(x) is increasing	f'(x) is positive
	f(x) is decreasing	f'(x) is negative
	f(x) is an extrema or flat	f'(x) is zero (critical point)
	f(x) is concave up	f''(x) is positive
	f(x) is concave down	f''(x) is negative
	The concavity of f(x) is changing	f''(x) is zero (point of inflection)
f'(x) (1st derivative)	f'(x) is positive	f(x) is increasing
	f'(x) is negative	f(x) is decreasing
	f'(x) is zero	f(x) is a min (- → +) or a max (+ → -)
	f'(x) is increasing	f''(x) is positive (f(x) is concave up)
	f'(x) is decreasing	f''(x) is negative (f(x) is concave down)
	f'(x) is an extrema or is flat	f''(x) is zero (f(x) is a point of inflection)
f''(x) (2nd derivative)	f''(x) is positive	f(x) is concave up
	f''(x) is negative	f(x) is concave down
	f''(x) is zero	f(x) is a point of inflection

$\frac{x}{y}$ = x in respect to y

crossections = squares perpendicular to $\sqrt{3}x = y$





$$x_n = \frac{\sqrt{3}}{2}y_0 + \frac{1}{2}x_0$$
$$y_n = \frac{1}{2}y_0 - \frac{\sqrt{3}}{2}x_0$$
$$x_0 = -\frac{\sqrt{3}}{2}y_n + \frac{1}{2}x_n$$
$$y_0 = \frac{1}{2}y_n + \frac{\sqrt{3}}{2}x_n$$

$\sqrt{3}\,x = \frac{\pi}{3}\alpha$

$-\frac{1}{\sqrt{3}}x = \frac{5\pi}{6}\alpha$

$\frac{y_0}{x_n} = \frac{\pi}{6}\alpha$

$\frac{x_0}{x_n} = -\frac{\pi}{3}\alpha$

$$y_0 = \sqrt[4]{x_0}$$

$$\frac{1}{2}y_n + \frac{\sqrt{3}}{2}x_n = \sqrt[4]{-\frac{\sqrt{3}}{2}y_n + \frac{1}{2}x_n}$$

solve for y_n

$$y_n = \left(\frac{\sqrt[4]{8\sqrt{3}\,x_n + 1} + \sqrt{3}x_n + 1}{3}\right)$$

intersects @ x_0 is @ $0, \sqrt{3}$

$$\sqrt{(\sqrt{3}-0)^2 + (\sqrt{3}\cdot\sqrt{3} - \sqrt{3}\cdot 0)^2}$$

$$\int_0^{2\sqrt{3}} \left| \left(\frac{\sqrt{8\sqrt{3}\,x_n + 1} + \sqrt{3}x_n + 1}{3}\right)^2 \right| dx$$

$\sqrt{3+9}$ $\quad \sqrt{12}$

$2\sqrt{3}$

$$= \frac{3^{3/2}}{20}$$

intersects @ $0, 2\sqrt{3} \sim x_n$

$f(\log_a(u)) = \frac{1}{u' \cdot \ln(a)}$

$$\frac{d}{dx}\left[\log_a(u)\right] = \frac{1}{u \ln(a)} \cdot u' = \frac{d}{dx}\left[\frac{\ln(u)}{\ln(a)}\right]$$

so a'?

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x(\ln e)}$$

$$\frac{d}{dx}\left[a^u\right] = a^u \ln(a) \cdot u'$$

$$\cancel{\frac{d}{dx}\left[a^u\right] = a^u \cdot \ln(a) \cdot u'}$$

$$\int a^x \, dx = a^x \left(\frac{1}{\ln(a)}\right) + C$$

$$\int e^x \, dx = e^x\left(\frac{1}{\ln e}\right) + C$$

$$\frac{\ln(1+\frac{h}{u})}{\ln(a)} \cdot \frac{1}{h} \quad \lim_{h\to 0} \frac{\ln((1+\frac{h}{u})^{\frac{1}{h}})}{\ln(a)} \quad \frac{\ln(e)}{\ln(a)} = \frac{1}{\ln(a)}$$

$f(x) = k\sqrt{x} - \ln x$

$$\frac{\log_a(\frac{u+h}{u})}{h} \quad \frac{\log_a(1+\frac{h}{u})}{h}$$

$$\lim_{h\to 0}\left(\frac{\log_a(u+h) - \log_a(u)}{h}\right)$$

$$\frac{d}{dx}\left[\frac{\ln(u)}{\ln(a)}\right] = \frac{-\frac{1}{a}a' \cdot \ln(u) + \frac{1}{u}u' \cdot \ln(a)}{\ln(a)^2} = \frac{1}{u\ln(a)} \cdot u'$$

$$\frac{\log_a(u+h)}{h} - \frac{\log_a(u)}{h}$$

$a=2$ $$\frac{-\frac{1}{2} \cdot \ln(u) + \frac{1}{u}u' \cdot \ln(2)}{\ln(2)^2}$$

$$\frac{\left(\frac{\ln(u+h)}{\ln(a)}\right)}{h} - \frac{\left(\frac{\ln(u)}{\ln(a)}\right)}{h}$$

$$\frac{-\frac{1}{2} \cdot \ln(u) + \frac{1}{u}u' \cdot .69314}{(.693)^2}$$

$$\frac{\left(\frac{1}{\ln(a)}\right)}{h}\left(\ln(u+h) - \ln(u)\right)$$

$$\frac{-\frac{1}{2} \cdot \ln(u)}{(.693)^2} + \frac{\frac{1}{u}u' \cdot .69314}{(.693)^2}$$

$$\lim_{h\to 0} \frac{h}{\ln(a)}\left(\ln(u+h) - \ln(u)\right)$$

$$\frac{\frac{1}{u}u'}{\frac{.693}{\ln(2)}} - \frac{\ln(u)}{2\ln(2)^2}$$

$$h^2 \cdot \frac{1}{\ln(a)} \cdot \frac{1}{h}\left(\ln\left(\frac{u+h}{u}\right)\right)$$

$$\ln(1+\frac{h}{u})$$

$$\lim_{h\to 0} h^2 \cdot \frac{1}{\ln(a)} \cdot \left(\ln((1+\frac{h}{u})^h)\right)$$

$$\lim_{h\to 0} h^2 \cdot \frac{1}{\ln(a)}$$

Worksheet
Volumes of solids—Disk and washer method #2
AP Calculus

Name Davi Lamy

Find the volume of the solid formed by the equations:

$\sqrt{dx^2+dy^2}$

$dy = \frac{f'(x)}{dx} \cdot dx$

$\int \sqrt{(f'(x)\cdot dx)^2+dx^2}$

$\int \sqrt{f'(x)^2 \cdot dx^2 + dx^2}$

$\int \sqrt{(f'(x)^2+1)(dx^2)}$

$\int \sqrt{f'(x)^2+1} \cdot \sqrt{dx^2}$

$\int \sqrt{f'(x)^2+1} \cdot dx$

1.) $y = x^2$, $y = 0$, $x = 2$, is rotated about:

a.) the x-axis $\frac{32\pi}{5}$

b.) the y-axis 8π

c.) the line $y = 4$ $\frac{224\pi}{15}$

d.) the line $x = 2$ $\frac{8\pi}{3}$

2.) $y = 1 + \sqrt{x}$, $y = 1$, $x = 4$ is rotated about:

a.) the x-axis $\frac{56\pi}{3}$

b.) the y-axis $\frac{128\pi}{5}$

c.) the line $y = 3$ $\frac{40\pi}{3}$

d.) the line $x = 6$ $\frac{192\pi}{5}$

3.) $y = x^2$ and $y = \sqrt[3]{x}$ is rotated about:

a.) the x-axis $\frac{2\pi}{5}$

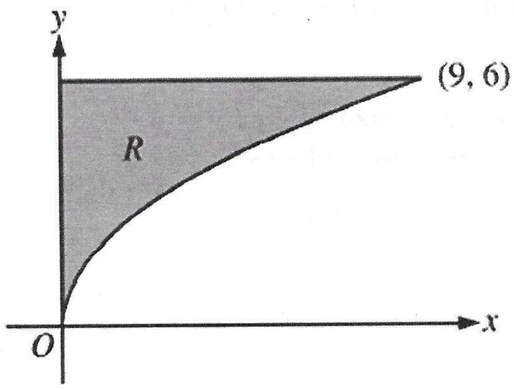
b.) the y-axis $\frac{5\pi}{14}$

c.) the line $y = 1$ $\frac{13\pi}{30}$

7-2 QUIZ AP CALCULUS AB NAME: David Crisby

You may NOT use your calculator on this problem.

length of line $= \int_a^b \left(\sqrt{f'(x)^2+1}\right)dx$



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

a) $\int_0^9 (6-2\sqrt{x})dx$

$6x - \frac{2x^{3/2}}{\frac{3}{2}}\Big|_0^9 = 6\cdot 9 - \frac{2\cdot 27}{\frac{3}{2}} \approx 18$

b) $\pi\int_0^9 \left((7-2\sqrt{x})^2 - (7-6)^2\right)dx$

~~X(i) → [illegible]~~

~~Y(i) →~~

$x_i \to x$ of ~~at~~ pair ~~spot~~ i

$y_i \to y$ ~~at~~ of pair i

~~F(x) =~~ $$g_n(x) = \prod_{i=1}^{n} (x - x_i)$$

$$F_1(x) = y_1$$

$$\lambda_n = \frac{y_{n+1} - F_n(x_{n+1})}{g_n(x_{n+1})}$$

$$F_n = F_{n-1}(x) + \lambda_{n-1} \cdot g_{n-1}(x)$$

$$F_n(x) = \left(F_{n-1}(x)\right) + \left(\frac{y_n - F_{n-1}(x_n)}{\prod_{i=1}^{n-1}(x_n - x_i)}\right) \cdot \prod_{i=1}^{n-1}(x - x_i)$$

1 2 3 4

1, 2   2, 5   4, 6   7, 2

4, 2

2, 3

$$g_n(x) = \prod_{i=1}^{n} (x - x_i)$$

$$F_1(x) = 2 = y_1 \qquad g_1(x) = (x - x_1) = x - 1$$

$$\lambda_1 = \frac{y_2 - F_1(x_2)}{g_1(x_2)} = \frac{5-2}{2-1} = \frac{3}{1}$$

$$F_2(x) = F_1(x) + \lambda_1 g_1(x)$$

$$F_2(x) = 2 + 3 \cdot (x-1)$$

$$g_2(x) = (x-1)(x-2)$$

$$\lambda_2 = \frac{6 - F_2(4)}{g_2(4)} = \frac{-5}{6}$$

$$F_3(x) = (2 + 3 \cdot (x-1)) + \left(\frac{-5}{6}(x-1)(x-2)\right)$$

$$g_3(x) = (x-1)(x-2)(x-4)$$

$$\lambda_3 = \frac{2 - F_3(7)}{g_3(7)} = \frac{7}{90}$$

$$F_4(x) = F_3(x) + \lambda_3 g_3(x) = f(x)$$

$$f(x) = \left((2 + 3 \cdot (x-1)) + \left(\frac{-5}{6}(x-1)(x-2)\right)\right) + \frac{7}{90}(x-1)(x-2)(x-4)$$

how to find Surface formula for region rotated about axis

Circ

$$A(x) = \int \theta \cdot C(x)\, dx$$

Surface area =

length that line $= \sum_{0}^{\infty, ss=r}$

for Sphere $l = \sqrt{x^2+y^2+z^2}$

Circumference circle $= 2\pi r$

$c = \sqrt{a^2+b^2}$

Surface area of rotated object

$$\int \sum_{0}^{b,\infty, ss=ra} 2\pi f(x) \qquad \sum$$

$$\Delta C = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\int_a^b 2\pi f(x)$$

$$\Delta C = \sqrt{dx^2 + (f'(x)\,dx)^2}$$

$$\sqrt{dx^2\,(1 + f'(x)^2)}$$

Meter: m $\qquad f(x) = x\,m$

Shape:

$\sqrt{1-x^2}$ $\qquad \sqrt{1-x^2}$

$\frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x)$

length of $f(x)$ $= \int \sqrt{1+f'(x)^2}\, dx$

m^2

$m \cdot m$

$$\int \sqrt{}$$

$$\cancel{\int \sqrt{1 + A'(f(x))^2}\, dx}$$

$$\int \sqrt{1 + \left(\frac{-x}{1-x^2}\right)^2}\, dx$$

$\sqrt{m^2}$

m

Surface area $= \int \sqrt{1 + A''(f(x))^2}\, dx$

$$\int \sqrt{1 + \frac{x^2}{1 - 2x^2 + x^4}}\, dx$$

$\int m$

Circ $= 2\pi r \qquad A = \pi r^2$

$(x - x^2)^2$

m^2

$\sqrt{1-x^2}$

$\frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x$

$f(x) = \sqrt{1-x^2} \cdot x$

$$\int$$

$\frac{x}{\sqrt{x}}$

$$\sum_{0}^{\infty, ss=r} \int \sqrt{1+f'(x)^2}\, dx \cdot r$$

$\frac{x^2}{x}$

$\sqrt{x}$

Surface area $\frac{?}{x} \iint \sqrt{1+f'(x)^2}\, dx\, dy$

infinite Sum of ~~constant~~ shape

are constant shape function

when shape function's the radius of $f(x)$

Surface area $= \int \sqrt{1 + S_f'(f(x))^2}\, dx$

S_f = Shape function

$$\sum_{n}^{b, ss=h} \sum$$

$$\sum_{h=a}^{b, ss=h} \left(\int_{\text{Solve}(S(h)=0,\, h)[0]}^{\text{Solve}(S(h)=0,\, h)[1]} h\sqrt{1 + S'(x)^2}\, dx \right)$$

~~more~~ more in page 2 of 2nd assignment

$s^{-1}(n)$ = inverse of $s(n)$

$s(n)$ is shape func.

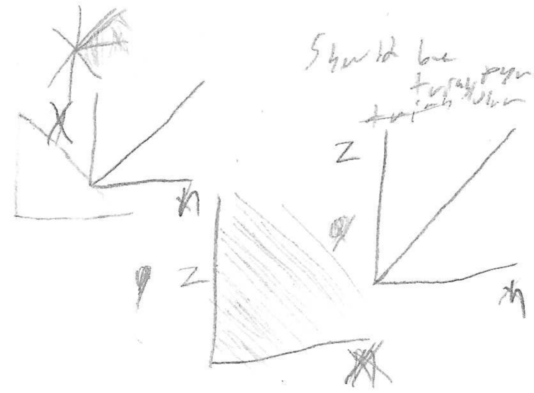
$$\int_a^b \int_{f(n)\cdot s_{(0)}^{-1}(0)[0]}^{f(n)\cdot s^{-1}(0)[1]} f(n)\sqrt{1+s'(x)^2}\,dx\,dn = \text{Surface area of function with a shape [illegible]}$$

Summation ~~of~~ of summation of hypotenuses, with hypotenuses growing by $f(x)$

$f(n) = n$

$s(x) = x$

$$\int_0^1 \int_0^{s(n)} n \cdot \sqrt{1+1^2}\,dx\,dn$$

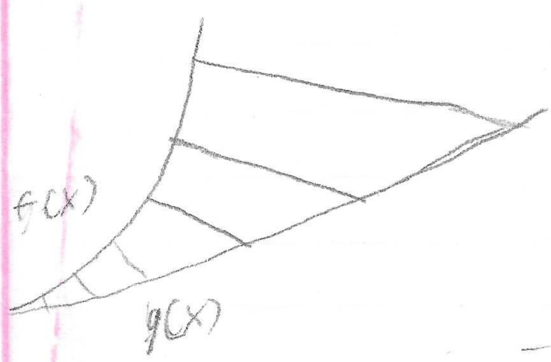


$$\int_a^b f(n) \int_c^d f \cdot \sqrt{1+s'(x)^2}\,dx\,dn$$

$$\int_a^b \left(f(n) \int_c^d \left(\sqrt{1+s'(x)^2}\right) dx\right) dn$$

not sure what c and d should be

Cross sections are square



C_x = Current X

- find line perpendicular to f(x) at x_i = P(x)
- find intercept of P(x) and g(x)
- Calculate length between intercept of P(x) and f(x), and intercept of P(x) and g(x)
- square result & multiply by ΔX
- repeat infinitely
- sum 'em up

$$P(x) - f(C_x) = -\frac{1}{f'(x)}(x - C_x)$$

$$P(x) = -\frac{1}{f'(x)}(x - C_x) + f(C_x)$$

$P(x) = g(x)$ x_i = the point that this happens

$\sqrt{(f(C_x) - g(x_i))^2 + (C_x - x_i)^2}$ = length between the two points

$$= \int \left((f(C_x) - g(x_i))^2 + (C_x - x_i)^2\right) dC_x$$

$$f(x,y) = \pm\sqrt{y^2 - f(x)^2} \qquad \underset{\pi \int f(x)^2\,dx}{\overset{g(x)}{\parallel}} = \int_{z=-f(x,0)}^{z=f(x,0)} f(x,y)$$

$2x+1 = f(x)$

$g(x)$

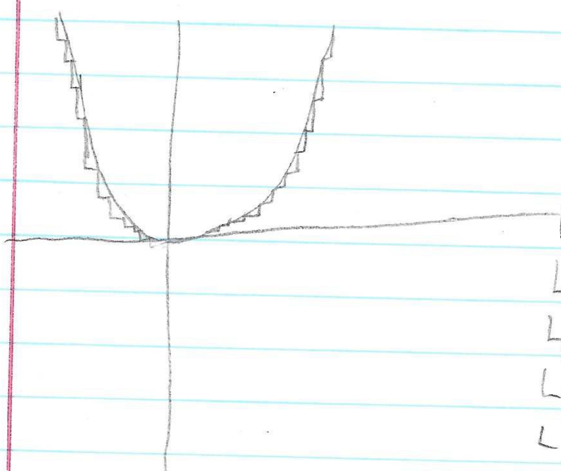
$$\pi \int f(x)^2\,dx = \iint f(x,y)\,dA$$

if $f(x)$ is continous

length of line on interval $[A,B] = \sum_{X=A}^{B} \left(\sqrt{(f(x+h)-f(x))^2+h^2}\right)$; $h \to 0$

$$\sum_{X=A}^{B\cdot\frac{1}{h}} \left(\sqrt{(f(xh+h)-f(xh))^2+h^2}\right)$$

Sum of the hypotenuses at ~~the~~ infinitly small distances.



step size $= .25$

$f(x) = x^2$

interval $= [-1,1]$

$L = G(x) = \sqrt{(f(x+.25)-f(x))^2+.25^2}$

$L_1 = \sqrt{(f(-1+.25)-f(-1))^2+.25^2} = .50389$

$L_2 = \sqrt{(f(-.75+.25)-f(-.75))^2+.25^2} = .40019$

$L_3 = G(-.5) = .3125$

$L_4 = G(-.25) = .25769$

$L_5 = G(0) = .25769$

$L_6 = G(.25) = .3125$

$L_7 = G(.5) = .40019$

$L_8 = G(.75) = .50389$

$L_T = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8$

$L_T = 2.94854$

$$\sum_{X=-1\cdot\frac{1}{.25}}^{1\cdot\frac{1}{.25}-1} G(X\cdot .25)$$

if $f(x)$ is continous

length of line $[A,B]$ on $f(x) =$

$$\sum_{X=A\cdot\frac{1}{h}}^{B\cdot\frac{1}{h}-1} \sqrt{(f(h(x+1))-f(xh))^2+h^2}\ ;\ h \to 0 \qquad \sum_{X=A\cdot\frac{1}{h}}^{B\cdot\frac{1}{h}-1} G(x\cdot h),\ \text{stepsize} = h$$

Sec 001-2442 | 0P 28 | Orsay, David   2

Q1 cont

47 $\sum F_x = m a_x$ $F^g_{e,O} = -9.8 \cdot .2\,kg$

$F^f_{ramp,O} + F^g_{e,O,x} = m a_x$ $a_{rot} = \frac{a_x}{r}$

$\sum T = \cancel{I\alpha} \; I a_{rot} = F^s_{r,O} \cdot \underset{R}{.05m} = I \frac{a_x}{r}$

$F^s_{r,O} = \frac{2}{5} m \cdot \cancel{R^2} \cdot \frac{a_x}{\cancel{R^2}}$

$\frac{2}{5} m a_x + mg \sin(\frac{\pi}{6}) = m a_x$

$\dfrac{+ -9.8\frac{m}{s^2} \sin(\frac{\pi}{6})}{\frac{2}{5}} = a_x = -8.17 \frac{m}{s^2}$

3. **(10 points)** Consider the surface S in $\mathbb{R}^3$ given by the equation $y - x^2 = 0$.

(a) Describe the intersection of S with the xy-plane (that is, the plane with $z = 0$). What kind of curve is it?

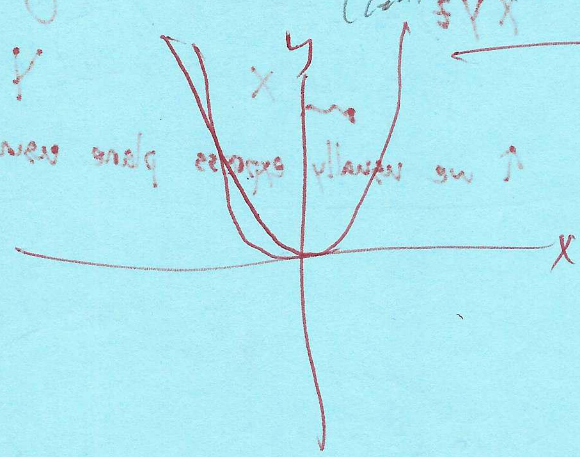
$y - x^2 = 0$

$y = x^2$

it be parabola ~~when~~ when

$z = 0$

(center at origin)



we usually express plane using x,y,z variables not a,b,c.

5

(b) Describe the intersection of S with the xz-plane (that is, the plane with $y = 0$). What kind of curve is it?

$0 - x^2 = 0$

$x = \pm 0 = 0$

it is a straight line where

~~$\{(a,b) \in \mathbb{R}^2 : a = 0,$~~ ? ?

$\{0\} \times \mathbb{R}$ is the points (x, z)

$\{(0, 0, z) : z \in \mathbb{R}\} = z\text{-axis}$

+3.5

8.5

David Is on Campuss.

$\lim_{x \to \infty}$

$= 1$

$$\frac{\cancel{f(x)}}{\cancel{f(x)}}$$

OGL M1 P3 cyesite logic

english: there exists a function op(a,b) such that it ~~can be used~~ a composition of op, a, and b can create all other f(a,b), a,b in an arbitrary finite set U, and op must be closed in U. for op to be an OGLG, turn it by the above ~~if else~~ ~~[illegible]~~ ~~op~~ to do this $OP(A,A) \not\equiv A$ and $OP(A,B) \not\equiv A$ and $OP(A,B) \not\equiv B$

```
OP_N(A,B) = if (A=B)
        ~~return~~ return (A-1)%N
    else
        return 0.

Maxima:   OP(a,b,n) := if a=b then
                 mod((a-1), n)
              else
                 0

test(f,n) := genmatrix(lambda([i,j], f(i,j)), n,n)   //prints truth table
```

f (a,b)

from MIP 4OGL

C_7 A	0	0	0	0
C_6 B	0	0	0	0
	0	0	2	0
	0	0	0	0

C_9

C_7 B	0	1	1	1
C_7 A	1	1	1	1
	1	1	2	1
	1	1	1	3

C_8

C_5 A	1	3	3	3
C_5 B	3	3	3	3
C_{10}	3	3	3	3
	3	3	3	3

C_6 A	0	3	3	3
C_6 B	3	3	3	3
	3	3	2	3
	3	3	3	3

C_4

OGL m2 p5

~~Let $U \subseteq Z$~~ : Let $U_p = \{x \in Z : 0 \leq x \leq p\}$

$Op_p : U \times U \to U$ for arbitrary $p \in \{x \in Z : x > 1\}$

$Op_p(a,b) =$ if $(a=b)$
$(a-1) \bmod (p)$
else
0

Op_p onto: Chose arbitrary $c \in U$

$c = Op_p(a,b)$ $m = \text{mod}$

$c =$ if $(a=b)$
$(a-1) \bmod (p)$
else
0

$((a+b) \bmod (c) + d) \bmod (c)$ ~~$c = Op_p$~~

$\stackrel{?}{=} (a+b+d) \bmod (c)$

$c = Op_p((c+1) \bmod (p), (c+1) \bmod (p))$

$c =$ if $(c(c+1)m(p) = (c+1)m(p))$
$(c(c+1)m(p)-1)m(p)$
else 0

$a \bmod b \equiv c$

~~$a = cb$ $c \in Z$~~

$c = a + bm$ $m \in Z$

$c = (c(c+1)m(p)-1)m(p)$

$c \equiv (c(c+1) + p \cdot m) - 1) \bmod (p)$

$c \equiv c \bmod (p)$

Op_p onto.

want

AB	0	1	2
0	0	2	2
1	0	1	0
2	1	1	0

AB	0	1	2
0	1	0	0
1	1	2	1
2	2	2	1

$\xrightarrow{a}$ $\xrightarrow{b}$

	0	1	2
0	0	2	2
1	0	1	0
2	1	1	0

OGL MIP2 (?-gate logic)

~~$(\forall a \in U)(OP(A,A) \not\equiv A)$~~

$(\forall a \in U)(\forall b \in U)\ (OP(A,B) \not\equiv A) \wedge (OP(A,B) \not\equiv B)$

U is the set of options i.e. binary $\{T, F\}$, trinary $\{2, 1, 0\}$

$(\forall a \in U)(\forall b \in U)(\forall f(a,b) \in \mathbb{F}_2(U))(\exists\ op(a,b) \in \mathbb{F}_2(U))$

$\mathbb{F}_2(U)$ is the set of function in U with arity of 2.

swap these two

$(\forall a,b \in U)(\exists\ op(a,b) \in \mathbb{F}_2(U))((\forall f(a,b) \in \mathbb{F}_2(U))(\exists$ a composition of op, called $g)(g(a,b) \equiv f(a,b)))$

furthermore

$(\forall c \in U)(OP(c,c) \not\equiv c) \wedge (\forall d,e \in U)((OP(d,e) \not\equiv d) \wedge (OP(d,e) \not\equiv e))$

~~there exists a function $op_{(a,b)}$ for the set of~~

an OP of size N

$A \backslash B$	0	1	2	3	...	
0	N	0	0	0	...	0
1	0	0	0	0	...	0
2	0	0	1	0	...	0
3	0	0	0	2	...	0
⋮	⋮	⋮	⋮	⋮	⋱	
⋮	0	0	0	0	...	A−1

When $A = B$

$OP(A, B) = (A-1)\ \%\ N$

else

0

I think this truth table works for all N

$OP_N(A,A) \equiv (A-1)\ \%\ N$

~~$OP_N(OP(A,A), OP(B,B)) \equiv OP_{N-1}(A,B),\ A,B \in (N-1)$~~

if I continue[?] ... to get to

$A \backslash B$	0	1	2	3	...
0	N−1				
1		0			
2			1		
3				2	... N−1

, called OP_{n-1}

then I should be able to say that if OP_n ~~function~~ is OGl, then OP_{n-1} is an OGl for $N-1$, and ~~if~~ ~~going down~~ OP_{n-2} is OGl, ..., OP_2 is OGl.

OGL m2 p6

Want

A\B	0	1	2
0	1	2	0
1	1	2	0
2	2	1	1

Given

B

A\B	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2

A

A\B	0	1	2
0	0	0	0
1	1	1	1
2	2	2	2

0 2 2

Op

A\B	0	1	2
0	2	0	0
1	0	0	0
2	0	0	1

0 1 0
0 1 0
0 0 0

0 0 0
0 1 0
2 0 0

a
2 0 2
2 0 2
0 2 2

b
2 0 0
2 0 1
0 2 2

	0	1	2
0	1	2	0
1	1	2	0
2	2	1	1

lookat

OGL m3 P3

Thm 1 scratch maxima:

```
OR(a,b) := block([], if a >= b then return(a) else
        return(b));

Pass(a,b,base) := block([], if a = base then return(b) else return(0));

f(a,b) := [[1,2,3],[4,5,6],[7,8,9]][a+1][b+1];

g(a,base) := f(base,a);

h(A,B) := OR(OR(Pass(A,g(B,0),0), Pass(A,g(B,1),1)), Pass(A,g(B,
        2),2));
```

---

thm 1 1/2 scratch:

Pass$_n$(v) = ~~if (n=v)~~
~~return v~~
~~else~~
~~0~~

we want OR_3

$A \backslash B$	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

A B ⓪ A→ OP_3 call C

a_{00}	a_{01}	a_{02}
a_{10}	a_{11}	a_{12}
a_{20}	a_{21}	a_{22}

= A→

$A \backslash B$	0	1	2
0	a_{00}	…	…
1	…	…	…
2	…	…	…

g_0

Pass$_0$

a_0 0 0

$\overline{A}$

$A \backslash B$	0	1	2
0	0	0	0
1	1	1	1
2	2	2	2

$\overline{B}$

$A \backslash B$	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2

OP_3

$A \backslash B$	0	1	2
0	2	0	0
1	0	0	0
2	0	0	1

OR

$\overline{B}$ ① $\overline{A}$ ② $\overline{B}$ ③ ③ $\overline{A}$ ④ (4) $\overline{A}$ (5) (2) (4) (6) (I) $\overline{B}$ (J)

$\overline{A}$→ 2 0 0 | 0 2 2 | 2 0 1 | 0 2 0 | 2 0 2 | 0 0 1 | 0 0 1

0 0 0 → | 0 0 0 → | 2 0 0 | 0 0 0 | 0 0 0 | 2 2 2 | 2 0 0

0 0 1 | 0 0 0 | 2 0 0 | 1 0 0 | 0 0 0 | 2 2 2 | 0 0 1

(7) $\overline{0}$ $\overline{2}$ $\overline{1}$ (D) $\overline{A}$ (F) (F) (F) (G) (G) (F) (H) (H) (H) (I)

1 2 2 | 0 0 0 → | 2 2 2 → | 1 1 1 | 0 0 0 | 2 2 2 | 0 0 0 | 2 2 2

2 2 2 → | 0 0 0 | 2 2 2 | 1 1 1 | 0 0 0 | 2 2 2 | 1 1 1 | 0 0 0

2 2 0 | 0 0 0 → | 2 2 2 → | 1 1 1 | 1 1 1 | 0 0 0 | 0 0 0 | 2 2 2

(8) (9) (8) $\overline{0}$ (C) (D) (E)

0 1 1 $\overline{A}$→ 2 0 0 | 2 0 0 → | 1 2 2 → | 0 1 1

1 1 1 | 0 0 0 | 0 0 0 | 2 2 2 | 1 1 1

1 1 2 → | 0 0 1 | 0 0 0 → | 2 2 2 → | 1 1 1

OGL m.3 P4

$U = Z_2 = \{0, 1\}$

want

B	0	1
0	0	1
1	1	1

~~OR~~ OP

$A \backslash B$	0	1
0	1	0
1	0	0

$\bar{A} \backslash B$	0	1
0	0	0
1	1	1

$\bar{B} \backslash A$	0	1
0	0	1
1	0	1

X $\bar{A} \to$ 1 1   0 0
X $\vec{A} \to$ 0 0   0 0   1 0
0 1

X $\bar{B} \to$ 1 0   1 1
$\vec{B} \to$ 1 0 $\to$ 0 1   0 1
0 0

$\bar{A} \to$ 1 0 $\to$ 0 1 ✓
$\bar{B} \to$ 0 0 $\to$ 1 1

want
010

OP(x,x)
x = 0 1 2
2 0 1

$\bar{A}$ ~~0 1 2~~
0 1 2

~~$\bar{A} \to$ 2 0 1 $\to$ 1 2 0~~

012 $\to$ OP(A,A)
012   2 0 1
012   2 ~~0~~ 1
2 ~~0~~ 1

0 1 2 $\xrightarrow{A}$ op(A,A) $\to$ 1 2 0
2 0 1 $\to$
A   0 1 0
B 0 0 0 $\to$ 2 2 2

Let $U = Z_n$, $Z_n = \{[0], [1], [2], \ldots, [n]\}$
$n > 1$

$OP: U \times U \to U$

OP([a][b]) = if ([a] = [b])
return [a−1]
else
return [0]

$= ([a]=[b])?\ [a-1] : [0]$

OP(OP([a],[b]),[b]) =
if (~~@~~ OP([a],[b]) == [b])
return ~~&~~ [OP([a],[b]) − 1]
else
return [0]

= if (~~if~~ ([a]==[b])? [a−1] : [0]) == [b])
[a−1]−1
else
[0]

= ((([a]==[b])? [a−1]==[b] : [0] ~~#~~ == [b])?
[a−1]−[0]

= (([a]==[b])? ([a−1]==[b]? [a−2] : [0]) :
([0]==[b]? [a−2] : [0]))

= ((([a]==[b])? [0] =

Z_3

	0	1	2
0	2	0	0
1	0	0	1
2	0	0	1

X $\bar{A} \to$ 2 0 0 $\to$ 1 2 2
$B \to$ 0 0 0   2 2 2
0 0 1 $\to$ 2 2 0

A 000
222

B 200
001
001

B 012
012
012

000
200
200

OGL m 4 P1 isolate and combine

OP_{n+1}

A\B	0	1	2	---	n
0	n	0	0	---	0
1	0	0	0	---	0
2	0	0	1	---	0
:	0	0	0	A-1	
n	0	0	0	...	n-1

Combine (CO)

A\B	0	1	2	---	B	n
0	0	1	2	---	B	n
1	1	1	*		*	*
2	2	*	2		*	
A	A	*	*	A		
n	n	*	*	---		n

$\bar{A}_0$

A\B	0	1	---	n
0	0	0	---	0
1	1	1	---	1
A	A	...		A
n	n	---		n

$\bar{B}_0$

	0	1	---	B	---	n
0	0	1	---	B	---	n
0	0	1	---	B	---	n
A	0	1	---	B	---	n
n	0	1	---	B	---	n

$\bar{A}_0$ and $\bar{B}_0$ are quasi gates, they exist to make notation consistent, they are not functions that use OP only.

$\bar{A}_m$ is $\bar{A} \to \to \dots (m+1) \to$
$\bar{A} \to \to \dots (m+1) \to$

$\bar{A}_m = $ ~~OP~~ ~~$\bar{A}_{m-1}$~~ ~~OP($\bar{A}$~~
$OP(\bar{A}_{m-1}, \bar{A}_{m-1})$

$\bar{B}_m = OP(\bar{B}_{m-1}, \bar{B}_{m-1})$

$\bar{0} = OP(\bar{A}_0, \bar{A}_1)$

$\bar{m} = $ if$(m \neq 0)$
$OP(\overline{(m+1)}, \overline{(m+1)})$
else
$\bar{0}$

Let $U = Z_n$.
$= \{[x] \in Z : 0 \leq x \leq n\}$

+ − are cyclical.

$T =$ is some intermediary table function

to get $\overline{(a,b,c)}$

A\B	0	1	---	b	---	n
0	0	0	0	0	0	0
1	0	0				0
a	0	0	...	c		0
n	0	0	...	0	...	0

$\overline{(a,b,c)}(x,y) = $ if$(x = a \wedge y = b)$
return c
else
return 0

$\overline{(a,b,c)} = \begin{matrix}\bar{A}_a \to \\ \bar{B}_b \to\end{matrix} T \xrightarrow{\bar{n}} T \dots$ ~~Combing~~ Combining incising with $\bar{J}$ until $\bar{c}$ and t changes, $t \in U$ or ~~#~~ OR it itself with itself.

if $(c) = 0$
return $\bar{0}$

to get gate take W, find each $\overline{(a,b,c)}$, then combine each $\overline{(a,b,c)}$ with COmbine.

since you ~~can~~ set ~~any time Combine~~ any time combine is called at least one of its arguments is 0,

combine$(a,b) = $ if $(a = 0)$
b
if$(b=0)$
a
else
*

Combine (CO)

A\B	0	1	2	--	B	---	n
0	0	1	2	--	B	---	n
1	1	*	*	--	*	--	*
2	2	*	*	--	*	--	*
A	A	*	*	--	*	--	*
n	n	*	--	--	*	--	*

* is any thing in Z_n

OGL m4 P2

OP_4

want combine X is anything.

	0	1	2	3
0	3	0	0	0
1	0	0	0	0
2	0	0	1	0
3	0	0	0	2

	0	1	2	3
0	0	1	2	3
1	1	X	X	X
2	2	X	X	X
3	3	X	X	X

$\overline{A_0}$: 0000 / 1111 / 2222 / 3333 $\xrightarrow{\overline{3}}$ $\overline{A_1}$: 3333 / 0000 / 1111 / 2222 $\rightarrow$ $\overline{A_2}$: 2222 / 3333 / 0000 / 1111 $\rightarrow$ $\overline{A_3}$: 1111 / 2222 / 3333 / 0000

$\overline{0}$: $\overline{A_0}\rightarrow$ 0000 / 0000, $\overline{A_1}\rightarrow$ 0000 / 0000 $\rightarrow$ $\overline{3}$: 3333 / 3333 / 3333 / 3333 $\rightarrow$ $\overline{2}$: 2222 / 2222 / 2222 / 2222

$\overline{B_0}$: 0123 / 0123 / 0123 / 0123 $\rightarrow$ $\overline{B_1}$: 3012 / 3012 / 3012 / 3012 $\rightarrow$ $\overline{B_2}$: 2301 / 2301 / 2301 / 2301 $\rightarrow$ $\overline{B_3}$: 1230 / 1230 / 1230 / 1230

$\overline{1}$: $\overline{2}\rightarrow$ 1111, $\overline{2}\rightarrow$ 1111 / 1111 / 1111

630* / 3XXX / 0XXX / *XXX

*230 / 2XXX / 3XXX / 0XXX

CO

*230 / 2XXX / 3XXX / 0XXX

$\overline{F_{00}}$: $\overline{A_0}\rightarrow$ 3000 / 0000, $\overline{B_0}\rightarrow$ 0010 / 0002 $\rightarrow$ $\overline{F_{01}}$: 2333 / 3333 / 3303 / 3331 $\rightarrow$ $\overline{F_{02}}$: 1222 / 2222 / 2232 / 2220 $\rightarrow$ $\overline{F_{03}}$: 0111 / 1111 / 1121 / 1113

$\overline{F_{10}}$: $\overline{A_1}$ 2000 / 0300, $\overline{B_1}$ 0000 / 0001 $\rightarrow$ $\overline{F_{11}}$: 1333 / 3233 / 3333 / 3330 $\rightarrow$ $\overline{F_{12}}$: 0222 / 2122 / 2222 / 2223 $\rightarrow$ $\overline{F_{13}}$: 3111 / 1011 / 1111 / 1112

$\overline{F_{20}}$: $\overline{A_2}$ 1000 / 0200, $\overline{B_2}$ 0030 / 0000 $\rightarrow$ $\overline{F_{21}}$: 0333 / 3133 / 3323 / 3333 $\rightarrow$ $\overline{F_{22}}$: 3222 / 2022 / 2212 / 2222 $\rightarrow$ $\overline{F_{23}}$: 2111 / 1311 / 1101 / 1111

$\overline{F_{30}}$: $\overline{A_3}$ 0000 / 0100, $\overline{B_3}$ 0020 / 0003 $\rightarrow$ $\overline{F_{31}}$: 3333 / 3033 / 3313 / 3332 $\rightarrow$ $\overline{F_{32}}$: 2222 / 2322 / 2202 / 2221 $\rightarrow$ $\overline{F_{33}}$: 1111 / 1211 / 1131 / 1110

$\overline{(0,0,0)} = \overline{0}$

$\overline{(0,1,1)}$: $\overline{A_0}\rightarrow$ 03 $\overline{3}\rightarrow$ 02 $\overline{2}\rightarrow$ 0100, $\overline{B_1}\rightarrow$

$\overline{(0,2,2)}$: $\overline{A_0}\rightarrow$, $\overline{B_2}\rightarrow$ $\overline{F_{00}}$ 0030 / 0000 / 1000 / 0200 $\xrightarrow{\overline{3}}$ 0020 / 0000 / 0000 / 0000

$\overline{(0,3,3)}$: $\overline{A_0}\rightarrow$, $\overline{B_3}\rightarrow$ $\overline{F_{21}}$ 3330 / 1333 / 3233 / 3333 $\xrightarrow{\overline{0}}$ 0003 / 0000 / 0000 / 0000

bottom want

$\overline{(1,0,1)}$: $\overline{A_1}\rightarrow$, $\overline{B_0}\rightarrow$ $\overline{F_{10}}$ 0001 / 2000 / 0300 / 0000 $\xrightarrow{\overline{2}}$ 0000 / 1000 / 0000 / 0000

$\overline{(2,0,2)}$: $\overline{A_2}\rightarrow$, $\overline{B_0}$ $\overline{F_{00}}$ 0010 / 0002 / 3000 / 0000 $\xrightarrow{\overline{3}}$ 0000 / 0000 / 2000 / 0000

$\overline{(3,0,3)}$: $\overline{A_3}\rightarrow$, $\overline{B_0}$ $\overline{F_{21}}$ 3133 / 3323 / 3333 / 0333 $\xrightarrow{\overline{0}}$ 0000 / 0000 / 0000 / 3000

~~if~~ $\overline{(a,b,c)} =$ if $(c \neq n-1)$ if $(c=0)$ return $\overline{0}$

$\overline{A_a}\rightarrow$, $\overline{B_b}\rightarrow$ $\overline{F_{(2-c)0}}$ $\overline{T}$ $\xrightarrow{\overline{c+1}}$ W

else

$\overline{A_a}\rightarrow$, $\overline{B_b}\rightarrow$ $\overline{F_{(c-1)1}}$ $\overline{T}$ $\xrightarrow{\overline{0}}$ W

return W

~~wrong, works for~~ Z_4