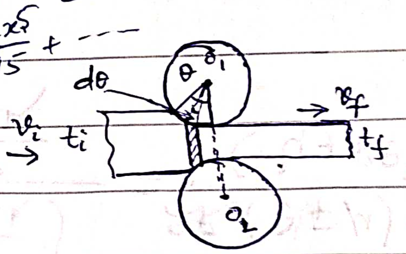
$$\sin\theta = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \cdots$$

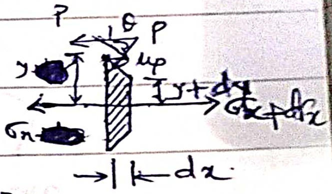
t_i = Initial thickness $\qquad \cos\theta = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \cdots$

t_f = final thickness $\qquad \tan\theta = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$

θ = Angle of bite

$d\theta$ = Angle of bite for small crosssection of strip.

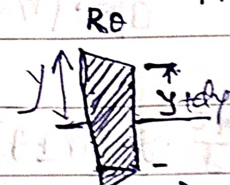




w = width (Assume unity)

v_i = Entry velocity, v_f = Exit velocity

$v_i < v < v_f$.



P = Pressure Exerted by rolls on the strip. $\qquad y = \frac{t_f}{2} + \frac{R\theta^2}{2}$

$$2(y+dy)\times(\sigma_x + d\sigma_x) - 2y\sigma_x + 2PR\,d\theta\,\sin\theta - 2\mu P\cos\theta \times R\,d\theta = 0$$

Since θ = very small.

$$2(y+dy)(\sigma_x + d\sigma_x) - 2y\sigma_x + 2PR\theta\,d\theta - 2\mu PR\,d\theta = 0$$

Neglecting higher order term

$$\cancel{2y\sigma_x} + 2y\,d\sigma_x + 2\sigma_x\,dy + \cancel{2\,d\sigma_x\,dy} - \cancel{2y\sigma_x} + 2PR\theta\,d\theta - 2\mu PR\,d\theta = 0$$

$$\Rightarrow \cancel{2}y\,d\sigma_x + \cancel{2}\sigma_x\,dy + \cancel{2}PR\theta\,d\theta - \cancel{2}\mu PR\,d\theta = 0$$

$$\Rightarrow \cancel{2}\,d(\sigma_x \cdot y) + RP(\theta\,d\theta - \mu\,d\theta) = 0$$

$$\Rightarrow \frac{d}{d\theta}(y\sigma_x) + RP(\theta - \mu) = 0 \Rightarrow \frac{d}{d\theta}(y\sigma_x) - (\mu - \theta)RP = 0$$

Principal stress $\sigma_x = \sigma_1$, $-P = \sigma_3$ $\qquad \sigma_2 = \frac{1}{2}(\sigma_x - P)$ $\left\{ \sigma_2 = \frac{\sigma_1 + \sigma_3}{2} \right.$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6K^2 \qquad \left\{ K = \text{shear yield st} \right.$$

Substituting the value of σ_1, σ_2 & σ_3 -

$$\left[\sigma_x - \left\{\frac{1}{2}(\sigma_x - P)\right\}\right]^2 + \left[\frac{1}{2}(\sigma_x - P) - (-P)\right]^2 + \left[-P - \sigma_x\right]^2 = 6K^2$$

$$\Rightarrow \boxed{P + \sigma_x = 2K} \Rightarrow \underline{\sigma_x = 2K - P}$$

$\frac{d}{d\theta}\left(y\cdot(2k-p)\right) - (\mu-\theta)RP = 0$

As we know that friction force changes its direction after the element ~~reaches~~ crossed the neutral point.

Then eqn becomes:

$$\frac{d}{d\theta}\left[(2k-P)y\right] - \left[(\pm\mu-\theta)RP\right] = 0$$

+ve sign indicates region before neutral point

-ve sign indicates region after neutral point.

$2ky\frac{d}{d\theta}\left(1-\frac{P}{2k}\right) + (\theta\mp\mu)RP = 0 \Rightarrow 2y\frac{d}{d\theta}\left(1-\frac{P}{2k}\right) + (\theta\mp\mu)R\cdot\frac{P}{k} = 0$ (dividing by k both side)

if θ = small then y can be expressed in the form

$$y = \frac{t_f}{2} + \frac{R\theta^2}{2}$$

$$\cancel{2\left(\frac{t_f}{2}+\frac{R\theta^2}{2}\right)\frac{d}{d\theta}} - (t_f + R\theta^2)\frac{d}{d\theta}\left(+\frac{P}{2k}\right) + 2(\theta\mp\mu)R\cdot\frac{P}{2k} = 0$$

$$\Rightarrow \cancel{\frac{d\left(\frac{P}{2k}\right)}{\frac{P}{2k}}}\quad (t_f + R\theta^2)\frac{d}{d\theta}\left(\frac{P}{2k}\right) = 2(\theta\mp\mu)\cdot R\cdot\cancel{\frac{P}{2k}}$$

$$\Rightarrow \frac{d\left(\frac{P}{2k}\right)}{\frac{P}{2k}} = \frac{2R(\theta\mp\mu)\cdot d\theta}{(t_f + R\theta^2)}$$

Integrating both sides

Integrating both sides.

$$\int \frac{d\left(\frac{P}{2K}\right)}{\left(\frac{P}{2K}\right)} = \int \frac{2R\theta\, d\theta}{(t_f + R\theta^2)} \mp \int \frac{2R\mu \cdot d\theta}{t_f + R\theta^2} + C_1$$

or $$\ln\left(\frac{P}{2K}\right) = \ln\left(t_f + R\theta^2\right) \mp 2\mu \sqrt{\frac{R}{t_f}} \tan^{-1}\left(\sqrt{\frac{R}{t_f}} \cdot \theta\right) + \ln\left(\frac{C}{2R}\right)$$ where C being another constant)

Assume $\lambda = 2\sqrt{\frac{R}{t_f}} \tan^{-1}\sqrt{\frac{R}{t_f}}\theta$

or

$$y = \frac{1}{2}\left(t_f + R\theta^2\right).$$

$$\ln\left(\frac{P}{2K}\right) = \ln(2y) \mp \mu\lambda + \ln\frac{C}{2R}$$

$$\Rightarrow \ln\left(\frac{P}{2K}\right) = \ln\left(2y \cdot \frac{C}{2R}\right) \mp \mu\lambda$$

$$= \ln\left(C \cdot \frac{y}{R}\right) \mp \mu\lambda$$

$$\Rightarrow \cancel{\frac{P}{2K} = C \cdot \frac{y}{R}} \Rightarrow \ln\left(\frac{\frac{P}{2K}}{C \cdot \frac{y}{R}}\right) = \mp\mu\lambda$$

$$\Rightarrow \frac{P}{2K} = C \cdot \frac{y}{R} \cdot e^{\mp\mu\lambda}$$

we know that
~~Applying~~ $(\sigma_x + P) = 2K$

$$\Rightarrow \frac{\sigma_x}{2K} + \frac{P}{2K} = 1$$

$$\Rightarrow \frac{P}{2K} = 1 - \frac{\sigma_x}{2K}$$

applying begining Condition

$$\Rightarrow \frac{P_1}{2K} = 1 - \frac{\sigma_{x_i}}{2K}$$

~~Substitue~~ $\frac{P}{2K} = C \times \frac{y}{R} e^{\mp \mu \lambda}$

$$\Rightarrow \frac{P_1}{2K} = C \frac{y}{R} \cdot e^{\mp \mu \left(2\sqrt{\frac{R}{t_f}} \cdot \tan^{-1}\sqrt{\frac{R}{t_f}}\,\theta\right)}$$

$$\Rightarrow \frac{P_1}{2K} = 1 - \frac{\sigma_{x_i}}{2K} = C \times \frac{y}{R} \cdot e^{-\mu \lambda_i}$$

where $\lambda_i = 2\sqrt{\frac{R}{t_f}} \cdot \tan^{-1}\left(\sqrt{\frac{R}{t_f}}\,\theta_i\right)$

$$=$$

$$C_1 \frac{(t_f + R\theta^2)}{2R} = C_1\left(\frac{t_f}{2R} + \frac{\theta^2}{2}\right)$$

$$= C_1 \frac{t_f}{2R}$$

$$\frac{p_i}{2k} = C^- \frac{t_i}{2R} \cdot e^{-\mu\lambda_i}$$

$$\frac{y}{R} = \frac{1}{R}\left(\frac{t_f}{2R} + \frac{R\theta^2}{2}\right) = \frac{1}{2R}\left(t_f + R\theta^2\right) = \frac{t_f}{2R} + \frac{\theta^2}{2}$$

C^- is Constant ~~ke~~ before the neutral point is reached.

hence $$C^- = \frac{2R}{t_i}\left(1 - \frac{\sigma_{xi}}{2k}\right) \cdot e^{+\mu\lambda_i}$$

for the region beyond the neutral point

$$\frac{p}{2k} = C \cdot \frac{y}{R} \cdot e^{+\mu\lambda}$$

$$\Rightarrow \frac{p_f}{2K} = C^+ \cdot \left(\frac{t_f}{2R}\right) \cdot e^{+\mu\lambda_f} = \left(1 - \frac{\sigma x_f}{2k}\right)$$

$$C^+ = \frac{2R}{t_f}\left(1 - \frac{\sigma x_f}{2k}\right) \cdot e^{-\mu\lambda_f}$$, where

$$= \frac{2R}{t_f}\left(1 - \frac{\sigma x_f}{2k}\right)$$

$$\lambda = 2\sqrt{\frac{R}{t_f}} \tan^{-1}\left(\sqrt{\frac{R}{t_f}}\theta_f\right)$$

$= 0$ because $\theta_f = 0$

$\sin \frac{0}{1} = 0.$

$\therefore e^{-\mu\lambda_f} = 1$

now substituting the value of C^- & C^+ in eqn. above

$$\frac{p}{2k} = C^- \cdot \frac{y}{R} \cdot e^{\mp\mu\lambda}$$

$$\boxed{\left(\frac{p}{2k}\right)_{before} = \frac{2y}{t_i}\left(1 - \frac{\sigma_{x_i}}{2k}\right) \cdot e^{-\mu\lambda}}$$

$$\left(\frac{p}{2k}\right)_{before} = \frac{2R}{t_f}\left(1 - \frac{\sigma_{x_i}}{2k}\right) e^{+\mu\lambda_i} \cdot \frac{y}{R} \cdot e^{-\mu\lambda}$$

$$= \frac{2y}{t_f}\left(1 - \frac{\sigma_{x_i}}{2k}\right) \cdot e^{\mu(\lambda_i - \lambda)}$$

$$\left(\frac{p}{2k}\right)_{after} = \frac{2y}{t_f} \cdot \left(1 - \frac{\sigma_{x_f}}{2k}\right) \cdot e^{\mu\lambda}$$

$$\lambda_n = \frac{1}{2}\left[\frac{1}{\mu}\ln\left\{\frac{t_f}{t_i}\left(\frac{1 - \frac{\sigma_{x_i}}{2k}}{1 - \frac{\sigma_{x_f}}{2k}}\right)\right\} + \lambda_i\right]$$

Determination of Roll separating force.

$$F = \int_0^{\theta_i} PR\cos\theta\, d\theta = \int_0^{\theta_i} PR\, d\theta \quad \text{since } d\theta = \text{small}$$

$$F = \int_0^{\theta_n} P_{after}\, R\, d\theta + \int_{\theta_n}^{\theta_i} P_{before}\, R\, d\theta$$