

Ex-III(A)

4/10 $X = \{(x_1, x_2) \mid x_1, x_2 \leq 1; x_1, x_2 \geq 0\}$ is not a convex set

Ex-IV(A)

(6) Maximize $Z = 2x_1 + 3x_2$,
 Subject to $x_1 + x_2 \leq 1$,
 $3x_1 + x_2 \leq 4$,
 $x_1, x_2 \geq 0$

Sol:-

The given problem introducing slack and surplus variables we can be put as,

$$Z = 2x_1 + 3x_2 + 0 \cdot x_3 + 0 \cdot x_4$$

Sub:-

$$x_1 + x_2 + x_3 = 1$$

$$3x_1 + x_2 + 0 \cdot x_3 + x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Simplex Tables, Table - 1

			G_j	2	3	0	0	Min ratio b/a_2
C_B	b	x_B	b	a_1	a_2	a_3	a_4	
0	a_3	x_3	1	1	1	1	0	1
0	a_4	x_4	4	3	1	0	1	4
$Z_j - G_j$				-2	-3	0	0	

a_2 is the entering vector, 1 is the key vector, a_3 is the departing vector.

Maximize $Z = 3x_1 + x_2 + 3x_3$

s.t

$$2x_1 + x_2 + x_3 \leq 2$$

$$x_1 + 2x_2 + 3x_3 \leq 5$$

$$2x_1 + 2x_2 + x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0$$

Sol:-

The given problem introducing slack and surplus variable we can be put as,

$$Z = 3x_1 + x_2 + 3x_3 + 0x_4 + 0x_5 + 0x_6$$

$$s.t \quad 2x_1 + x_2 + x_3 + x_4 + 0x_5 + 0x_6 = 2$$

$$x_1 + 2x_2 + 3x_3 + 0x_4 + x_5 + 0x_6 = 5$$

$$2x_1 + 2x_2 + x_3 + 0x_4 + 0x_5 + x_6 = 6$$

$$x_1, \dots, x_6 \geq 0$$

Simplex Table:- Table - 1

			C_j	3	1	3	0	0	0	Min ratio
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	$\frac{b}{a_1}$
0	a_4	x_4	2	<u>2</u>	1	1	1	0	0	1
0	a_5	x_5	5	1	2	3	0	1	0	$\frac{5}{1}$
0	a_6	x_6	6	2	2	1	0	0	1	3
$Z_j - C_j$				-3	-1	-3	0	0	0	

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Table-2

C_j				3	1	3	0	0	0	Min ratio (b/a_j)
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	
3	a_1	x_1	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	2
0	a_5	x_5	4	0	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	1	0	$\frac{8}{5}$
0	a_6	x_6	4	0	1	0	-1	0	1	
$Z_j - C_j$				0	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	0	0	

$$3 - \frac{1}{2} = \frac{5}{2}$$

$$\frac{3}{2} - 1 = \frac{1}{2}$$

$$\frac{3}{2} - 3 = -\frac{3}{2}$$

$$\frac{3}{2} \times \frac{2}{5} - \frac{1}{2} \times \frac{2}{5}$$

Table-3

C_j				3	1	3	0	0	0	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	
3	a_1	x_1	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	$-\frac{1}{5}$	0	
3	a_3	x_3	$\frac{8}{5}$	0	$\frac{3}{5}$	1	$-\frac{1}{5}$	$\frac{2}{5}$	0	
0	a_6	x_6	4	0	1	0	-1	0	1	
$Z_j - C_j$										

$$Z_j - C_j \geq 0$$

$$R_2 - \frac{1}{2} R_1 =$$

$$= \frac{3}{5} - \frac{1}{2} \times \frac{1}{5} = \frac{3}{5} - \frac{1}{10} = \frac{6-1}{10} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore x_1 = \frac{1}{5}, \quad x_2 = 0, \quad x_3 = \frac{8}{5}, \quad x_6 = 4$$

$$\therefore \max Z = 3 \times \frac{1}{5} + 1 \times 0 + 3 \times \frac{8}{5}$$

$$= \frac{3}{5} + 0 + \frac{24}{5} = \frac{27}{5} \quad \text{Ans}$$

H/O Ex-4(11) (10) Minimize $Z = x_1 - 3x_2 + 2x_3$

s.t.

$$\begin{aligned} 3x_1 - x_2 + 2x_3 &\leq 7, \\ -2x_1 + 4x_2 &\leq 12, \\ -4x_1 + 3x_2 + 8x_3 &\leq 10, \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

(12) Maximize $Z = 2x_1 + 4x_2 + x_3 + x_4$

s.t.

$$\begin{aligned} x_1 + 3x_2 + x_4 &\leq 4, \\ 2x_1 + x_2 &\leq 3, \\ x_2 + 4x_3 + x_4 &\leq 3, \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

Sol:- The given problem introducing slack and surplus variables we can be put as,

$$Z = 2x_1 + 4x_2 + x_3 + x_4 + 0x_5 + 0x_6 + 0x_7$$

s.t.

$$x_1 + 3x_2 + x_4 + x_5 + 0x_6 + 0x_7 = 4$$

$$2x_1 + x_2 + 0x_3 + x_6 + 0x_7 = 3$$

$$-x_2 + 4x_3 + x_4 + 0x_5 + 0x_6 + x_7 = 3$$

$$x_1, \dots, x_7 \geq 0$$

Simplex Tables :- Table-1

C_j				2	4	1	1	0	0	0	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	Min ratio b/a_2
0	a_5	x_5	4	1	3	0	1	1	0	0	$\frac{4}{3}$
0	a_6	x_6	3	2	1	0	0	0	1	0	3
0	a_7	x_7	3	0	1	4	1	0	0	1	3
$Z_j - C_j$				-2	-4	-1	-1	0	0	0	

Talele-2

				C_j	2	4	1	1	0	0	0	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	Minratio b/a_3	
4	a_2	x_2	$\frac{4}{3}$	$\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{1}{3}$	0	0		
0	a_6	x_6	$\frac{5}{2}$	$\frac{5}{2}$	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	1	0		
0	a_7	x_7	$\frac{5}{2}$	$-\frac{1}{3}$	0	4	$\frac{2}{3}$	$-\frac{1}{3}$	0	1	$\frac{5}{12}$	
$Z_j - C_j$				$-\frac{2}{3}$	0	-1	$\frac{1}{3}$	$\frac{4}{3}$	0	0		

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Talele-3

				C_j	2	4	1	1	0	0	0	Minratio (b/a_1)
C_B	B	x_B	b		a_1	a_2	a_3	a_4	a_5	a_6	a_7	
4	a_2	x_2	$\frac{4}{3}$		$\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{1}{3}$	0	0	4
0	a_6	x_6	$\frac{5}{3}$	$\frac{5}{3}$	0	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	1	0	1
1	a_3	x_3	$\frac{5}{12}$	$\frac{1}{12}$	0	1	$\frac{1}{6}$	$-\frac{1}{12}$	0	0	$\frac{1}{4}$	
$Z_j - C_j$					$-\frac{19}{12}$	0	0	$\frac{1}{2}$	$\frac{5}{4}$	0	$\frac{1}{4}$	

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Talele-4

			C_j	2	4	1	1	0	0	0	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	
4	a_2	x_2	1	0	1	0	$\frac{2}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	0	
2	a_1	x_1	1	1	0	0	$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{3}{5}$	0	
1	a_3	x_3	$\frac{1}{2}$	0	0	1	$\frac{9}{20}$	$-\frac{1}{2}$	$\frac{1}{20}$	$\frac{1}{4}$	
$Z_j - C_j$				0	0	0	$\frac{7}{20}$	$\frac{7}{10}$	$\frac{9}{20}$	$\frac{1}{4}$	

$$\begin{aligned}
 & \frac{4}{3} - \frac{1}{12} - 2 \\
 & = \frac{16}{12} - \frac{1}{12} - 2 \\
 & = \frac{15}{12} - 2 \\
 & = \frac{5}{4} - 1 \\
 & = \frac{5}{4} - \frac{4}{4} \\
 & = \frac{1}{4} \\
 & \frac{10}{60} - \frac{1}{20} - 1 \\
 & = \frac{10}{60} - \frac{3}{60} - 1 \\
 & = \frac{7}{60} - 1 \\
 & = \frac{7}{60} - \frac{60}{60} \\
 & = \frac{7-60}{60} \\
 & = \frac{-53}{60} \\
 & \frac{1}{60} - \frac{1}{12} - 1 \\
 & = \frac{1}{60} - \frac{5}{60} - 1 \\
 & = \frac{1-5}{60} - 1 \\
 & = \frac{-4}{60} - 1 \\
 & = \frac{-1}{15} - 1 \\
 & = \frac{-1}{15} - \frac{15}{15} \\
 & = \frac{-1-15}{15} \\
 & = \frac{-16}{15} \\
 & \frac{32}{20} - \frac{1}{2} - 1 \\
 & = \frac{32}{20} - \frac{10}{20} - 1 \\
 & = \frac{22}{20} - 1 \\
 & = \frac{11}{10} - 1 \\
 & = \frac{11}{10} - \frac{10}{10} \\
 & = \frac{1}{10}
 \end{aligned}$$

$$\therefore z_j - c_j \geq 0$$

$$\therefore x_1 = 1, x_2 = 1, x_3 = \frac{1}{2}, x_4 = 0$$

$$\begin{aligned} \therefore \text{Max } z &= 2x_1 + 4x_2 + x_3 + x_4 \\ &= 2 \times 1 + 4 \times 1 + \frac{1}{2} + 0 \\ &= 2 + 4 + \frac{1}{2} \\ &= \frac{13}{2} \quad \text{Ans} \end{aligned}$$

(13) Maximize, $z = 3x_1 + 4x_2 + x_3 + 5x_4$

s.t. $8x_1 + 3x_2 + 2x_3 + 2x_4 \leq 10,$

$2x_1 + 5x_2 + x_3 + 4x_4 \leq 5,$

$x_1 + 2x_2 + 5x_3 + x_4 \leq 6,$

$x_1, x_2, x_3, x_4 \geq 0.$

Sol:- The given problem introducing slack and surplus variables we can be put as,

$$z = 3x_1 + 4x_2 + x_3 + 5x_4 + 0x_5 + 0x_6 + 0x_7$$

s.t. $8x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 + 0x_6 + 0x_7 = 10,$

$2x_1 + 5x_2 + x_3 + 4x_4 + 0x_5 + x_6 + 0x_7 = 5$

$x_1 + 2x_2 + 5x_3 + x_4 + 0x_5 + 0x_6 + x_7 = 6,$

$x_1, \dots, x_7 \geq 0$

Simplex Tables:- Table-1.

			c_j	3	4	1	5	0	0	0	Min
c_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	ratio (b/a_j)
0	a_5	x_5	10	8	3	2	2	1	0	0	5
0	a_6	x_6	5	2	5	1	4	0	1	0	5/4
0	a_7	x_7	6	1	2	5	1	0	0	1	6
$z_j - c_j$				-3	-4	-1	-5	0	0	0	

(10) Minimize $Z = x_1 - 3x_2 + 2x_3$
 s.t $3x_1 - x_2 + 2x_3 \leq 7$,
 $-2x_1 + 4x_2 \leq 12$,
 $-4x_1 + 3x_2 + 8x_3 \leq 10$,
 $x_1, x_2, x_3 \geq 0$.

Sol.:- The given problem introducing slack and surplus variable we can be put as,

$$3x_1 - x_2 + 2x_3 + x_4 + 0.x_5 + 0.x_6 = 7$$

$$-2x_1 + 4x_2 + 0.x_3 + x_5 + 0.x_6 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + 0.x_4 + 0.x_5 + x_6 = 10$$

Let, $Z' = -Z$

$$\therefore \max(Z') = \max(-Z)$$

$$= -x_1 + 3x_2 - 2x_3 + 0.x_4 + 0.x_5 + 0.x_6$$

Simplex Tables :- Table - 1

C_j				-1	3	-2	0	0	0	Min ratio
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	b/a_2
0	a_4	x_4	7	3	-1	2	1	0	0	-7
0	a_5	x_5	12	-2	4	0	0	1	0	3
0	a_6	x_6	10	-4	3	8	0	0	1	$\frac{10}{3}$
$Z_j - C_j$				+1	-3	2	0	0	0	

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For the next table a_2 is the entering vector, 4 is the key vector and a_5 is the departing vector.

Table-2

				C_j	-1	3	-2	0	0	0	Ratio
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	b/a_1	
0	a_4	x_4	10	$\frac{5}{2}$	0	2	1	$\frac{1}{4}$	0	4	
3	a_2	x_2	3	$-\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	-6	$-\frac{3}{2}$
0	a_6	x_6	1	$-\frac{5}{2}$	0	8	0	$-\frac{1}{4}$	1	$-\frac{2}{5}$	$-\frac{3}{2} + 1$
$Z_j - C_j$				$-\frac{1}{2}$	0	2	0	$\frac{3}{4}$	0		$\frac{3}{2} + 1$

Table-3

For the next table a_1 is the entering vector, $\frac{5}{2}$ is the key vector and a_4 is the departing vector.

Table-3

				C_j	-1	3	-2	0	0	0	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6		
-1	a_1	x_1	4	1	0	$\frac{4}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0		
3	a_2	x_2	5	0	1	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	0		
0	a_6	x_6	11	0	0	10	1	$\frac{1}{2}$	1		
$Z_j - C_j$				0	0	$\frac{12}{5}$	$\frac{1}{5}$	$\frac{4}{5}$	0		

$Z_j - C_j \geq 0$ For all j . Hence the optimality condition has been satisfied, the optimal solution is $x_1 = 4, x_2 = 5, x_3 = 0$.

$$\begin{aligned} \text{Max}(Z) \\ \therefore \text{Min}(Z') &= -4 + 3 \times 5 - 2 \times 0 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \therefore \text{Min}(Z) &= -\text{Max}(Z') \\ &= -11 \end{aligned}$$

The optimal solutions are, $x_1 = 4, x_2 = 5, x_3 = 0$

Ex. 6. Make a graphical representation of the set of constraints in the following L. P. P. :

$$\begin{aligned} &\text{Maximize} \quad z = 2x_1 + x_2 \\ &\text{subject to} \quad x_1 + 3x_2 \leq 15, \\ &\quad \quad \quad 3x_1 - 4x_2 \leq 12, \\ &\quad \quad \quad x_1, x_2 \geq 0 \end{aligned}$$

and find the corner points of the region of feasible solutions.

We consider first the inequations for the constraints as equations and draw on a graph paper the straight lines

$$AB, \text{ given by } x_1 + 3x_2 = 15$$

$$\text{and } CD, \text{ given by } 3x_1 - 4x_2 = 12.$$

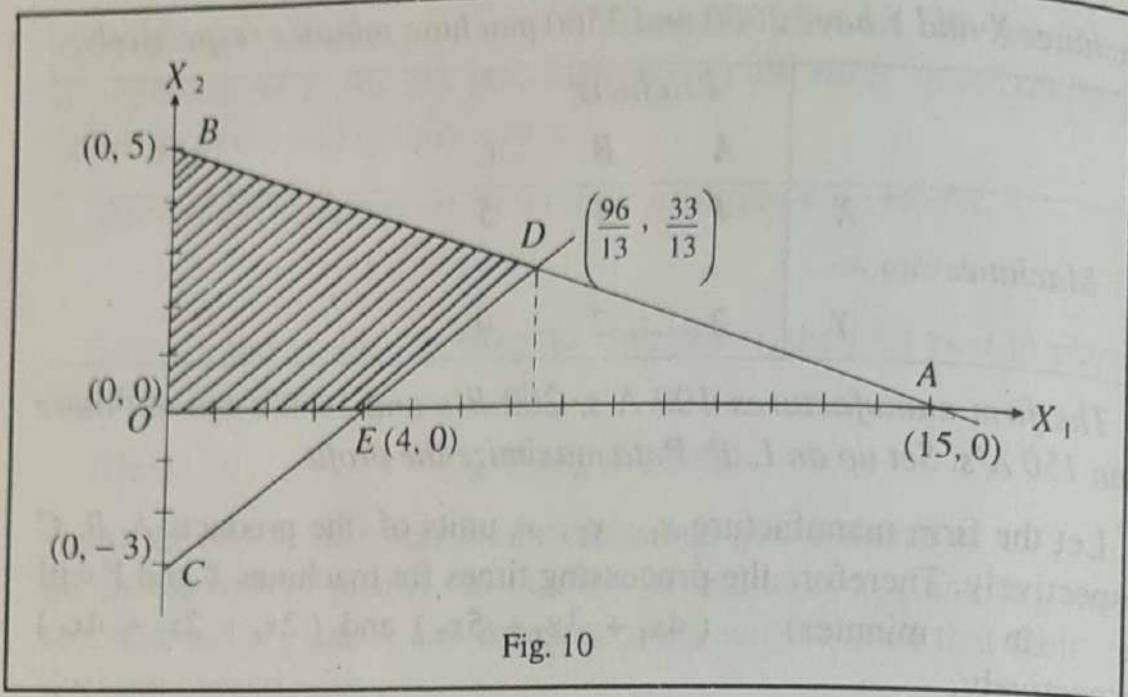


Fig. 10

The sign of the inequations attached to the constraints and the non-negativity conditions $x_1, x_2 \geq 0$ give the region of feasible solution for the given constraints and it is shown by the shaded region of the graph.

The corner points of the feasible region are seen to be $B(0, 5)$, $D(\frac{96}{13}, \frac{33}{13})$, $E(4, 0)$ and $O(0, 0)$.

Ex. 7. Make the graphical representation of the set of constraints in the following L. P. P. :

$$\text{Maximize } z = 10x_1 + 15x_2$$

$$\text{subject to } x_1 + x_2 \geq 2,$$

$$3x_1 + 2x_2 \leq 6,$$

$$x_1, x_2 \geq 0$$

and find the extreme points of the region of feasible solutions. Find also the maximum value of the objective function.

We first draw the graphs of the given constraints considering them as equations. Thus the straight lines on the graph are

$$x_1 + x_2 = 2,$$

$$3x_1 + 2x_2 = 6.$$

The signs of the constraints together with the non-negativity conditions of the variables ($x_1, x_2 \geq 0$) give the feasible region as

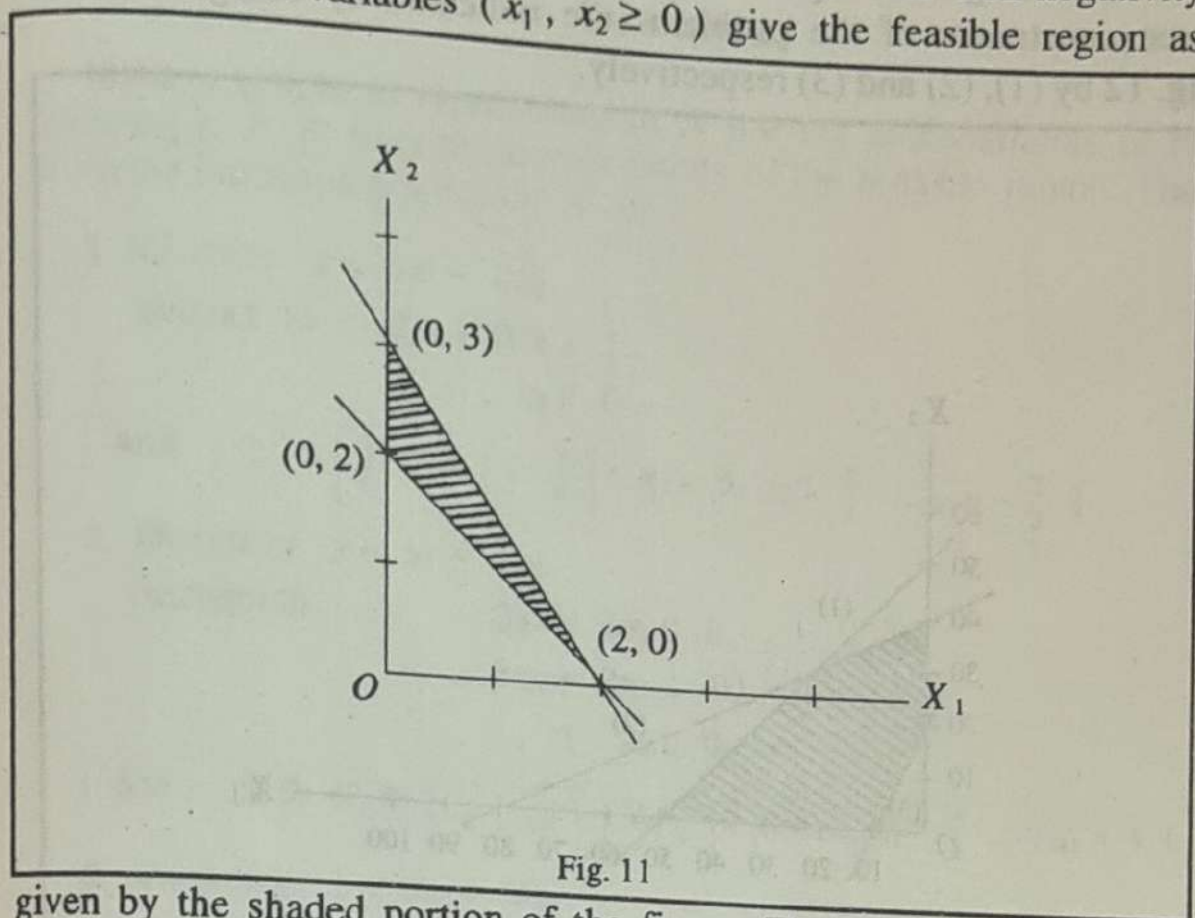


Fig. 11

given by the shaded portion of the figure. The corner points of the feasible region are (0, 2), (0, 3) and (2, 0).

The value of the objective function ($10x_1 + 15x_2$) at the corner point (0, 2) is 30, at the corner point (0, 3) is 45 and at the corner point (2, 0) is 20.

Hence the maximum value of the objective function is 45 at the corner point $x_1 = 0, x_2 = 3$, which is thus the optimal solution.

Ex. 8. Make a graphical representation of the set of constraints in the following L. P. P. :

$$\begin{aligned} &\text{Maximize } z = 4x_1 + 3x_2 \\ &\text{subject to } x_1 + x_2 \leq 50, \\ &\quad x_1 + 2x_2 \leq 80, \\ &\quad 2x_1 + x_2 \geq 20, \\ &\quad x_1, x_2 \geq 0. \end{aligned}$$

Find the corner points of the feasible region.

Hence find the maximum value of the objective function.

(iv) A manufacturer of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20000 bottles of A and 40000 bottles of B, but there are only 45000 bottles into which either of the medicines can be put. Furthermore it takes three hours to prepare enough material to fill 1000 bottles of A, it takes one hour to prepare enough material to fill 1000 bottles of B and there are 66 hours available for this operation. The profit is Rs. 8.00 per bottle of A and Rs. 7.00 per bottle of B.

Formulate this as a linear programming problem to maximize the profit.

Let the manufacturer produce x_1 thousand bottles of medicine A and x_2 thousand bottles of medicine B.

Time for preparing 1000 bottles of A is 3 hours and that for preparing 1000 bottles of B is 1 hour. Hence the total time necessary to prepare x_1 thousand bottles of A and x_2 thousand bottles of B will be $(3x_1 + x_2)$ hours.

But the maximum time available is 66 hours ; hence

$$3x_1 + x_2 \leq 66 . \quad \dots \quad (1)$$

Again only 45000 bottles are available for filling the medicines A and B ; hence $x_1 + x_2 \leq 45 . \quad \dots \quad (2)$

Since sufficient ingredients are available to make 20 thousand bottles of A and 40 thousand bottles of B; the constraints in this respect will be

$$x_1 \leq 20 , \quad \dots \quad (3)$$

$$x_2 \leq 40 . \quad \dots \quad (4)$$

Also the numbers of bottles must be non-negative ; hence

$$x_1 , x_2 \geq 0 \quad \dots \quad (5)$$

The rates of profit for medicines A and B are respectively Rs. 8.00 and Rs. 7.00 per bottle. Hence the total profit from x_1 thousand bottles of A and x_2 thousand bottles of B medicines are

$$8 \times 1000x_1 + 7 \times 1000x_2.$$

Let this profit be denoted by z , which is to be maximized. Thus we get the following L. P. P. :

$$\text{Maximize } z = 8000x_1 + 7000x_2$$

subject to the constraints

$$3x_1 + x_2 \leq 66,$$

$$x_1 + x_2 \leq 45,$$

$$x_1 \leq 20,$$

$$x_2 \leq 40$$

and $x_1, x_2 \geq 0.$

(v) For a 24-hour restaurant the following waitresses are required :

Time of day	Minimum number of waitresses
1 — 5	5
5 — 9	7
9 — 13	10
13 — 17	8
17 — 21	12
21 — 1	4

Each waitress works eight consecutive hours per day. The problem is to find a linear programming model to find the smallest number of waitresses required to comply with the above requirements.

Let $x_j, j = 1, 2, 3, 4, 5, 6$ be the number of waitresses starting at the beginning of the j -th period. We find also that the same set of waitresses cannot work for more than two periods of four hours each as that would defeat the restriction of their working for not more than eight hours per day. For the period (1—5), x_1 is the number of starting

waitresses and x_6 waitresses were working in the sixth period (21—1). Thus, for the first period, $x_1 + x_2 \geq 5$. So for others.

Hence the problem becomes

$$\text{Minimize } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

subject to

$$x_1 + x_6 \geq 5,$$

$$x_2 + x_1 \geq 7,$$

$$x_3 + x_2 \geq 10,$$

$$x_4 + x_3 \geq 8,$$

$$x_5 + x_4 \geq 12,$$

$$x_6 + x_5 \geq 4,$$

$$x_j \geq 0; \text{ for all } j.$$

(vi) A company has two grades of inspectors, I and II, who are to be assigned for a quality control inspection. It is required that at least 2000 pieces be inspected per 8 hour day. Grade I inspectors can check pieces at the rate of 50 per hour with an accuracy of 97%. Grade II inspectors can check pieces at the rate of 40 per hour with an accuracy of 95%. The wage rate of Grade I inspectors is Rs. 4.50 per hour and that of Grade II is Rs. 2.50 per hour. Each time an error is made by an inspector, the cost to the company is one rupee. The company has available, for the inspection job, 10 grade I and 5 Grade II inspectors. Formulate the problem to minimize the total cost of inspection.

Let x and y be the number of Grade I and II inspectors respectively engaged for the work. The constraint on availability of two types of inspectors ensures $x \leq 10$ and $y \leq 5$ (1)

Grade I inspectors can check 50 pieces per hour and Grade II inspectors can check 40 pieces per hour. Hence, for a 8 hour day, they can check $\{(8 \times 50)x + (8 \times 40)y\}$ pieces which should be greater than or equal to 2000 as required by the company. Hence

$$(8 \times 50)x + (8 \times 40)y \geq 2000,$$

$$\text{that is, } 5x + 4y \geq 25.$$

... (2)

The cost to be minimized is the total of wages and cost due to inspection error. They are

Rs. 4.50 + Rs. $\left\{1 \times \left(\frac{3}{100} \times 50\right)\right\}$ = Rs. 6 per hour, for Grade I inspectors and

Rs. 2.50 + Rs. $\left\{1 \times \left(\frac{5}{100} \times 40\right)\right\}$ = Rs. 4.50 per hour, for Grade II inspectors.

Thus the objective function which is to be minimized is

$$6 \times 8x + 4.5 \times 8y.$$

Hence the required *L. P. P.* is

$$\text{Minimize } z = 48x + 36y$$

subject to $5x + 4y \geq 25$,

$$x \leq 10,$$

$$y \leq 5 \text{ and } x, y \geq 0.$$

Thus to formulate the *L. P. P.* the unknown decision variables are to be first identified. They are to be assigned symbols like x , y , z , etc. or x_1 , x_2 , x_3 , etc. The restrictions of the problem are to be expressed then as linear equations or inequations with the decision variables. The objective function will then be identified and put as a linear function of the decision variables and equated to the symbol z signifying its value.