

$$\frac{d^2\phi}{dx^2}=-\frac{\rho}{\epsilon_r}$$

$$\phi'(0)+\phi(0)=5$$

$$\phi(3)=2$$

$$\rho=1$$

$$\epsilon_r(x)=\begin{cases}10&\text{dla }x\in[0,1]\\5&\text{dla }x\in(1,2]\\1&\text{dla }x\in(2,3]\end{cases}$$

Wyprowadzenie sformułowania wariancyjnego

$$\phi''(x)=-\frac{\rho}{\epsilon_r}$$

$$\phi''(x)v(x)=-\frac{\rho}{\epsilon_r}v(x)$$

$$\int_0^3\phi''(x)v(x)\,dx=\int_0^3-\frac{\rho}{\epsilon_r}v(x)dx$$

Z zasady całkowania przez części:

$$[\phi'(x)v(x)]_0^3-\int_0^3\phi'(x)v'(x)dx=\int_0^3-\frac{\rho}{\epsilon_r}v(x)dx$$

$$\phi'(3)v(3)-\phi'(0)v(0)-\int_0^3\phi'(x)v'(x)dx=\int_0^3-\frac{\rho}{\epsilon_r}v(x)dx$$

$$\phi'(3)=0\text{ ponieważ }\phi(3)=2$$

$$-\phi'(0)v(0)-\int_0^3\phi'(x)v'(x)dx=\int_0^3-\frac{\rho}{\epsilon_r}v(x)dx$$

Korzystając z podanych danych:

$$-(5-\phi(0))v(0)-\int_0^3\phi'(x)v'(x)dx=\int_0^3-\frac{\rho}{\epsilon_r}v(x)dx$$

$$\phi(0)v(0)-5v(0)-\int_0^3\phi'(x)v'(x)dx=\int_0^3-\frac{\rho}{\epsilon_r}v(x)dx$$

$$\phi(0)v(0)+\int_0^3\phi'(x)v'(x)dx=-\int_0^3\frac{\rho}{\epsilon_r}v(x)dx+5v(0)$$

$$B(\phi,v)=L(v)$$

$$\phi=\hat{\phi}+w$$

$$\hat{\phi}=2e_n$$

$$B(\hat{\phi}+w,v)=L(v)$$

$$B(w,v)=L(v)-B(\hat{\phi},v)$$

$$B(w,v)=L(v)-B(2e_n,v)$$

$$B(w,v)=L(v)-2B(e_n,v)$$

$$B(w,v)=\hat{L}(v)$$