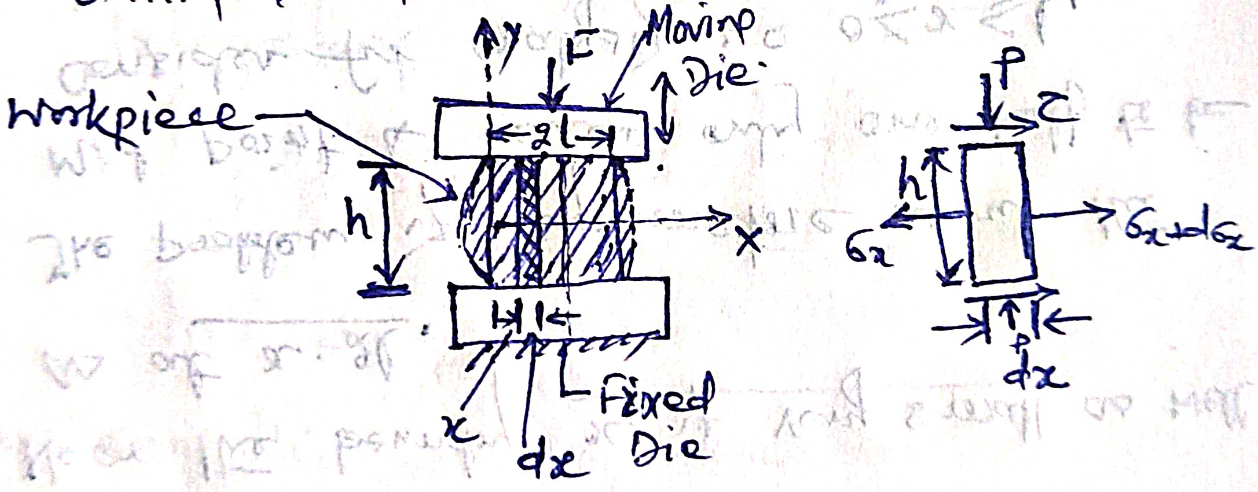
Assumptions

1. The forging force F attains its max^m value at the end of operation.
2. The Co-efficient of friction remains Constant
3. The thickness of work-piece is small as Compare to its other dimension & variation stress field in y-direction is negligible.
4. Length of strip is much more than the width assume to be unity
5. The entire ~~process~~ workpiece is in the plastic state during the process.



Resolving the forces in x direction

$h(\sigma_x + d\sigma_x) + 2\tau\, dx - h\sigma_x = 0$

$\Rightarrow \cancel{h\sigma_x} + h\, d\sigma_x + 2\tau\, dx - \cancel{h\sigma_x} = 0$

$\Rightarrow h\, d\sigma_x + 2\tau\, dx = 0$ — (1)

where τ is the frictional stress.

As per yied criterion

−p & σ_x are principal stress.

$\sigma_x - \sigma_y = 2k$

$\Rightarrow \sigma_x + p = 2k$

$d\sigma_x + dp = 2k \Rightarrow d\sigma_x = -dp$

Substituting the value of $d\sigma_x$ in eqn (1).

$$h(-dp) + 2\tau dx = 0$$

$$h\,dp = 2\tau dx \Rightarrow dp = \frac{2\tau}{h} dx \quad \text{(11)}$$

$\cancel{\frac{dp}{dx}} \cancel{\frac{dq}{k}}$

$\cancel{\Rightarrow dx}$

Near the boundry x is very small as well as at $x = 2l$.

The problem is symmetric about the Mid point & hence only one half to be Consider for analysis. i.e $0 \leq x \leq l$.

A sliding between workpiece & Die takes place to allow for the required expansion of the workpiece. However, beyond the certain value of x (Region between $0 \leq x \leq l$) say x_s there is no sliding & there after striking force in rest of the zone $x_s \leq x \leq l$

Now for region $0 \leq x \leq x_s$

$$\tau = \mu p$$

for region $x_s \leq x \leq l$

$$\tau = k$$

Now $0 \le x \le x_c$

$$\tau = \mu p$$

eqn (11) becomes

$$dp = \frac{2\tau}{h}\,dx$$

$$\Rightarrow dp = \frac{2\mu p}{h}\,dx$$

$$\Rightarrow \frac{dp}{p} = \frac{2\mu}{h}\,dx$$

Integrating both side

$$\Rightarrow \boxed{\int \frac{dp}{p} = \frac{2\mu}{h}\int dx}$$

$$\Rightarrow \ln p = \frac{2\mu}{h}x + C_1 \quad \text{—— (111)}$$

applying the boundary condition

at $x = 0$, $\sigma_x = 0$,

$$\therefore \sigma_x + p = 2k$$

$$\Rightarrow 0 + p = 2k$$

$$\Rightarrow p = 2k$$

equation (111) becomes $\ln p = \frac{2\mu}{h}\times 0 + C_1$

$$\Rightarrow C_1 = \ln p = \ln(2k)$$

Substituting the value of C_1 in eqn (iii)

$$\ln P = \frac{2\mu}{h}x + \ln(2k)$$

$$\Rightarrow \ln P - \ln(2k) = \frac{2\mu}{h}x$$

$$\Rightarrow \ln\left(P/2k\right) = \frac{2\mu}{h}x$$

$$\Rightarrow \frac{P}{2k} = e^{\frac{2\mu}{h}x}$$

$$\Rightarrow \boxed{P = 2k\, e^{\frac{2\mu}{h}x}} \quad \text{(IV)}$$

for region $x_s \le x \le l$

$$dp = \frac{2\tau}{h}dx$$

Integrating both side

$$\int dP = \frac{2\tau}{h}\int dx$$

$$\Rightarrow P = \frac{2\tau}{h}x + C_2 \quad \text{(V)}$$

if $P = P_s$ at $x = x_s$ Then :

$$P_s = \frac{2\tau}{h}x_s + C_2$$

$$C_2 = P_s - \frac{2\tau}{h} x_s$$

Substituting the value of C_2 in eqn (V)

$$P = \frac{2\tau}{h} x + P_s - \frac{2\tau}{h} x_s$$

$$\Rightarrow P - P_s = \frac{2\tau}{h}(x - x_s) \quad \text{——— (VI)}$$

As per equation (IV) P becomes P_s

$$\therefore \boxed{P_s = 2k e^{\frac{2\mu}{h} x_s}}$$

$$\mu P_s = 2k\mu \, e^{\frac{2\mu}{h} x_s} \qquad \Bigg| \quad \begin{matrix} \mu P_s = \tau \\ k = \tau \end{matrix}$$

$$\therefore \cancel{\tau} = 2\cancel{\tau}\mu \, e^{\frac{2\mu}{h} x_s}$$

$$\Rightarrow \boxed{e^{\frac{2\mu}{h} x_s} = \frac{1}{2\mu}}$$

$$\Rightarrow \frac{2\mu}{h} x_s = \ln\left(\frac{1}{2\mu}\right)$$

$$\Rightarrow \boxed{x_s = \frac{h}{2\mu} \ln\left(\frac{1}{2\mu}\right)}$$

Substituting the value of P_s & x_s in eqn (B):

$$P - P_s = \frac{2\tau}{h}(x - x_s) = \frac{2\tau}{h}x - \frac{2\tau}{h}x_s$$

$$\Rightarrow P = 2k\,e^{\frac{2\mu}{h}x_s} + \frac{2\tau}{h}x - \frac{2\tau}{h}x_s$$

$$= \cancel{2}k \cdot \frac{1}{\cancel{2}\mu} + \frac{2\tau}{h}x - \frac{\cancel{2}\tau}{\cancel{h}} \cdot \frac{\cancel{h}}{\cancel{2}\mu} \cdot \ln\left(\frac{1}{2\mu}\right)$$

$$= \cancel{\frac{\tau}{2}}$$

$|\, k = \tau$

$$= \frac{\tau}{\mu} + \frac{2\tau}{h}x - \frac{\tau}{\mu}\ln\left(\frac{1}{2\mu}\right)$$

$$= \frac{\tau}{\mu}\left(1 - \ln\left(\frac{1}{2\mu}\right)\right) + \frac{2\tau}{h}x$$

$$= 2\tau\left[\frac{1}{2\mu}\left\{1 - \ln\left(\frac{1}{2\mu}\right)\right\} + \frac{x}{h}\right]$$

$$= 2k\left[\frac{1}{2\mu}\left\{1 - \ln\left(\frac{1}{2\mu}\right)\right\} + \frac{x}{h}\right]$$

For total forging force per unit length of the workpiece

$$F = 2\left[\int_0^{x_s} P_1\,dx + \int_{x_s}^{l} P_2\,dx\right]$$

Substituting the value of p_s & x_s in eqn (13)

$$p - p_s = \frac{2\tau}{h}(x - x_s) = \frac{2\tau}{h}x - \frac{2\tau}{h}x_s$$

$$\Rightarrow p = 2k\, e^{\frac{2\mu}{h}x_s} + \frac{2\tau}{h}x - \frac{2\tau}{h}x_s$$

$$= \cancel{2}k \cdot \frac{1}{\cancel{2}\mu} + \frac{2\tau}{h}x - \frac{2\tau}{h}x_s$$

$$= \frac{\tau}{\mu} + \frac{2\tau}{h}x - \frac{\cancel{2}\tau}{\cancel{h}}\cdot\frac{\cancel{h}}{\cancel{2}\mu}\cdot\ln\left(\frac{1}{2\mu}\right)$$

$$= \frac{\tau}{\mu} + \frac{2\tau}{h}x - \frac{\tau}{\mu}\ln\left(\frac{1}{2\mu}\right) \qquad | k = \tau$$

$$= \frac{\tau}{\mu}\left(1 - \ln\left(\frac{1}{2\mu}\right)\right) + \frac{2\tau}{h}x$$

$$= 2\tau\left[\frac{1}{2\mu}\left\{1 - \ln\left(\frac{1}{2\mu}\right)\right\} + \frac{x}{h}\right]$$

$$= 2k\left[\frac{1}{2\mu}\left\{1 - \ln\left(\frac{1}{2\mu}\right)\right\} + \frac{x}{h}\right]$$

for total forging force per unit length of the workpiece

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