MP345: Maths Methods Assignment 2

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Q1 $y'' - ay' + 6x^2y = 0$

$$a=6 \quad b=3$$

$$y'' - 6y' + 3x^2y = 0$$

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 = \sum_{n=0}^{\infty} a_n \underline{X^n}$$

$$y' = a_1 + 2a_2X + 3a_3X^2 + 4a_4X^3 = \sum_{n=1}^{\infty} n a_n X^{n-1}$$

$$n-1=p \rightarrow n=p+1 \qquad = \sum_{p=0}^{\infty} (p+1)\, a_{p+1} X^p$$

$$\boxed{= \sum_{n=0}^{\infty} (n+1)\, a_{n+1} \underline{X^n}}$$

$$yx^2 = a_0X^2 + a_1x^3 + a_2x^4 + a_3x^5 = \sum_{n=0}^{\infty} a_n X^{n+2}$$

$$n+2=p \rightarrow n=p-2 \qquad \sum_{p=2}^{\infty} a_{p-2} X^p = \boxed{\sum_{n=2}^{\infty} a_{n-2} \underline{X^n}}$$

$$y'' = 2a_2 + 6a_3X + 12a_4x^2 + 20a_5x^3 = \sum_{n=2}^{\infty} n(n-1)\, a_n X^{n-2}$$

$$n-2=p \rightarrow n=p+2 \qquad = \sum_{p=0}^{\infty} (p+2)(p+1)\, a_{p+2} X^p$$

$$= \boxed{\sum_{n=0}^{\infty} (n+2)(n+1)\, a_{n+2} \underline{X^n}}$$

$$y' = a_0 + 2a_2X + \sum_{n=2}^{\infty} (n+1)\, a_{n+1} X^n$$

$$y'' = 2a_2 + 6a_3X + \sum_{n=2}^{\infty} (n+2)(n+1)\, a_{n+2} X^n$$

$$yx^2 = \sum_{n=2}^{\infty} a_{n-2} X^n$$

$$y'' - 6y' + 3x^2 y = 0$$

$$\underbrace{2a_2 + 6a_3 x - 6(a_0 + 2a_2 x)}_{\gamma} + \sum_{n=2}^{\infty} \dots\dots = 0$$

$$\gamma + \sum_{n=2}^{\infty} \Big[\underbrace{(n+2)(n+1)a_{n+2} - 6((n+1)a_{n+1}) + 3(a_{n-2})}_{=0}\Big] x^n$$

β

$$\gamma \rightarrow 2a_2 + 6a_3 x - 6(a_0 + 2a_2 x) = 0$$

$$2a_2 + 6a_3 x = 6a_0 + 12a_2 x$$

$$\div 2 \quad a_2 + 3a_3 x = 3a_0 + 6a_2 x$$

$$3a_3 \cancel{x} = 6a_2 \cancel{x} \qquad a_3 = 2a_2$$

$$a_2 = 3a_0$$

$$a_3 = 2a_2 = 6a_0$$

$$\beta \rightarrow (n+2)(n+1)a_{n+2} - 6((n+1)a_{n+1}) + 3(a_{n-2}) = 0$$

$$(n+2)(n+1)a_{n+2} = 6(n+1)a_{n+1} - 3(a_{n-2})$$

$$a_{n+2} = \frac{6(n+1)a_{n+1} - 3(a_{n-2})}{(n+2)(n+1)}$$

$$\boxed{a_{n+2} = \frac{6a_{n+1}}{(n+2)} - \frac{3a_{n-2}}{(n+2)(n+1)} \qquad a_3 = 2a_2 = 6a_0}$$

Recursion Relation.

try One solution $\rightarrow$ $\boxed{a_1 = 2 \quad a_0 = 1}$ $\quad a_2 = 3a_0 \Rightarrow$

$a_2 = 3 \qquad a_3 = 2a_2 \Rightarrow$

$a_3 = 6$

Rearrange Recursion Relation:

$$6(n+1)a_{n+1} = 3a_{n-2} + (n+2)(n+1)a_{n+2}$$

$$a_{n+1} = \frac{1}{2}\left(\frac{a_{n-2}}{(n+1)}\right) + \frac{1}{6}(n+2)a_{n+2}$$

$$3a_{n-2} = 6(n+1)a_{n+1} - (n+2)(n+1)a_{n+2}$$

$$a_{n-2} = 2(n+1)a_{n+1} - \frac{1}{3}(n+2)(n+1)a_{n+2}$$

$a_1 = 2$

$a_0 = 1 \quad a_2 = 3 \quad a_3 = 6$

$$a_{n+2} = \frac{6a_{n+1}}{(n+2)} - \frac{3a_{n-2}}{(n+2)(n+1)}$$

$$a_4 = \frac{6a_3}{(2+2)} - \frac{3(a_0)}{(2+2)(2+1)}$$

$$a_4 = \frac{6(6)}{4} - \frac{3(1)}{4(3)}$$

$$a_4 = 9 - \frac{1}{4}$$

$$a_4 = \frac{35}{4}$$

$$y_0 = 1 + 2x + 3x^2 + 6x^3 + \frac{35}{4}x^4 + \ldots\ldots$$

try $y_1 \rightarrow \quad a_0 = 3 \quad a_1 = 10$

$a_2 = 9 \quad a_3 = 18$

$n = 2$

a_{n+2}

$$a_4 = \frac{6a_3}{(2+2)} - \frac{3(a_0)}{(2+2)(2+1)}$$

$$a_4 = \frac{6(18)}{4} - \frac{3(3)}{12}$$

$$a_4 = \frac{108}{4} - \frac{9}{12} = \frac{105}{4}$$

$$y_1 = 3 + 10x + 9x^2 + 18x^3 + \frac{105}{4}x^4 + \ldots$$

general solution is then

$$y(x) = c_1 y_0(x) + c_2 y_1(x)$$

c_1, c_2 constants

y_0, y_1 linearly independent.

$a=6 \qquad b=3$

Q2. $y(0)=a=6 \qquad y'(0)=b=3$

$$y = c_1\left(1+2x+3x^2+6x^3+\tfrac{35}{4}x^4+\ldots\right)$$
$$+ c_2\left(3+10x+9x^2+18x^3+\tfrac{105}{4}x^4+\ldots\right)$$

$y(0)=6$

$$6 = c_1\left(1+2(0)+3(0)^2+6(0)^3+\tfrac{35}{4}(0)^4\right)$$
$$+ c_2\left(3+10(0)+9(0)^2+18(0)^3+\tfrac{105}{4}(0)^4\right)$$

$$6 = c_1 + 3c_2$$

$$y'(0) = 3 = c_1\left(2+6(0)+18(0)^2+35(0)^3\right)$$
$$+ c_2\left(10+18(0)+54(0)^2+105(0)^3\right)$$

$$3 = 2c_1 + 10c_2$$
$$6 = c_1 + 3c_2$$

$$3 = \cancel{2c_1} + 10c_2$$
$$-12 = \cancel{-2c_1} - 6c_2$$

$$-9 = 4c_2$$
$$c_2 = -9/4$$

$$c_1 = 6 - 3c_2 = 6 - 3(-9/4) = 12\tfrac{3}{4} = 51/4$$
$$c_1 = 51/4$$

In [3]:

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#Question 2:

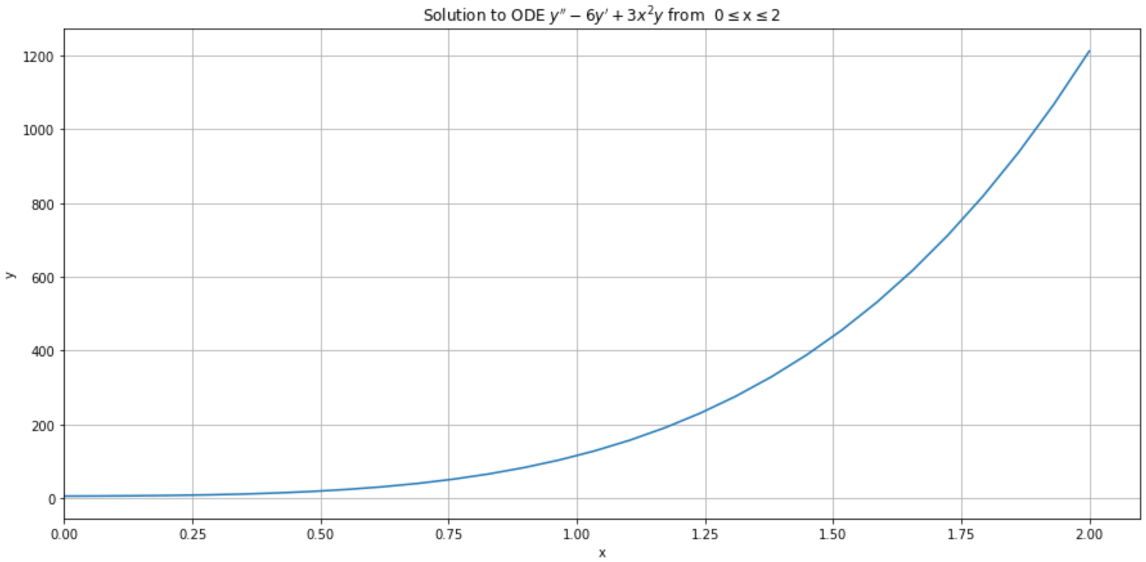

#plotting my solution to the ODE in Q1 subject to initial conditions y(0) = 6 and y'(0)
= 3

import numpy as np
import matplotlib.pyplot as plt
import matplotlib

#define array of x values along horizontal axis
x = np.linspace(0,2,30)

#define solution
y = (51/4)*(1 + 2*x + 3*x**2 + 6*x**3 + (35/4)*x**4) + (-9/4)*(3 + 10*x + 9*x**2 + 18*x
**3 + (105/4)*x**4)

#plot y against x
fig= plt.figure(figsize=(15,7))
plt.xlim(0,2.1)
plt.plot(x, y)
plt.title("Solution to ODE $y'' - 6y' + 3x^2y$ from  0$\leq$x$\leq$2")
plt.xlabel("x")
plt.ylabel("y")
plt.grid()
plt.show()
```



In [4]:

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#Question 3:

y_0 = np.zeros(100) #set array of 100 zeros to hold a_n values for y_0
y_0[0] = 1          #define first 5 a_n values
y_0[1] = 2
y_0[2] = 3
y_0[3] = 6
y_0[4] = (35/4)

y_1 = np.zeros(100) #set array of 100 zeros to hold a_n values for y_1
y_1[0] = 3          #define first 5 a_n values
y_1[1] = 10
y_1[2] = 9
y_1[3] = 18
y_1[4] = (105/4)

for n in range(3,98):   ## recursion relation for y_0 to assign all a_n values from a_5 to a_99
    y_0[n+2] = (6*(y_0[n+1])/(n+2)) - (3*(y_0[n-2])/((n+2)*(n+1)))

for n in range(3,98):   ## recursion relation for y_1 to assign all a_n values from a_5 to a_99
    y_1[n+2] = (6*(y_1[n+1])/(n+2)) - (3*(y_1[n-2])/((n+2)*(n+1)))


#Multiply all of y_0 and y_1 by C1 and C2
C1 = 51/4
C2 = -9/4

y_0 = y_0 * C1
y_1 = y_1 * C2

#define y - the general solution and x
y = np.zeros(100)
x = np.linspace(0,2,100)
y_values = np.zeros(100)


for i in range(100):      #y= y0 +y1 (coefficients are being added here)
    y[i] = y_0[i] + y_1[i]

for j in range(100):       #y terms are multiplied by corresponding x terms (1,x,x^2 .. etc)
    total = 0              #and then y values for graph are computed by summing y terms at each x
    for i in range(100):
        total = total + y[i]*(x[j]**i)
    y_values[j] = total



#plot y against x
fig= plt.figure(figsize=(15,7))
plt.xlim(0,2.1)
plt.plot(x, y_values)
plt.title("Solution to ODE $y'' - 6y' + 3x^2y$ for 100 terms of power series from  0$\leq$x$\leq$2")
plt.xlabel("x")
plt.ylabel("y")
```

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plt.grid()
plt.show()
```