half derivative M3 P1

$$\frac{d^r x}{dx} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0) \cdot x^{(n-r)}}{\Gamma(n-r)} \quad r < 1 \qquad \Gamma(x) = \frac{1}{\Gamma(x)}$$

$$e^x = \frac{d^r e^x}{dx} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n-r}}{\Gamma(n-r)} \; ; r < 1$$

$\Gamma(x) =$

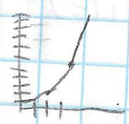
$\text{stdev}(f(x), g(x), a, b) = \sqrt{\int_a^b (f(x) - g(x))^2 dx}$

$\frac{d^r \sin(x)}{dx} @ x=0 \overset{?}{=} \sin(2\pi r)$ or $\cos(2\pi r)$

$\Gamma(x) = (\lceil x \rceil)!$ $\quad \text{stdev}(e^x, \frac{d^{\frac{1}{2}} e^x}{dx}, 0, 8) = 1337.58$

$\Gamma(x) = (\lfloor x \rfloor)!$ $\quad$ " $= 3665.713$

$\Gamma(x) = \Gamma(x+1)$ $\quad$ " $= .963057$

$\Gamma(x) = (\lceil x \rceil! - \lfloor x \rfloor!)(x - \lfloor x \rfloor) + \lfloor x \rfloor! =$ straight lines  $= 773.9103$

$\Gamma(x) = (\lceil x \rceil)!$ $\quad$ " $= 1164.317$

$\Gamma(x) =$ curves (x^2)

$\text{lines}(x) = (\lceil x \rceil! - \lfloor x \rfloor!)(x - \lfloor x \rfloor) + \lfloor x \rfloor!$

$\int_0^x \left(D(\text{lines}(x+1), x) - D(\text{lines}(x-1), x) \right) C$

connect $2x$ @ $x=1$ with $6x$ @ $x=2$ with parabola

must start with slope one at and end with slope 2.

$\int \left(\frac{D(2x) - D(6x)}{1 - 2} (x-1) + 2(1) \right) dx$

$2x$ @ $x=1$

$6x$ @ $x=2$

$(2ax + b)D(1) = 2D(2x)$

$(2ax + b)D(2) = D(6x)$

$2a + b = 2$ $\qquad 2 - 2a = b$

$4a + b = 6$ $\qquad 4a + 2 - 2a = 6$

$2a = 6 - 2$

$2a = 4$

$a = 2$

$4 + b = 2$

$b = -2$

$P_1 = (\cancel{D(\text{lines}(x-1))}, (x-1, D(\text{lines}(x-1))))$ $\quad \cancel{D(ax^2 + bx + c)(a)} = L_1$

$P_2 = (x+1, D(\text{lines}(x+1)))$

line than 2 points: $y - \cancel{P_{1,y}} = \left(\frac{P_{2,y} - P_{1,y}}{P_{2,x} - P_{1,x}}\right)(x - P_{1,x}) + P_{1,y}$

half derivative part 2

$f(x) = x^2$

$c = 1$

$x = 0$

$$f(x) = \frac{-2(x-1)}{0!} + 2(x-1) + 2\cdot(x-1)^2$$

$$= (x-1)\cdot\frac{1}{2} + \frac{f^{\frac{1}{2}}(0)\,(x-1)^{\frac{1}{2}}}{\frac{1}{2}!\cdot 2} + 2(x-1) + (x-1)^2$$

$$f(0) = 0 = -\frac{1}{2} + \frac{f^{\frac{1}{2}}(0)\,(-1)^{\frac{1}{2}}}{\frac{1}{2}!\cdot 2} + \underset{-1}{-2} + 1$$

$$0 = -\frac{1}{2} + \rightarrow$$

$$\cancel{-\frac{3}{2} \quad \frac{1}{2} = \frac{f^{\frac{1}{2}}(0)\,(-1)^{\frac{1}{2}}}{\frac{1}{2}!\cdot 2}}$$

$$\frac{1}{1} = \frac{1}{1} = \frac{1}{-1}$$

$$\cancel{\frac{1}{2}!\,(1) = f^{\frac{1}{2}}(0) = \frac{1}{2}!\cdot -1}$$

$$\cancel{-\frac{1}{2} = \frac{f^{\frac{1}{2}}(0)\,(-1)^{\frac{1}{2}}}{\frac{1}{2}!\cdot 2} + \frac{1}{2}}$$

$$\frac{3}{2} = \frac{f^{\frac{1}{2}}(0)\,(-1)^{\frac{1}{2}}}{\frac{1}{2}!\cdot 2}$$

$$\frac{3\cdot\frac{1}{2}!}{i} = f^{\frac{1}{2}}(0) \qquad f^{\frac{1}{2}}(0) = -3i\cdot\frac{1}{2}!$$

$$f_1^{(1/2)}(0) = -3i\cdot\frac{1}{2}!$$

Multi Variable Taylors theorem r/m1

Thm: let f be a function, and v be a vector, furthermore, f take v-length arguments. ~~let f be~~

$$T_f(v) = \sum_{n=0}^{\infty} \frac{\nabla^n f \overset{@\langle 0,0\rangle}{\cdot} (v)^n}{n!}$$

Ex: $f(x,y) = x^2 + y^2$

$$T_f(x,y) = \frac{(x^2+y^2)\overset{@(x=0,y=0)}{} \cdot (\langle x,y\rangle)^0}{0!} + \frac{\langle 2x, 2y\rangle \overset{@(0,0)}{} \cdot (\langle x,y\rangle)^1}{1!}$$

$$\frac{0!}{} = 0 \qquad \frac{\langle 0,0\rangle \cdot \langle x,y\rangle}{1!} = 0$$

$$+ \frac{\langle \langle 2,0\rangle, \langle 0,2\rangle\rangle \overset{@(0,0)}{\cdot} (\langle x,y\rangle)^2}{2!}$$

$$= \frac{\langle 2,0\rangle \cdot \langle x^2,y^2\rangle + \langle 0,2\rangle \cdot \langle x^2,y^2\rangle}{2}$$

$$= \frac{2x^2 + 0y^2 + 0x^2 + 2y^2}{2}$$

$$= x^2 + y^2 = f(x,y)$$

$$f(x_0+x, y_0+y) \approx f(x_0,y_0) + \nabla f(x_0,y_0) \cdot (x,y) + \frac{1}{2}\begin{bmatrix} x & y \end{bmatrix} Hf(x_0,y_0) \begin{bmatrix} x \\ y \end{bmatrix} + \cdots$$

$$\lim_{x \to x_0} \frac{\int_{-\infty}^{\infty} (x-r)\, dr}{\int_{-\infty}^{\infty} (x-r)\, dr}$$

$$\lim_{\Delta r_i \to 0} \sum_{i=1}^{n} (x-r_i)\, \Delta r_i$$

$$\lim_{\Delta t \to 0} \sum_{-\infty}^{\infty} t = \int_{-\infty}^{\infty} t\, dt$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} t\, dt = \boxed{\int_{0}^{\infty} \int_{0}^{\infty} t\, dt}$$

$$\int$$

half derivative MY P1

$$\frac{d^r x}{dx} = \sum_{n=\lceil r \rceil}^{\infty} \frac{f^{(n)}(0) \cdot x^{n-r}}{\Gamma(n-r)} \qquad ?<r<? \qquad \Gamma(x) = \frac{1}{\Gamma(x)}$$

maybe $-\infty < r < \infty$

$$e^x = \frac{d^n e^x}{dx} = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=\lceil r \rceil}^{\infty} \frac{x^{n-r}}{\Gamma(n-r)}$$

or $$\frac{d^n f(x)}{dx} = \sum_{n=\lfloor r \rfloor}^{\infty} \frac{f^{(n)}(0) \cdot x^{n-r}}{\Gamma(n-r)}$$

$$\frac{d^n f(x)}{dx} = \frac{f(0)}{\Gamma(r)} + \sum_{n=1}^{\infty} \frac{f^{(n)}(0) \cdot x^{n-r}}{\Gamma(n-r)} \; ; r<1$$

or $$\frac{d^n f(x)}{dx} = f(0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(0) \cdot x^{n-r}}{\Gamma(n-r)} \; ; r<1$$

$$e^x = \cancel{\sum} \; \cancel{\Gamma(1)} + \sum_{n=1}^{\infty} \frac{f^{(n)}(0) \cdot x^{n-r}}{\Gamma(n-r)} \cdot \Gamma(n-r) \quad \text{Cont form} \quad M_2$$

ans matrix

$$A = \begin{bmatrix} e^1-1 \\ e^2-1 \\ \vdots \\ e^j-1 \end{bmatrix} = [e^j-1]$$

var matrix

$$V = \begin{bmatrix} \Gamma(1-r) \\ \Gamma(2-r) \\ \vdots \\ \Gamma(j-r) \end{bmatrix} = [\Gamma(j-r)]$$

Coeff matrix

$$C = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ j \end{array} \begin{bmatrix} 1^{(1-r)} & 1^{(2-r)} & \cdots & 1^{(k-r)} \\ 2^{(1-r)} & 2^{(2-r)} & \cdots & 2^{(k-r)} \\ \vdots & \vdots & \ddots & \vdots \\ j^{(1-r)} & j^{(2-r)} & \cdots & j^{(k-r)} \end{bmatrix} = \begin{bmatrix} j^{(k-r)} \end{bmatrix}$$

$k \rightarrow$, $j \downarrow$

trends toward 0

Half derivative part 3

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = \frac{f^{0}(a)}{0!}\cdot\frac{1}{2} + \frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2} + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad *(x-a)^{\frac{1}{2}}$$

$$f(x) - \frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2} = \frac{f^{0}(a)}{0!}\cdot\frac{1}{2} + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad *(x-a)^{\frac{1}{2}}$$

$$-\frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2} = \frac{f^{0}(a)}{0!}\cdot\frac{1}{2} - f(x) + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad *(x-a)^{\frac{1}{2}}$$

$$f^{\frac{1}{2}}(a) = \frac{-2\cdot\frac{1}{2}!}{(x-a)^{\frac{1}{2}}}\left(-f(x) + \frac{f(a)}{2} + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n\right)$$

$f(x) = x^2$

$a = 0$

$x = 1$

$f'(x) = 2x$

$f''(x) = 2$

$$f_1^{\frac{1}{2}}(0) = \frac{-2\cdot\frac{1}{2}!}{(1-0)^{\frac{1}{2}}}\left(-1 + 0 + \frac{2\cdot 0}{1!}(1-0) + \frac{2}{2!}(1-0)^2\right)$$

$$0 \quad + 1$$

$$f_1^{\frac{1}{2}}(0) = 0$$

$a=0, x=2$

$$f_2^{\frac{1}{2}}(0) = \frac{-2\cdot\frac{1}{2}!}{\sqrt{2-0}}\left(-4 + 0 + 0 + \frac{2}{2!}(2-0)^2\right)$$

$$= 0 \qquad 4$$

$a=1, x=2$

$$f_2^{\frac{1}{2}}(1) = \frac{-2\cdot\frac{1}{2}!}{\sqrt{2-1}}\left(-4 + \frac{1^2}{2} + \frac{2\cdot 1}{1!}(2-1) + \frac{2}{2!}(2-1)^2\right)$$

$$\frac{1}{2} + 2 + 1$$

$$3\frac{1}{2}$$

$$= \frac{-\cancel{2}\cdot\frac{1}{2}!}{\sqrt{2-1}}\left(\cancel{-}\frac{1}{2}\right)$$

$$= \frac{1}{2}!$$

$a=1, x=3$

$$f_3^{\frac{1}{2}}(1) = \frac{-2\cdot\frac{1}{2}!}{\sqrt{3-1}}\left(-9 + \frac{1^2}{2} + \frac{2\cdot 1}{1!}(3-1) + \frac{2}{2!}(3-1)^2\right)$$

$$\frac{1}{2} \qquad 4 \qquad 4$$

$$-9 + 8\frac{1}{2}$$

$$= \frac{-2\cdot\frac{1}{2}!}{\sqrt{3-1}}\left(-\frac{1}{2}\right)$$

$$f_3^{\frac{1}{2}}(1) = \frac{\frac{1}{2}!}{\sqrt{2}}$$

half derivative Part 1

$$f(x) = \sum_{n=0}^{\infty, \frac{1}{2}=1} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) \approx \frac{f^0(a)}{0!}\cdot\frac{1}{2} + \frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2} + \sum_{n=1}^{\infty, \frac{1}{2}=1} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$(x-a)^0 \qquad (x-a)^{\frac{1}{2}}$$

$$(x-a)^{\frac{1}{2}}\frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2} \approx -f(x) + f^0(a)\cdot\frac{1}{2} + \sum_{n=1}^{\infty, \frac{1}{2}=1} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f^{\frac{1}{2}}(a) \approx \frac{\frac{1}{2}!\cdot 2}{(x-a)^{\frac{1}{2}}}\left(-f(x) + \frac{f(a)}{2} + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n\right)$$

$$f(x) = x^2 = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)\cdot(x-1)^n}{n!} = \frac{\overset{x^2}{1^2}\cdot(x-1)}{0!} + \frac{\overset{2x}{2\cdot 1}\cdot(x-1)}{1!} + \frac{2\cdot(x-1)^2}{2!}$$

$c=1$

$x=2$

$$x^2 = \frac{1^2\cdot(x-1)}{0!}\cdot\frac{1}{2} + \frac{f^{\frac{1}{2}}(1)}{\frac{1}{2}!}(x-1)^{\frac{1}{2}}\cdot\frac{1}{2} + 2(x-1)$$

$$4 = \frac{2-1}{2} + \frac{f^{\frac{1}{2}}(1)(2-1)^{\frac{1}{2}}}{\frac{1}{2}!}\cdot\frac{1}{2} + (x-1)^2 + 2 + 1$$

$$4 = \frac{1}{2} + \frac{f^{\frac{1}{2}}(1)}{\frac{1}{2}!\cdot 2} + 3$$

$$4 = 3\frac{1}{2} + \frac{f^{\frac{1}{2}}(1)}{\frac{1}{2}!\cdot 2}$$

$f^{(\frac{1}{2})}(x) =$

$$-\frac{f^{\frac{1}{2}}(a)}{\frac{1}{2}!}\cdot\frac{1}{2} + f(x) = \frac{f^0(a)}{0!}\cdot\frac{1}{2} + \sum_{n=1}^{\infty} f^{n}(a) \ldots$$

$f^{(\frac{1}{2})}_{c=1}(2)$

$$2\cdot\frac{1}{2}\left(\frac{f^0(a)}{0!}\cdot\frac{1}{2} - f(x) + \sum f^n(\ldots)\right)$$

$$\frac{1}{2} = \frac{f^{\frac{1}{2}}(1)}{\frac{1}{2}!\cdot 2}$$

$$\frac{1}{2}! = f^{\frac{1}{2}}(1)$$

$$\frac{1}{2}! = \frac{d^{\frac{1}{2}}(x^2)}{dx^{\frac{1}{2}}}(1)$$

$$\boxed{f^{(\frac{1}{2})}_1(2) = \frac{1}{2}!}$$

plug in $x=2$

$$(x!) = \left(\sum_{n=0}^{\infty} \frac{\left(\sum_{i=1}^{x} (\ln(i))\right)^n}{n!} \right) = e^{\sum_{i=1}^{x} (\ln(i))}$$

Eln P3

$$2! = \sum_{n=0}^{\infty} \frac{\left(\sum_{i=1}^{2} (\ln(i))\right)^n}{n!}$$

$$= \left(\sum_{i=1}^{2} (\ln(i))\right)^0 + \sum_{i=1}^{2} (\ln(i)) + \frac{\left(\sum_{i=1}^{2} (\ln(i))\right)^2}{2!} + \cdots$$

$$= 1 + \ln(1) + \ln(2) + \frac{\ln(2)^2}{2!} + \frac{\ln(2)^3}{3!} + \cdots$$

$$= 1.9897 + \cdots$$

$$= e^{\ln(2)}$$

$$\prod_{n=0}^{a} f(x) = e^{\sum_{n=0}^{a} \ln(f(x))}$$

$$\sum_{n=0}^{a} f(x) = \ln\left(\prod_{n=0}^{a} e^{f(x)}\right)$$

$$x! \approx \sum_{n=0}^{\infty} \frac{\left(\int_1^x \ln(i)\,di\right)^n}{n!}$$

$$2! \approx \sum_{n=0}^{\infty} \frac{\left(\int_1^2 (\ln(i))\,di\right)^n}{n!}$$

$$2! \approx \sum_{n=0}^{\infty} \frac{(2\ln(2)-1)^n}{n!}$$

$$2! \approx e^{(2\ln(2)-1)}$$

$$2! \approx 1.4715$$

Maybe not

$$x! \approx e^{\int_1^x (\ln(i))\,di} \; ; x > 0$$

$$e^{\int_1^x (\ln(i))\,di} \neq \Gamma(x+1) \quad x \notin I$$

E 1n p 4

$$f(x) = e^{\sum_{i=1}^{x} \log(i)}$$

$$h(x) = e^{\int_1^x \log(i)\,di}$$

$$0 \overset{?}{<} \lim_{x \to \infty} \left(\frac{f(x)}{h(x)} \right) \overset{?}{<} \infty$$

$$\lim_{x \to \infty} \frac{e^{\sum_{i=1}^{x} \ln(i)}}{e^{\int_1^x \ln(i)}} = \lim_{x \to \infty}$$

$HCOOH + NaOH \rightarrow NaCOOH + H_2O$

M_i ~~.2~~ ? -.1105

L_i .02 .03543

Q_i ~~.004~~ .003915

$H_3PO_4 + 3NaOH \rightarrow 3H_2O(l) + Na_3PO_4$

L .026 .02463

.120

& .0029556

~~Na_2CO_7HC~~

$NaHCO_3 + HCl \rightarrow NaCl + H_2CO_3$

$H_2CO_3 \rightarrow H_2O + CO_2$

g 2.5 ~~2.5~~ ?

$$e^{\sum_{n=0}^{\infty} \ln\left(\frac{f^{(n)}(a)}{n!}(x-a)^n\right)} \overset{?}{=} \prod_{n=0}^{\infty}\left(\frac{f^{(n)}(a)}{n!}(x-a)^n\right)$$

Eln ?!

$$e^{(\ln(f(a)\ldots) + \ln(f'(a)(x-a)) + \ldots}$$

$$e^{(\ln(f(a)\cdot f'(a)(x-a) \cdot \ldots)}$$

$$f(a)\cdot f'(a)(x-a)\cdot \frac{f''(a)}{2!}(x-a)^2 \cdot \ldots$$

Alternate Principal Normal unit vector M1P1

$$\vec{N}(t) \overset{?}{=} \lim_{h \to 0} \frac{\vec{r}(t) \times (\vec{r}(t-h) \times \vec{r}(t))}{|\vec{r}(t) \times (\vec{r}(t-h) \times \vec{r}(t))|}$$

$$\vec{N}(t) = \frac{\frac{T'(t)}{dt}}{\left\|\frac{T'(t)}{dt}\right|} = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} \qquad \vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{r}(t) \times (\vec{r}(t-h) \times \vec{r}(t)) \underset{h \to 0}{\overset{?}{\propto}} \vec{T}'(t)$$

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = \langle \underset{-1}{2-4-9}, \underset{2}{(4-6)}, \underset{-1}{3-4} \rangle \qquad \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\frac{d\,f(x)}{dx} = \lim_{h \to 0} \left(\frac{f(x) - f(x-h)}{h} \right)$$

$$\vec{r}(t) \times (\vec{r}(t-h) \times \vec{r}(t)) = CR(t) = [a(t), b(t), c(t)]$$

$$a(t) = -x(t)z(t)z(t-h) - x(t)y(t)y(t-h) + (z(t)^2 + y(t)^2)x(t-h)$$
$$b(t) = -y(t)z(t)z(t-h) + (z(t)^2 + x(t)^2)y(t-h) - x(t)y(t)x(t-h)$$
$$c(t) = (y(t)^2 + z(t)^2)z(t-h) - y(t)z(t)y(t-h) - x(t)z(t)x(t-h)$$

$$|CR(t)| = \sqrt{z(t)^2 + y(t)^2 + x(t)^2} \cdot \text{Sqrt}(((y(t)^2 + x(t)^2)z(t-h)^2 + (-2y(t)z(t)y(t-h) - 2x(t)z(t)x(t-h))z(t-h) + (z(t)^2 + x(t)^2)y(t-h)^2 - 2x(t)y(t)x(t-h)y(t-h) + (z(t)^2 + y(t)^2)x(t-h)^2)$$

$$\frac{\vec{v}(t)}{|\vec{v}(t)|} \text{ (small vector of unit length)} \qquad \left|\frac{\vec{v}(t)}{|\vec{v}(t)|}\right| = 1$$

half derivative M_2 P 6

$$\frac{d^r x}{dx^r} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0) \cdot x^{(n-r)}}{\tilde{1}(n-r)} \qquad r<1$$

b/c $\frac{d^1(x^0)}{dx^1} = 0 \neq x^{-1}$

$\tilde{1}(x) = \frac{1}{\tilde{1}(x)}$

Tilda pipe

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n-r}}{\tilde{1}(n-r)} \; ; r<1$$

$$e^x = x^{(0-r)} \cdot \tilde{1}(0-r) + x^{(1-r)} \tilde{1}(1-r) + \dots \qquad r<1$$

$$e^1 = 1^{(0-r)} \cdot \tilde{1}(0-r) + 1^{(1-r)} \tilde{1}(1-r) + \dots$$

$$e^2 = 2^{(0-r)} \cdot \tilde{1}(0-r) + 2^{(1-r)} \tilde{1}(1-r) + \dots$$

answer matrix

$$A = \begin{array}{c} j=1 \\ j=2 \\ j=3 \\ \vdots \\ j=\infty \end{array} \begin{bmatrix} e^1 \\ e^2 \\ \vdots \\ e^j \end{bmatrix} = [e^j]$$

Variable matrix / this is what were solving for

$$V = \begin{array}{c} j \\ j=1 \\ j=2 \\ \vdots \\ j=\infty \end{array} \begin{bmatrix} \tilde{1}(0-r) \\ \tilde{1}(1-r) \\ \vdots \\ \tilde{1}(j-1-r) \end{bmatrix} = [\tilde{1}(j-1-r)]$$

$[f(j)]$ means it is that column vector with values of $f(0), f(1), \dots f(j)$

Coefficient matrix

$r<1$

$$C = \begin{array}{c} j=1 \\ j=2 \\ \vdots \\ j=\infty \end{array} \begin{array}{c} \begin{array}{cccc} k=1 & k=2 & & k=\infty \end{array} \\ \begin{bmatrix} 1^{(0-r)} & 1^{(1-r)} & \dots & 1^{(k-1-r)} \\ 2^{(0-r)} & 2^{(1-r)} & \dots & 2^{(k-1-r)} \\ \vdots & \vdots & \ddots & \vdots \\ j^{(0-r)} & j^{(1-r)} & \dots & j^{(k-1-r)} \end{bmatrix} \end{array} = [j^{(k-1-r)}]$$

```
maxima: ans: genmatrix(lambda([j,k], 2.e^j), size,1)
        coef: genmatrix(lambda([j,k], j^(k-1-r)), size, size)
        Vars: linsolve_by_lu(coef, ans)
```

can't go over twelve with my machine

```
GenPipeTilde(r,size):=1/float(linsolve_by_lu(genmatrix(lambda(\n
[j,k],j^(k-1-r)),size,size), genmatrix(lambda([j,k],2.e^j),size,1)))
```

$(a+1)^6$

$(a+b)^0 = 1 \quad (a+b)^1 = a+b \quad (a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 = (a^2+2ab+b^2)(a+b)$

$a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$

$a^3+3a^2b+3ab^2+b^3$

coefficients

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

powers

0,0

1,0 0,1

2,0 1,1 0,2

3,0 2,1 1,2 0,3

algebraic [illegible] element $= z_n$

[illegible] case $\times$ $a \cdot z_n =$

a^n

case $z_n + a =$

$a \cdot z_n$

$z_n = a z_{n-1} + b \quad z_0 = c \rightarrow$

$c \cdot a^n + b \cdot a^n - b$

$\sum_{z=0}^{n-1} a^z + b$

$a z_{n-1} + c$

$+ b \sum_{z=0}^{n-1} a^z$

$b \sum_{z=0}^{n-1} a^z$

$a_n = 2a_{n-1} + 2 \qquad a_0 = 2$

$0 = 2 \cdot 0 + 2 = 4$

$1 = 2 \cdot 2 + 2 = 6 \to 8 \qquad 2^n + 2n$

$2 = 2 \cdot 6 + 2 = 14 \to 16 \quad = 2(2 \cdot 2 + 2) + 2$

$3 = 2 \cdot 14 + 2 = 30 \to 32 \quad = 2(2 \cdot (2 \cdot 2 + 2) + 2) + 2$

$4 = 2 \cdot 30 + 2 = 62$

$2^{n+2} - 2$

$a_{n-1} = b$

$2b + 2$

$\ln(2)\ln(b) + \ln(2)$

$a_0 = 2$

$2b + 2 \to 2^{n+2} - 2$

$\ln(b^2 \cdot 2) \quad \ln(\ln(2^b \cdot 2))$

$\ln(2^{b+1}$

$a_n = 2a_{n-1} + 1 \qquad a_0 = 3$

$0 = 3$

$1 = 2 \cdot 3 - 1 = 5 \to 2$

$2 = 2 \cdot 5 - 1 = 9 \to 4$

$3 = 2 \cdot 9 - 1 = 17 \to 8$

$4 = 2 \cdot 17 - 1 = 33 \to 16$

$2^{n+1} + 1$

$a_0 = 0$

$0 = 0 \to -1$

$1 = 2 \cdot 0 - 1 = -1 \to -2$

$2 = 2 \cdot -1 - 1 = -3 \to -4$

$3 = 2 \cdot -3 - 1 = -7 \to -8$

$4 = 2 \cdot -7 - 1 = -15$

$-2^n + 1$

$0 = 1 \to 1$

$1 = 2 \to 2$

$2 = 4 \to 2$

$\ln(a + b) = \ln(a) \overset{?}{=} \ln(b) \qquad \ln(a) \overset{+}{\neq} \ln(b)$

$ELN(x^2 + 3x^4 - 2 + e^x) = \ln$

$ELN(0) = \ln(0) = 0$

$ELN(1) = \ln(1) = 1$

$ELN(x) = e^{\ln(x)} = x$

$ELN(x+1) = e^{\ln(x) + \ln(1)} = e^{\ln(1 \cdot x)} = x$

$ELn(x+2) = e^{\ln(x) + \ln(2)} = e^{\ln(2x)} = 2x$

$ELN(2x) = e^{\ln(2) \cdot \ln(x)} = e^{\ln(x^{\ln(2)})}$

$= e^{\ln(2^{\ln(x)})} = x^{\ln(2)}$

$= 2^{\ln(x)}$

$\prod_{n=1}^{\infty} (2$

Let Lasg. notation

$$f(n) = \prod_{n=1}^{\infty} \left(1 + \frac{-1^{n+1}}{n}\right) = \sqrt{2}$$

∞

$n=1$

$$\left(1+\frac{1}{1}\right)^{\left(1-\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots}$$

$\sqrt{2}$, 1.5874, $\sqrt{2}$, 1.5157

$$f(n+1) = f(n)^{\left(1+\frac{-1^{n+1}}{n}\right)}$$

$$f(0) = \left(1 + \frac{\pm 1}{1}\right)$$

$n + (-1)^n$

$$\prod_{n=1}^{\infty} \left(1 + \frac{(-1)^{(n+1)}}{xn}\right)$$, ~~[illegible]~~

2, $2^{\wedge}\frac{1}{2}$, $2^{\wedge}\frac{1}{2} \wedge \frac{4}{3}$, $2^{\wedge}\frac{1}{2} \wedge \frac{4}{3} \wedge \frac{3}{4}$, $2 \wedge \frac{1}{2} \wedge \frac{4}{3} \wedge \frac{3}{4} \wedge \frac{5}{5}$

$2^{\wedge}\frac{4}{6}$

$2^{\wedge}\frac{2}{3}$, $2^{\wedge}\frac{2}{3} \wedge \frac{3}{4}$

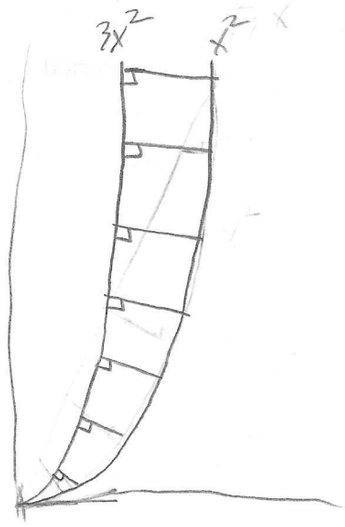
$e^{f(\ln(x))}$

$f(x) = 5x + 3$

$5 \ln x + 3$

$\ln x^5 \cdot 3$

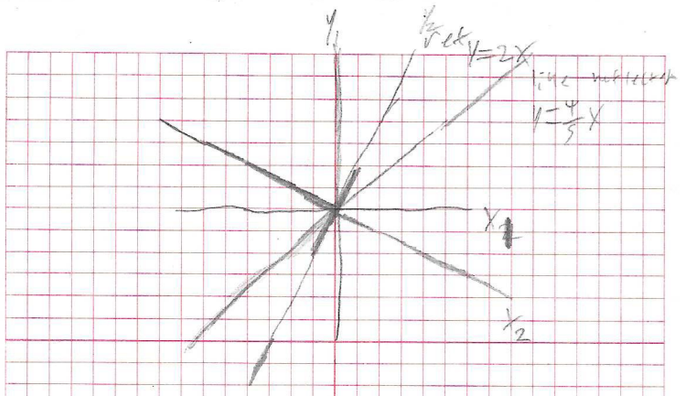
$x^5 \cdot 3$



first cross sectional area

cross sectional area given, perpendicular to

x^2



$\frac{a}{b}$ = a with respect to / a measured with b

$$x_2 = \sin\left(-\tan\left(\frac{\Delta x_2}{x_1}\right)\right)y_1 + \cos\left(\operatorname{atan}\left(\frac{\Delta x_2}{x_1}\right)\right)x_1$$

$$y_2 = \sin\left(\operatorname{atan}\left(\frac{\Delta y_2}{x_1}\right)\right)y_1 + \cos\left(\operatorname{atan}\left(\frac{\Delta y_2}{x_1}\right)\right)x_1$$

$$x_1 = \sin\left(\operatorname{atan}\left(\frac{\Delta x_1}{x_2}\right)\right)y_2 + \cos\left(\operatorname{atan}\left(\frac{\Delta x_1}{x_2}\right)\right)x_2$$

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testing flipping over given line

$y_2 = 2x$ $\qquad\qquad$ $f(x) = x^2$

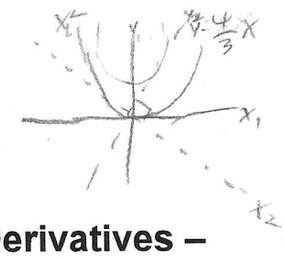
$$y_2 = x_1 \cdot \overset{\frac{1}{\sqrt{5}}}{\cos(\operatorname{atan}(2))} + y_1 \cdot \overset{\frac{2}{\sqrt{5}}}{\sin(\operatorname{atan}(2))}$$

$$x_2 = x_1 \cdot \sin(\operatorname{atan}(2)) - y_1 \cdot \cos(\operatorname{atan}(2))$$

$$x \cdot \frac{1}{\sqrt{5}} + y_1 \cdot \frac{2}{\sqrt{5}} = (x_1 \cdot \frac{2}{\sqrt{5}} - y_1 \cdot \frac{1}{\sqrt{5}})$$

$y = x^2$

$G(y) = f(x)$ ~~f(x)~~ $G(y) = f(x)$



$x_1 = \frac{\sqrt{3}}{2} x_2 + \frac{1}{2} y_2$

$y_1 = -\frac{1}{2} x_2 + \frac{\sqrt{3}}{2} y_2$

$\left(-\frac{1}{2} x_2 + \frac{\sqrt{3}}{2} y_2\right) =$

$\left(\frac{\sqrt{3}}{2} x + \frac{1}{2} y\right)^2$

Applications of Derivatives – Summary for interpreting graphs

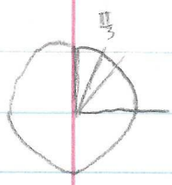
Review: What do the derivatives tell us?

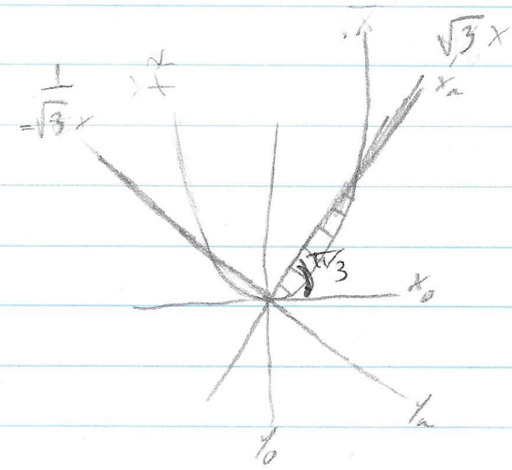
Derivative	What does it tell us?
1st	The rate of change of the function: instantaneous slope of the tangent to the curve
2nd	The rate of change of the slope of the tangent to the curve (1st derivative): concavity of the curve

On a graph of:	(What to look for) If:	(What it means) then:
f(x) (original function)	f(x) is increasing	f'(x) is positive
	f(x) is decreasing	f'(x) is negative
	f(x) is an extrema or flat	f'(x) is zero (critical point)
	f(x) is concave up	f''(x) is positive
	f(x) is concave down	f''(x) is negative
	The concavity of f(x) is changing	f''(x) is zero (point of inflection)
f'(x) (1st derivative)	f'(x) is positive	f(x) is increasing
	f'(x) is negative	f(x) is decreasing
	f'(x) is zero	f(x) is a min (- → +) or a max (+ → -)
	f'(x) is increasing	f''(x) is positive (f(x) is concave up)
	f'(x) is decreasing	f''(x) is negative (f(x) is concave down)
	f'(x) is an extrema or is flat	f''(x) is zero (f(x) is a point of inflection)
f''(x) (2nd derivative)	f''(x) is positive	f(x) is concave up
	f''(x) is negative	f(x) is concave down
	f''(x) is zero	f(x) is a point of inflection

$\frac{x}{y}$ = x in respect to y

cross sections = squares perpendicular to $\sqrt{3}x = y$





$x_a = \frac{\sqrt{3}}{2}y_0 + \frac{1}{2}x_0$

$y_a = \frac{1}{2}y_0 - \frac{\sqrt{3}}{2}x_0$

$x_0 = -\frac{\sqrt{3}}{2}y_a + \frac{1}{2}x_a$

$y_0 = \frac{1}{2}y_a + \frac{\sqrt{3}}{2}x_a$

$\sqrt{3}\,x = \frac{\pi}{3}\alpha$

$-\frac{1}{\sqrt{3}}x = \frac{5\pi}{6}\alpha$

$\frac{y_0}{x_a} = \frac{\pi}{6}\alpha$

$\frac{x_0}{x_a} = -\frac{\pi}{3}\alpha$

$y_0 = \sqrt{x_0}$

$\frac{1}{2}y_a + \frac{\sqrt{3}}{2}x_a = \sqrt{-\frac{\sqrt{3}}{2}y_a + \frac{1}{2}x_a}$

solve for y_a

$y_a = \left(\frac{\sqrt{8\sqrt{3}\,x_a + 1} + \sqrt{3}x_a + 1}{3}\right)$

intersects @ y_0 is @ $0, \sqrt{3}$

$0 = x_0$

$= -\sqrt{3}$

$\sqrt{(\sqrt{3}-0)^2 + (\sqrt{3}\cdot\sqrt{3} - \sqrt{3}\cdot 0)^2}$

$\sqrt{3+9}$ $\quad$ $\sqrt{12}$

$2\sqrt{3}$

$\int_0^{2\sqrt{3}} \left| \left(\frac{\sqrt{8\sqrt{3}\,x_a + 1} + \sqrt{3}x_a + 1}{3}\right)^2 \right| dx$

$= \frac{3^{3/2}}{20}$

intersects @ $0, 2\sqrt{3} \sim x_a$

$f(\log_a(u)) = \frac{1}{u' \ln(a)}$

$$\frac{d}{dx}\left[\log_a(u)\right] = \frac{1}{u' \ln(a)} \cdot u' = \frac{d}{dx}\left[\frac{\ln(u)}{\ln(a)}\right]$$

how a'?

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x(\ln e)}$$

$$\frac{d}{dx}\left[a^u\right] = a^u \ln(a) \cdot u'$$

$$\cancel{\frac{d}{dx}\left[a^u\right] = a^u \cdot \ln(a) \cdot u'}$$

$$\int a^x \, dx = a^x \left(\frac{1}{\ln(a)}\right) + C$$

$$\int e^x \, dx = e^x \left(\frac{1}{\ln e}\right) + C$$

$$\frac{\ln(1+\frac{h}{a})}{\ln(a)} \cdot \frac{1}{h} \qquad \lim_{h \to 0} \frac{\ln\left(\left(1+\frac{h}{a}\right)^{\frac{1}{h}}\right)}{\ln(a)} \qquad \frac{\ln(e)}{\ln(a)} = \frac{1}{\ln(a)}$$

$$f(x) = 1 + \sqrt{x} - \ln x$$

$$\frac{\log_a\left(\frac{u+h}{u}\right)}{h} \qquad \frac{\log_a\left(1+\frac{h}{u}\right)}{h}$$

$$\lim_{h \to 0}\left(\frac{\log_a(u+h) - \log_a(u)}{h}\right)$$

$$\frac{\log_a(u+h)}{h} - \frac{\log_a(u)}{h}$$

$$\left(\frac{\frac{\ln(u+h)}{\ln(a)}}{h}\right) - \left(\frac{\frac{\ln(u)}{\ln(a)}}{h}\right)$$

$$\left(\frac{1}{h \ln(a)}\right)\left(\ln(u+h) - \ln(u)\right)$$

$$\lim_{h \to 0} \frac{h}{\ln(a)}\left(\ln(u+h) - \ln(u)\right)$$

$$h^2 \cdot \frac{1}{\ln(a)} \cdot \frac{1}{h}\left(\ln\left(\frac{u+h}{u}\right)\right)$$

$$\ln\left(1+\frac{h}{u}\right)$$

$$\lim_{h \to 0} h^2 \cdot \frac{1}{\ln(a)} \cdot \left(\ln\left(\left(1+\frac{h}{u}\right)^h\right)\right)$$

$$\lim_{h \to 0} h^2 \cdot \frac{1}{\ln(a)}$$

$$\frac{d}{dx}\left[\frac{\ln(u)}{\ln(a)}\right] = \frac{-\frac{1}{a} a' \cdot \ln(u) + \frac{1}{u} u' \cdot \ln(a)}{\ln(a)^2} = \frac{1}{u \ln(a)} \cdot u'$$

$$a = 2 \quad \frac{-\frac{1}{2} \cdot \ln(u) + \frac{1}{u} u' \cdot \ln(2)}{\ln(2)^2}$$

$$\frac{-\frac{1}{2} \cdot \ln(u) + \frac{1}{u} u' \cdot .69314}{(.693)^2}$$

$$\frac{-\frac{1}{2} \cdot \ln(u)}{(.693)^2} + \frac{\frac{1}{u} u' \cdot .69314}{(.693)^2}$$

$$\frac{\frac{1}{u} u'}{\ln(2)} - \frac{\ln(u)}{\ln(2)^2}$$

Worksheet
Volumes of solids—Disk and washer method #2
AP Calculus

Name Pritang

Find the volume of the solid formed by the equations:

$\sqrt{dx^2+dy^2}$

$dy = \frac{f'(x)}{dx} \cdot dx$

$\int \sqrt{(f'(x)\cdot dx)^2+dx^2}$

$\int \sqrt{f'(x)^2 \cdot dx^2 + dx^2}$

$\int \sqrt{(f'(x)^2+1)(dx^2)}$

$\int \sqrt{f'(x)^2+1} \cdot \sqrt{dx^2}$

$\int \sqrt{f'(x)^2+1} \cdot dx$

1.) $y = x^2$, $y = 0$, $x = 2$, is rotated about:

a.) the x-axis $\frac{32\pi}{5}$

b.) the y-axis 8π

c.) the line $y = 4$ $\frac{224\pi}{15}$

d.) the line $x = 2$ $\frac{8\pi}{3}$

2.) $y = 1 + \sqrt{x}$, $y = 1$, $x = 4$ is rotated about:

a.) the x-axis $\frac{56\pi}{3}$

b.) the y-axis $\frac{128\pi}{5}$

c.) the line $y = 3$ $\frac{40\pi}{3}$

d.) the line $x = 6$ $\frac{192\pi}{5}$

3.) $y = x^2$ and $y = \sqrt[3]{x}$ is rotated about:

a.) the x-axis $\frac{2\pi}{5}$

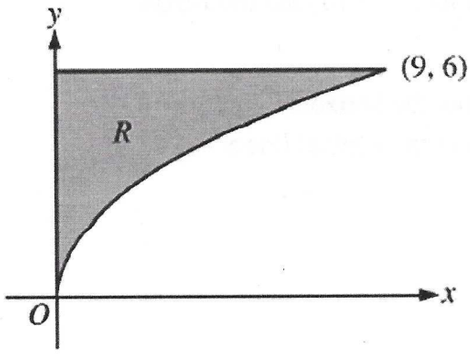
b.) the y-axis $\frac{5\pi}{14}$

c.) the line $y = 1$ $\frac{13\pi}{30}$

7-2 QUIZ **AP CALCULUS AB** **NAME:** David Cosby

You may NOT use your calculator on this problem.

length of line $= \int_a^b (\sqrt{f'(x)^2+1})dx$



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rotation whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

a) $\int_0^9 (6 - 2\sqrt{x})dx$

$6x - \frac{2x^{3/2}}{3/2}\Big|_0^9 = 6 \cdot 9 - \frac{227}{3/2} \approx 18$

b) $\pi \int_0^9 (7 - 2\sqrt{x})^2 - (7-6)^2 dx$

~~$X(i) \to$~~ $x_i \to x$ at point i

~~$Y(\ ) \to$~~ $y_i \to y$ at point i

~~$f(x) =$~~

$$g_n(x) = \prod_{i=1}^{n} (x - x_i)$$

$$F_1(x) = y_1$$

$$\lambda_n = \frac{y_{n+1} - F_n(x_{n+1})}{g_n(x_{n+1})}$$

$$F_n = F_{n-1}(x) + \lambda_{n-1} \cdot g_{n-1}(x)$$

$$F_n(x) = (F_{n-1}(x)) + \left(\frac{y_n - F_{n-1}(x_n)}{\prod_{i=1}^{n-1} (x_n - x_i)} \right) \cdot \prod_{i=1}^{n-1} (x - x_i)$$

1	2	3	4
1, 2	2, 5	4, 6	7, 2

~~4, 2~~

~~2, 3~~

$$g_n(x) = \prod_{i=1}^{n} (x - x_i)$$

$F_1(x) = 2 = y_1$ $\qquad g_1(x) = (x - x_1) = x - 1$

$$\lambda_1 = \frac{y_2 - F_1(x_2)}{g_1(x_2)} = \frac{5-2}{2-1} = \frac{3}{1}$$

$$F_2(x) = F_1(x) + \lambda_1 g_1(x)$$

$$F_2(x) = 2 + 3 \cdot (x-1)$$

$$g_2(x) = (x-1)(x-2)$$

$$\lambda_2 = \frac{6 - F_2(4)}{g_2(4)} = \frac{-5}{6}$$

$$F_3(x) = (2 + 3 \cdot (x-1)) + \left(\frac{-5}{6}(x-1)(x-2)\right)$$

$$g_3(x) = (x-1)(x-2)(x-4)$$

$$\lambda_3 = \frac{2 - F_3(7)}{g_3(7)} = \frac{7}{90}$$

$$F_4(x) = F_3(x) + \lambda_3 g_3(x) = f(x)$$

$$f(x) = \left((2 + 3 \cdot (x-1)) + \left(\frac{-5}{6}(x-1)(x-2)\right)\right) + \frac{7}{90}(x-1)(x-2)(x-4)$$

How to find Surface formula for region rotated about axis

Circ

Surface area =

$A(x) = \int \cdot C(x)\,dx$

length of line = $\sum_{0}^{\infty, ss=r}$

for sphere $1 = \sqrt{x^2+y^2+z^2}$

circumf of circle = $2\pi r$

Surface of revolution object

$\int \sum_{0}^{b, \infty, ss=dh} 2\pi f(x)$ $\sum$

$\int_a^b 2\pi f(x)$

$C = \sqrt{a^2+b^2}$

$\Delta C = \sqrt{\Delta x^2 + \Delta y^2}$

$\Delta C = \sqrt{\Delta x^2 + (f'(x)\,\Delta x)^2}$

$\sqrt{\Delta x^2 (1 + f'(x))}$

Meter = m $f(x) = x\,m$

Shape = $\sqrt{1-x^2}$ $\sqrt{1-x^2}$

$\frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x)$

m^2

$m \cdot m$

$\sqrt{m^2}$

m

$\int m$

m^2

$\sqrt{1-x^2}$ $\frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x$

$\frac{x}{\sqrt{x}}$

$\frac{x^2}{x}$

$\sqrt{x}$

$\int\sqrt{\quad}$

$\int\sqrt{1+\left(\frac{-x}{1-x^2}\right)^2}\,dx$

$\int\sqrt{1+\frac{x^2}{1-2x^2+x^4}}\,dx$

$(1-x^2)^2$

length of $f(x)$ $= \int \sqrt{1+f'(x)^2}\,dx$

~~$\int\sqrt{1+A'(f(x))^2}\,dx$~~

Surface area $= \int \sqrt{1+A''(f(x))^2}\,dx$

Circ $= 2\pi r$ $A = \pi r^2$

$f(x) = \sqrt{1-x^2} \cdot x$

$\int \sum_{0}^{\infty, ss=r} \int \sqrt{1+f'(x)^2}\,dx \cdot r$

Surface area $\overset{?}{=} \int\int \sqrt{1+f'(x)^2}\,dx\,dy$

infinite sum of ~~length~~ shape are length shape function when shape function is a function of $f(x)$

Surface area $= \int \sqrt{1+S_f'(f(x))^2}\,dx$

S_f = shape function

$\sum_{n}^{b, ss=h} \sum$

$\sum_{n=a}^{b, ss=h} \left(\int_{\text{solve}(S(n)=0,n)[0]}^{\text{solve}(S(n)=0,n)[1]} n\sqrt{1+S'(n)^2}\,dx \right)$

~~more~~ more in page 2 of ~~final~~ assignment

$$\int_a^b \int_{f(x)\cdot g^{-1}(0)[0]}^{f(x)\cdot g^{-1}(0)[1]} f(x)\sqrt{1+g'(x)^2}\,dx\,dx = \text{surface area of}$$

$g^{-1}(x)$ = inverse of $g(x)$

function with a shape orthogonal ~~the top~~

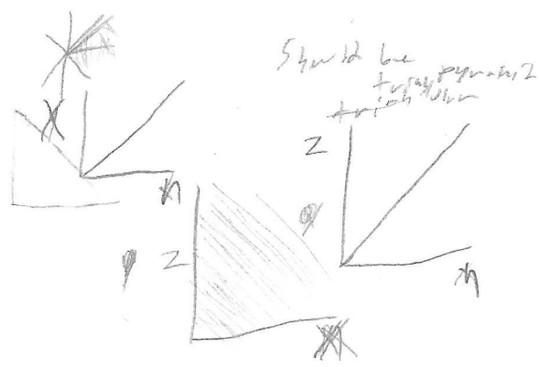
$g(x)$ is symmetric

Summation ~~of~~ of summation of hypotenuses, with hypotenuses growing $f(x)$

$f(x) = x$

$g(x) = x$

$$\int_0^1 \int_0^{g(x)} x\cdot\sqrt{1+1^2}\,dx\,dx$$

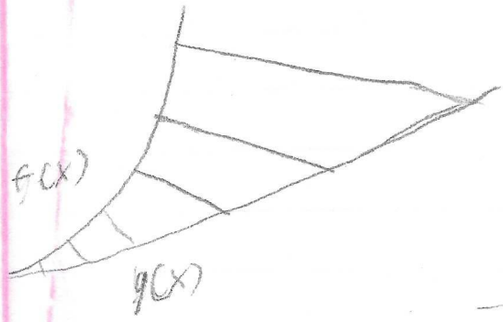


$$\int_a^b f(x)\int_c^d \sqrt{1+g'(x)^2}\,dx\,dx$$

$$\int_a^b \left(f(x)\int_c^d \left(\sqrt{1+g'(x)^2}\right)dx\right)dx$$

not sure what c and d should be

Cross sections are square



C_x = Current x

- find line perpendicular to f(x) at x_i = P(x)

- find intersection of P(x) and g(x)
- Calculate length between intersection P(x) and f(x), and intersection of P(x) and g(x)
- square was multiplied by Δx
- repeat infinitely
- sum them up

$$P(x) - f(C_x) = -\frac{1}{f'(C_x)}(x - C_x)$$
$$P(x) = -\frac{1}{f'(C_x)}(x - C_x) + f(C_x)$$

$P(x) = g(x)$ $\qquad$ x_i = the point that this happens

$\sqrt{(f(C_x) - g(x_i))^2 + (C_x - x_i)^2}$ = length between the two points

$$= \int \left((f(C_x) - g(x_i))^2 + (C_x - x_i)^2\right) dC_x$$

$$f(x,y) = \pm\sqrt{y^2 - f(x)^2} \qquad \pi \int \overset{g(x)}{\overset{\shortparallel}{f(x)^2}}\, dx = \int_{z=-f(x,0)}^{z=f(x,0)} f(x,y)$$

$2x+1 = f(x)$

$g(x)$

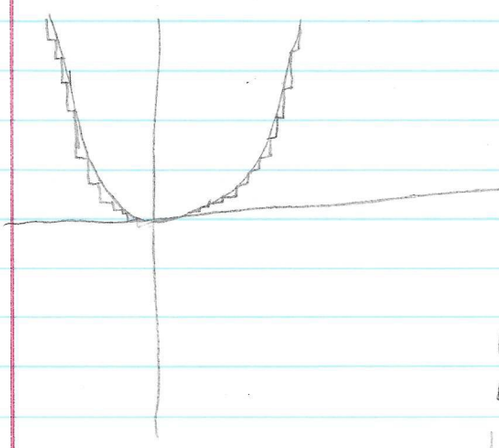
$$\pi \int f(x)^2\, dx = \iint f(x,y)\, dA$$

if $f(x)$ is continous

length of line on interval $[A,B] = \sum_{x=A}^{B} \left(\sqrt{(f(x+h)-f(x))^2 + h^2}\right)$; $h \to 0$

$$\sum_{x=A}^{B\cdot\frac{1}{h}} \left(\sqrt{(f(xh+h)-f(xh))^2 + h^2}\right)$$

Sum of the hypotenuses of infinitely small distances.



step size $= .25$

$f(x) = x^2$

interval $= [-1,1]$

$L = G(x) = \sqrt{(f(x+.25)-f(x))^2 + .25^2}$

$L_1 = \sqrt{(f(-1+.25)-f(-1))^2 + .25^2} = .50389$

$L_2 = \sqrt{(f(-.75+.25)-f(-.75))^2 + .25^2} = .40019$

$L_3 = G(-.5) = .3125$

$L_4 = G(-.25) = .25769$

$L_5 = G(0) = .25769$

$L_6 = G(.25) = .3125$

$L_7 = G(.5) = .40019$

$L_8 = G(.75) = .50389$

$L_T = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8$

$L_T = 2.14854$

$$\sum_{x=-1\cdot\frac{1}{.25}}^{1\cdot\frac{1}{.25}-1} G(x\cdot .25)$$

if $f(x)$ is continous

length of line $[A,B]$ on $f(x) =$

$$\sum_{x=A\cdot\frac{1}{h}}^{B\cdot\frac{1}{h}-1} \sqrt{(f(h(x+1)) - f(xh))^2 + h^2} \; ; h \to 0$$

$$\sum_{x=A\cdot\frac{1}{h}}^{B\cdot\frac{1}{h}-1} G(x\cdot h)$$, step size $= h$

Q 1 cont

h) $\sum F_x = ma_x$ $\qquad F^g_{g,0} = -9.8 \cdot .2\,kg$

$F^f_{rmp,0} + F^g_{g,0,x} = ma_x$ $\qquad a_{rot} = \frac{a_x}{r}$

$\sum \tau = \cancel{mOC}\, I\, a_{rot} = F^s_{r,0} \cdot \overset{R}{.05\,m} = I \frac{a_x}{r}$

$F^s_{r,0} = \frac{2}{5} m \cdot \cancel{R^2} \cdot \frac{a_x}{\cancel{R^2}}$

$\frac{2}{5} m a_x + mg \sin(\frac{\pi}{6}) = ma_x$

$\frac{+-9.8\frac{m}{s^2} \sin(\frac{\pi}{6})}{\frac{2}{5}} = a_x = -8.17\frac{m}{s^2}$

3. **(10 points)** Consider the surface S in $\mathbb{R}^3$ given by the equation $y - x^2 = 0$.

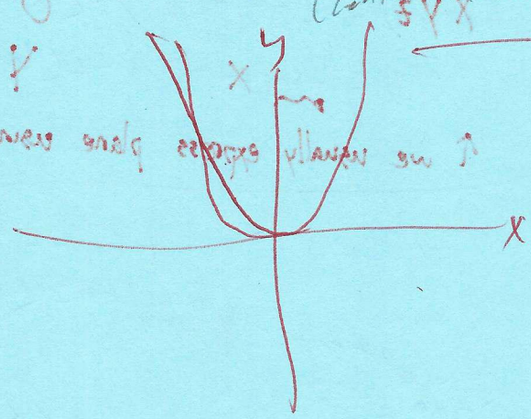
(a) Describe the intersection of S with the xy-plane (that is, the plane with $z = 0$). What kind of curve is it?

$y - x^2 = 0$

$y = x^2$

it be parabola when $z=0$

(centered at origin)



5

(b) Describe the intersection of S with the xz-plane (that is, the plane with $y = 0$). What kind of curve is it?

$0 - x^2 = 0$

$x = \pm 0 = 0$

+3.5

it is a straight line where

~~$\{(a,b) \in \mathbb{R}^2 : a = 0,$~~ ??

$\{0\} \times \mathbb{R}$ is the points

(x, z)

$\{(0, 0, z) : z \in \mathbb{R}\} = z$-axis

line.

8.5

Fig. 2
1.5
1.0
0.5
0
Cmp 0.55

$$\lim_{x \to \mathbb{R}} \left[\frac{\sum_{-\infty}^{\infty} (x+n)@n}{\sum_{-\infty}^{\infty} (x+n)@n} \right] = 1 \qquad \frac{f(\infty)}{f(\infty)}$$

CLAVIER INSTRUCTIONAL No. 150056

OGL MIP3 overstate logic

emphasis: there exists a function op(a,b) such that if ~~can be used~~ a composition of op, a, and b can create all atoms f(a,b), a, b in an arbitrary finite set U, and op must be closed in U, then op is an OGLG, then it has to be the above, ~~it also~~ ~~should be true~~ ~~op~~ is to do this op(A,A) ≠ A and op(A,B) ≠ A and op(A,B) ≠ B.

```
opN(A,B) := if (A=B)
                return (A-1)%N
            else
                return 0.
```

Maximum:

```
op(a,b,n) := if a=b then
                 mod((a-1), n)
             else
                 0
```

```
test(f,n) := gen matrix(lambda([i,j], f(i,j)), n,n)
```

print truth table.

f (a,b)

from MIP4 OGL

C_7 A	0	0	0	0
C_6 B	0	0	0	0
	0	0	2	0
	0	0	0	0
				C_4

C_7 B	0	1	1	1
C_7 A	1	1	1	1
	1	1	2	1
	1	1	1	3
			C_8	

C_5 A	1	3	3	3
C_5 B	3	3	3	3
C_{10}	3	3	3	3
	3	3	3	3

C_6 A	0	3	3	3
C_6 B	3	3	3	3
	3	3	2	3
	3	3	3	3
			C_4	

OGL m2 P5

~~Let $U \subseteq Z$~~ : Let $U_p = \{x \in Z : 0 \le x \le p\}$

for arbitrary $p \in \{Z : \{x \in Z : x > 1\}\}$

$OP_p : U \times U \to U$

$OP_p(a,b) = \text{if } (a=b)$
$\quad (a-1) \bmod (p)$
$\text{else } 0$

OP_p onto: Choose arbitrary $c \in U$

$c = OP_p(a,b)$ $\qquad m = \text{mod}$

$c = \text{if } (a=b)$
$\quad (a-1) \bmod (p)$
$\text{else } 0$

$c = OP_p((c+1) \bmod (p), (c+1) \bmod (p))$

$c = \text{if } ((c+1)_m(p) = (c+1)_m(p))$
$\quad ((c+1)_m(p) - 1)_m(p)$
$\text{else } 0$

$c = ((c+1)_m(p) - 1)_m(p)$

$c = ((c+1) + p \cdot m) - 1) \bmod (p)$

$c = c \bmod (p)$

OP_p onto.

$((a+b) \bmod (c) + d) \bmod (c) \; c = OP_p$
$\overset{?}{=} (a+b+d) \bmod (c)$

$a \bmod b \equiv c$
~~$a = cb \; c \in Z$~~
$c = a + bm \quad m \in Z$

want

AB	0	1	2
0	0	2	2
1	0	1	0
2	1	1	0

AB	0	1	2
0	1	0	0
1	1	2	1
2	2	2	1

$\xrightarrow{a,\ b}$

	0	1	2
0	0	2	2
1	0	1	0
2	1	1	0

OGL MIPZ (negate logic)

$(\forall a \in U)(op(A,A) \neq A)$

$(\forall a \in U)(\forall b \in U)\ (op(A,B) \neq A) \wedge (op(A,B) \neq B)$

U is the set of opins i.e. binary [T, F], trinary [2, 1, 0]

$(\forall a \in U)(\forall b \in U)(\forall f(a,b) \in \mathbb{F}_2(U))(\exists\ op(a,b) \in \mathbb{F}_2(U))$

$\mathbb{F}_2(U)$ is the set of function in U without arity 2.

swap these two

$(\forall a,b \in U)(\exists\ op(a,b) \in \mathbb{F}_2(U))((\forall f(a,b) \in \mathbb{F}_2(U))(\exists$ a composition of op, called g) $(g(a,b) \equiv f(a,b)))$

further more

$(\forall c \in U)(op(c,c) \not\equiv c) \wedge ((\forall d,e \in U)((op(d,e) \not\equiv d) \wedge (op(d,e) \not\equiv e)))$

~~there exists a function op(a,b) for the set of~~

an op of size N

A \ B	0	1	2	3	----
0	N	0	0	0	---0
1	0	0	0	0	---0
2	0	0	1	0	---0
3	0	0	0	2	---0
⋮	⋮	⋮	⋮	⋮	⋮
	0	0	0	0	---A-1

N-1

when $A = B$

$op(A,B) = (A-1) \% N$

else

0

I think this truth table works for all N

$op_N(A,A) \equiv (A-1) \% N$

~~$op_N(op(A,A), op(B,B)) \equiv op_{N-1}(A,B)$; $A, B \in (N-1)$~~

if something not set

, called op_{N-1}

A \ B	0	1	2	3	----
0	N-1				
1		0			
2			1		
3					
⋮					

N-1

then I should be able to say that

if op_n ~~[illegible]~~ is OGL, then ~~op_{n-1}~~ op_{n-1} is an OGL for N+1, and ~~if [illegible]~~ op_{n-2} is OGL

---- op_2 is OGL.

OGL m2 p6

want

A \ B	0	1	2
0	1	2	0
1	1	2	0
2	2	1	1

Grim

A \ B	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2

A \ B	0	1	2
0	0	0	0
1	1	1	1
2	2	2	2

0 2 2

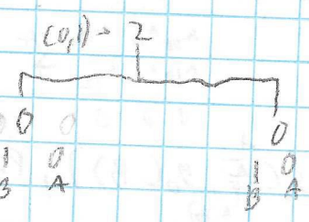
A \ B	0	1	2
0	2	0	0
1	0	0	0
2	0	0	1

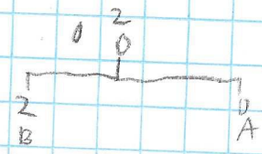
	0	1	2
0	1	2	0
1	1	2	0
2	2	1	1

2	0	2
2	0	2
0	2	2
2	0	0
2	0	1
0	2	2

0	1	0
0	1	0
0	0	0
0	0	0
0	1	0
2	0	0

lookat





OGL m3 P3

Thm 1 scriptn maxima:

OR(a,b) := block([], if a >= b then return(a) else return(b));

Pass(a,b,base) := block([], if a = base then return(b) else return(a));

f(a,b) := [[1,2,3],[4,5,6],[7,8,9]][a+1][b+1];

g(a,base) := f(base,a);

h(A,B) := OR(OR(Pass(A, g(B,0),0), Pass(A, g(B,1),1)), Pass(A, g(B, 2),2));

---

thm 1 $\frac{1}{2}$ scriptn:

$Pass_n(u) =$ if $(n=u)$ return u else 0

g_0

$Pass_0$

a_b 0 0

OR_3

want

A^B	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

A B C

$\begin{matrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{matrix} =$

$A \to$ $\overset{B}{\downarrow}$ OP_3 call C

A^B	0	1	2
0	a_{00}	-	.
1	.	.	.
2	.	.	.

$\bar{A}$

A^B	0	1	2
0	0	0	0
1	1	1	1
2	2	2	2

$\bar{B}$

A^B	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2

OP_3

A^B	0	1	2
0	2	0	0
1	0	0	0
2	0	0	1

$\bar{B}$ OR ①

$\bar{A} \to$

2	0	0
0	0	0
0	0	1

$\bar{A}$ ②

0	2	2
0	0	0
0	0	0

$\bar{B}$ ③

2	0	1
2	0	0
2	0	0

③ $\bar{A}$ ④

0	2	0
0	0	0
1	0	0

(4) $\bar{A}$ (5)

2	0	2
0	0	0
0	0	0

(2) (4) (6)

0	0	1
2	2	2
2	2	2

(1) $\bar{B}$ (3)

0	0	1
2	0	0
0	0	1

(7)

1	2	2
2	2	2
2	2	0

$\bar{O}$

0	0	0
0	0	0
0	0	0

$\bar{2}$

2	2	2
2	2	2
2	2	2

$\bar{1}$

1	1	1
1	1	1
1	1	1

(D) $\bar{A}$ (F)

0	0	0
0	0	0
1	1	1

(F) (F) (G)

2	2	2
2	2	2
0	0	0

(6) (F) (H)

0	0	0
1	1	1
0	0	0

(H) (H) (I)

2	2	2
0	0	0
2	2	2

(8) (9) (8) $\bar{O}$ (C) (D) (E)

$\bar{A} \to$

0	1	1
1	1	1
1	1	2

2	0	0
0	0	0
0	0	1

2	0	0
0	0	0
0	0	0

1	2	2
2	2	2
2	2	2

0	1	1
1	1	1
1	1	1

OGL m.3 P4

$U = Z_2 = \{0,1\}$

want

B \ A	0	1
0	0	1
1	1	1

OP

A \ B	0	1
0	1	0
1	0	0

$\bar{A}$ \ B	0	1
0	0	0
1	1	1

$\bar{B}$ \ A	0	1
0	0	1
1	0	1

$\bar{A} \to 1\ 1$

$\bar{A} \to 0\ 0$

$\bar{B} \to 1\ 0$

$\bar{B} \to 1\ 0$

$\bar{A} \to 1\ 0 \to 0\ 1$ ✓

$\bar{B} \to 0\ 0 \to 1\ 1$

Z_3

	0	1	2
0	2	0	0
1	0	0	0
2	0	0	1

$\bar{A} \to 2\ 0\ 0 \to 1\ 2\ 2$

$B \to 0\ 0\ 0 \quad 2\ 2\ 2$

$0\ 0\ 1 \to 2\ 2\ 0$

want

$0\ 1\ 0$

OP(A,A)

x =	0	1	2
	2	0	1

$\bar{A}$	0	1	2
	0	1	2

~~$\bar{A} \to 2\ 0\ 1 \to 1\ 2\ 0$~~

$0\ 1\ 2 \to OP(A,A)$

0 1 2	2	0	1
0 1 2	2	0	1
	2	0	1

$0\ 1\ 2 \to OP(A,A) \to 1\ 2\ 0$

$2\ 0\ 1 \to$

$B\ 0\ 0\ 0 \to 2\ 2\ 2 \quad 0\ 1\ 0$

Ex $U = Z_n$, $Z_n = \{[0],[1],[2],\dots,[n-1]\}$

$n > 1$

$OP: U \times U \to U$

$OP([a],[b]) =$ if $([a]=[b])$
return $[a-1]$
else
return $[0]$

$= ([a]=[b])?\ [a-1] : [0]$

$OP(OP([a],[b]),[b]) =$
if $(OP([a],[b]) == [b])$
return $[OP([a],[b])-1]$
else
return $[0]$

$=$ if $(\text{if } ([a]==[b])?\ [a-1]:[0]) == [b])$
$[a-1]-1$
else
$[0]$

$= (([a]==[b])?\ [a-1]==[b] : [0] == [b])?$
$[a-1]:[0]$

$= (([a]==[b])?\ ([a-1]==[b])?[a-2]:[0]) :$
$([0]==[b]?\ [a-2]:[0]))$

$= (([a]==[b])?\ [0] :$

OGL my PI

isolate and combine

OP_{n+1}

A \ B	0	1	2	...	n
0	n	0	0		0
1	0	0	0		0
2	0	0	1		0
⋮	0	0	0	... A-1	⋮
n	0	0	0	...	n-1

Combine (CO)

A \ B	0	1	2	...	B	...	n
0	0	1	2				n
1	1	1	*		*		*
2	2	*	2		*		⋮
⋮	A	*	*	...	A		⋮
n	n	**		...			n

$\bar{A}_0$

A \ B	0	1	...	n
0	0	0	...	0
1	1	1	...	1
⋮	⋮			⋮
A	A	...		A
n	n	...		n

$\bar{B}_0$

A \ B	0	1	...	B	...	n
0	0	1	...	B	...	n
1	0	1				⋮
⋮	⋮	⋮		⋮		⋮
A	0	1	...	B	...	n
⋮	⋮			⋮		⋮
n	0	1	...	B	...	n

$\bar{A}_0$ and $\bar{B}_0$ are quasi gates, they exist to make initial conditions, they are not functions that use op, cons.

$\bar{A}_m$ is $\bar{A} \to \to \dots (m-1) \to$

$\bar{A} \to \to \dots (m-1) \to$

$\bar{A}_m = op(\bar{A}_{m-1}, \bar{A}_{m-1})$

$\bar{B}_m = op(\bar{B}_{m-1}, \bar{B}_{m-1})$

$\bar{0} = op(\bar{A}_0, \bar{A}_1)$

$\bar{m} = if(m \neq 0)$
$op(\overline{(m+1)}, \overline{(m+1)})$
else
$\bar{0}$

Let $U = Z_n$.
$= \{[x] \in Z : 0 \le x \le n\}$

$+$ $-$ are cyclical.

T = sum interpreting table function

to get $\overline{(a,b,c)}$

A \ B	0	1	...	b	...	n
0	0	0	0	0	0	0
1	0	0	...	...		0
⋮	⋮					0
a	0	0	...	c		0
⋮	⋮					0
n	0	0	...	0	...	0

$\overline{(a,b,c)}(x,y) = if(x = a \wedge y = b)$
return c
else
return 0

$\overline{(a,b,c)} = \bar{A}_a \to \quad T \quad \bar{n} \to \quad T$ ... combining with $\bar{T}$ will go out, it changes, $\wedge \in U$
or $\bar{B}_b \to$ or it itself with itself

if (c) = 0
return $\bar{0}$

to get sum take W, for each $\overline{(a,b,c)}$, we combine each $\overline{(a,b,c)}$ with CO mbine.

Since you can not ~~any time combine~~ any time combine is called at least once it's ignore is 0,

combine(a,b) = if (a = 0)
b
if (b = 0)
a
else
*

Combine (CO)

A \ B	0	1	2	...	B	...	n
0	0	1	2		B		n
1	1	*	*		*		*
2	2	*	*		*		*
⋮	⋮			⋱			⋮
A	A	*	*		*		*
⋮	⋮				⋮		⋮
n	n	*			*		*

* is any thing in Z_n

OGL mY P2

OP_4

want combine × isomorphism.

$\bar{A}_0$ $\bar{A}_1$ $\bar{A}_2$ $\bar{A}_3$

$\bar{B}_0$ $\bar{B}_1$ $\bar{B}_2$ $\bar{B}_3$

$\bar{0}$ $\bar{3}$ $\bar{2}$

$\bar{1}$

$\bar{F}_{00}$ $\bar{F}_{01}$ $\bar{F}_{02}$ $\bar{F}_{03}$

$\bar{F}_{10}$ $\bar{F}_{11}$ $\bar{F}_{12}$ $\bar{F}_{13}$

$\bar{F}_{20}$ $\bar{F}_{21}$ $\bar{F}_{22}$ $\bar{F}_{23}$

$\bar{F}_{30}$ $\bar{F}_{31}$ $\bar{F}_{32}$ $\bar{F}_{33}$

$\overline{(0,0,0)} = \bar{0}$

$\overline{(0,1,1)}$: $\bar{A}_0 \to$ $\bar{B}_1 \to$

$\overline{(0,2,2)}$: $\bar{A}_0 \to$ $\bar{B}_2 \to$ $\bar{F}_{00}$

$\overline{(0,3,3)}$: $\bar{A}_0 \to$ $\bar{B}_3 \to$ $\bar{F}_{21}$

↑ boundary with

$\overline{(1,0,1)}$: $\bar{A}_1 \to$ $\bar{B}_0 \to$ $\bar{F}_{10}$

or

$\overline{(2,0,2)}$: $\bar{A}_2 \to$ $\bar{B}_0 \to$ $\bar{F}_{00}$

$\overline{(3,0,3)}$: $\bar{A}_3 \to$ $\bar{B}_0 \to$ $\bar{F}_{21}$

~~if~~ $\overline{(c,h,0)}$ = if (c ≠ h, -1) if (c = 0) return $\bar{0}$

$\bar{A}_6 \to$ $\bar{B}_6 \to$ $\bar{F}_{(c-2-c)0}$ T $\overline{c+1} \to W$

else

$\bar{A}_6 \to$ $\bar{B}_6 \to$ $\bar{F}_{(c-1)1}$ T $\bar{0} \to W$

return W

~~[illegible]~~ Z_4