# Maths Methods HW1

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## Question 1:

We are provided an inhomogeneous 2nd Order ODE modelling Damped Harmonic Motion as follows:

$$m\ddot{y} + c\dot{y} + ky = F_0 cos(\omega_0 t)$$

and we must provide values of $m, c, k, F_0$ and $\omega_0$ to the equation and solve it for a solution that is similar to the graph of the overdamped solution given in the question.

I chose values as follows:

$$9\ddot{y} + 3\dot{y} + 6y = \frac{1}{2}cos(t)$$

I attempt to get a general solution $y_h + y_p$ where $y_h$ is the homogeneous solution and $y_p$ is the particular solution to the equation.

I start by trying to identify $y_h$. I try $y_h = e^{rt}$ and end up solving a quadratic equation which has complex roots. The general solution for the Homogeneous Equation here is of the form:

$$y = c_1 e^{\alpha t} cos(\beta t) + c_2 e^{\alpha t} sin(\beta t)$$

inputting my solutions I obtain:

$$y_h = c_1 e^{\frac{-1}{6}t} cos\left(\frac{\sqrt{23}}{6}\right) t) + c_2 e^{\frac{-1}{6}t} sin\left(\frac{\sqrt{23}}{6}t\right)$$

To Obtain $y_p$ first I made an informed guess and decided to guess that $y_p$ is of the form $ACos(t) + Bsin(t)$. I computed the first 2 derivatives of this and substituted it into the original inhomogeneous ODE. I found that $A = \frac{-1}{12}$ and $B = \frac{1}{12}$. Now I have the particular solution $y_p = \frac{-1}{12}Cos(t) + \frac{1}{12}Sin(t)$.

I can now write the General Solution:

$$y = c_1 e^{\frac{-1}{6}t} cos\left(\frac{\sqrt{23}}{6}\right) t) + c_2 e^{\frac{-1}{6}t} sin\left(\frac{\sqrt{23}}{6}t\right) - \frac{1}{12}Cos(t) + \frac{1}{12}Sin(t)$$

Setting $y(0) = 0$ and $\dot{y}(0) = 1$ I can determine the constants $c_1 and c_2$

$$c_1 = \frac{1}{12}$$
$$c_2 = 1.164$$

My final solution is then:

$$y = \frac{1}{12}e^{\frac{-1}{6}t}cos\left(\frac{\sqrt{23}}{6}\right)t) + 1.164e^{\frac{-1}{6}t}sin\left(\frac{\sqrt{23}}{6}t\right) - \frac{1}{12}Cos(t)$$
$$+ \frac{1}{12}Sin(t)$$

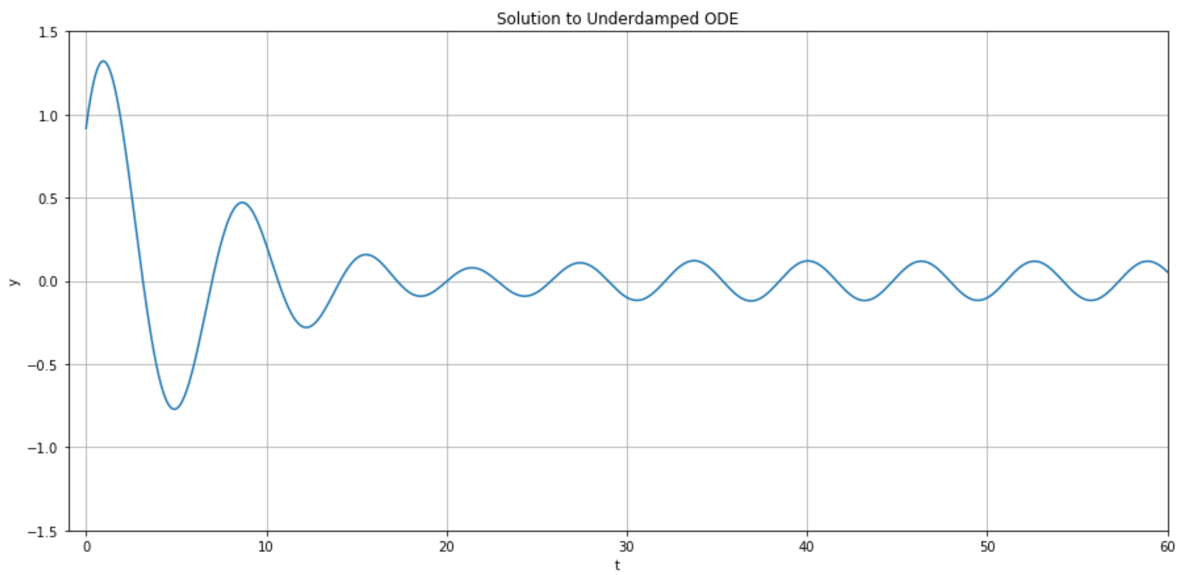
In [50]:

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib

#define array of t values for time along horizontal axis
t = np.linspace(0,200,5000)

#define solution
y = (np.cos(((np.sqrt(23))/6)*t) + (1.16)*np.sin(((np.sqrt(23))/6)*t))*np.exp((-1/6)*t) - (

#plot y against t
fig= plt.figure(figsize=(15,7))
plt.ylim(-1.5,1.5)
plt.xlim(-1,60)
plt.plot(t, y)
plt.title("Solution to Underdamped ODE")
plt.xlabel("t")
plt.ylabel("y")
plt.grid()
plt.show()
```




## Question 2:

The first part of question 2 is to guess the form of one solution of the ODE:

$$3x^2y'' + xy' - y = 0$$

I guessed that $y_1$ is of the form $y_1 = x$ which solves the equation.

I calculated y_2 using Theorem 1.5:

$$y_2 = y_1 \int \frac{1}{{y_1}^2} e^{-\int P}$$

solving this integral I found $y_2$ to be

$$y_2 = -\frac{3}{4} x^{-\frac{1}{3}}$$

then the general solution to this homogeneous ODE is:

$$c_1 x + c_2 x^{-1/3}$$

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In the next part of Question 2 the same ODE is to be solved but here the right hand side is equal to $x^r$, where I chose $r$ to be $-1$ so we have:

$$3x^2 y'' + xy' - y = x^{-1}$$

I used the Wronskian to determine if $y_1$ and $y_2$ were linearly independent or not.

$$W = (-\frac{1}{3} x^{-\frac{4}{3}}) - x^{-\frac{1}{3}}(1)$$
$$W = -\frac{4}{3} x^{-\frac{1}{3}} \neq 0$$

Since $W \neq 0$, $y_1$ and $y_2$ are linearly independent. Now that we know the homogeneous solutions are linearly independent we can find $y_p$ using the following identity:

$$y_p = -y_1 \int \frac{y_2 r}{W} + y_2 \int \frac{y_1 r}{W}$$

where $r$ is the right hand side of the equation ($x^{-1}$) and $W$ is the Wronskian I calculated.

I obtained $y_p = \frac{1}{4} x ln(x) - \frac{9}{16} x$

the general solution I obtained for this equation is then:

$$y = c_1 x + c_2 x^{\frac{-1}{3}} + (\frac{1}{4} ln(x) - \frac{9}{16}) x$$