13 CHAPTER Simulation

13.1 INTRODUCTION

The technique of simulation has long been used by the designers and analysts in physical sciences and it promises to become an important tool for tackling the complicated problems of managerial decision-making. Scale models of machines have been used to simulate the plant layouts and models of aircrafts have been tested in wind tunnels to determine their aerodynamic characteristics. Simulation, which can appropriately be called management laboratory, determines the effect of a number of alternate policies without disturbing the real system. It helps in selecting the best policy with the prior assurances that its implementation will be beneficial.

Probably the first important application of simulation was made by John Von Neumann and Stanislaw Ulam for studying the tedius behaviour of neutrons in a nuclear shielding problem which was too complex for mathematical analysis. With the remarkable success of the technique on neutron problem, it became popular and found many applications in business and industry. Development of digital computers in early 1950s is further responsible for the rapid progress made by the simulation techniques. The range of simulation application varies from simple queuing models to models of large integrated systems of production.

13.2 WHEN TO USE SIMULATION?

In the foregoing chapters we have discussed a number of operations research tools and techniques for solving various types of managerial decision-making problems. Techniques like linear programming, dynamic programming, queuing theory, network models, etc., are not sufficient to tackle all the important managerial problems requiring data analysis. Each technique has its own limitations.

Linear programming models assume that the data do not alter over the planning horizon. It is one time decision process and assumes average values for the decision variables. If the planning horizon is long, say 10 years, the multiperiod linear programming model may deal with the yearly averaged data, but will not take into account the variations over the months and weeks with the result that month to month and week to week variations are left uncovered. Other important limitation of linear programming is that it assumes the data to be known with certainty. In many real situations the uncertainties about the data are such that they cannot be ignored. In case the uncertainty relates to only a few variables, the sensitivity analysis can be applied to determine its effect on the decision. But, in situations, where uncertainty pervades the entire model, the sensitivity analysis may become too cumbersome and computationly difficult to determine the impact of uncertainty on the recommended plan.

Dynamic programming models, however, can be used to determine optimal strategies, by taking into account the uncertainties and can analyse multiperiod planning problems. In other words, this technique is free from the two main limitations of linear programming. But it has its own shortcomings. Dynamic programming models can be used to tackle very simple situations involving only a few variables. If the number of state variables is a bit larger, the computation task becomes quite complex and involved.

Similar limitations hold good for other mathematical techniques like dynamic stochastic models such as inventory and waiting line situations. Only small scale systems are amenable to these models; moreover, by making a number of assumptions the systems are simplified to such an extent that in many cases the results obtained are only rough approximations.

From the above discussion we conclude that when the characteristics such as uncertainty, complexity, dynamic interaction between the decision and subsequent event, and the need to develop detailed procedures and finely divided time intervals combine together in one situation, it becomes too complex to be solved by any of the techniques of mathematical programming and probabilistic models. It must be analysed by some other kind of quantitative technique which may give quite accurate and reliable results. Many new techniques are coming up, but, so far, the best available is simulation.

In general, the simulation technique is a dependable tool in situations where mathematical analysis is either too complex or too costly.

13.3 WHAT IS SIMULATION?

Simulation is an imitation of reality. A children cycling park, with various crossings and signals, is a simulated model of the city traffic system. In the laboratories a number of experiments are performed on simulated models to determine the behaviour of the real system in true environments. A simple illustration is the testing of an aircraft model in a wind tunnel from which we determine the performance of the actual aircraft under real operating conditions. Planetarium shows represent a beautiful simulation of the planet system. Environments in a geological garden and in a museum of natural history are other examples of simulation.

In all these examples, it has been tried to imitate the reality to see what might happen under real operating conditions. This imitation of reality which may be in the physical form or in the form of mathematical equations may be called *simulation*.

The simple examples cited above are of simulating the reality in physical form, and are referred to as (*environmental*) *analogue simulation*. For the complex and intricate problems of managerial decision-making, the analogue simulation may not be practicable, and actual experimentation with the system may be uneconomical. Under such circumstances, the complex system is formulated into a mathematical model for which a computer programme is developed, and the problem is solved by using high speed electronic computer, and hence it is named as *computer simulation* or *system simulation*.

With this background it will now be in order to define simulation. According to one definition "*simulation is a representation of reality through the use of a model or other device which will react in the same manner as reality under a given set of conditions.*" Simulation has also been defined as "*the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation.*" According to Donald G. Malcolm, *a simulated model may be defined as one which depicts the working of a large scale system of men, machines, materials and information operating over a period of time in a simulated environment of the actual real world conditions. According to Shannon simulation is the process of designing a model of the real system by conducting experiments with this model for the purpose of understanding the behaviour of the operation of the system.*

13.4 ADVANTAGES OF THE SIMULATION TECHNIQUE

The simulation technique, when compared with the mathematical programming and standard probability analysis, offers a number of advantages over these techniques; a few important among them can be summarized as follows:

1. Many important managerial decision problems are too intricate to be solved by mathematical programming and experimentation with the actual system, even if possible, is to too costly and risky. Simulation offers the solution by allowing experimentation with model of the

system without interfering with the real system. Simulation is, thus, often a bypass for complex mathematical analysis.

2. Through simulation, management can foresee the difficulties and bottlenecks which may come up due to the introduction of new machines, equipment or process. It, thus, eliminates the need of costly trial and error methods of trying out the new concept on real methods and equipment.
3. Simulation has the advantage of being relatively free from mathematics and, thus, can be easily understood by the operating personnel and non-technical managers. This helps in getting the proposed plans accepted and implemented.
4. Simulation models are comparatively flexible and can be modified to accommodate the changing environments of the real situation.
5. Computer simulation can compress the performance of a system over several years and involving large calculations into a few minutes of computer running time.
6. The simulation technique is easier to use than mathematical models and is considered quite superior to the mathematical analysis.
7. Simulation has advantageously been used for training the operating and managerial staff in the operation of complex plans. It is always advantageous to train people on simulated models before putting into their hands the real system. Simulated exercises have been developed to impart the trainee sufficient exercise and experience. A simulated exercise familiarizes the trainee with the data required and helps in judging what information is really important. Due to his personal involvement into the exercise, the trainee gains sufficient confidence, and moreover becomes familiar with data processing on electronic computer.
8. Once a simulation model has been constructed, it may be used time and again to analyse different situations.
9. It is a valuable and convenient method of breaking down a complicated system into subsystems and then studying each of these subsystems individually or jointly with others.

13.5 LIMITATIONS OF THE SIMULATION TECHNIQUE

In spite of all the advantages claimed by the simulation technique, many operations research analysts consider it a method of last resort and use it only when all other techniques fail. If a particular type of problem can be well represented by a mathematical model, the analytical approach is considered to be more economical, accurate and reliable. Further, in very large and complex problems simulation may suffer from the same deficiencies as other mathematical models. In brief, the simulation technique suffers from the following limitations:

1. Simulation does not produce optimum results. When the model deals with uncertainties, the results of simulation are only reliable approximations subject to statistical errors.
2. Quantification of the variables is another difficulty. In a number of situations it is not possible to quantify all the variables that affect the behaviour of the system.
3. In very large and complex problems, the large number of variables and the interrelationships between them make the problem very unwieldy and hard to program. The number of variables may be too large and may exceed the capacity of the available computer.
4. Simulation is, by no means, a cheap method of analysis. In a number of situations, such as corporate planning, simulation is comparatively costlier and time consuming.
5. Other important limitations stem from too much tendency to rely on the simulation models. This results in application of the technique to some simple problems which can more appropriately be handled by other techniques of mathematical programming.

13.6 APPLICATIONS OF SIMULATION

Simulation is quite versatile and commonly applied technique for solving decision problems. It has been applied successfully to a wide range of problems of science and technology as given below:

1. In the field of basic sciences, it has been used to evaluate the area under a curve, to estimate the value of π, in matrix inversion and study of particle diffusion.
2. In industrial problems including the shop floor management, design of computer systems, design of queuing systems, inventory control, communication networks, chemical processes, nuclear reactors and scheduling of production processes.
3. In business and economic problems, including customer behaviour, price determination, economic forecasting, portfolio selection and capital budgeting.
4. In social problems, including population growth, effect of environment on health and group behaviour.
5. In biomedical systems, including fluid balance, distribution of electrolyte in human body and brain activities.
6. In the design of weapon systems, war strategies and tactics.
7. In the study of projects involving risky investments.

A town has six wards and they contain 170, 510, 640, 75, 250 and 960 houses respectively. Make a random selection of 8 houses using the table of random numbers. Explain the procedure adopted by you. [*C.A. May, 1989*]

Solution

Since the total number of houses is 2,605, first of all random numbers 0–2,604 are allocated in proportion to the number of houses in each of the six wards as shown in the table below:

TABLE 13.1

Allocation of random numbers to the houses				
(1) *Ward no.*	(2) *No. of houses*	(3) *Cumulative no. of houses*	(4) *Range*	(5) *Random nos. fitted*
1	170	170	0-169	
2	510	680	170-679	0590 (7), 0354 (8)
3	640	1,320	680-1,319	1128 (2), 0764 (4), 1292 (5)
4	75	1,395	1,320-1,394	1340 (6)
5	250	1,645	1,395-1,644	
6	960	2,605	1,645-2,604	2181 (1), 1749 (3)

The first random number picked up from the random number table is 2181 (first 4-digits of the first random number in table C.1). Since it lies within the interval 1,645-2,604, it is fitted against ward number 6 in column 5. The next random number is 1128, which lies in the interval 680-1,319 and is, therefore, fitted against ward number 3 in column 5. The next random number 7112 is > 2,604 in column 4 and is, therefore, dropped from consideration. In this manner the following random numbers are either fitted in column 5 or dropped; 6557 (D or dropped), 4199 (D), 3545 (D), 1749 (F3), 9103 (D), 0764 (F4), 3493 (D), 1292 (F5), 4397 (D), 3807 (D), 4984

(D), 1340 (F6), 0590 (F7), 9566 (D), 7615 (D), 8508 (D), 6970 (D), 5799 (D), 6343 (D), 4165 (D), 0354 (F5). We stop because 8 houses have been selected. The eight houses selected belong to ward number 6, 3, 6, 3, 3, 4, 2 and 2 respectively.

EXAMPLE 13.7-5

A bakery keeps stock of a popular brand of cake. Daily demand based on past experience is given below:

Daily demand	*0*	*15*	*25*	*35*	*45*	*50*
Probability	*0.01*	*0.15*	*0.20*	*0.50*	*0.12*	*0.02*

Consider the following sequence of random numbers:

48, 78, 09, 51, 56, 77, 15, 14, 68 and 09.

(i) Using the sequence, simulate the demand for the next 10 days.

(ii) Find the stock situation if the owner of the bakery decides to make 35 cakes every day. Also estimate the daily average demand for the cakes on the basis of the simulated data.

[*P.T.U. B.Tech. (Mech.) Dec., 2011; B.E., 2001; Nellore MBA, 2001; P.U.BBA, 2001*]

Solution

(*i*) The simulated demand for the cakes for the next 10 days can be obtained from the table below.

TABLE 13.2

Allocation of random numbers to demand of cakes				
Demand	*Probability*	*Cumulative probability*	*Random number interval*	*Random numbers fitted*
0	0.01	0.01	00	
15	0.15	0.16	01 – 15	09(3), 15(7), 14(8), 09(10)
25	0.20	0.36	16 – 35	
35	0.50	0.86	36 – 85	48(1), 78(2), 51(4), 56(5), 77(6), 68(9)
45	0.12	0.98	86 – 97	
50	0.02	1.00	98 – 99	

In order to simulate the demand, the number 00 is assigned to zero demand, numbers 01–15 are assigned to demand of 15 cakes, 16 – 35 are assigned to demand of 25 cakes and so on. The given 10 random numbers are fitted in the last column corresponding to the ranges against the demand values. These random numbers give the demand for cakes for next 10 days. Serial no. of these random numbers are shown in the parentheses.

∴ Number of cakes demanded in the next 10 days are :

35, 35, 15, 35, 35, 35, 15, 15, 35 and 15 respectively.

(*ii*) The stock situation for various days if the decision is made to make 35 cakes everyday is given in the table below:

TABLE 13.3

Day	*Demand*	*No. of cakes made*	*Stock*
1	35	35	–
2	35	35	–
3	15	35	20
4	35	35	20
5	35	35	20
6	35	35	20
7	15	35	40
8	15	35	60
9	35	35	60
10	15	35	80

$\therefore$ Average daily demand $= \dfrac{1}{10}$ [35 + 35 + 15 + 35 + 35 + 35 + 15 + 15 + 35 + 15]

$= \dfrac{270}{10} = 27$ cakes.

EXAMPLE 13.7-6

A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds, whose probability distribution is as given below:

Production/day :	*196*	*197*	*198*	*199*	*200*	*201*	*202*	*203*	*204*
Probability :	*0.05*	*0.09*	*0.12*	*0.14*	*0.20*	*0.15*	*0.11*	*0.08*	*0.06*

The finished mopeds are transported in a specially designed three-storeyed lorry that can accommodate only 200 mopeds. Using the following 15 random numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54 and 10, simulate the process to find out

(i) what will be the average number of mopeds waiting in the factory?

(ii) what will be the number of empty spaces in the lorry?

[*C.A. Dec., 1990; P.T.U. B.Tech., 2001*]

Solution

The random numbers are established as in the table below:

TABLE 13.4

Production/day	*Probability*	*Cumulative probability*	*Random number interval*
196	0.05	0.05	00 – 04
197	0.09	0.14	05 – 13
198	0.12	0.26	14 – 25
199	0.14	0.40	26 – 39
200	0.20	0.60	40 – 59
201	0.15	0.75	60 – 74
202	0.11	0.86	75 – 85
203	0.08	0.94	86 – 93
204	0.06	1.00	94 – 99

Based on the 15 random numbers given, we simulate the production per day in the table below.

TABLE 13.5

Day no.	*Random number*	*Production per day*	*No. of mopeds waiting*	*Empty spaces in the lorry*
1	82	202	2	–
2	89	203	5	–
3	78	202	7	–
4	24	198	5	–
5	53	200	5	–
6	61	201	6	–
7	18	198	4	–
8	45	200	4	–
9	04	196	–	–
10	23	198	–	2
11	50	200	–	–
12	77	202	2	–
13	27	199	1	–
14	54	200	1	–
15	10	197	–	2

∴ Average number of mopeds waiting in the factory

$$= \frac{1}{15}[2+5+7+5+5+6+4+4+2+1+1] = 2.8.$$

$$\text{Average number of empty spaces in the lorry} = \frac{4}{15} = 0.27.$$

EXAMPLE 13.7-7 (Queuing Problem)

Two persons X and Y work on a two-station assembly line. The distributions of activity times at their stations are

TABLE 13.6

Time in seconds	*Time frequency for X*	*Time frequency for Y*
10	*4*	*2*
20	*7*	*3*
30	*10*	*6*
40	*15*	*8*
50	*35*	*12*
60	*18*	*9*
70	*8*	*7*
80	*3*	*3*

(a) Simulate operation of the line for eight items.

(b) Assuming Y must wait until X completes the first item before starting work, will he have to wait to process any of the other seven items ? What is the average waiting time of items for Y. Use the following random numbers:

For X : 83, 70, 02, 12, 59, 46, 54 and 03.
For Y : 51, 99, 84, 81, 15, 36, 12 and 54.

(c) Determine the inventory of items between the two stations.

(d) What is the average production rate?

[R.T.M. Nagpur U. B.Tech. June, 2006; Kuru. U. B.E. (Mech.) June, 2012; 1993]

Solution

(*a*) Table below shows the cumulative frequency distribution for X. Eight random numbers given for person X are also fitted. The serial numbers of random numbers are shown in the parentheses.

TABLE 13.7

Time in seconds	*Time frequency for X*	*Cumulative frequency*	*Range*	*Random numbers fitted*
10	4	4	00 - 03	02(3), 03(8)
20	7	11	04 - 10	
30	10	21	11 - 20	12(4)
40	15	36	21 - 35	
50	35	71	36 - 70	70(2), 59(5), 46(6), 54(7)
60	18	89	71 - 88	83(1)
70	8	97	89 - 96	
80	3	100	97 - 99	

Thus the eight times for X are 60, 50, 10, 30, 50, 50, 50 and 10 seconds respectively. Likewise, the eight times for Y are derived from his cumulative distribution below.

TABLE 13.8

Time in seconds	Time frequency for Y	Cumulative frequency (i)	(ii)=2× (i)	Range	Random numbers fitted
10	2	2	4	00-03	
20	3	5	10	04-09	
30	6	11	22	10-21	15(5), 12(7)
40	8	19	38	22-37	36(6)
50	12	31	62	38-61	51(1), 54(8)
60	9	40	80	62-79	
70	7	47	94	80-93	84(3), 81(4)
80	3	50	100	94-99	99(2)

Thus the eight times for Y are 50, 80, 70, 70, 30, 40, 30 and 50 seconds respectively. Note that the cumulative frequency has been multiplied by 2 to make it 100.

(*b*) The above times for persons X and Y are used to calculate the waiting time, if any.

TABLE 13.9

Item no.	Person X		Person Y		Waiting time on the part of Y	Waiting time on the part of item
	Time in	Time out	Time in	Time out		
1	0	60	60	110	60	—
2	60	110	110	190	—	—
3	110	120	190	260	—	70
4	120	150	260	330	—	110
5	150	200	330	360	—	130
6	200	250	360	400	—	110
7	250	300	400	430	—	100
8	300	310	430	480	—	120

Thus person Y will not have to wait for the remaining seven items.

$$\text{Average waiting time of items} = \frac{0+0+70+110+130+110+100+120}{8} = \frac{640}{8} = 80 \text{ secs.}$$

(*c*) In all there are 6 items waiting between the two stations.

(*d*) Total time taken to process 8 items = 480 secs = 8 minutes.

$$\therefore \text{ Average production rate} = \frac{8}{8} = 1 \text{ item/minute.}$$

EXAMPLE 13.7-8 (Queuing Problem)

A dentist schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time needed to complete the work.

TABLE 13.10

Category	Time required (minutes)	Probability of category
Filling	45	0.40
Crown	60	0.15
Cleaning	15	0.15
Extraction	45	0.10
Checkup	15	0.20

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival times, starting at 8 A.M. Use the following random numbers for handling the above problem: 40, 82, 11, 34, 25, 66, 17 and 79. [*P.T.U. B.Tech. April, 2012; R.T.M. Nagpur U. B.E. (Mech.) 2011; I.T., 2009; June, 2005; C.A. (Final) Nov., 1990*]

Solution

The time taken by the dentist to treat the eight patients arriving in four hours at the clinic is calculated in the table below.

TABLE 13.11

Category	*Time (minutes)*	*Probability*	*Cumulative probability*	*Random no. interval*	*Random no. fitted*
Filling	45	0.40	0.40	00-39	11(3), 34(4), 25(5), 17(7)
Crown	60	0.15	0.55	40-54	40(1)
Cleaning	15	0.15	0.70	55-69	66(6)
Extraction	45	0.10	0.80	70-79	79(8)
Checkup	15	0.20	1.00	80-99	82(2)

Thus the times taken by the dentist to treat the eight patients are 60, 15, 45, 45, 45, 15, 45 and 45 minutes respectively.

Let us simulate the dentist's clinic (for eight patients) starting at 8 A.M.

TABLE 13.12

Patient no.	*Arrival time*	*Dentist's treatment*		*Waiting time on the part of the patient*	*Idle time for the dentist*
		Starts	*Ends*		
1	8.00	8.00	9.00	–	–
2	8.30	9.00	9.15	30	–
3	9.00	9.15	10.00	15	–
4	9.30	10.00	10.45	30	–
5	10.00	10.45	11.30	45	–
6	10.30	11.30	11.45	60	–
7	11.00	11.45	12.30	45	–
8	11.30	12.30	13.15	60	–

$\therefore$ Average waiting time for the patients $= \dfrac{1}{8}\ [30 + 15 + 30 + 45 + 60 + 45 + 60]$

$$= \frac{285}{8} = 35.625 \text{ minutes.}$$

Average idleness of the dentist $=$ Nil.