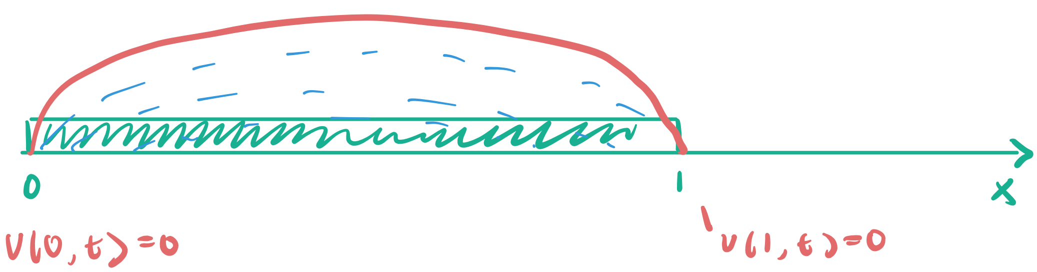
· similar to Assignment 1, but unknown function

← heat equation

$u(x,t)$ = temperature on

$u(x,0) = u_0(x)$ - initial



$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial t}(x_i, t_k) = \frac{\partial^2 u}{\partial x^2}(x_i, t_k) + \cancel{f(x,t)}$$

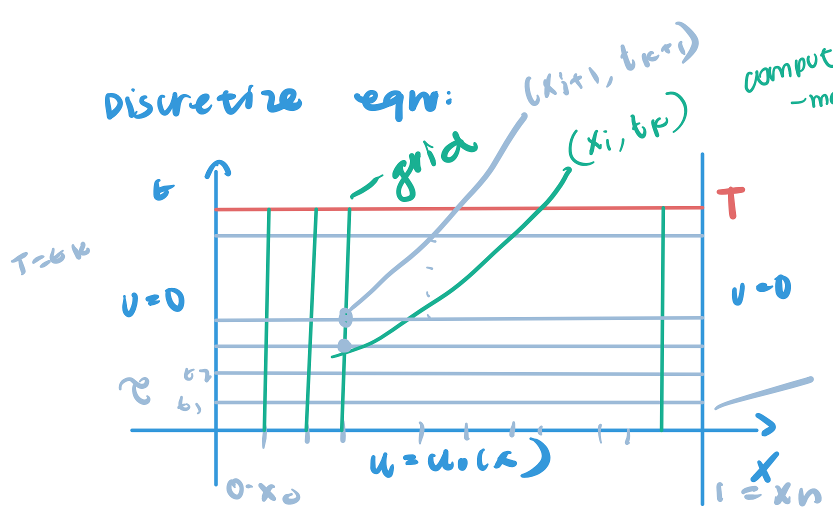
don't have function - external forces

NO EXTERNAL HEATING IN THE system

Heat equation - numerical implementation

Discretize eqn:

computed solution - matrix



τ much smaller than h

$h = 1/n$

- introduce grid

Assignment 1

$$u''(x) \approx \frac{u(x_{i+1}) - 2u(x_i) + u(x_{i-1})}{h^2}$$

now

variable

frozen

, use for any t

$$\frac{\partial^2 u}{\partial x^2}(x_i, t) \approx \frac{u(x_{i+1}, t) - 2u(x_i, t) + u(x_{i-1}, t)}{h^2}$$

variable

frozen

, use for any t

$$\frac{\partial^2 u}{\partial x^2}(x_i, t_k) \approx \frac{u(x_{i+1}, t_k) - 2u(x_i, t_k) + u(x_{i-1}, t_k)}{h^2}$$

$$\frac{\partial u}{\partial t}(x_i, t_k) \approx \frac{u(x_i, t_{k+1}) - u(x_i, t_k)}{k}$$

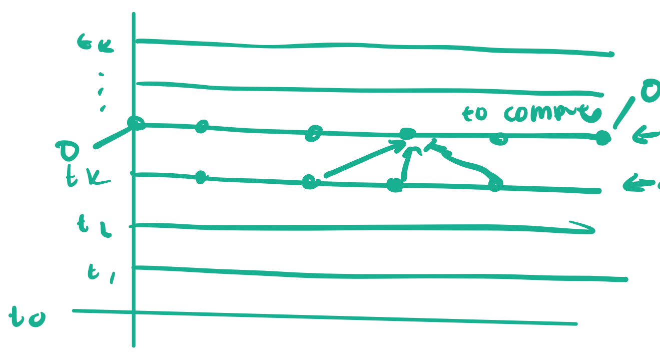
$$\frac{u(x_i, t_{k+1}) - u(x_i, t_k)}{k} \approx \frac{u(x_{i+1}, t_k) - 2u(x_i, t_k) + u(x_{i-1}, t_k)}{h^2}$$

unknown — known

$$\frac{U_i^{k+1} - U_i^{k}}{k} = \frac{U_{i+1}^{k} - 2U_i^{k} + U_{i-1}^{k}}{h^2}$$

↑ explicit euler

How to implement



time t_0 initial condition

← compute $U_i^0 = u_0(x_i)$

← given $= \sin(\pi x^2)$

define lambda for initial condition

for $k = 0 : K-1$:

given computed soln U_i^k

compute U_i^{k+1}

?

$i = 1, \ldots, n-1$

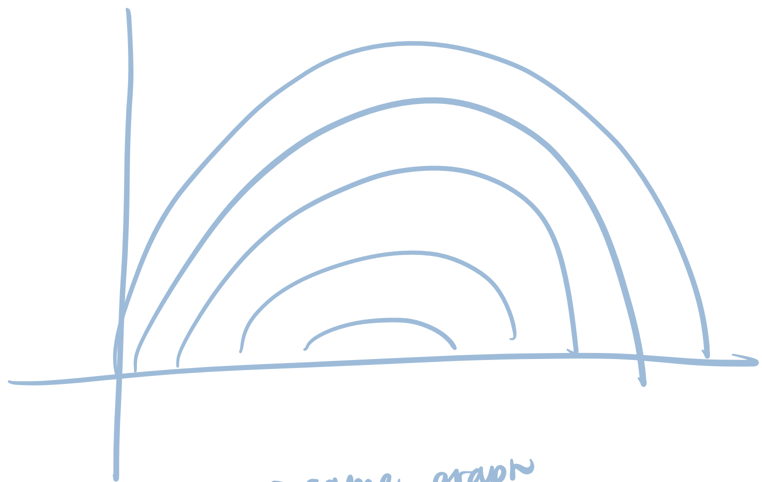
$$\left.\begin{aligned} U_0^{k+1} &= 0 \\ U_{n+1}^{k+1} &= 0 \end{aligned}\right\} \text{B.C.}$$

loop in time (i.e. k) is required

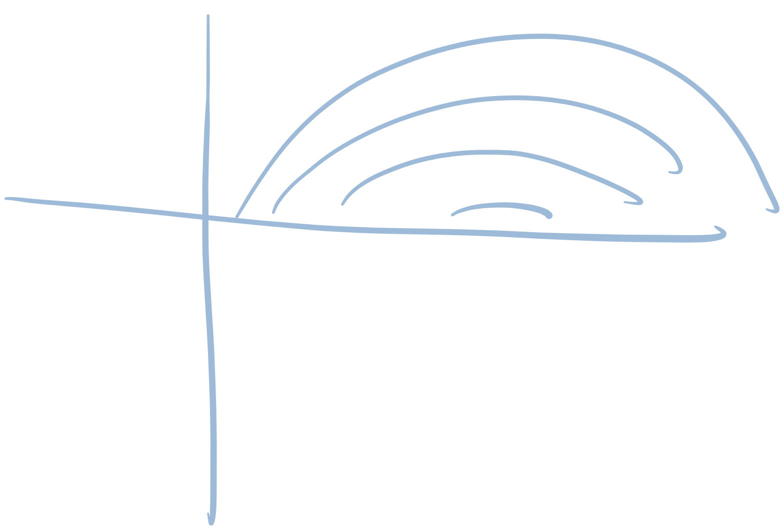
loop in (i) is NOT - vector operations

try to implement w/o loop

Plots



- same graph



sometimes numerically methods unstable
based on timestamp