12 CHAPTER Inventory Models

This chapter presents the kind of analysis which develops mathematical models of inventory processes. Efforts will be made to develop not a single general model but a wide variety of models each for a specific situation.

An *Inventory* consists of *usable but idle resources* such as men, machines, materials or money. When the resource involved is a material, the inventory is also called *'stock'*. An inventory problem is said to the exist if either the resources are subjected to control or if there is at least one such cost that decreases as inventory increases. The objective is to minimize total (actual or expected) cost. However, in situations where inventory affects demand, the objective may also be to maximize profit.

12.1 NECESSITY FOR MAINTAINING INVENTORY

Though inventory of materials is an idle resource (since the materials lie idle and are not to be used immediately), almost every organisation must maintain it for efficient and smooth running of its operations. Without it no business activity can be performed, whether it is a service organisation like a hospital or a bank or it is a manufacturing or trading organisation. If an enterprise has no inventory of materials at all, on receiving a sales order it will have to place order for purchase of raw materials, wait of their receipt and then start production. The customer will thus have to wait for a long time for the delivery of the goods and may turn to other suppliers resulting in loss of business for the enterprise. Most organisations have 20 to 25 per cent of the total funds devoted to inventory. It may even increase to 70 per cent in case of pharmaceutical, chemical and paints industries. Maintaining an inventory is necessary because of the following reasons :

1. It helps in smooth and efficient running of an enterprise. It decouples the production from customers and vendors and simplifies the otherwise complex organisation for manufacture and reduces the co-ordination effort.
2. It provides service to the customer at a short notice. Timely deliveries can fetch more goodwill and orders.
3. In the absence of inventory, the enterprise may have to pay high prices because of piecemeal purchasing. Maintaining of inventory may earn price discount because of bulk purchasing. It also takes advantage of favourable market.
4. It reduces product costs since there is an added advantage of batching and long, uninterrupted production runs.
5. It acts as a buffer stock when raw materials are received late and shop rejections are too many.
6. Process and movement inventories (also called pipeline stocks) are quite necessary in big enterprises where significant amounts of times are required to tranship items from one location to another.
7. Bulk purchases will entail less orders and, therefore, less clerical costs. This applies to goods produced within the organisation as well. Less orders, as a result of larger lots, will entail lesser machine setups and other associated costs.
8. An organisation may have to deal with several customers and vendors who are not necessarily near it. Inventories, therefore, have to be built to meet the demand at least during the transit time.

9. It helps in maintaining economy by absorbing some of the fluctuations when the demand for an item fluctuates or is seasonal.

However, too often inventories are wrongly used as a substitute for management. For example, if there are large finished goods inventories, inaccurate sales forecasting by marketing deptt. may never be apparent. Similarly, a production foreman who has large in-process inventories may be able to hide his poor planning since there is always something to manufacture. Furthermore, inventory means unproductive 'tied up' capital of the enterprise. The capital could be usefully utilised in other ventures as well. With large inventory, there is always likelihood of obsolescence too. Also maintenance of inventory costs additional money to be spent on personnel, equipment, insurance, etc. Thus excess inventories are not at all desirable. This necessitates controlling the inventories in the most useful way.

Causes of Poor Inventory Control

1. Overbuying without regard to the forecast or proper estimate of demand to take advantage of favourable market.
2. Overproduction or production of goods much before the customer requires them.
3. Overstocking may also result from the desire to provide better service to the customers. Bulk production to cut down production costs will also result in large inventories.
4. Cancellation of orders and minimum quantity stipulations by the suppliers may also give rise to large inventories.

12.1-1 Classifications of Inventories

Inventories are generally classified into the following types :

1 Direct Inventories

They include items that are directly used for production and are classified as :

(*a*) *Production Inventory* : Items such as raw materials, components and subassemblies used to produce the final product.

(*b*) *Work-in-Process Inventory* : Items in semi-finished form or products at different stages of production.

(*c*) *Finished Goods Inventory* : This includes the final products ready for dispatch to consumers or distributors.

(*d*) *MRO Inventory* : Maintenance, repair and operating items such as spare parts and consumable stores that do not go into the final product but are consumed during the production process.

(*e*) *Miscellaneous Inventory* : All other items such as scrap, obsolete and unsaleable products, stationery and other items used in office, factory and sales department, etc.

2 Indirect Inventories

Indirect inventories may be classified as :

(*a*) *Transit or Pipeline Inventories* : Also called *movement inventories*, they consist of items that are currently under transportation *e.g.*, coal being transported from coalfields to a thermal plant.

(*b*) *Buffer Inventories* : They are required as protection against the uncertainties of supply and demand. A company may well know the average demand of an item that it needs; however, the actual demand may turn out to be quite different–it may well exceed the average value. Similarly, the average delivery period (lead time) may be known but due to some unforeseen reasons, the actual delivery period could be much more. Such situations require extra stock of the item to reduce the number of stock-outs or back-orders. This extra stock in excess of the average demand during the lead time is called *buffer stock* (or *safety stock* or *cushion stock*).

(*c*) *Decoupling Inventories* : They are required to decouple or disengage the different parts of the production system. For an item that requires processing on a series of different machines with different processing times, it is a must to have decoupling inventories of the item in between the various machines for smooth and continuous production. The decoupling inventories act as shock absorbers in case of varying work-rates, machine breakdowns or failures, etc.

(*d*) *Seasonal Inventories* : Some items have seasonal demands *e.g.*, demand of woollen textiles in winter, coolers and air conditioners in summer, raincoats in rainy season, etc. Inventories for such items have to be maintained to meet their high seasonal demand.

(*e*) *Lot Size Inventories* : Items are usually purchased in lots to

(*i*) avail price discounts

(*ii*) reduce transportation and purchase costs

(*iii*) minimize handling and receiving costs.

Lot size or *cycle inventories* are, therefore, held by purchasing items in lots rather than their exact quantities required. For example, a textile industry may buy cotton in bulk during cotton season rather than buying it everyday.

(*f*) *Anticipation Inventories* : They are held to meet the anticipated demand. Purchasing of crackers well before Diwali, fans before the approaching summer, piling up of raw material in the face of imminent transporters' strike are examples of anticipation inventories.

12.2 INVENTORY COSTS

The four costs considered in inventory control models are :

1. Purchase costs
2. Inventory carrying or stock holding costs
3. Procurement costs (for bought-outs) or setup costs (for made-ins) and
4. Shortage costs (due to disservice to the customers).

12.2-1 Purchase Costs

It is the price that is paid for purchasing/producing an item. It may be constant per unit or may vary with the quantity purchased / produced. If the cost / unit is constant, it does not affect the inventory control decision. However, the purchase cost is definitely considered when it varies as in quantity discount situations.

12.2-2 Inventory Carrying Costs (or Stock Holding Costs or Holding Costs or Storage Costs)

They arise on account of maintaining the stocks and the interest paid on the capital tied up with the stocks. They vary directly with the size of the inventory as well as the time for which the item is held in stock. Various components of the stockholding cost are :

1. *Cost of money or capital tied up in inventories.* This is, by far, the most important component. Money borrowed from the banks may cost interest of about 12%. But usually the problem is viewed in a slightly different way *i.e.*, how much the organisation would have earned, had the capital been invested in an alternative project such as developing a new product, etc. It is generally taken somewhere around 15% to 20% of the value of the inventories.
2. *Cost of storage space.* This consists of rent for space. Besides space expenses, this will also include heating, lighting and other atmospheric control expenses. Typical values may vary from 1 to 3% .
3. *Depreciation and deterioration costs.* They are especially important for fashion items or items undergoing chemical changes during storage. Fragile items such as crockery are liable to damage, breakage, etc. 0.2% to 1% of the stock value may be lost due to damage and deterioration.
4. *Pilferage cost.* It depends upon the nature of the item. Valuables such as gun metal bushes and expensive tools may be more tempting, while there is hardly any possibility of heavy casting or forging being stolen. While the former must be kept under lock and key, the latter may be simply dumped in the stockyard. Pilferage cost may be taken as 1% of the stock value.

5. *Obsolescence cost.* It depends upon the nature of the item in stock. Electronic and computer components are likely to be fast outdated. Changes in design also lead to obsolescence. It may be possible to quantify the percentage loss due to obsolescence and it may be taken as 5% of the stock value.
6. *Handling costs.* These include all costs associated with movement of stock, such as cost of labour, overhead cranes, gantries and other machinery used for this purpose.
7. *Record-keeping and administrative cost.* There is no use of keeping stocks unless one can easily know whether or not the required item is in stock. This signifies the need of keeping funds for record-keeping and necessary administration.
8. *Taxes and Insurance.* Most organisations have insurance cover against possible loss from theft, fire, etc. and this may cost 1% to 2% of the invested capital.

Inventory carrying cost C_1 is expressed either as per cent/unit time (*e.g.*, 20% per year) or in terms of monetary value/unit /unit time (*e.g.*, ₹ 5/unit/ year).

Example : If the average stock during a year is of value ₹ 20,000, the inventory carrying costs, being, say, equal to 20%, amount to ₹ $20{,}000 \times \frac{20}{100}$ = ₹ 4,000.

12.2-3 Procurement Costs or Setup Costs

These include the fixed cost associated with placing of an order or setting up a machinery before starting production. They include costs of purchase, requisition, follow up, receiving the goods, quality control, cost of mailing, telephone calls and other follow up actions, salaries of persons for accounting and auditing, etc. *Also called order costs or replenishment costs*, they are assumed to be independent of the quantity ordered or produced but directly proportional to the number of orders placed. At times, however, these costs may not bear any simple relationship to the number of orders. More than one stock item may be ordered on one set of the documents; the clerical staff is not divisible and without the existing staff increasing or decreasing, there may be considerable scope for changing the number of orders. In such a case, the acquisition cost relationship may be quadratic or stepped instead of a straight line. They are expressed in terms of ₹/order or ₹/setup.

12.2-4 Shortage Costs or Stock-out Costs

These costs are associated with either a delay in meeting demands or the inability to meet it at all. Therefore, shortage costs are usually interpreted in two ways. In case the unfilled demand can be filled at a later stage (backlog case), these costs are proportional to quantity that is short as well as the delay time and are expressed as ₹/unit back ordered/unit time (*e.g.* ₹ 7/unit/year). They represent loss of goodwill and cost of idle equipment. In case the unfilled demand is lost (no backlog case), these costs become proportional to only the quantity that is short. These result in cancelled orders, lost sales, profit and even the business itself.

It follows from the above discussion that if the purchase cost is constant and independent of the quantity purchased, it is not considered in formulating the inventory control policy. The total variable inventory cost in this case is given by

Total variable inventory cost = Carrying cost + Ordering cost + Shortage cost.

However, if the unit cost depends upon the quantity purchased *i.e.*, price discounts are available, the purchase cost is definitely considered in formulating the inventory control policy. The total inventory cost in this case is then given by

Total inventory cost = Purchase cost + Carrying cost + Ordering cost + Shortage cost.

12.3 INVENTORY CONTROL PROBLEM

The inventory control problem consists of determination of three basic factors :

1. When to order (produce or purchase) ?
2. How much to order ?
3. How much safety stock should be kept ?

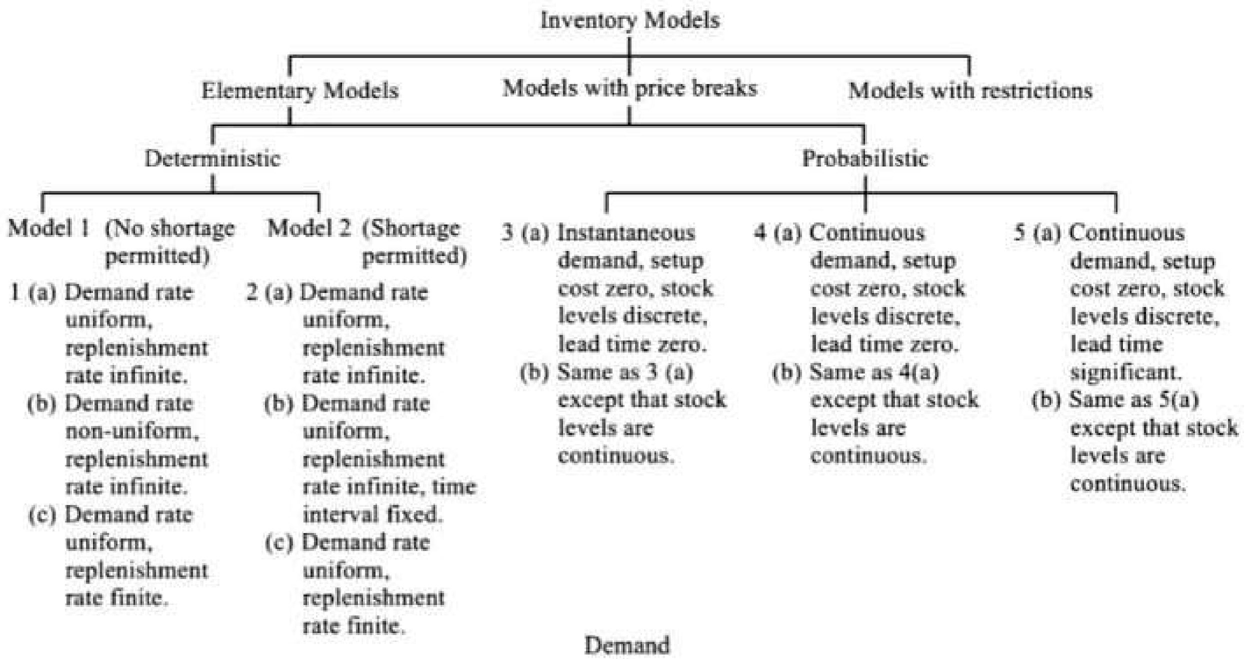
When to order. This is related to the *lead time* (also called *delivery lag*) of an item. Lead time may be defined as the time interval between the placement of an order for an item and its receipt in stock. It may be replenishment order on an outside firm or within the works. There should be enough stock for each item so that customers' orders can be reasonably met from this stock until replenishment. This stock level, known as *reorder level*, has, therefore, to be determined for each item. It is determined by balancing the cost of maintaining these stocks and the disservice to the customer if his orders are not filled in time.

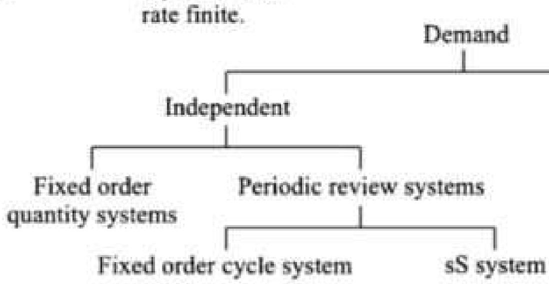
How much to order. As already discussed, each order has associated with it the ordering cost or acquisition cost. To keep it low, the number of orders should be as few as possible *i.e.*, the order size should be large. But large order size would imply high inventory carrying cost. Thus the problem of how much to order is solved by compromising between the acquisition costs and inventory carrying costs.

How much should be the safety stock. This is important to avoid overstocking while ensuring that no stock-outs take place.

The inventory control policy of an organisation depends upon the demand characteristics. The demand for an item may be independent or dependent. For instance, the demand for the different models of television sets manufactured by a company does not depend upon the demand of any other item, while the demand of its various components will depend upon the demand (and hence sale) of the television sets and may be arithmetically computed from the latter. The independent demand is usually ascertained by extrapolating the past demand history *i.e.*, by *forecasting*. The order level can be fixed from the demand forecasts and the lead time. Thus while in the case of dependent demand, simple arithmetic computations are enough to ascertain requirement of the components; in the case of independent demand items, statistical forecasting techniques have to be employed. The discussion of these forecasting techniques will be taken up later in this chapter. The family tree drawn in the next section gives an idea of the various inventory control policies.

12.4 CLASSIFICATION OF FIXED ORDER QUANTITY INVENTORY MODELS





Fixed order quantity systems will now be discussed in detail. The periodic review systems will be taken up briefly towards the end of this chapter.

12.5 INVENTORY MODELS WITH DETERMINISTIC DEMAND

It is extremely difficult to formulate a single general inventory model which takes into account all variations in real systems. In fact, even if such a model were developed, it may not be analytically solvable. Thus inventory models are usually developed for some specific situations.

In this section we shall deal with situations in which demand is assumed to be fixed and completely known. Models for such situations are called *economic lot size models or economic order quantity models.*

12.5-1 Model 1 (a) Classical EOQ Model (Demand Rate Uniform, Replenishment Rate Infinite)

This is one of the simplest inventory models. A stockist has order to supply goods to customers at a uniform rate R per unit time. Hence demand is fixed and known. No shortages are allowed, consequently, the cost of shortage, C_2 is infinity. He places an order with the manufacturer every t time units, where t is fixed; and the ordering cost per order is C_3. Replenishment time is negligible *i.e.*, replenishment rate is infinite so that replenishment is instantaneous (lead time is zero). The holding cost is assumed to be proportional to the amount of inventory as well as the time inventory is held. Thus the cost of holding inventory I for time T is C_1IT, where C_1 is the cost of holding one unit in inventory for a unit of time. The cost coefficients C_1, C_2 and C_3 are assumed to be constants. The stockist's problem is to determine

(*i*) How frequently he should place the order.

(*ii*) How many units should be ordered in each order.

This model is illustrated schematically in figure 12.1.

If orders are placed at intervals t, a quantity $q = Rt$ must be ordered in each order. Since the stock in small time dt is $Rtdt$, the stock in time period t will be

$$\int_0^t R\,t.dt = \frac{1}{2}Rt^2 = \frac{1}{2}qt = \text{Area of inventory triangle OAP.}$$

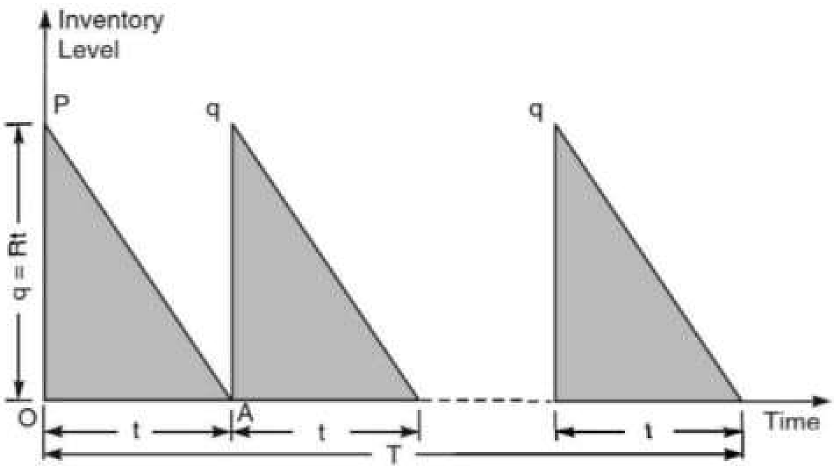


Fig. 12.1. Inventory situation for model 1 (a).

$\therefore$ Cost of holding inventory during time $t = \frac{1}{2}C_1Rt^2$.

Ordering cost to place an order $= C_3$.

$\therefore$ Total cost during time $t = \frac{1}{2}C_1Rt^2 + C_3$.

$\therefore$ Average total cost per unit time, $C(t) = \frac{1}{2}C_1Rt + \frac{C_3}{t}$. ...(12.1)

C will be minimum if $\frac{dC(t)}{dt} = 0$ and $\frac{d^2C(t)}{dt^2}$ is positive.

Differentiating equation (12.1) w.r.t. 't',

$$\frac{dC(t)}{dt} = \frac{1}{2} C_1R - \frac{C_3}{t^2} = 0, \text{ which gives } t = \sqrt{\frac{2C_3}{C_1R}}.$$

Differentiating equation (12.1) twice w.r.t. 't'

$\frac{d^2C(t)}{dt^2} = \frac{2C_3}{t^3}$, which is positive for value of t given by the above equation.

Thus C(t) is minimum for optimal time interval,

$$t_0 = \sqrt{\frac{2C_3}{C_1R}}. \qquad \text{...(12.2)}$$

Optimum quantity q_0 to be ordered during each order,

$$q_0 = R\,t_0 = \sqrt{\frac{2C_3R}{C_1}}, \qquad \text{...(12.3)}$$

which is known as the *optimal lot size (or economic order quantity) formula* due to R.H. Wilson. It is also called *Wilson's or square root formula or Harris lot size formula.*

Any other order quantity will result in a higher cost.

The resulting minimum average cost per unit time,

$$C_0(q) = \frac{1}{2}C_1R.\sqrt{\frac{2C_3}{C_1R}} + C_3.\sqrt{\frac{C_1R}{2C_3}}$$

$$= \frac{1}{\sqrt{2}}\sqrt{C_1C_3R} + \frac{1}{\sqrt{2}}\sqrt{C_1C_3R} = \sqrt{2C_1C_3R}. \qquad \text{...(12.4)}$$

This cost curve has the lowest value (Fig. 1.1) just above the intersection of the two cost curves viz, ordering cost curve and carrying cost curve. At the intersection point the two costs are equal.

Also the total minimum cost per unit time, including the cost of the item

$$= \sqrt{2C_1C_3R} + CR, \qquad \text{...(12.4a)}$$

where C is the cost/unit of the item.

Equation (12.1) can be written in an alternative form by replacing t by q/R as

$$C(q) = \frac{1}{2}C_1q + \frac{C_3R}{q}. \qquad \text{...(12.5)}$$

The average inventory is $\frac{q_0+0}{2} = \frac{q_0}{2}$ and is, thus, time independent.

It may be realized that some of the assumptions made are not satisfied in actual practice. For instance, it is seldom that a customer demand is known exactly and that replenishment time is negligible.

Corollary 1. In the above model if the order cost is $C_3 + bq$ instead of being fixed, where b is the order cost per unit item, we can prove that there is no change in the optimum order quantity due to the changed order cost.

Proof. The average cost per unit time, $C(q) = \frac{1}{2}C_1q + \frac{R}{q}(C_3 + bq)$. [From equation (12.5)]

For the minimum cost $\frac{dC(q)}{dq} = 0$ and $\frac{d^2C(q)}{dq^2}$ is positive.

i.e., $$\frac{1}{2}C_1 - \frac{RC_3}{q^2} = 0 \text{ or } q = \sqrt{\frac{2RC_3}{C_1}},$$

and $\dfrac{d^2C(q)}{dq^2} = \dfrac{2RC_3}{q^3}$, which is necessarily positive for above value of q.

$$\therefore \qquad q_0 = \sqrt{\frac{2C_3R}{C_1}}, \text{ which is same as equation (12.3).}$$

Hence there is no change in optimum order quantity as a result of change in the order cost.

Corollary 2. In model 1 (*a*) discussed above, the lead time has been assumed to be zero. Most practical problems, however, have a positive lead time L from the time the order for the item is placed until it is actually delivered. The ordering policy of the above model, therefore, must satisfy the reorder point.

If L is the lead time in days and R is the inventory consumption rate in units per day, the total inventory requirements during the lead time = LR. Thus we should place an order q as soon as the stock level becomes LR. This is called reorder point p = LR.

In practice, this is equivalent to continuously observing the level of inventory until the reorder point is obtained. That is why the economic lot size model is also called *continous review model*. Figure 12.2 shows the reorder points.

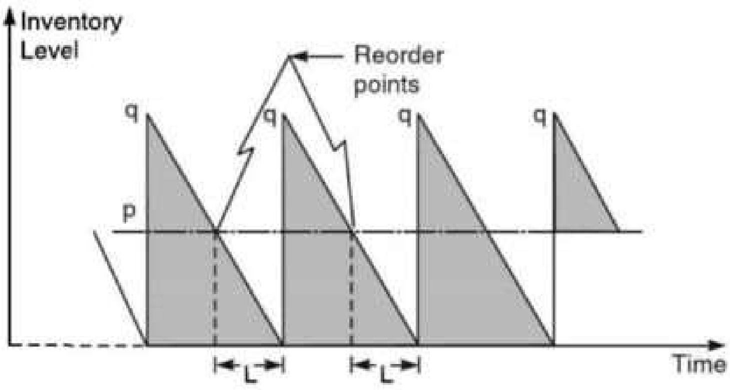


Fig. 12.2. Inventory situation with lead time.

If a buffer stock B is to be maintained, reorder point will be

$$p = B + LR. \qquad \text{...(12.6)}$$

Furthermore, if d days are required for reviewing the system,

$$p = B + LR + \frac{Rd}{2} = B + R\left(L + \frac{d}{2}\right). \qquad \text{...(12.7)}$$

Assumptions in E.O.Q. Formula

Following simplifying assumptions have been made while deriving the economic order quantity formula :

1. Demand is known and uniform (constant).
2. Shortages are not permitted ; as soon as the stock level becomes zero, it is instantaneously replenished.
3. Replenishment of stock is instantaneous or replenishment rate is infinite.
4. Lead time is zero. The moment the order is placed, the quantity ordered is received.
5. Inventory carrying cost and ordering cost per order remain constant over time. The former is linearly related to the quantity ordered and the latter to the number of orders.
6. Cost of the item remains constant over time. There are no price-breaks or quantity discounts.
7. The item is purchased and replenished in lots or batches.
8. The inventory system pertains to a single item.

Limitations of (Objections to) E.O.Q. Formula

The E.O.Q. formula has a number of limitations. It has been highly controversial since a number of objections have been raised regarding its validity. Some of them are

1. In practice the demand is neither known with certainty nor it is uniform. If the fluctuations are mild, the formula can be applicable but for large fluctuations it loses its validity. Dynamic E.O.Q. models, instead, may have to be applied.
2. The ordering cost is difficult to measure. Also it may not be linearly related to the number of orders as assumed in the derivation of the model. The inventory carrying rate is still more difficult to measure and even to define precisely.
3. It is difficult to predict the demand. Present demand may be quite different from the past history. Hardly any prediction is possible for a new product to be introduced in the market.
4. The E.O.Q. model assumes instantaneous replenishment of the entire quantity ordered. In practice, the total quantity may be supplied in parts. E.O.Q. model is not applicable in such a situation.
5. Lead time may not be zero unless the supplier is next-door and has sufficient stock of the item, which is rarely so.
6. Price variations, quantity discounts and shortages may further ivalidate the use of the E.O.Q. formula.

However, the flatness of the total cost curve around the minimum (Fig. 1.1) is an answer to many objections. Even if we deviate from E.O.Q. within reasonable limits, there is no substantial change in cost. For example, if because of inaccuracies and errors, we have selected an order quantity 20% more (or less) than q_0, the increase in total cost will be less than 2%.

EXAMPLE 12.5-1

A stockist has to supply 12,000 units of a product per year to his customer. The demand is fixed and known and the shortage cost is assumed is to be infinite. The inventory holding cost is ₹ 0.20 per unit per month and the ordering cost per order is ₹ 350. Determine

(i) The optimum lot size q_0

(ii) optimum scheduling period t_0

(iii) minimum total variable yearly cost.

Solution

Supply rate, $$R = \frac{12,000}{12} = 1,000 \text{ units/month},$$

C_1 = ₹ 0.20 per unit per month, C_3 = ₹ 350 per order.

$$(i) \quad q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 350 \times 1,000}{0.20}} = 1,870 \text{ units/order.}$$

$$(ii) \quad t_0 = \sqrt{\frac{2C_3}{C_1R}} = \sqrt{\frac{2 \times 350}{0.20 \times 1,000}} = 1.87 \text{ months} = 8.1 \text{ weeks between orders.}$$

$$(iii) \quad C_0 = \sqrt{2C_1C_3R} = \sqrt{2 \times .20 \times 12 \times 350 \times (1,000 \times 12)} = ₹\ 4,490 \text{ per year.}$$

EXAMPLE 12.5-2

A particular item has a demand of 9,000 units/year. The cost of one procurement is ₹100 and the holding cost per unit is ₹ 2.40 per year. The replacement is instantaneous and no shortages are allowed. Determine

(i) the economic lot size,

(ii) the number of orders per year,

(iii) the time between orders,

(iv) the total cost per year if the cost of one unit is ₹ 1. [NIIFT Mohali, 2000]

Solution

$$R = 9{,}000 \text{ units/year},$$
$$C_3 = ₹\ 100/\text{procurement}, \ C_1 = ₹\ 2.40/\text{unit/year}.$$

(*i*) $$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 100 \times 9{,}000}{2.40}} = 866 \text{ units/procurement.}$$

(*ii*) $$n_0 = \frac{1}{t_0} = \sqrt{\frac{C_1R}{2C_3}} = \sqrt{\frac{2.40 \times 9{,}000}{2 \times 100}} = \sqrt{108} = 10.4 \text{ orders/year.}$$

(*iii*) $$t_0 = \frac{1}{n_0} = \frac{1}{10.4} = 0.0962 \text{ years} = 1.15 \text{ months between procurement.}$$

(*iv*) $$C_0 = 9{,}000 \times 1 + \sqrt{2C_1C_3R}$$
$$= 9{,}000 + \sqrt{2 \times 2.40 \times 100 \times 9{,}000}$$
$$= 9{,}000 + 2{,}080 = ₹\ 11{,}080/\text{year.}$$

EXAMPLE 12.5-3

A stockist has to supply 400 units of a product every Monday to his customers. He gets the product at ₹ 50 per unit from the manufacturer. The cost of ordering and transportation from the manufacturer is ₹ 75 per order. The cost of carrying inventory is 7.5% per year of the cost of the product. Find

(*i*) *the economic lot size,*
(*ii*) *the total optimal cost (including the capital cost),*
(*iii*) *the total weekly profit if the item is sold for ₹ 55 per unit.*

[*NIIFT Mohali, 2001; P.U. B.Com. Sept., 2005*]

Solution

$$R = 400 \text{ units/week},$$
$$C_3 = ₹\ 75/ \text{ per order},$$
$$C_1 = 7.5\% \text{ per year of the cost of the product}$$
$$= ₹\ \left(\frac{7.5}{100} \times 50\right) \text{ per unit per year}$$
$$= ₹\ \left(\frac{7.5}{100} \times \frac{50}{52}\right) \text{ per unit per week}$$
$$= ₹\ \frac{3.75}{52} \text{ per unit per week.}$$

(*i*) $$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 75 \times 400 \times 52}{3.75}} = 912 \text{ units/order.}$$

(*ii*) $$C_0 = 400 \times 50 + \sqrt{2C_1C_3R}$$
$$= 20{,}000 + \sqrt{\frac{2 \times 3.75}{52} \times 75 \times 400}$$
$$= 20{,}000 + 65.80 = ₹\ 20{,}065.80 \text{ per week.}$$

(*iii*) Profit $P = 55 \times 400 - C_0 = 22{,}000 - 20{,}065.80 = ₹\ 1{,}934.20$ per week.

EXAMPLE 12.5-4

A stockist purchases an item at the rate of ₹ 40 per piece from a manufacturer. 2,000 units of the item are required per year. What should be the order quantity per order if the cost per order is ₹ 15 and the inventory charges per year are 20 paise ? [*J.N.T.U. Hyderabad B.Tech. Nov., 2010*]

Solution

$$R = 2{,}000 \text{ units/year},$$
$$C_3 = ₹\ 15 \text{ / order},$$
$$I = \text{Re. } 0.20 \text{ / year} \therefore\ C_1 = CI = ₹\ 0.20 \times 40 = ₹\ 8\text{/unit/year.}$$

$$\therefore \qquad q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 15 \times 2{,}000}{8}} = 87 \text{ units/order.}$$

EXAMPLE 12.5-5

The demand for a commodity is 100 units per day. Every time an order is placed, a fixed cost of ₹ 400 is incurred. Holding cost is ₹ 0.08 per unit per day. If the lead time is 13 days, determine the economic lot size and the reorder point.

Solution

$$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 400 \times 100}{0.08}} = 1{,}000 \text{ units.}$$

Length of cycle, $$t_0 = \frac{1{,}000}{100} = 10 \text{ days.}$$

As the lead time is 13 days and cycle length is 10 days, reodering should occur when the level of inventory is sufficient to satisfy the demand for 13–10 = 3 days.

$\therefore$ Reorder point = 100 × 3 = 300 units.

It may be noted that the 'effective' lead time is taken equal to 3 days rather than 13 days. It is because the lead time is longer than t_0.

EXAMPLE 12.5-6

(a) Calculate the E.O.Q. in units and total variable cost for the following items, assuming an ordering cost of ₹ 5 and a holding cost of 10%.

Item	*Annual demand*	*Unit price (₹)*
A	*800 units*	*0.02*
B	*400 units*	*1.00*
C	*392 units*	*8.00*
D	*13,800 units*	*0.20*

(b) For the above problem, compute E.O.Q. in ₹ as well as in years of supply. Also calculate the E.O.Q. frequency for each of the four items.

Solution

(a) $$q_0 = \sqrt{\frac{2C_3R}{C_1}},\ C_0 = \sqrt{2C_1C_3R}.$$

Item A $$q_0 = \sqrt{\frac{2 \times 5 \times 800}{0.02 \times \frac{10}{100}}} = \sqrt{\frac{800}{0.002}} = 2{,}000 \text{ units,}$$

$$C_0 = \sqrt{2 \times 5 \times 800 \times 0.02 \times \frac{10}{100}} = ₹\ 4.$$

Item B $$q_0 = \sqrt{\frac{2 \times 5 \times 400}{1.00 \times \frac{10}{100}}} = 200 \text{ units,}$$

$$C_0 = \sqrt{2 \times 5 \times 400 \times 1.00 \times \frac{10}{100}} = ₹\ 20.$$

Item C
$$q_0 = \sqrt{\frac{2 \times 5 \times 392}{8.00 \times \frac{10}{100}}} = 70 \text{ units},$$

$$C_0 = \sqrt{2 \times 5 \times 392 \times 8.00 \times \frac{10}{100}} = ₹\ 56.$$

Item D
$$q_0 = \sqrt{\frac{2 \times 5 \times 13{,}800}{0.20 \times \frac{10}{100}}} = 2{,}627 \text{ units},$$

$$C_0 = \sqrt{2 \times 5 \times 13{,}800 \times 0.20 \times \frac{10}{100}} = ₹\ 52.54.$$

(*b*) *E.O.Q. in ₹*

for item A : $2{,}000 \times 0.02 = 40$,
for item B : $200 \times 1 = 200$,
for item C : $70 \times 8 = 560$,
and for item D : $2{,}627 \times 0.20 = 525.40$.

E.O.Q. in years of supply

for item A : $\frac{2000}{800} = 2.5$ years,

for item B : $\frac{200}{400} = 0.5$ year,

for item C : $\frac{70}{392} = 0.18$ year,

and for item D : $\frac{2{,}627}{13{,}800} = 0.19$ year.

E.O.Q. frequency (number of orders per year)

for item A : $\frac{1}{1.25} = 0.4$,

for item B : $\frac{1}{0.5} = 2$,

for item C : $\frac{1}{0.18} = 5.6$,

and for item D : $\frac{1}{0.19} = 5.25$.

EXAMPLE 12.5-7

(*a*) *Compute the E.O.Q. and the total variable cost for the following :*

Annual demand	:	*25 units,*
unit price	:	*₹ 2.50,*
order cost	:	*₹ 4.00,*
storage rate	:	*1% per year,*
interest rate	:	*12% per year,*
obsolescence rate	:	*7% per year.*

(*b*) *Compute the order quantity and the total variable cost that would result if an incorrect price of ₹ 1.60 were used for the item.* [NIIFT Mohali, 2001; P.T.U. MBA May, 2002]

Solution

(*a*)
$$C_1 = ₹\ \frac{(1+12+7)}{100} \times 2.50 = ₹\ 0.50 \text{ per unit per year.}$$

$$\therefore \quad q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 4 \times 25}{0.50}} = 20 \text{ units,}$$

$$C_0 = \sqrt{2C_1C_3R} = \sqrt{2 \times 4 \times 25 \times 0.50} = ₹\ 10.$$

(*b*)
$$q = \sqrt{\frac{2 \times 4 \times 25}{\frac{20}{100} \times 1.60}} = 25 \text{ units. This is non-optimal size.}$$

$$\text{Ordering cost} = \frac{C_3R}{q} = \frac{4 \times 25}{25} = ₹\ 4.$$

$$\text{Stock holding cost} = \frac{1}{2}C_1 . q = \frac{1}{2}.\left(\frac{20}{100} \times 2.50\right) \times 25 = ₹\ 6.25.$$

Note that for calculating the stock holding cost, correct price is to be used. A clectrical error does not mean that the stock value changes. Price paid for the stocks shall still be ₹ 2.50 even though a less price of ₹ 1.60 is taken for E.O.Q. computations.

∴ Total variable cost/year = ₹ 10.25.

EXAMPLE 12.5-8

ABC manufacturing company purchases 9,000 parts of a machine for its annual requirement, ordering one month's usage at a time. Each part costs ₹ 20. The ordering cost per order is ₹ 15, and the carrying charges are 15% of the average inventory per year.

You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?

[*SVSM PGDM, 2009; P.T.U.B.E. (Mech.) May, 2006; P.U.B.Com. April, 2006; Sept., 2004*]

Solution

Here, $R = 9{,}000$ parts/year,

$$\therefore \quad q = \frac{9{,}000}{12} = 750 \text{ parts.}$$

$$C = ₹\ 20 \text{ part, } C_3 = ₹\ 15 \text{ /order,}$$

$$C_1 = ₹\ 20 \times \frac{15}{100} = ₹\ 3 \text{ / part/ year.}$$

$$\text{Total annual variable cost} = \frac{q}{2}.C_1 + \frac{R}{q}.C_3$$

$$= ₹\ \left[\frac{750}{2} \times 3 + \frac{9{,}000}{750} \times 15\right] = ₹\ 1{,}305.$$

$$q_0 = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 9{,}000 \times 15}{3}} = 300 \text{ units.}$$

$$\text{Total annual variable cost} = \sqrt{2RC_1C_3} = \sqrt{2 \times 9{,}000 \times 3 \times 15} = ₹\ 900.$$

Hence if the company purchases 300 units each time and places 30 orders in the year, the net saving to the company will be ₹ (1,305 – 900) = ₹ 405 a year.