

# Deformation Processing - Rolling

ver. 1

Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton

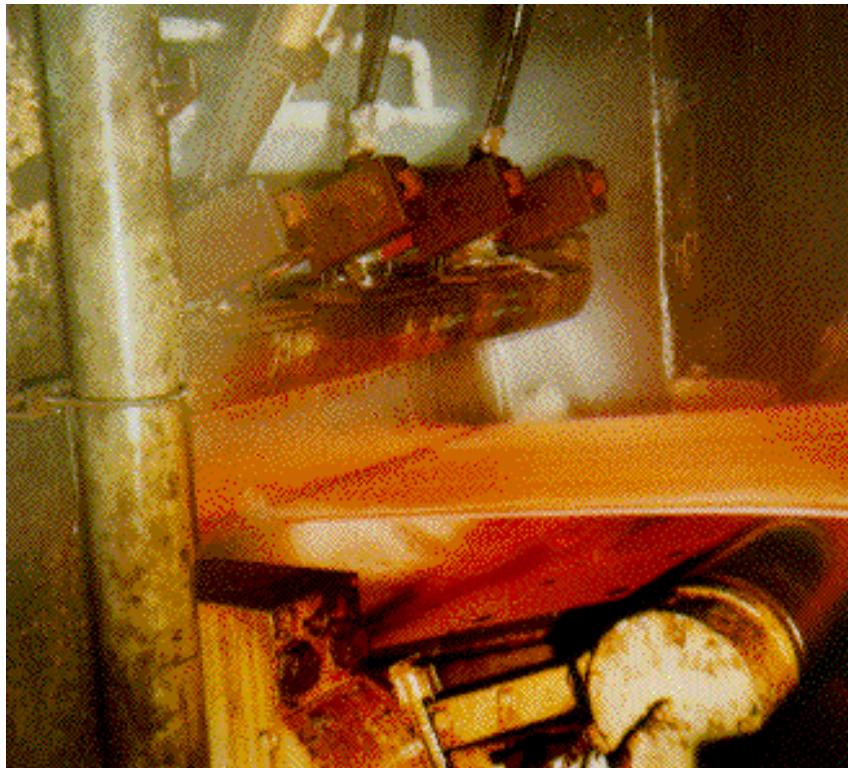


# Overview

- Process
- Equipment
- Products
- Mechanical Analysis
- Defects



# Process



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



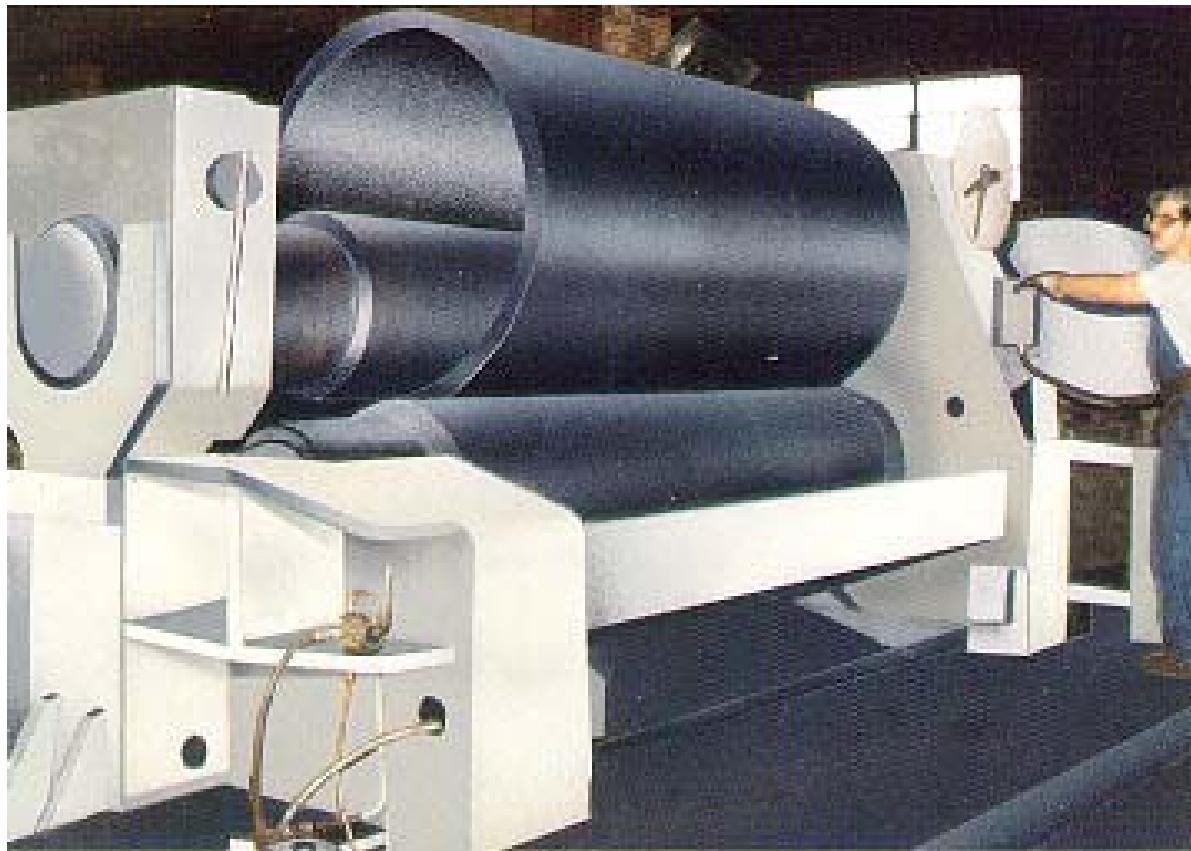
# Process



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Process



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Ring Rolling



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



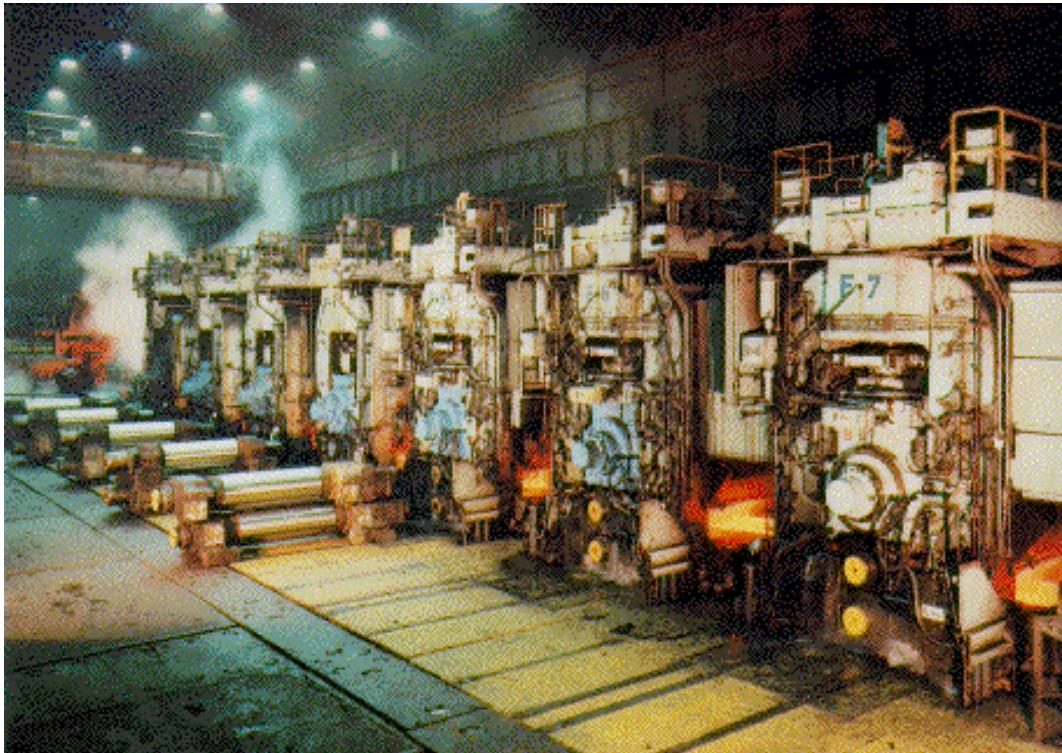
# Equipment



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Equipment

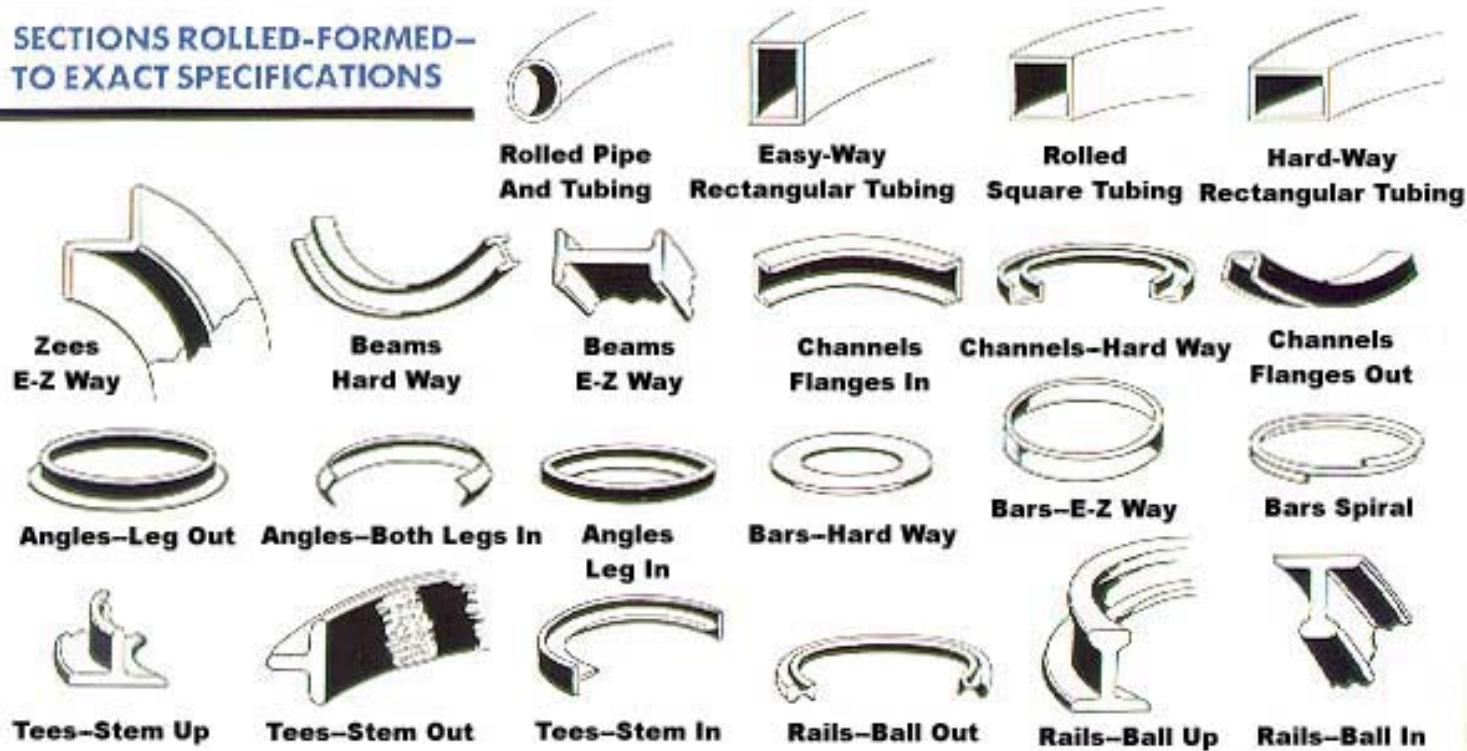


Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Products

## SECTIONS ROLLED-FORMED- TO EXACT SPECIFICATIONS



# Products

- Shapes
  - I-beams, railroad tracks
- Sections
  - door frames, gutters
- Flat plates
- Rings
- Screws



# Products

- A greater volume of metal is rolled than processed by any other means.



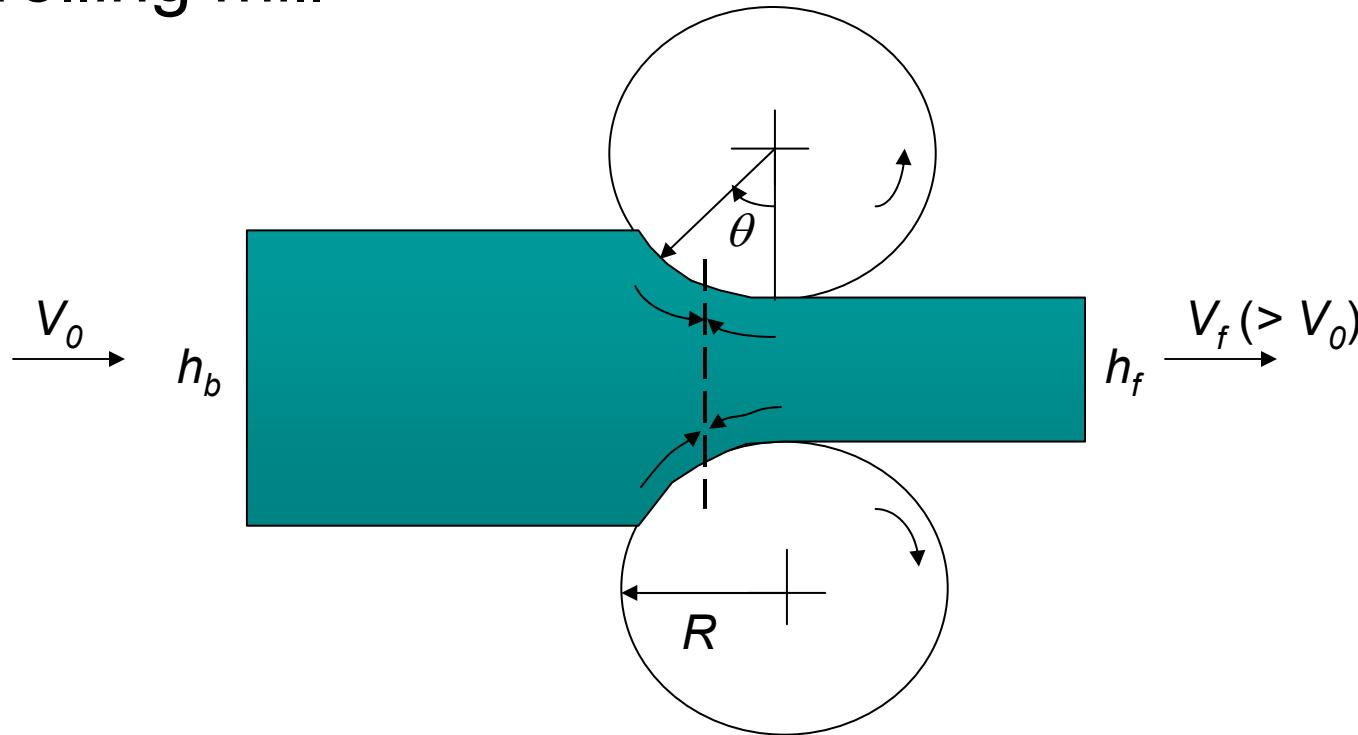
# Rolling Analysis

- Objectives
  - Find distribution of roll pressure
  - Calculate roll separation force (“rolling force”) and torque
  - Processing Limits
  - Calculate rolling power



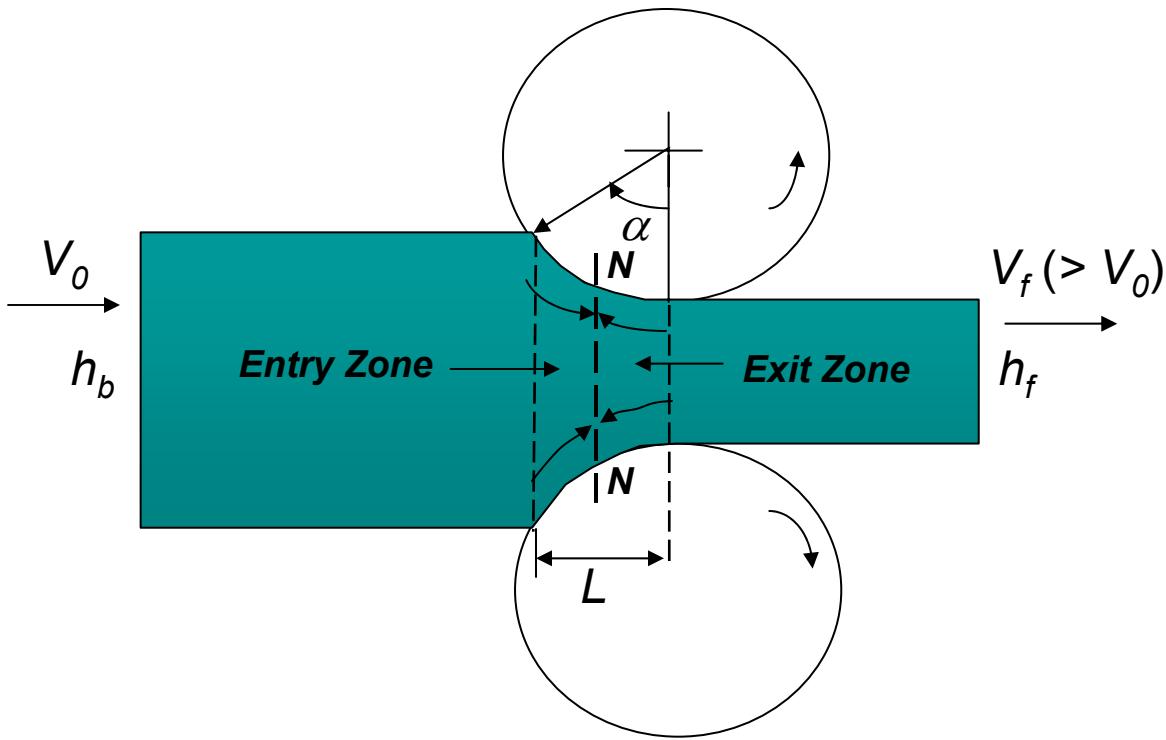
# Flat Rolling Analysis

- Consider rolling of a flat plate in a 2-high rolling mill



*Width of plate  $w$  is large  $\rightarrow$  plane strain*

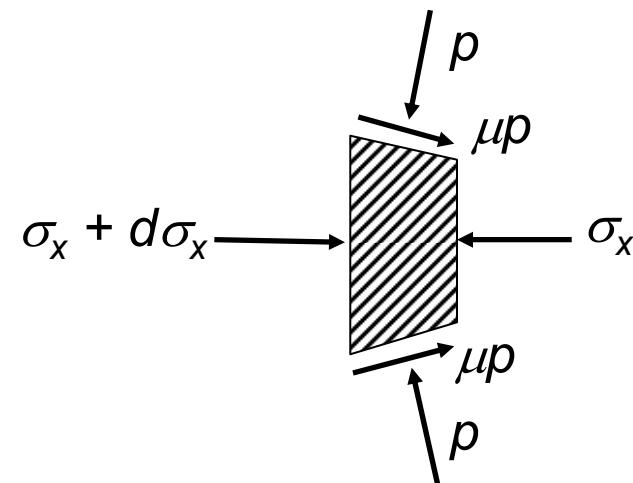
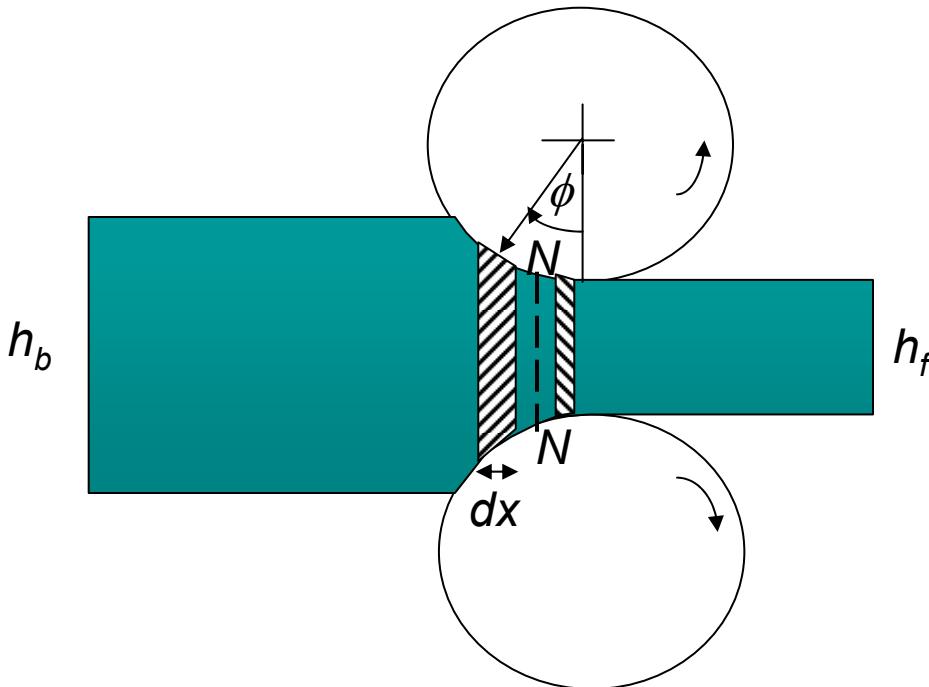
# Flat Rolling Analysis



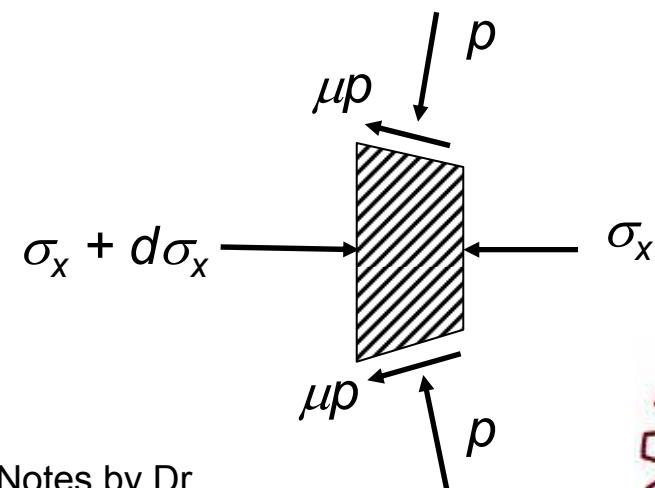
- Friction plays a critical role in enabling rolling  $\rightarrow \mu \geq \tan \alpha$  cannot roll without friction; for rolling to occur
- Reversal of frictional forces at neutral plane (NN)

# Flat Rolling Analysis

Stresses on Slab in Entry Zone



Stresses on Slab in Exit Zone



# Equilibrium

- Applying equilibrium in x (top entry, bottom exit)

$$(\sigma_x + d\sigma_x) \cdot (h + dh) - 2pR \cdot d\phi \cdot \sin \phi \pm 2\mu pR \cdot d\phi \cdot \cos \phi - \sigma_x h = 0$$

Simplifying and ignoring HOTs

$$\frac{d(\sigma_x h)}{d\phi} = 2pR \cdot (\sin \phi \mp \mu \cos \phi)$$



# Simplifying

- Since  $\alpha \ll 1$ , then  $\sin\phi = \phi$ ,  $\cos\phi = 1$

$$\frac{d(\sigma_x h)}{d\phi} = 2pR \cdot (\phi \mp \mu)$$

- Plane strain, von Mises

$$p - \sigma_x = 1.15 \cdot Y_{flow} \equiv Y'_{flow}$$



# Differentiating

- Substituting

$$\frac{d[(p - Y'_{flow}) \cdot h]}{d\phi} = 2pR \cdot (\phi \mp \mu)$$

- or

$$\frac{d}{d\phi} \left[ Y'_{flow} \cdot \left( \frac{p}{Y'_{flow}} - 1 \right) \cdot h \right] = 2pR \cdot (\phi \mp \mu)$$



# Differentiating

$$Y'_{flow} \cdot h \cdot \frac{d}{d\phi} \left( \frac{p}{Y'_{flow}} \right) + \left( \frac{p}{Y'_{flow}} - 1 \right) \cdot \frac{d}{d\phi} (Y'_{flow} \cdot h) = 2pR \cdot (\phi \mp \mu)$$

Rearranging, the variation  $Y'_{flow} \cdot h$  with respect to  $\phi$  is small compared to the variation  $p / Y'_{flow}$  with respect to  $\phi$  so the second term is ignored

$$\frac{\frac{d}{d\phi} \left( \frac{p}{Y'_{flow}} \right)}{\frac{p}{Y'_{flow}}} = \frac{2R}{h} (\phi \mp \mu)$$

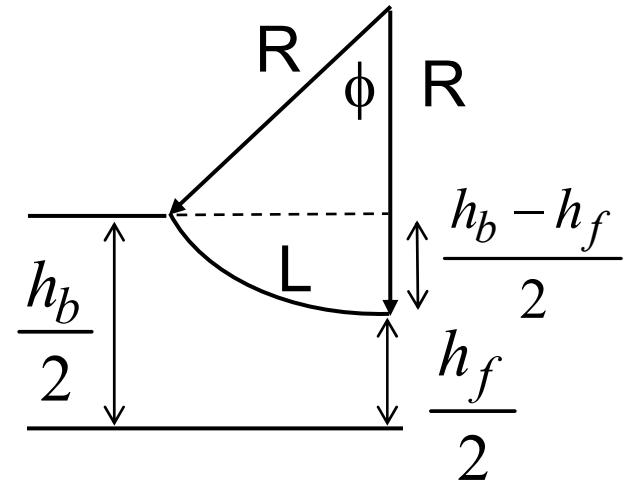
Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Thickness

$$h = h_f + 2R \cdot (1 - \cos \phi)$$

from the definition  
of a circular segment



or, after using a Taylor's series expansion, for small  $\phi$

$$\cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \dots$$

0

$$h = h_f + R \cdot \phi^2$$

# Substituting and integrating

$$\int \frac{d\left(\frac{p}{Y'_{flow}}\right)}{\frac{p}{Y'_{flow}}} = \int \frac{2R}{h_f + R \cdot \phi^2} (\phi \mp \mu) \cdot d\phi$$

In[1]:=  $\int \frac{2 \text{ R } (\phi - \mu)}{\text{hf} + \text{R } \phi^2} \text{ d}\phi$

Out[1]=  $2 \text{ R} \left( -\frac{\mu \text{ ArcTan}\left[\frac{\sqrt{\text{R}} \phi}{\sqrt{\text{hf}}}\right]}{\sqrt{\text{hf}} \sqrt{\text{R}}} + \frac{\text{Log}\left[\text{hf} + \text{R } \phi^2\right]}{2 \text{ R}} \right)$

$$\ln \frac{p}{Y'_f} = \ln \frac{h}{R} \mp 2\mu \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right) + \ln C$$



# Eliminating $\ln()$

$$p = C \cdot Y'_{flow} \cdot \frac{h}{R} \exp(\mp \mu H)$$

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$



# Entry region

- at  $\phi = \alpha$ ,  $H = H_b$ ,

$$p = C \cdot Y'_{flow} \cdot \frac{h}{R} \exp(-\mu H)$$

$$C = \frac{R}{h_b} \exp(\mu H_b) \quad p = Y'_{flow} \frac{h}{h_b} \exp(\mu [H_b - H])$$

$$p = (Y'_{flow} - \sigma_{xb}) \frac{h}{h_b} \exp(\mu [H_b - H]) \quad \text{With back tension} = (Y'_{flow} - \sigma_{xb})$$

$$H_b = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \alpha \sqrt{\frac{R}{h_f}} \right)$$

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$



# Exit region

at  $\phi = 0$ ,  $H = H_f = 0$ ,

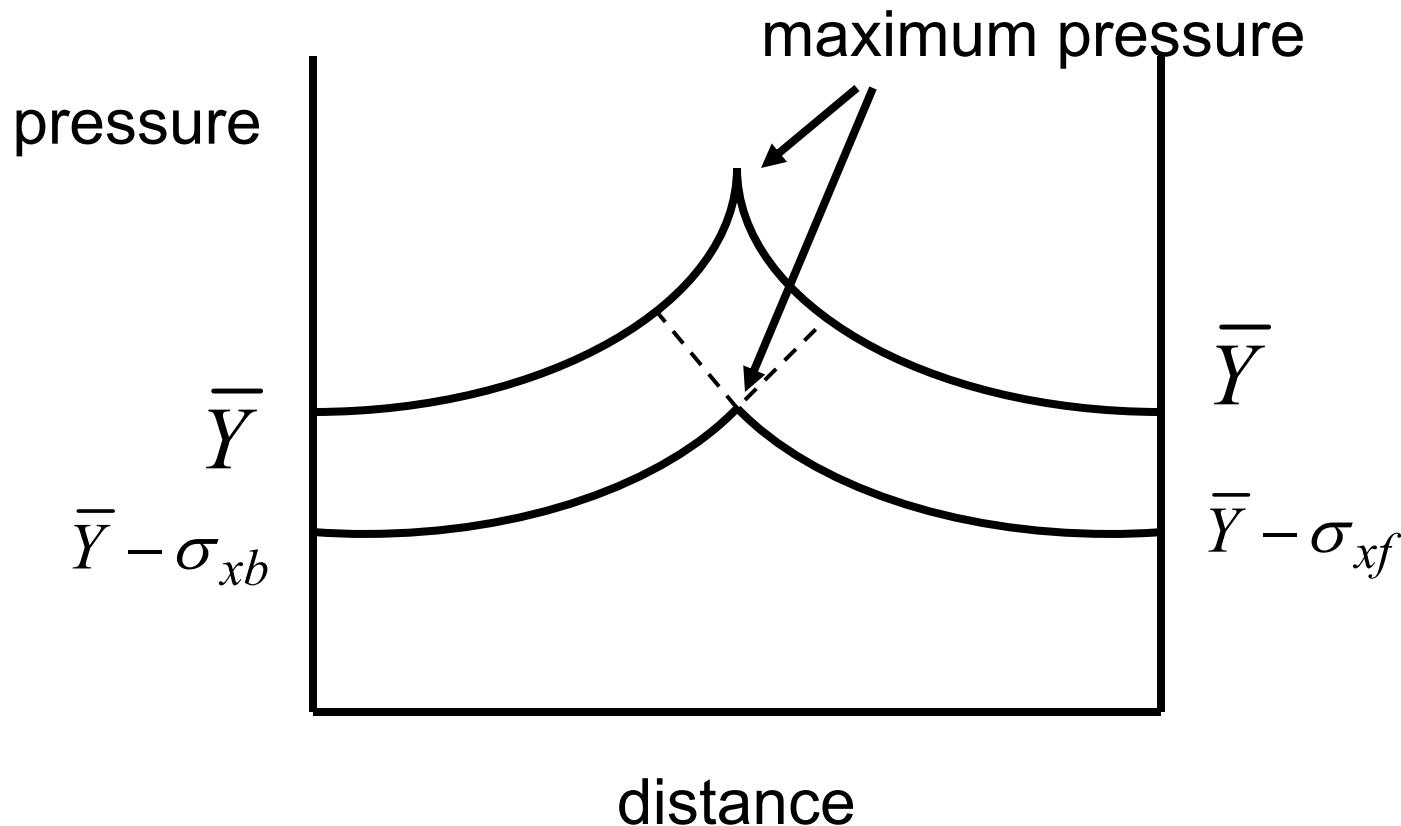
$$C = \frac{R}{h_f} \quad p = \left(Y'_{flow}\right) \frac{h}{h_f} \exp(\mu H)$$

$$p = \left(Y'_{flow} - \sigma_{xf}\right) \frac{h}{h_f} \exp(\mu H) \quad \text{With forward tension}$$

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$

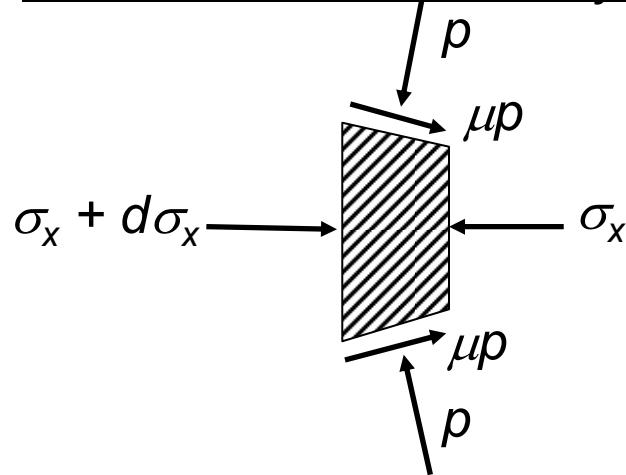


# Effect of back and front tension

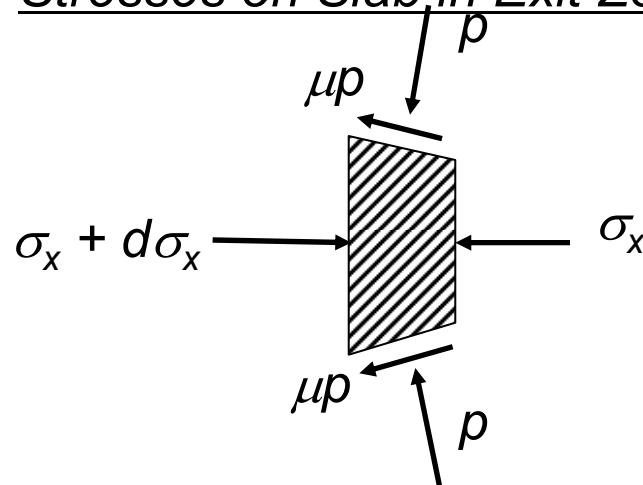


# Flat Rolling Analysis Results – without front and back tension

Stresses on Slab in Entry Zone



Stresses on Slab in Exit Zone



Using slab analysis we can derive roll pressure distributions for the entry and exit zones as:  $h_0$  and  $h_b$  are the same thing

$$p = \frac{2}{\sqrt{3}} Y_f \frac{h}{h_0} e^{\mu(H_0 - H)}$$

Entry Zone

$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \sqrt{\frac{R}{h_f}} \phi \right)$$

$$H_0 = H @ \phi = \alpha$$

$$p = \frac{2}{\sqrt{3}} Y_f \frac{h}{h_f} e^{\mu H}$$

Exit Zone

# Average rolling pressure – per unit width

$$p_{ave,entry} = -\frac{1}{R(\alpha - \phi_n)} \int_{\alpha}^{\phi_n} p_{entry} R d\phi; \quad p_{ave,exit} = \frac{1}{R\phi_n} \int_0^{\phi_n} p_{exit} R d\phi$$

# Rolling force

- $F = p_{ave,entry} \times \text{Area}_{entry} + p_{ave,exit} \times \text{Area}_{exit}$



# Force

- An alternative method

$$F = \int_{\phi_n}^{\alpha} w \cdot p_{entry} \cdot R \cdot d\phi + \int_0^{\phi_n} w \cdot p_{exit} \cdot R \cdot d\phi$$

- again, very difficult to do.



# Force - approximation

$$F / \text{roller} = L w p_{ave}$$

$$L \approx \sqrt{R \Delta h}$$

$$\Delta h = h_b - h_f$$

$$p_{ave} = f\left(\frac{h_{ave}}{L}\right)$$



# Derivation of “L”

circular segment

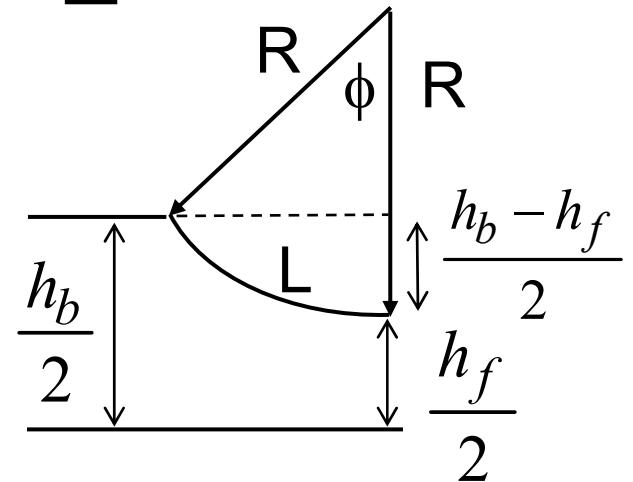
$$h = h_f + 2R \cdot (1 - \cos \phi)$$

Taylor's expansion

$$\cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \dots$$

$$h = h_f + R \cdot \phi^2$$

$$R \cdot \phi = L$$



# Derivation of “L”

setting  $h = h_b$  at  $\phi = \alpha$ , substituting, and rearranging

$$h_b - h_f = \Delta h = R \cdot \left( \frac{L}{R} \right)^2$$

or

$$L = \sqrt{R \cdot \Delta h}$$



# Approximation based on forging plane strain – von Mises

$$p_{ave} = 1.15 \cdot \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$

average flow stress:  
due to shape of element



# Small rolls or small reductions

$$\Delta = \frac{h_{ave}}{L} \gg 1$$

- friction is not significant ( $\mu \rightarrow 0$ )

$$p_{ave} = 1.15 \cdot \bar{Y}_{flow} \left( 1 + \frac{\cancel{\mu L}}{2h_{ave}} \right) \rightarrow 0$$

$$p_{ave} = 1.15 \cdot \bar{Y}_{flow}$$



# Large rolls or large reductions

$$\Delta \equiv \frac{h_{ave}}{L} \ll 1$$

- Friction is significant (forging approximation)

$$p_{ave} = 1.15 \cdot \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$



# Force approximation: low friction

$$\Delta \equiv \frac{h_{ave}}{L} \gg 1$$

$$F_{\text{roller}} = 1.15 \cdot Lw \bar{Y}_{\text{flow}}$$



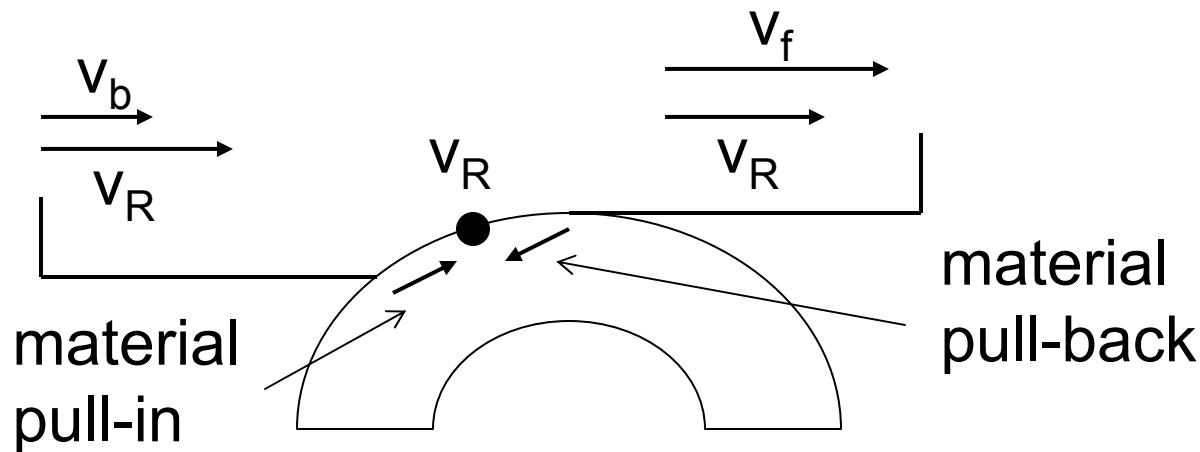
# Force approximation: high friction

$$\Delta \equiv \frac{h_{ave}}{L} \ll 1$$

$$F_{\text{roller}} = 1.15 \cdot Lw \bar{Y}_{\text{flow}} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$

# Zero slip (neutral) point

- Entrance: material is pulled into the nip
  - roller is moving faster than material
- Exit: material is pulled back into nip
  - roller is moving slower than material



# System equilibrium

- Frictional forces between roller and material must be in balance.
  - or material will be torn apart
- Hence, the zero point must be where the two pressure equations are equal.

$$\frac{h_b}{h_f} = \frac{\exp(\mu H_b)}{\exp(2\mu H_n)} = \exp(\mu(H_b - 2H_n))$$



# Neutral point

$$H_n = \frac{1}{2} \left( H_b - \frac{1}{\mu} \ln \frac{h_b}{h_f} \right)$$

$$\phi_n = \sqrt{\frac{h_f}{R}} \tan \left( \frac{H_n}{2} \sqrt{\frac{h_f}{R}} \right)$$

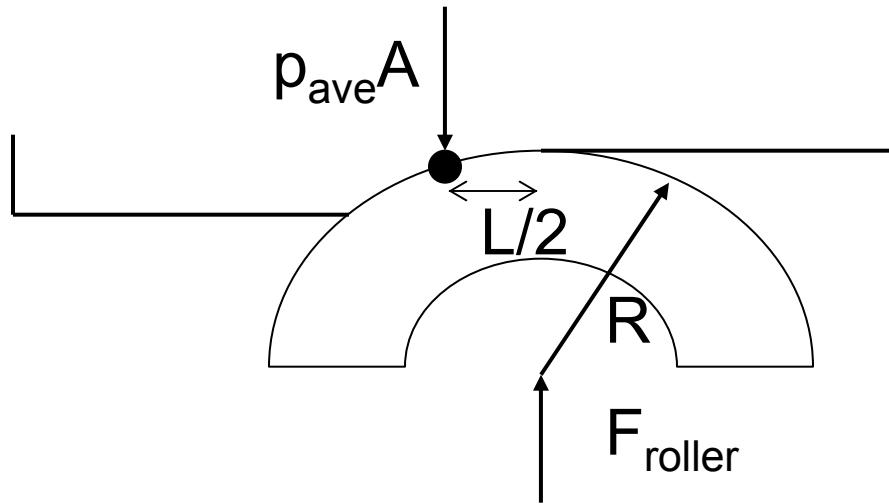
# Torque

$$L \approx \sqrt{R\Delta h}$$

$$\Delta h = h_b - h_f$$

$$\sum F_y = 0$$

$$\therefore F_{roller} = p_{ave} A$$



$$T = \int_{\phi_n}^{\alpha} w \mu p R^2 d\phi - \int_0^{\phi_n} w \mu p R^2 d\phi$$

entry                    exit

$$Torque / roller = r \cdot F_{roller} = \frac{L}{2} \cdot F_{roller} = \frac{F_{roller} L}{2}$$

Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Power

$$\text{Power / roller} = T\omega = F_{\text{roller}}L\omega / 2$$

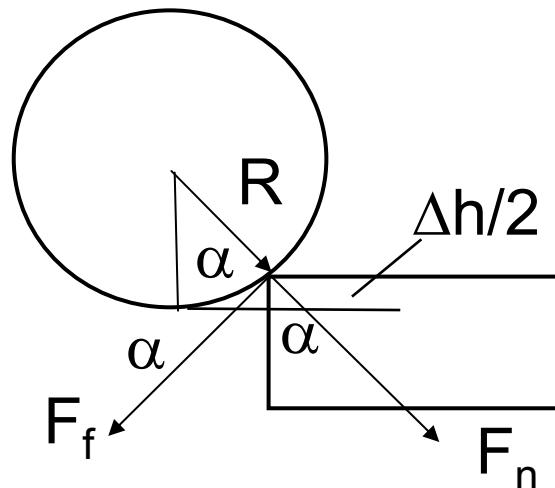
$$\omega = 2\pi N$$

$$N = [\text{rev/min}]$$



# Processing limits

- The material will be drawn into the nip if the horizontal component of the friction force ( $F_f$ ) is larger, or at least equal to the opposing horizontal component of the normal force ( $F_n$ ).



$$F_f \cos \alpha \geq F_n \sin \alpha$$

$$F_f = \mu \cdot F_n$$

$$\tan \alpha = \mu$$

$\mu$  = friction coefficient

# Processing limits

Also

$$\cos \alpha = \frac{R - \frac{\Delta h}{2}}{R} = 1 - \frac{\Delta h}{2R}$$

and  $\Delta h \ll R$        $\sin \alpha = \sqrt{1 - \cos^2 \alpha}$

$$\sin \alpha = \sqrt{1 - 1 + \frac{\Delta h}{2R} - \left(\frac{\Delta h}{2R}\right)^2} \quad \sin \alpha \approx \sqrt{\frac{\Delta h}{R}}$$

$$\tan \alpha = \sqrt{\frac{\frac{\Delta h}{R}}{1 - \frac{\Delta h}{R} + \left(\frac{\Delta h}{2R}\right)^2}} \approx \sqrt{\frac{\Delta h}{R - \Delta h}} \approx \sqrt{\frac{\Delta h}{R}}$$



# Processing limits

So, approximately

$$(\tan \alpha)^2 = \mu^2 = \frac{\Delta h}{R}$$

Hence, maximum draft

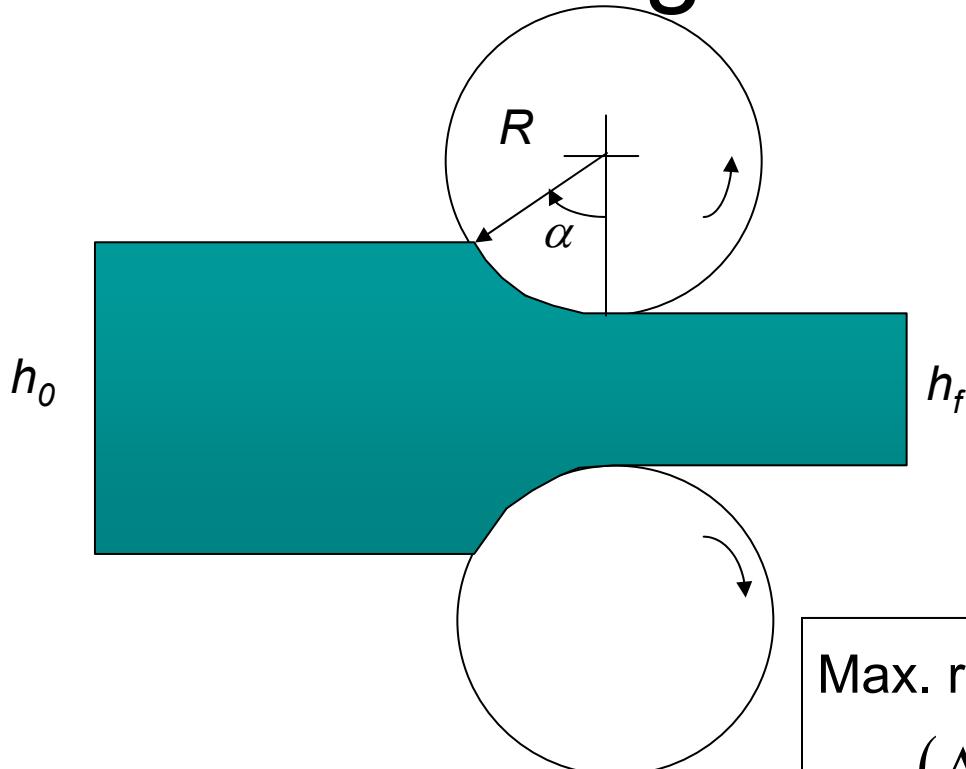
$$\Delta h_{\max} = \mu^2 R$$

Maximum angle of acceptance

$$\phi_{\max} = \alpha = \tan^{-1} \mu$$



# Processing Limits



Max. reduction in thickness

$$(\Delta h)_{\max} = \mu^2 R$$

Max. angle of acceptance

$$\phi_{\max} = \alpha = \tan^{-1} \mu$$

# Cold rolling (below recrystallization point) strain hardening, plane strain – von Mises

$$2\tau_{flow} = 1.15 \cdot \bar{Y}_{flow} = 1.15 \cdot \frac{K\varepsilon^n}{n+1}$$

average flow stress:  
due to shape of element



# Hot rolling – (above recrystallization point) strain rate effect, plane strain - von Mises

- Average strain rate

$$\dot{\bar{\varepsilon}} = \frac{\bar{\varepsilon}}{t} = \frac{V_R}{L} \ln\left(\frac{h_b}{h_f}\right)$$

$$2\tau_{flow} = 1.15 \cdot \bar{Y}_{flow} = 1.15 \cdot C \cdot \dot{\bar{\varepsilon}}^m$$



average flow stress:  
due to shape of element

# Example 1.1

- Cold roll a 5% Sn-bronze
- Calculate force on roller
- Calculate power
- Plot pressure in nip (no back or forward tension)



# Example 1.2

- $w = 10 \text{ mm}$
- $h_b = 2 \text{ mm}$
- height reduction = 30% ( $h_f = 0.7 h_b$ )
  - $h_f = 1.4 \text{ mm}$
- $R = 75 \text{ mm}$
- $v_R = 0.8 \text{ m/s}$
- mineral oil lubricant ( $\mu = 0.1$ )
- $K = 720 \text{ MPa}$ ,  $n = 0.46$



# Example 1.3

- Maximum draft:

$$\begin{aligned}\Delta h_{\max} &= \mu^2 R \\ &= (0.1)^2 \cdot 75 = 0.75 \text{ mm}\end{aligned}$$

$$\begin{aligned}\Delta h_{\text{actual}} &= h_b - h_f = 2 - 1.4 \\ &= 0.6 \text{ mm}\end{aligned}$$



# Example 1.4

- Maximum angle of acceptance

$$\phi_{\max} = \tan^{-1} \mu = \tan^{-1}(0.1) = 0.1 \text{ radian}$$

$$\alpha = \sqrt{\frac{(h_b - h_f)}{R}} = \sqrt{\frac{(2 - 1.4)}{75}}$$
$$= 0.089 \text{ rad} = 5.12^\circ$$



# Example 1.5

- Roller force:  $F = L w p_{ave}$
- $L = (R\Delta h)^{0.5} = [75 \times (2-1.4)]^{0.5}$   
 $= 6.7 \text{ mm}$
- $w = 10 \text{ mm}$
- $h_{ave} = (h_b + h_f) / 2 = 1.7 \text{ mm}$   
 $h_{ave} / L = 1.7 / 6.7 = 0.25 < 1$   
 $\therefore$  friction is important

$$F_{roller} = 1.15 \cdot L w \bar{Y}_{flow} \left( 1 + \frac{\mu L}{2h_{ave}} \right)$$



# Example 1.6

$$\varepsilon_f = \left| \ln\left(\frac{h_f}{h_b}\right) \right| = \left| \ln\left(\frac{1.4}{2}\right) \right| = 0.36$$

$$2\tau_{flow} = 1.15 \cdot \bar{Y} = 1.15 \cdot \frac{K\varepsilon_f^n}{n+1}$$
$$= 1.15 \cdot \frac{720 \cdot (0.36)^{0.46}}{1.46} = 354 \text{ MPa}$$



# Example 1.7

$$\begin{aligned} F_{\text{roller}} &= 1.15 \cdot L_w \bar{Y}_{\text{flow}} \left( 1 + \frac{\mu L}{2h_{\text{ave}}} \right) \\ &= 6.7 \times 10^{-3} \cdot 10 \times 10^{-3} \cdot 354 \times 10^6 \\ &\quad \times \left( 1 + \frac{0.1 \times 6.7}{2 \times 1.7} \right) \\ &= 28,392 \text{ } N = 3.2 \text{ } \textit{tons} \end{aligned}$$



# Example 1.8

$$\frac{Power (kW)}{roller} = T \times \omega = \frac{F \cdot L \cdot V_R}{2 \cdot R}$$

$$Power (kW)/roll = \frac{28,392 \cdot 6.7 \times 10^{-3} \cdot 0.8}{2 \cdot 0.075}$$
$$= 1.01 kW / roll = 1.35 hp$$



# Example 1.9

- Entrance

$$p = \left( Y'_{flow} - \sigma_{xb} \right) \frac{h}{h_b} \exp(\mu(H_b - H))$$

- Exit

$$p = \left( Y'_{flow} - \sigma_{xf} \right) \frac{h}{h_f} \exp(\mu(H))$$



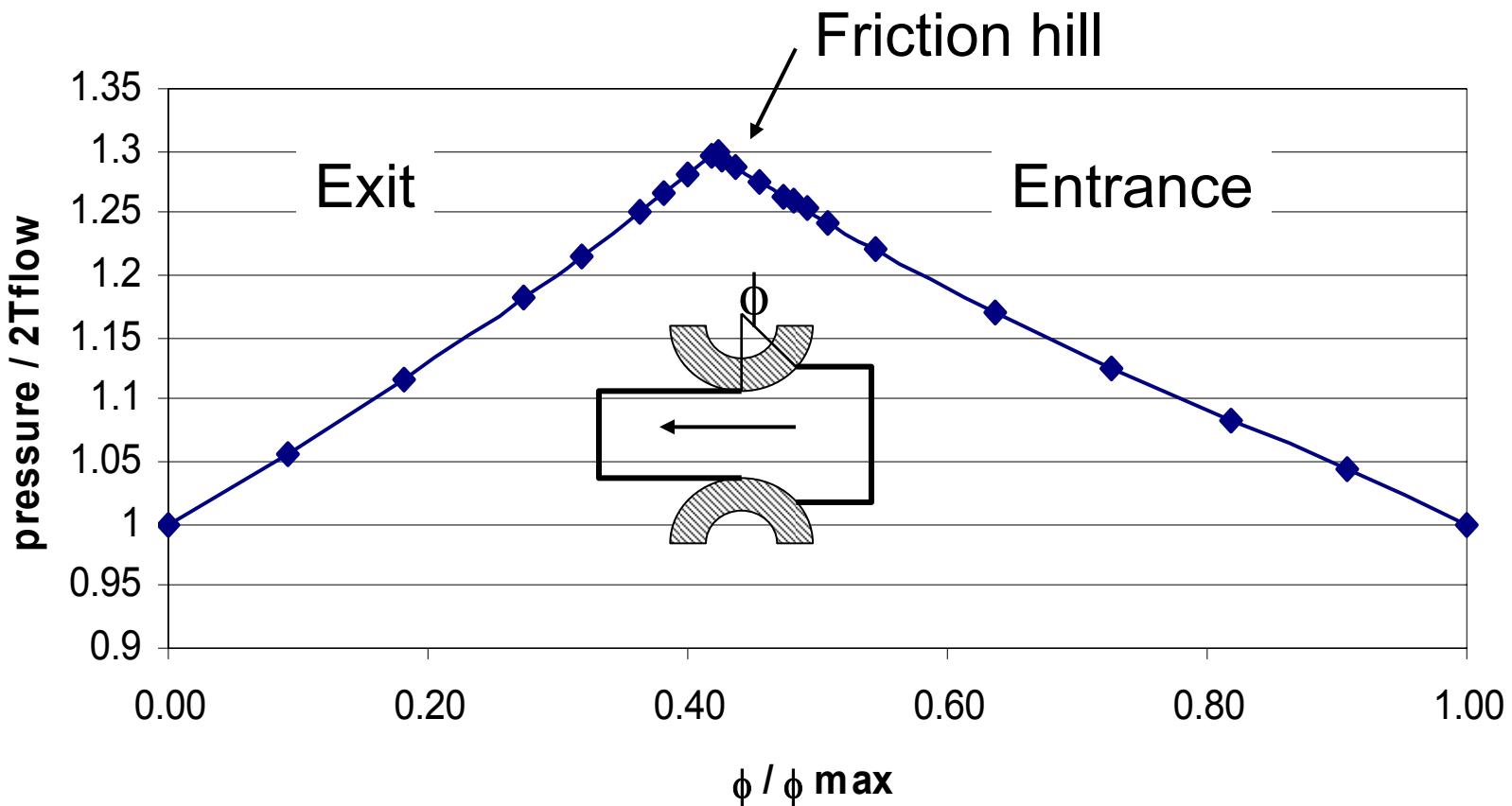
# Example 1.10

$$\phi = \sqrt{\frac{(h - h_f)}{R}}$$

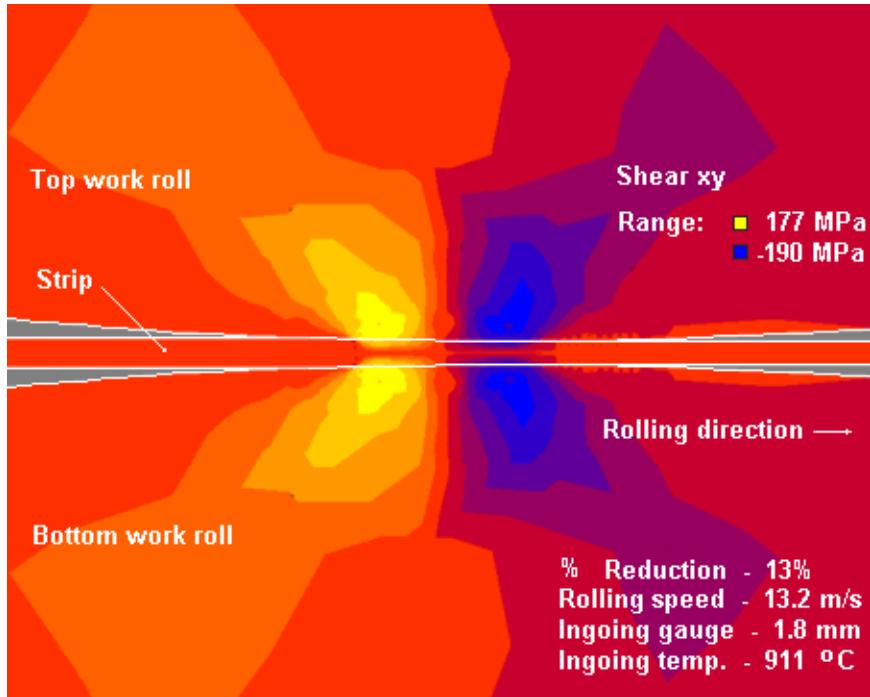
$$H = 2 \sqrt{\frac{R}{h_f}} \tan^{-1} \left( \phi \sqrt{\frac{R}{h_f}} \right)$$



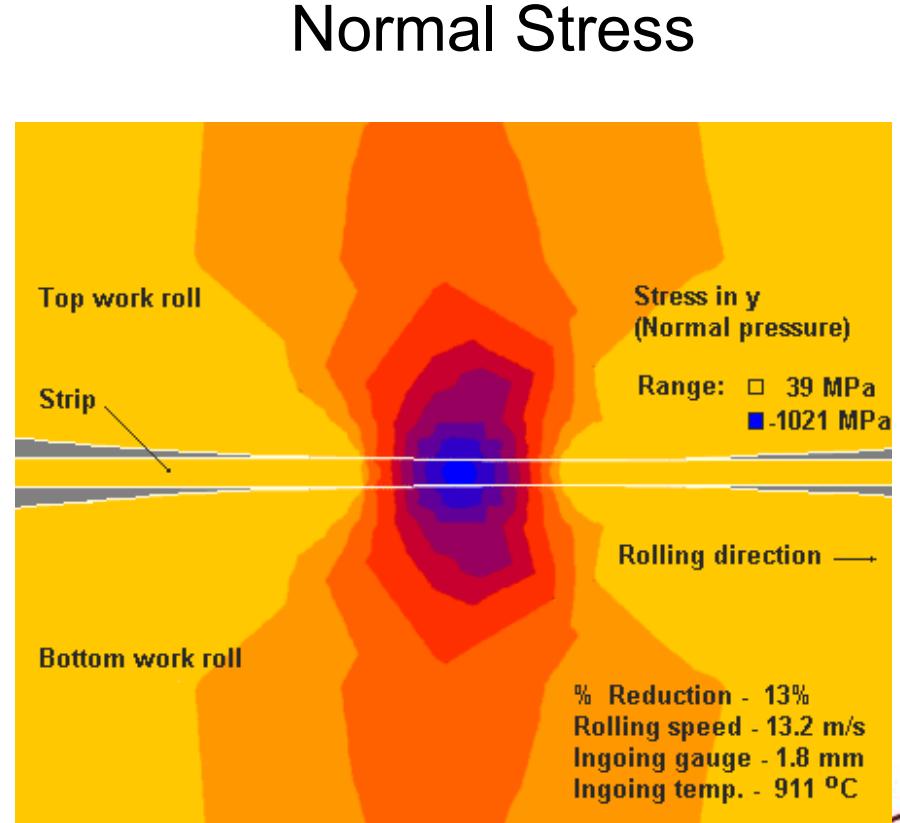
# Example 1.11



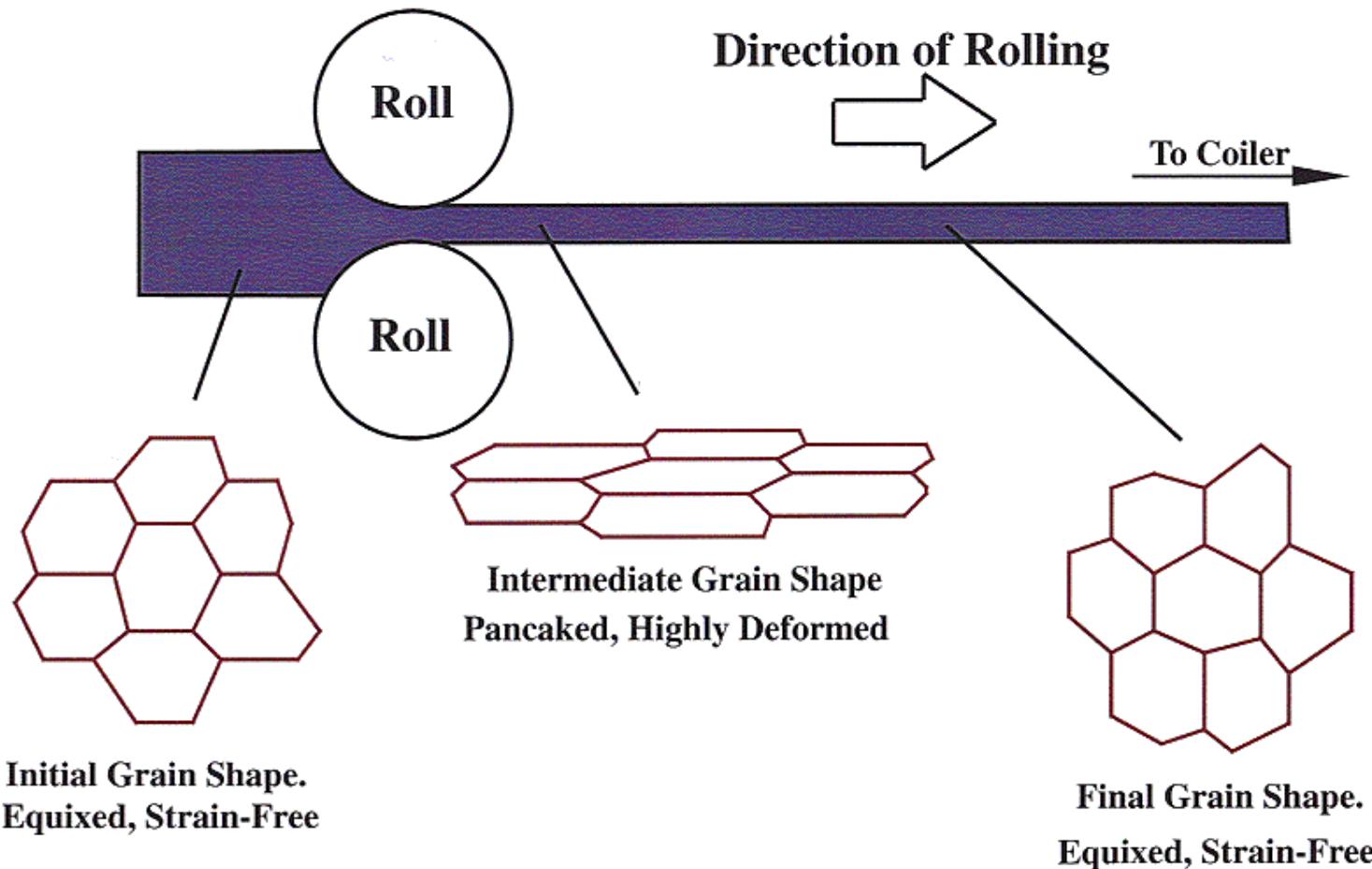
# Rolling



Shear stress



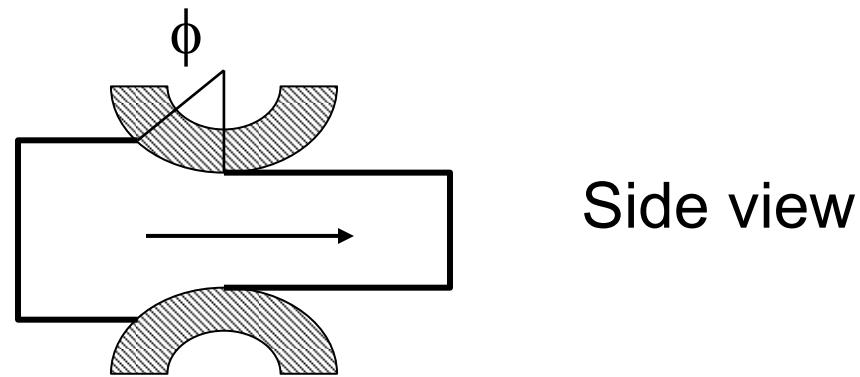
## EFFECT OF FINISH HOT-ROLLING ON THE STRIP SHAPE AND THE AUSTENITE GRAIN STRUCTURE.



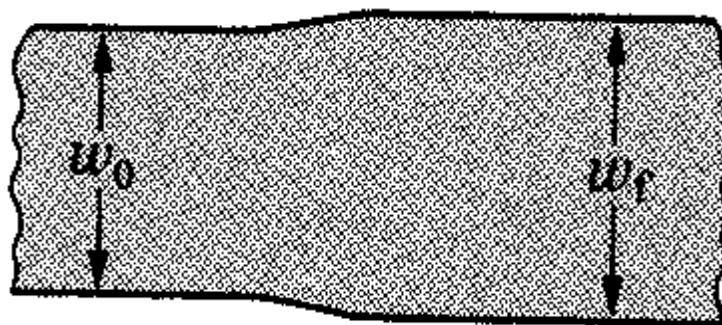
Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Widening of material



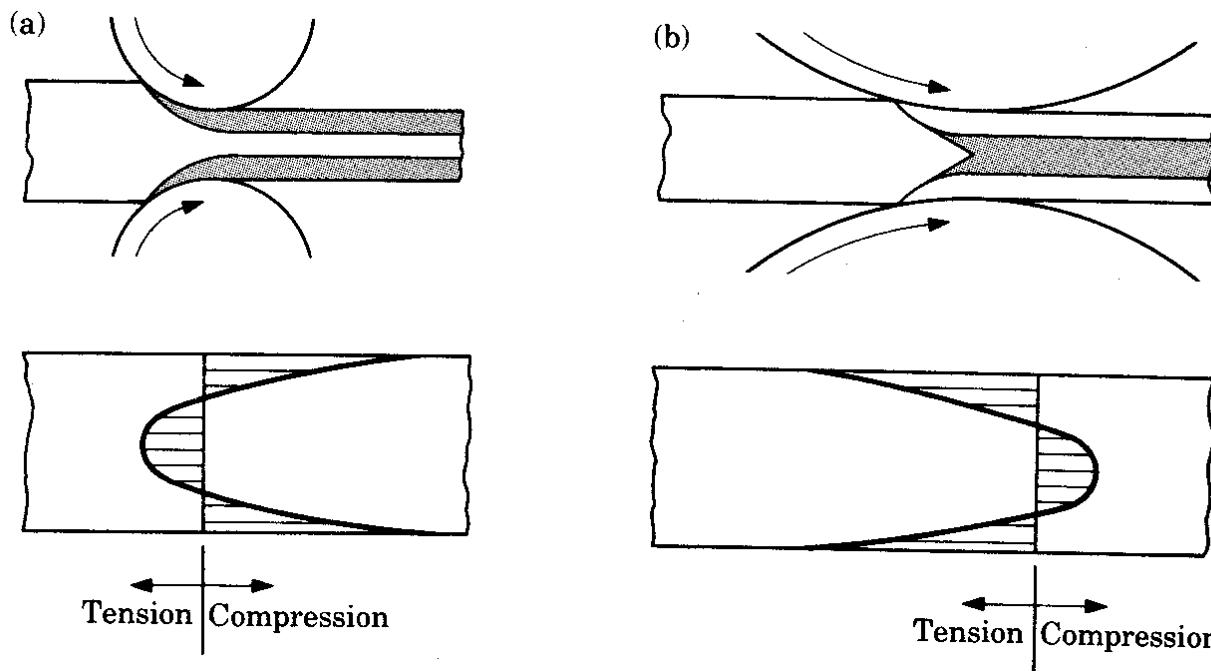
Side view



Top view

# Residual stresses - due to frictional constraints

- a) small rolls or small reduction (ignore friction)
- b) large rolls or large reduction (include friction)

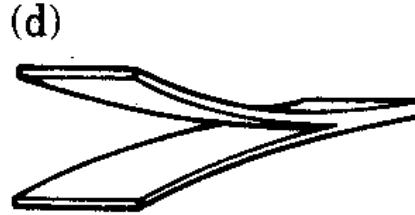
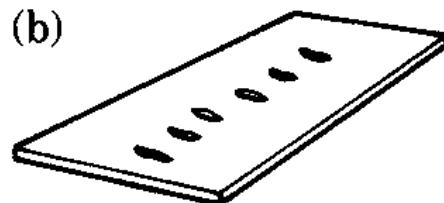
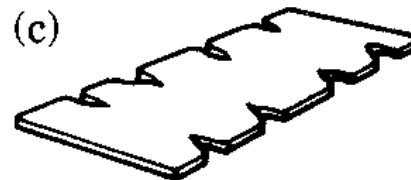


Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



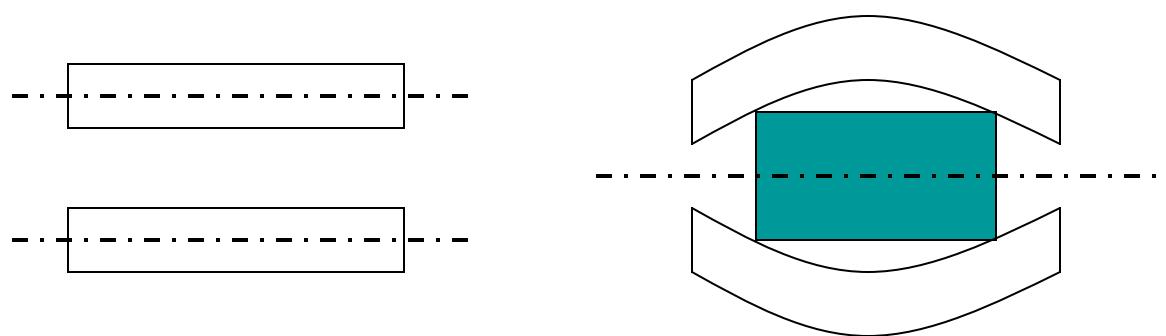
# Defects

- a) wavy edges
  - roll deflection
- b) zipper cracks
  - low ductility
- c) edge cracks
  - barreling
- d) alligatoring
  - piping, inhomogeniety

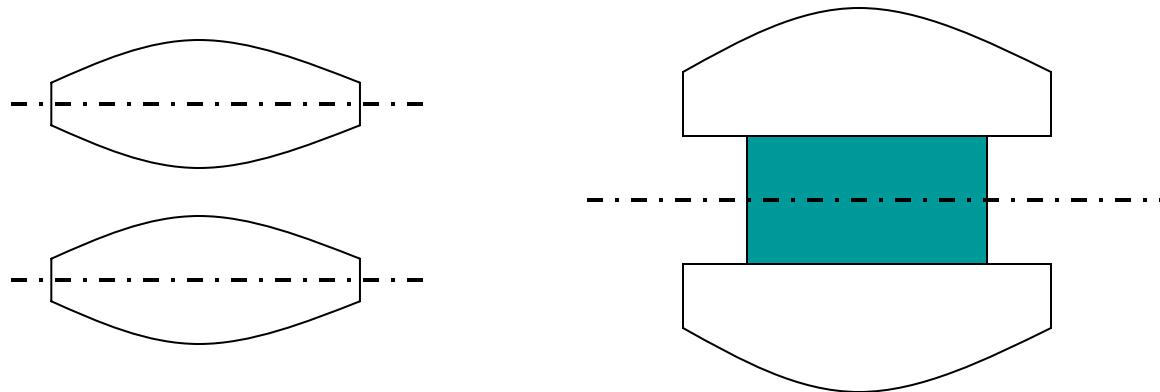


# Roll deflection

Rolls can deflect under load

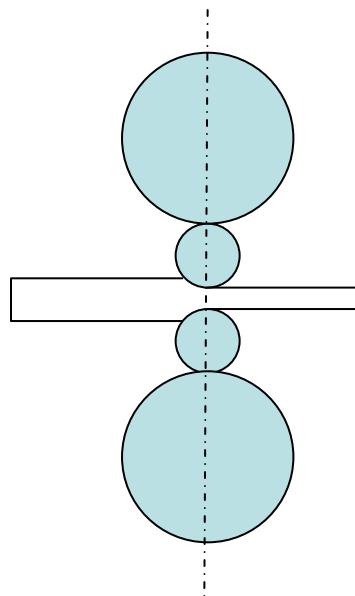


Rolls can be crowned

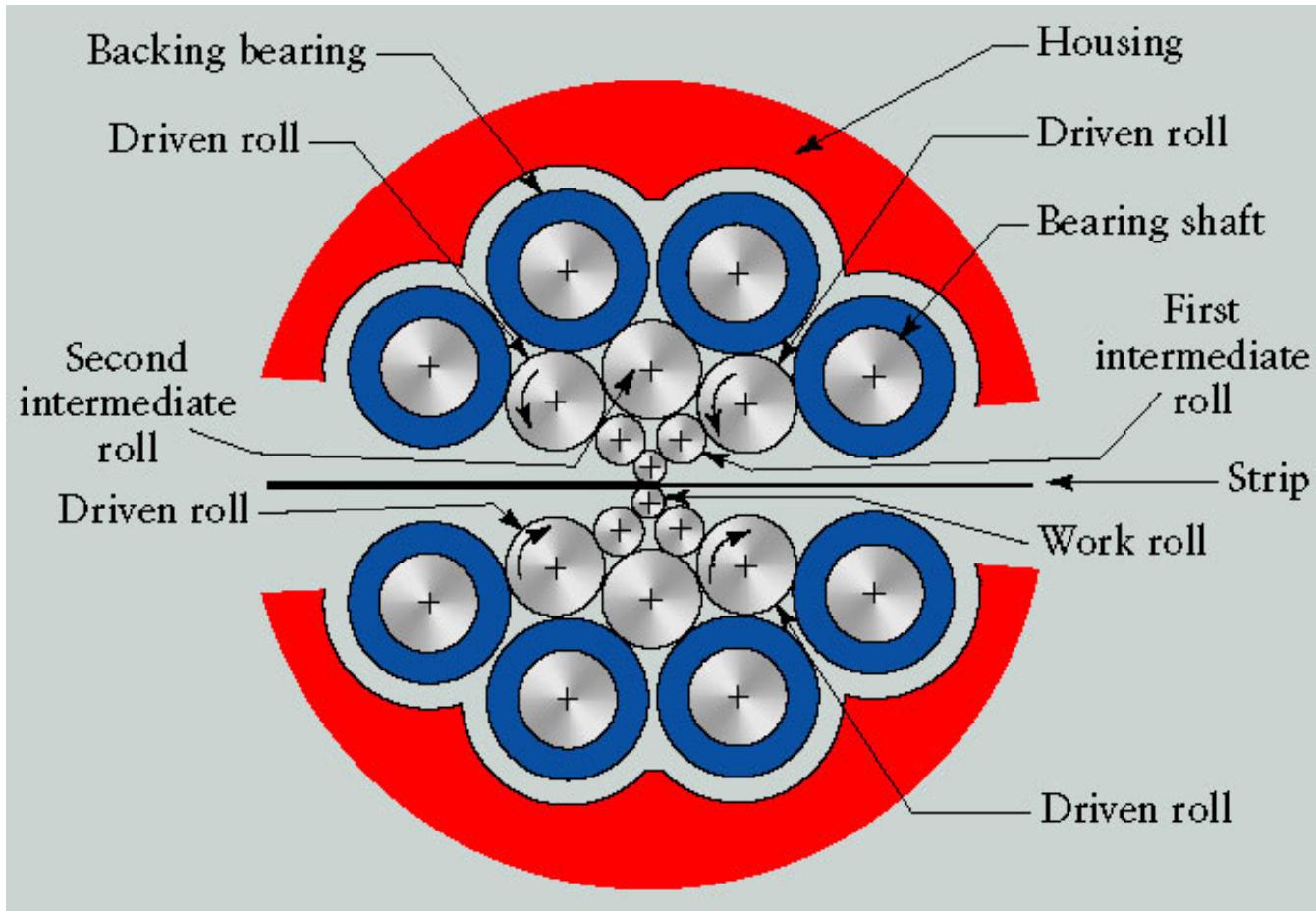


# Roll deflection

Rolls can be stacked for stiffness



# Method to reduce roll deflection



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton



# Summary

- Process
- Equipment
- Products
- Mechanical Analysis
- Defects



Prof. Ramesh Singh, Notes by Dr.  
Singh/ Dr. Colton

