# **California State University, Los Angeles**



ME 409 – Mechanical Engineering Analysis

Prepared by:

William Santiago (301579504)

David Arenas (300089950)

Edward Gonzalez (300593922)

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Professor: Usama Tohid, P.E.

## 1. Abstract

The purpose of this study is to analyze a non-homogenous system. To better understand this, the approach for calculating our results will be by building a mathematical model with a programming aid such as Matlab. This mathematical model is used towards the non-homogenous system which in this case is known as a mixture problem with two separate tanks. The method used in this study for calculating is known as the Method of Undetermined Coefficients. The findings reveal different equations or solutions when using the method of Undetermined Coefficients when changing the values of Tank 1 and Tank 2. Furthermore, a total of 50 combinations were made to fully understand the methods used and determining the amounts of salt for both Tank 1 and Tank2.

#### Nomenclatures:

Greek Symbol	Meaning	Unit
λ	Scalar (Real or Complex Number)	_

## 2. Introduction

One of the main applications of differential equations in general is known as Modeling. Modeling is the process of writing a differential equation to describe a physical situation that is occurring. Most of the differential equations that are used in industry for engineers are there because somebody, at some point modeled a situation and came up with the differential equation that we are using. There are times when the modeling problem is a non-homogenous system, enabling the undetermined coefficients method to be used when calculating the differential equation.

## 2.1 Overview and Purpose

In the modeling system problem that we will be solving starts with a substance that is being dissolved in a liquid. This liquid entering the tank may or may not contain

more of the substance that is being dissolved in it. The liquid leaving the tank will contain the substance dissolved in it. We know that if Q(t) gives the amount of the substance that is being dissolved in the liquid in the tank at any time t we want to develop a differential equation that when it is solved, it will give us an expression for Q(t).

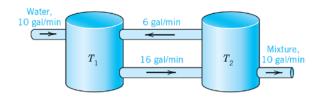
The main assumption that we are making in this problem is that the concentration of the substance in the liquid is uniform throughout the tank. Taking into consideration that there are two tanks that are being interconnected with liquid while potentially entering both and with an exit.

Our team has made the following assumptions for this situation involving two tanks. First of all, the inflow and outflow from each tank are equal, or in a simpler detail the volume in each tank is constant. Next, the concentration of contaminate in each tank will be the same at each point in the tank. Also, the concentration of contaminate in the outflow of the tank 1 will be the same as the concentration in tank 1, while the concentration of contaminate in the outflow from tank 2 will be the same as the concentration in tank 2. In addition, the outflow from tank 1 will be exiting tank 1 and reaching tank 2. The total liquid will be exiting the system completely.

As mentioned before, our team is dealing with a system containing two tanks. The first tank initially contains 200 gallons of water with 160 lb of salt within being dissolved. The second tank initially contains 100 gallons of pure water while the liquid is pumped throughout the system and the mixtures are kept uniform while being stirred. Our team proposed the idea for calculating the solution with the method of undetermined coefficients for this non-homogenous system.

#### 2.2 Statement of the Problem

1.) Tank T1 in Fig. 101 initially contains 200 gal of water in which 160 lb of salt are dissolved. Tank T2 initially contains 100 gal of pure water. Liquid is pumped through the system as indicated, and the mixtures are kept uniform by stirring. Find the amounts of salt y1(t) and y2(t) in T1 and T2, respectively.



- 2.) Repeat the same problem for which T1 contains: 50, 100, 150, 200, 250 gal of water and T2 contains 10, 20, 50, 200, 500 gal of water. (25 combinations)
- 3.) Repeat the same problem if the flow from tank T1 to T2 changes from 1,2,5,10,20 gal/min and T2 to T1 1,2,5,10,100 gal/min. (25 combinations)

## 3. Theory and Principles

In this study, most of our linear systems will consist of two linear Ordinary Differential Equations (ODEs) in two unknown functions  $y_1(t), y_2(t)$ ,

$$y_1' = a_{11} y_1 + a_{12} y_2$$

$$y_2' = a_{21} y_1 + a_{22} y_2$$

In addition, a linear system of n first-order Ordinary Differential Equations (ODEs) in n unknown functions is of the form

$$y_1' = a_{11} y_1 + a_{12} y_2$$

$$y_2' = a_{21} y_1 + a_{22} y_2$$

$$y_n' = a_{n1} y_1 + a_{n2} y_2 + a_{nn} y_n$$

#### 3.1 Matrices

The (constant or variable) coefficients form a **2x2 matrix A**, that are,

an array

$$A = \begin{bmatrix} a_{jk} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

In addition, the coefficients in the equation form an *n* matrix

$$A = \begin{bmatrix} a_{jk} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & a \\ a_{n1} & K_{n2} & \dots & a_{nn} \end{bmatrix}$$

The  $a_{11}, a_{12}, \cdots$  are called **entries**, the horizontal lines **rows**, and the vertical lines **columns**. Thus, in the equation, the first row is  $[a_{11}, a_{12}]$ , the second row is  $[a_{21}, a_{22}]$  and the first and second columns are

$$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$
 and  $\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}$ .

In the "double subscript notation" for various entries, the first subscript is that one that denotes the row and the second the column in which the entry stands. Similarly in the previous equation. The **main diagonal** is the diagonal  $a_{11}$   $a_{12}$   $\cdots$   $a_{nn}$ , hence  $a_{11}$   $a_{12}$ .

We shall need only **square matrices**, that is, matrices with the same number of rows and columns, as in the previous equations in this project.

#### 3.2 Vectors

A **column vector** x with n **components** such as these  $x_1 \cdots , x_n$  is of the form

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$
, thus if  $n = 2$ ,  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ 

Similarly, a row vector v is of the following form below,

$$\mathbf{v} = [v_1 \quad \cdots \quad v_n], \text{ thus if } n = 2, \qquad v = [v_1 \quad v_2]$$

## 3.3 Equality

Now we move on to calculations with matrices and vectors. Two  $n \times n$  matrices will be equal if and only if corresponding entries are equal as well. For example, let's take this for instance, Thus for =2, let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$
 and  $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ 

Then A = B if and only if

$$a_{11} = b_{11}, \quad a_{12} = b_{12}$$

$$a_{21} = b_{21}$$
,  $a_{22} = b_{22}$ 

Now, two column vectors (or two row vectors) are equal if and only if they both have n components and corresponding components are equal as well. For example, let

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$
 and  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ 

then v = x if and only if  $v_1 = x_1$ ,  $v_2 = x_2$ 

#### 3.4 Addition

This is performed by adding the corresponding entries (or components). The matrices must both be  $n \times n$ , and vectors must both have the same number of components. For example, for = 2,

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}, \quad \mathbf{v} + \mathbf{x} = \begin{bmatrix} v_1 + x_1 \\ v_2 + x_2 \end{bmatrix}$$

## 3.5 Matrix Multiplication.

The product C = AB (in this order) of two  $n \times n$  matrices  $A = [a_{jk}]$  and  $B = [b_{jk}]$  is the  $n \times n$  matrix  $C = [c_{jk}]$  with entries the following;

$$c_{jk} = \sum_{m=1}^{n} a_{jm} b_{mk}$$

$$j = 1, \dots, n$$
 and  $k = 1, \dots, n$ 

Let us keep caution with the following note; Matrix multiplication is **not commutative**  $AB \neq BA$ .

Furthermore, a multiplication of an  $n \times n$  matrix A by a vector x with n components is defined by the same rule: v = Ax is the vector with the n components

$$v_j = \sum_{m=1}^n a_{jm} x_m \qquad j = 1, \dots, r$$

#### 3.6 Differentiation

Now we will move on to Systems of ODEs as Vector Equations. The *derivative* of a matrix (or vector) with variable entries (or components) is obtained by differentiating each entry (or component). By using the

matrix multiplication and differentiation, we can now write the following;

$$y' = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \mathbf{A}\mathbf{y} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Similarly, the  $\boldsymbol{n}$   $\boldsymbol{x}$   $\boldsymbol{n}$  matrix  $\boldsymbol{A}$  and a column vector  $\boldsymbol{y}$  with  $\boldsymbol{n}$  components, namely,  $\boldsymbol{y}' = \boldsymbol{A}\boldsymbol{y}$  The vector equation is equivalent to two equations for the components, and these are precisely the two ODEs in the first equation.

## 3.7 Inverse Matrix

The  $n \times n$  unit matrix I is the  $n \times n$  matrix with main diagonal 1, 1,..., 1 and all of the other entries that are zero. If, for a given  $n \times n$  matrix A, there is a  $n \times n$  matrix B such that AB = BA = I, then A is called nonsingular and B is called the Inverse of A and is denoted by  $A^{-1}$  so the following is true;

$$AA^{-1} = A^{-1}A = I$$

The inverse will exist if the determinant det A of A is not zero. If A has not inverse, then we will call it a singular. For example is n = 2, then

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} a_{11} & -a_{12} \\ -a_{21} & a_{22} \end{bmatrix}$$

Where the determinant of A is

$$\det A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

#### 3.8 Eigenvalues, Eigenvectors

This next process is quite important and useful. The Eigenvalues and eigenvectors will be very important during the process of calculating our solution.

Let  $A = [a_{jk}]$  be an  $\mathbf{n} \times \mathbf{n}$  matrix. Consider the following equation;

$$Ax = \lambda x$$

where  $\lambda$  is a scalar (a real or complex number) to be determined and  $\mathbf{x}$  is a vector to be determined. Now, for every  $\lambda$ , a solution is =0. A scalar  $\lambda$  such that it holds for some vector  $\mathbf{x} \neq 0$  is called an **eigenvalue** of  $\mathbf{A}$ , and this

vector is called an **eigenvector** of **A** corresponding to this eigenvalue  $\lambda$ .

$$(A - \lambda I)x = 0$$

These are n linear algebraic equations in the n unknowns  $x_1 \cdots, x_n$  (the components of  $\mathbf{x}$ ). For these equations to have a solution  $\mathbf{x} \neq \mathbf{0}$ , the determinant of the coefficient matrix  $\mathbf{A} - \lambda \mathbf{I}$  must be zero. For example, we only need this only for  $\mathbf{z} = \mathbf{1}$ . Then the following is

$$\begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

In components,

$$(a_{11} - \lambda)x_1 + a_{12}x_2 = 0$$

$$a_{21}x_1 + (a_{22} - \lambda)x_2 = 0$$

Now,  $\mathbf{A} - \lambda \mathbf{I}$  is singular if and only if its determinant det  $(\mathbf{A} - \lambda \mathbf{I})$ , called the **characteristic determinant** of  $\mathbf{A}$  (also for general n), is zero. This will give us the following;

$$\det (\mathbf{A} - \lambda \mathbf{I}) = \begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix}$$
$$= (a_{11} - \lambda)(a_{22} - \lambda) - a_{12} a_{21}$$
$$= \lambda^2 (a_{11} + a_{22})\lambda + a_{11} a_{22} - a_{12} a_{21} = 0$$

This quadratic equation in  $\lambda$  is called the **characteristic equation** of **A**. Its solutions are the eigenvalues  $\lambda_1$  and  $\lambda_2$  of **A**. First determine these. Then use with  $\lambda=\lambda_1$  to determine an eigenvector  $\mathbf{x}^{(1)}$  of **A** corresponding to  $\lambda_1$ . Finally use with  $\lambda=\lambda_2$  to find an eigenvector  $\mathbf{x}^{(2)}$  of **A** corresponding to  $\lambda_2$ . Note that if  $\mathbf{x}$  is an eigenvector of **A**, so is kx with any  $k\neq 0$ .

## 3.9 Nonhomogeneous Linear Systems of ODEs

We will now move on to the situation where the linear system of the Ordinary Differential Equation is Nonhomogenous. Here we will use the method for solving nonhomogeneous linear systems of ODEs;

$$y' = Ay + g$$

where the vector g(t) is not identically zero. We assume g(t) and the entries of the  $n \times n$  matrix A(t) to be continuous on some interval J of the t-axis. From a

general solution  $y^{(h)}(t)$  of the homogeneous system  $y' = \mathbf{A}\mathbf{y}$  on J and a **particular solution**  $y^{(p)}(t)$  of  $y' = \mathbf{A}\mathbf{y}$ .

$$y = y^{(h)}(t) + y^{(p)}(t)$$

y is called a **general solution** of (1) on J because it includes every solution of  $y' = \mathbf{A}\mathbf{y}$  on J.

#### 3.10 Method of Undetermined Coefficients

Now for the actual use of the method. Just as for a single ODE, this method is suitable only if the entries of  $\bf A$  are constants and the components of  $\bf g$  are constants, positive integer powers of t, exponential functions, or cosines and sines. For instance, in such a case, a particular solution  $y^{(p)}$  is assumed in a form similar to  $\bf g$ ; for instance,  $y^{(p)} = \bf u + vt + wt^2$  if  $\bf g$  has components quadratic in t, with  $\bf u, v, w$  to be determined by substitution into  $y' = \bf Ay + g$ .

## 4. Data Analysis

## 4.1. Analysis of Calculation and Tables

## 4.1.1 Sample Calculation using Method of Undetermined Coefficients for Original Problem

The following calculation demonstrates a sample calculation or the original problem in finding the equations to determine the amount of salt in Tank 1 and Tank 2. This was done by hand before using the aid of Matlab to see if the solution would match. Verification was very crucial for our team. Here is our sample calculation:

#### Given:

 $T_1 = 200 \ gal$ , 160 lb salt, Inflow= 6 gal/min, Outflow = 16 gal/min

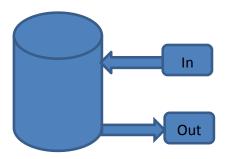
 $T_2 = 100 \ gal$ , pure water, Inflow= 16 gal/min, Outflow = 10 gal/min

#### Required:

$$y_1(t)$$
 and  $y_2(t)$  in  $T_1$  and  $T_2$ 

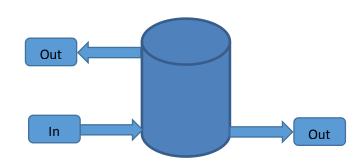
## Solution:

Free Body Diagram (FBD):



$$y_1' = \frac{Inflow}{min} - \frac{Outflow}{min} = \frac{6}{100}y_2 - \frac{16}{200}y_1$$
$$= -0.08y_1 + 0.06y_2$$

Free Body Diagram (FBD):



$$y_2' = \frac{Inflow}{min} - \frac{Outflow}{min} = \frac{16}{200}y_2 - \frac{6}{200}y_1 - \frac{10}{200}y_1$$

$$= -0.08y_1 + 0.06y_2$$

$$y' = \mathbf{A}\mathbf{y}$$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} -0.08 & 0.06 \\ 0.08 & -0.16 \end{bmatrix}$$

$$(\mathbf{A} - \lambda \mathbf{I}) = \mathbf{0}$$

$$\det (\mathbf{A} - \lambda \mathbf{I}) = \begin{bmatrix} -0.08 - \lambda & 0.06 \\ 0.08 & -0.16 - \lambda \end{bmatrix}$$

Eigenvalues

$$(-0.08 - \lambda)(-0.16 - \lambda) - (0.08)(0.06)$$

$$= 0.128 + 0.08\lambda + 0.16\lambda + \lambda^{2}$$

$$- 0.048$$

$$\lambda^{2} + 0.24\lambda + 0.008$$

$$\lambda = \frac{(-0.24) \pm \sqrt{(0.24)^2 - 4(1)(0.008)}}{2(1)}$$
$$= \frac{(-0.24) \pm \sqrt{0.0256}}{2(1)}$$
$$= \frac{(-0.24) \pm 0.16}{2(1)}$$

$$\lambda = -0.2$$
 and  $\lambda = -0.4$ 

Eigenvectors for  $\lambda = -0.2$ 

$$\begin{bmatrix} -0.08 - (-0.2) & 0.06 \\ 0.08 & -0.16 - (-0.2) \end{bmatrix}$$
$$\begin{bmatrix} -0.08 & 0.06 \\ 0.08 & -0.16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -0.08 & 0.06 & | & 0 \\ 0.08 & -0.16 & | & 0 \end{bmatrix} - > \begin{bmatrix} 0.12 & 0.06 \\ 0.0696 & 0.0448 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$0.12x_1 + 0.06x_2 = 0$$

$$\frac{0.12}{0.12}x_1 = \frac{0.06}{0.12}x_2$$

$$x_1 = -0.5 x_2 \qquad x_1 = 1$$

$$\frac{1}{-0.5} = \frac{-0.5}{-0.5}x_2 \qquad x_2 = -2$$

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix} for \lambda = -0.2$$

Eigenvectors for  $\lambda = -0.04$ 

$$\begin{bmatrix} -0.08 - (-0.04) & 0.06 \\ 0.08 & -0.16 - (-0.04) \end{bmatrix}$$

$$\begin{bmatrix} -0.04 & 0.06 \\ 0.08 & -0.12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -0.04 & 0.06 \\ 0.08 & -0.12 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} - > \begin{bmatrix} -0.0635 & -0.1152 \\ 0.08 & -0.12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$0.8x_1 - 0.12x_2 = 0$$

$$\frac{0.08}{0.08}x_1 = \frac{0.12}{0.08}x_2$$

$$x_1 = 1.5x_2 \qquad x_2 = 1$$

$$x_1 = 1.5(1) \qquad x_1 = 1.5$$

$$\begin{bmatrix} 1.5 \\ 1 \end{bmatrix} for \lambda = -0.04$$

$$y = c_1 x^{(1)} e^{\lambda_1 t} + c_2 x^{(2)} e^{\lambda_2 t}$$

$$y = c_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} e^{\lambda_2 t}$$

$$y = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-0.2t} + c_2 \begin{bmatrix} 1.5 \\ 1 \end{bmatrix} e^{-0.04t}$$

$$\begin{bmatrix} 160 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$$

$$160 = c_1 + 1.5c_2$$

$$0 = -2c_1 + c_2$$

$$c_2 = 2c_1$$

$$160 = c_1 + 1.5(2c_1)$$

$$\frac{160}{4} = \frac{4}{4}c_1$$

$$c_1 = 40$$

$$0 = -2c_1 + c_2$$

$$0 = -2(40) + c_2$$

$$c_2 = 80$$

$$y_1 = 40e^{-0.2t} + 120e^{-0.04t}$$

$$y_2 = -80e^{-0.2t} + 80e^{-0.04t}$$

## 4.1.2 Sample Table Data for Part 1

Eigenvalues	Vector	Vector	Constants	Constants
	V11	V22	for y1	for y2
-0.04	1	1	40	-80
-0.2	-2	0.6666	120	80
		66667		
T(min)	у1		T(min)	y2
0	0		0	160
1	11.364		1	148.0439
	69489			628
2	20.223		2	137.5867
	70403			634
3	27.048		3	128.3829
	70405			178

4       32.225       4       120.2304         18599       132         5       36.068       5       112.9628         6       38.834       6       106.4431         69193       118         7       40.734       7       100.5579         9422       275         8       41.940       8       95.21374         20153       517       9       90.33311         466       42.590       9       90.33311         466       685       10       42.798       466         10       42.798       10       85.85181         78102       685       11       81.71649         686       12       77.88272         23508       515       686         12       42.245       12       77.88272         23508       13       74.31340       75.758       888         14       40.831       14       70.97749       907         15       39.921       15       67.84887       907         16       38.922       16       64.90557       904         17       37.859       17       62.12896			T	ı	ı
5         36.068         5         112.9628         68           6         38.834         6         106.4431         118           7         40.734         7         100.5579         9422         275           8         41.940         8         95.21374         517           9         42.590         9         90.33311         466           10         42.798         10         85.85181         685           11         42.658         11         81.71649         686           12         42.245         12         77.88272         23508         515           13         41.619         13         74.31340         75758         888           14         40.831         14         70.97749         907         907         907         907         907         907         907         907         907         907         907         907         907         907         907         907         908         907         908         907         908         907         908         907         908         907         908         907         908         907         907         908         907         908	4	32.225		4	120.2304
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6       38.834       6       106.4431         69193       118         7       40.734       7       100.5579         9422       275         8       41.940       8       95.21374         20153       517         9       42.590       9       90.33311         19503       466       685         10       42.798       10       85.85181         78102       685       685         11       42.658       11       81.71649         66102       686       686         12       42.245       12       77.88272         23508       515       15         13       41.619       13       74.31340         75758       888       88         14       40.831       14       70.97749         901       017       15       39.921       15       67.84887         96542       15       67.84887       907         16       38.922       16       64.90557         41761       904       17       62.12896         18       36.754       18       59.50321         28	5			5	
69193       118         7       40.734       7       100.5579         9422       275         8       41.940       8       95.21374         20153       517       9       90.33311         19503       466       466       466         10       42.798       10       85.85181         685       11       81.71649       685         11       42.658       11       81.71649         66102       686       686         12       42.245       12       77.88272         23508       515       515         13       41.619       13       74.31340         75758       888       88         14       40.831       14       70.97749         901       017       15       39.921       15       67.84887         907       15       39.921       15       67.84887         907       16       38.922       16       64.90557         41761       904       17       62.12896         18       36.754       18       59.50321         28       28       18       59.50321 <tr< td=""><td>-</td><td>ł</td><td></td><td></td><td></td></tr<>	-	ł			
7       40.734       7       100.5579         9422       7       275         8       41.940       8       95.21374         517       9       42.590       9       90.33311         19503       466       466         10       42.798       10       85.85181         78102       685       11       81.71649         686       66102       686       686         12       42.245       12       77.88272         23508       515       515         13       41.619       13       74.31340         75758       888       88         14       40.831       14       70.97749         901       901       907       97         15       39.921       15       67.84887         907       15       67.84887       907         16       38.922       16       64.90557         41761       904       17       62.12896         19       37.623       19       57.01480         28268       19       57.01480       212         20       34.481       20       54.65210	6			6	
9422       275         8       41.940       8       95.21374         20153       9       90.33311         19503       466         10       42.798       10       85.85181         78102       685         11       42.658       11       81.71649         66102       686       666         12       42.245       12       77.88272         23508       515         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       15       67.84887         96542       907       15         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         96       49779       988         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         0602       125				_	
8       41.940       8       95.21374         9       42.590       9       90.33311         19503       466       466         10       42.798       10       85.85181         78102       685       11       81.71649         685       11       81.71649       686         12       42.245       12       77.88272         23508       515       515         13       41.619       13       74.31340         75758       888       70.97749         9201       15       67.84887         96542       15       67.84887         907       15       67.84887         907       15       67.84887         907       16       38.922       16       64.90557         41761       904       17       62.12896         49779       988       18       59.50321         18       36.754       18       59.50321         20       34.481       20       54.65210         125       21       33.337       21       52.40508         19573       23       31.077       23       48.22435      <	7			7	
20153       517         9       42.590       9       90.33311         19503       466         10       42.798       10       85.85181         78102       685         11       42.658       11       81.71649         686       686         12       42.245       12       77.88272         23508       515         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         907       917       67.84887         907       15       67.84887       907         15       39.921       15       67.84887         907       16       38.922       16       64.90557         41761       904       907         17       37.859       17       62.12896         49779       988       961         19       35.623       19       57.01480         65241       21       212         20       34.481       20       54.65210         19573       588         22       32.200       22       50.26		1		_	
9       42.590       9       90.33311         19503       466         10       42.798       10       85.85181         78102       685         11       42.658       11       81.71649         66102       686         12       42.245       12       77.88272         23508       515         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       907       15       67.84887         96542       907       907         16       38.922       16       64.90557         41761       904       907         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         19       35.623       19       57.01480         65241       212       20       34.481       20       54.65210         19573       588         22       32.200       22       50.26504         44574       3       3         23       31.0	8			8	
19503       466         10       42.798       10       85.85181         78102       685         11       42.658       11       81.71649         66102       686         12       42.245       12       77.88272         23508       515         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       017         15       39.921       15       67.84887         96542       907         16       38.922       16       64.90557         41761       904       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         2868       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       23       48.22435		1			
10       42.798       10       85.85181         78102       685         11       42.658       11       81.71649         66102       686         12       42.245       12       77.88272         23508       515         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       017         15       39.921       15       67.84887         96542       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         2868       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       23       48.22435         36       24       29.973       24 <t< td=""><td>9</td><td></td><td></td><td>9</td><td></td></t<>	9			9	
78102       685         11       42.658       11       81.71649         66102       686         12       42.245       12       77.88272         23508       515         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       15       67.84887         96542       907       907         15       39.921       15       67.84887         96542       16       64.90557         4079       907       907         16       38.922       16       64.90557         41761       904       904         17       62.12896       988         18       36.754       18       59.50321         28268       961       19       57.01480         65241       212       20       54.65210         20       34.481       20       54.65210         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3		1			
11       42.658       11       81.71649         66102       686       686         12       42.245       12       77.88272         23508       515       515         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       15       67.84887         96542       907       907         15       39.921       15       67.84887         96542       16       64.90557         40779       16       64.90557       904         17       37.859       17       62.12896         49779       98       18       59.50321         98       18       59.50321       961         19       35.623       19       57.01480         65241       212       20       54.65210         06602       125       21       33.337       21       52.40508         19573       23       48.22435       836         22       32.200       22       50.26504         44574       3       336         24       29.973	10			10	
66102       686         12       42.245       12       77.88272         23508       13       74.31340         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       017       017         15       39.921       15       67.84887         96542       907       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       19       57.01480         19       35.623       19       57.01480         65241       212       21         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3       33         23       31.077       23       48.22435 <td></td> <td>1</td> <td></td> <td></td> <td>685</td>		1			685
12       42.245       12       77.88272         23508       13       74.31340         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       15       67.84887         96542       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       19       57.01480         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       23       48.22435         22       32.200       22       50.26504         44574       3       23       31.077       23       48.22435         37643       24       46.27633       05111       62         25       28.891       25       44.415	11			11	
23508       515         13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       15       67.84887         96542       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       19       57.01480         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       23       48.22435         22       32.200       22       50.26504         44574       3       23       46.27633         24       29.973       24       46.27633         05111       62       25       28.891       25       44.41505         31953       082       26       42.63522       44.41      <		1			
13       41.619       13       74.31340         75758       888         14       40.831       14       70.97749         9201       017         15       39.921       15       67.84887         96542       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       961         19       35.623       19       57.01480         65241       21       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3       33         23       31.077       23       48.22435         37643       24       46.27633         24       29.973       24       46.27633         25       28.891       25       44.41505         <	12	42.245		12	
75758       888         14       40.831       14       70.97749         9201       017         15       39.921       15       67.84887         96542       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3       3         23       31.077       23       48.22435         37643       24       46.27633         05111       62         25       28.891       25       44.41505         31953       26       42.63522         0494       441         27       26		23508			515
14       40.831       14       70.97749         9201       15       67.84887         96542       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       21       52.40508         22       32.200       22       50.26504         44574       3       3         23       31.077       23       48.22435         37643       836         24       29.973       24       46.27633         05111       62         25       28.891       25       44.41505         31953       082         26       27.835       06       42.63522         0494       441 <td>13</td> <td>41.619</td> <td></td> <td>13</td> <td>74.31340</td>	13	41.619		13	74.31340
9201       017         15       39.921       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       28       50.26504         22       32.200       22       50.26504         44574       3       3         23       31.077       23       48.22435         37643       836         24       29.973       24       46.27633         05111       62         25       28.891       25       44.41505         31953       082         26       27.835       06       42.63522         0494       441         27       26.806       27       40.93212		75758			888
15       39.921       15       67.84887         96542       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3         23       31.077       23       48.22435         37643       23       48.22435         24       29.973       24       46.27633         05111       62         25       28.891       25       44.41505         31953       26       42.63522         0494       27       40.93212         27       26.806       27       40.93212         31558       632	14	40.831		14	70.97749
96542       907         16       38.922       16       64.90557         41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3         23       31.077       23       48.22435         37643       24       46.27633         05111       62         25       28.891       25       44.41505         31953       082         26       27.835       26       42.63522         0494       27       40.93212         31558       632         28       25.806       28       39.30148		9201			017
16       38.922	15	39.921		15	67.84887
41761       904         17       37.859       17       62.12896         49779       988         18       36.754       18       59.50321         28268       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3         23       31.077       23       48.22435         37643       24       46.27633         24       29.973       24       46.27633         05111       62         25       28.891       25       44.41505         31953       082         26       27.835       06       42.63522         0494       441         27       26.806       27       40.93212         31558       632         28       25.806       28       39.30148		96542			907
17       37.859 49779       17       62.12896 988         18       36.754 28268       18       59.50321 961         19       35.623 65241       19       57.01480 212         20       34.481 06602       20       54.65210 125         21       33.337 19573       21       52.40508 588         22       32.200 44574       22       50.26504 37643         23       31.077 37643       23       48.22435 836         24       29.973 05111       24       46.27633 62         25       28.891 31953       25       44.41505 082         26       27.835 0494       26       42.63522 441         27       26.806 31558       27       40.93212 632         28       25.806       28       39.30148	16	38.922		16	64.90557
49779       988         18       36.754       18       59.50321         28268       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3         23       31.077       23       48.22435         37643       836         24       29.973       24       46.27633         05111       62         25       28.891       25       44.41505         31953       082         26       27.835       06       42.63522         0494       441         27       26.806       27       40.93212         31558       632         28       25.806       28       39.30148		41761			904
18       36.754 28268       18       59.50321 961         19       35.623 65241       19       57.01480 212         20       34.481 20 54.65210 125       125         21       33.337 588       21 52.40508 588         22       32.200 22 50.26504 31.077 3643 836         23       31.077 3643 836         24       29.973 24 46.27633 62         25       28.891 31953 62         26       27.835 0494 441         27       26.806 27 40.93212 632         28       25.806 28 39.30148	17	37.859		17	62.12896
28268       961         19       35.623       19       57.01480         65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3         23       31.077       23       48.22435         37643       836         24       29.973       24       46.27633         05111       62         25       28.891       25       44.41505         31953       082         26       27.835       26       42.63522         441         27       26.806       27       40.93212         31558       632         28       25.806       28       39.30148		49779			988
19       35.623 65241       19       57.01480 212         20       34.481 06602       20       54.65210 125         21       33.337 19573       21       52.40508 588         22       32.200 44574       22       50.26504 37643         23       31.077 37643       23 836       48.22435 836         24       29.973 05111       24 62       46.27633 62         25       28.891 31953       25 0494       44.41505 441         27       26.806 31558       27 632       40.93212 632         28       25.806       28       39.30148	18	36.754		18	
65241       212         20       34.481       20       54.65210         06602       125         21       33.337       21       52.40508         19573       588         22       32.200       22       50.26504         44574       3       3         23       31.077       23       48.22435         37643       836         24       29.973       24       46.27633         05111       62         25       28.891       25       44.41505         31953       082         26       27.835       26       42.63522         0494       441         27       26.806       27       40.93212         31558       632         28       25.806       28       39.30148		28268			961
20       34.481 06602       20       54.65210 125         21       33.337 19573       21       52.40508 588         22       32.200 44574       22       50.26504 3         23       31.077 37643       23       48.22435 836         24       29.973 24 46.27633 62       24       46.27633 62         25       28.891 25 44.41505 31953 082         26       27.835 0494 441       26       42.63522 441         27       26.806 31558 26       27 40.93212 632         28       25.806 28       39.30148	19	35.623		19	57.01480
06602     125       21     33.337     21     52.40508       19573     588       22     32.200     22     50.26504       44574     3       23     31.077     23     48.22435       37643     836       24     29.973     24     46.27633       05111     62       25     28.891     25     44.41505       31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148		65241			212
21       33.337 19573       21       52.40508 588         22       32.200 44574       22       50.26504 3         23       31.077 37643       23       48.22435 336         24       29.973 62       24       46.27633 62         25       28.891 31953       25       44.41505 62         26       27.835 6494       26       42.63522 441         27       26.806 31558       27       40.93212 632         28       25.806       28       39.30148	20	34.481		20	54.65210
19573     588       22     32.200     22     50.26504       44574     3       23     31.077     23     48.22435       37643     836       24     29.973     24     46.27633       62       25     28.891     25     44.41505       31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148		06602			125
22     32.200       44574     3       23     31.077     23     48.22435       37643     836       24     29.973     24     46.27633       05111     62       25     28.891     25     44.41505       31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148	21			21	
44574       3         23       31.077       23       48.22435         37643       836         24       29.973       24       46.27633         05111       62         25       28.891       25       44.41505         31953       082         26       27.835       26       42.63522         0494       441         27       26.806       27       40.93212         31558       632         28       25.806       28       39.30148		19573			588
23     31.077     23     48.22435       37643     24     46.27633       24     29.973     24     46.27633       05111     62       25     28.891     25     44.41505       31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148	22	32.200		22	50.26504
37643     836       24     29.973     24     46.27633       05111     62       25     28.891     25     44.41505       31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148		44574			3
24     29.973     24     46.27633       05111     62       25     28.891     25     44.41505       31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148	23	31.077		23	48.22435
05111     62       25     28.891     25     44.41505       31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148		37643			836
25     28.891     25     44.41505       31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148	24	29.973		24	46.27633
31953     082       26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148		05111			62
26     27.835     26     42.63522       0494     441       27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148	25	28.891		25	44.41505
0494       441         27       26.806       27       40.93212         31558       632         28       25.806       28       39.30148		31953			082
27     26.806     27     40.93212       31558     632       28     25.806     28     39.30148	26	27.835		26	42.63522
31558     632       28     25.806     28     39.30148		0494			441
28 25.806 28 39.30148	27	26.806		27	40.93212
		31558			632
55447 99	28	25.806		28	39.30148
		55447			99

	1		
29	24.836	29	37.73944
	69009		39
30	23.897	30	36.24245
	23678		552
31	22.988	31	34.80728
	38298		338
32	22.110	32	33.43093
	05945		835
33	21.261	33	32.11065
	99471		096
34	20.443	34	30.84384
	76014		424
35	19.654	35	29.62811
	80656		095
36	18.894	36	28.46119
	49383		447
37	18.162	37	27.34097
	11485		272
38	17.456	38	26.26544
	91484		449
39	16.778	39	25.23271
	1069		794
40	16.124	40	24.24100
	88443		066
41	15.496	41	23.28859
	4311		122
42	14.891	42	22.37387
	9287		182
43	14.310	43	21.49530
	56337		198
44	13.751	44	20.65141
	53046		298
45	13.214	45	19.84080
	03827		298
46	12.697	46	19.06213
	31094		271
47	12.200	47	18.31412
	59054		165
48	11.723	48	17.59554
	13867		461
49	11.264	49	16.90522
	23755		857
50	10.823	50	16.24204
	19066		999
51	10.399	51	15.60493
	32324		212
52	9.9919	52	14.99284
	82377		276
53	9.6005	53	14.40479
	37		206

54	9.2243		54	13.83983
	77723			05
55	8.8629		55	13.29704
	16533			707
56	8.5155		56	12.77556
	86415			749
57	8.1818		57	12.27455
	40898			263
58	7.8611		58	11.79319
	53561			692
59	7.5530		59	11.33072
	17491			697
60	7.2569		60	10.88640
	44726			016
61	6.9724		61	10.45950
	65681			339
62	6.6991	T	62	10.04935
	2856			181
63	6.4364	T	63	9.655287
	98779			691
64	6.1841		64	9.276679
	58374			284
65	5.9417		65	8.912919
	05431			799
66	5.7087		66	8.563426
	53516			371
67	5.4849		67	8.227639
	31121			104
68	5.2698		68	7.905020
	81115			151
69	5.0632		69	7.595052
	60218			828
70	4.8647		70	7.297240
	38488			776
71	4.6739		71	7.011107
	98813			148
72	4.4907		72	6.736193
	36436			836
73	4.3146		73	6.472060
	58476			73
74	4.1454	T	74	6.218285
	83483			006
75	3.9829		75	5.974460
	40997			44
76	3.8267		76	5.740196
	71123			757
77	3.6767		77	5.515119
	24128			
78	3.5325		78	5.298866
	60043			926
-				

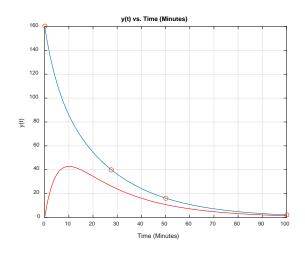
79	3.3940	79	5.091094
79	4829	79	428
90	1	80	
80	3.2609	80	4.891468 979
01	67315	0.1	
81	3.1331	81	4.699671
02	04237	02	097
82	3.0102	82	4.515393
02	5451	02	834
83	2.8922	83	4.338342
0.4	21599	0.4	281
84	2.7788	84	4.168233
0.5	1667	0.5	096
85	2.6698	85	4.004794
0.5	58285	0.5	051
86	2.5651	86	3.847763
	72115	07	595
87	2.4645	87	3.696890
	90663		434
88	2.3679	88	3.551933
	52996		129
89	2.2751	89	3.412659
	04489		71
90	2.1858	90	3.278847
	96577		303
91	2.1001	91	3.150281
	8652		775
92	2.0178	92	3.026757
	3717		389
93	1.9387	93	2.908076
	16759		476
94	1.8626	94	2.794049
	98683		119
95	1.7896	95	2.684492
	613		847
96	1.7194	96	2.579232
	87741		345
97	1.6520	97	2.478099
	65714		172
98	1.5872	98	2.380931
	87334		492
99	1.5250	99	2.287573
	48942		816
100	1.4652	100	2.197876
	50946		749

## 4.2 Data Analysis Plot Graphs

The following analysis was made with the aid of Matlab programming. The following graphs demonstrate the mathematical model for the differential equation. The

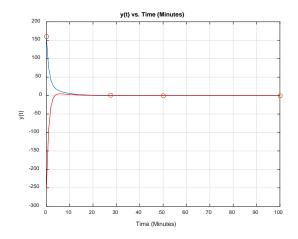
differential equation was then plotted to demonstrate the function over time. The team began with graphing a plot for the Part 1. Following up with the modeling system, the graphs were then made for Part 2 and Part 3 to demonstrate the change of gallons of water and change of flow of the water, respectively. The following graphs show the complete run for Part A since it was only 1 run. Next, the graphs for Part 2 and Part 3 only show for the first 5 runs, respectively. This was done to save time and demonstrate how the function will be changing over time.

## 4.2.1 Graph for Part 1

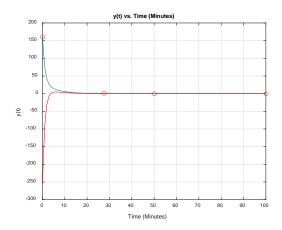


Graph 1: Part 1 1st Run

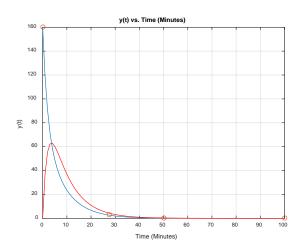
## 4.2.2 Graphs for Part 2



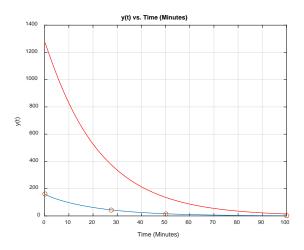
Graph 2: Part 2 1st Run



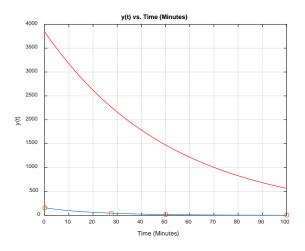
Graph 3: Part 2 2<sup>nd</sup> Run



Graph 4: Part 2 3rd Run

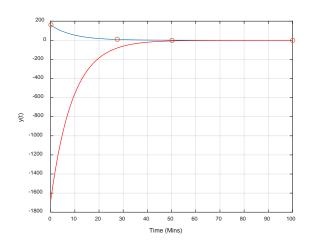


Graph 5: Part 2 4th Run

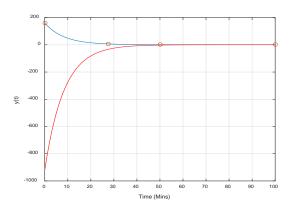


Graph 6: Part 2 5th Run

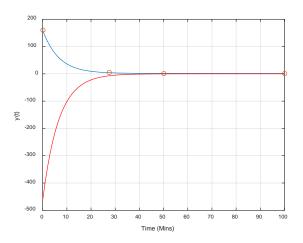
# 4.2.3 Graphs for Part 3



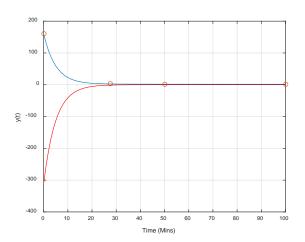
Graph 7: Part 3 1st Run



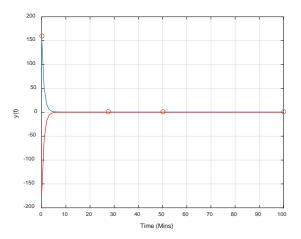
Graph 8: Part 3 2<sup>nd</sup> Run



Graph 9: Part 3 3rd Run



Graph 10: Part 3 4th Run



Graph 11: Part 3 5th Run

## 5. Results and Discussion

Our team managed to compute a differential equation model for the non-homogeneous system involving two tanks. The mixture problem was an ideal situation for our understanding of modeling systems. The eigenvectors and eigenvalues were shown to be quite different when solving each mathematical model. For Part 1, this was basic due to the fact that we had everything given in terms of what we needed to solve the non-homogenous system. For Part 2, we had to change the gallons for the tank1 and tank 2. When this was applied, the values were prompted to change. This concept was also seen in Part 3, when the flow of water was changed for tank 1 and tank 2. When the flow was changed as well, the values given to us were quite different as well. Each graph shows a differential equation for tank 1 and tank 2. The graphs show the differences between times for each tank and also it displays differences of each. At some point, both differential equations will intersect at a neutral point in which shows the mixture of liquid and a substance (salt) combine into one and thus revealing a mixture flow throughout the function. Overall, the graphs give us an understanding where the mixture is occurring and conclude this understanding of nonhomogenous system. The results give us a great ideal of information to see how this non-homogenous system behaves as a mathematical model in a differential equation.

#### 6. Conclusion

This cumulative study on modeling a differential equation for a non-homogeneous system involving a mixture of two tanks was a great opportunity to broaden our understanding. This situation gave us a broad idea of what modeling differential equation is like and how to show a functional mathematical model with the aid of programming such as Matlab. The values show different outcomes when the differential equation for the modeling system were created. This project allowed us to see how the differential equations behave when plotted and helps us view what is occurring over time. The overall task was completed for the first part with a great understanding. The team then proceeded to repeat the same problem when T1 contained different values

such as 50, 100, 150, 200, 250 gal of water and while T2 contained 10, 20, 50, 200, 500 gal of water. This allowed us to see different situations for our modeling systems. Once this was completed, the team repeated the same problem when the flow from tank T1 to T2 changes with different values such as 1,2,5,10,20 gal/min and while T2 to T1 with the following values 1, 2,5,10,100 gal/min. We did a total of 25 combinations for both the second and third parts of this project giving us a total of 50 combinations. Overall, the study of modeling a non-homogenous system was enlightening and can be concurrently applied practically in research and design within the engineering industry.

## 7. References

- [1] Kreyszig, Erwin. "Systems of ODEs. Phase Plane. Qualitative Methods." Advanced Engineering Mathematics. 10th ed. Hoboken, New Jersey: John Wiley & Sons Inc., 2011. 124-165. Print.
- [2] "Differential Equations Modeling." *Differential Equations Modeling*. Paul's Online Math Notes, n.d. Web. 04 June 2016. <a href="http://tutorial.math.lamar.edu/Classes/DE/SystemsModeling.aspx">http://tutorial.math.lamar.edu/Classes/DE/SystemsModeling.aspx</a>.
- [3] "Differential Equations Undetermined Coefficients." *Differential Equations - Undetermined Coefficients*. Paul's Online Math Notes, n.d. Web. 04 June 2016. <a href="http://tutorial.math.lamar.edu/Classes/DE/UndeterminedCoefficients.aspx">http://tutorial.math.lamar.edu/Classes/DE/UndeterminedCoefficients.aspx</a>.
- [4] "Differential Equations Modeling with First Order DE's." *Differential Equations Modeling with First Order DE's.* Paul's Online Math Notes, n.d. Web. 04 June 2016. <a href="http://tutorial.math.lamar.edu/Classes/DE/Modeling.as">http://tutorial.math.lamar.edu/Classes/DE/Modeling.as</a> px>.

## 8. Appendix

#### 8.1 Part 1 Matlab Code

## 8.1.1 Run 1

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
Y' = -a11(Y1) + a12(Y2)
b=6;
c=100;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y' = a21(Y1) + a22(Y2)
d=16;
f=200;
a21=d/f
q=10;
h=100;
i = 6;
a22 = -i/h-g/h
A = [al1 \ al2; a21 \ a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
% C1 + 1.5C2 = 160
% -2C1 + C2 = 0
J = [V22, V11]
b = [160;0];
C = inv(J)*b
% Y1 = 40 e^{(-0.04)}t + 120 e^{(-0.2)}t
% Y2 = 26.68 e^(-0.04)t - 240 e^(-
0.2)t
```

## 8.2 Part 2 Matlab Code

## 8.2.1 Run 1

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y' = -a11(Y1) + a12(Y2)
b=6;
c=10;
a12=b/c
d=16;
f = 50;
```

```
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y' = a21(Y1) + a22(Y2)
d=16;
f=50;
a21=d/f
q=10;
h=10;
i= 6;
a22 = -i/h - g/h
A = [all \ al2; a21 \ a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
% C1 + 1.5C2 = 160
% -2C1 + C2 = 0
J = [V22, V11]
b = [160;0];
C = inv(J)*b
% Y1 = 13.98 e^{(-0.1844)t} + 146.01
e^{(-1.7356)t}
% Y2 = 3.1615 e^(-0.1844)t - 344.49
e^{(-1.7356)}t
```

## 8.2.2 Run 2

```
% Tank 1
 % Y'= Inflow/Min - Outflow/Min
 % Y' = -a11(Y1) + a12(Y2)
 b=6;
 c = 20;
 a12=b/c
 d=16;
 f=50;
a11=-d/f
 % Tank 2
% Y'= Inflow/Min - Outflow/Min
 % Y' = a21(Y1) + a22(Y2)
 d=16;
 f=50;
 a21=d/f
 q = 10;
 h=20;
 i= 6;
 a22 = -i/h-g/h
 A = [a11 \ a12; a21 \ a22]
 P = poly(A)
 Eig = eig(A)
 [V,D] = eig(A);
 V1 = V(:,1);
```

V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) % C1 + 1.5C2 = 160 % -2C1 + C2 = 0 J = [V22,V11] b = [160;0]; C = inv(J)*b % Y1 = 31.01 e^(-0.1681)t + 128.98 e^(-0.9519)t % Y2 = 15.70 e^(-0.1681)t - 271.70 e^(-0.9519)t	<pre>b=6; c=200; a12=b/c d=16; f=50; a11=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2) d=16; f=50; a21=d/f</pre>
<pre>8.2.3 Run 3 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -all(Y1)+ al2(Y2) b=6; c=50; al2=b/c d=16; f=50; al1=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2) d=16; f=50; a21=d/f g=10; h=50; i= 6; a22= -i/h-g/h</pre>	g=10; h=200; i= 6; a22= -i/h-g/h A =[a11 a12;a21 a22] P = poly(A) Eig = eig(A) [V,D] = eig(A); V1 = V(:,1); V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) % C1 + 1.5C2 = 160 % -2C1 + C2 = 0 J = [V22,V11] b = [160;0]; C = inv(J)*b % Y1 = 18.0323 e^(-0.3549)t + 141.9677 e^(-0.0451)t % Y2 = -20.98 e^(-0.3549)t + 1300.99 e^(-0.0451)t
<pre>A =[all al2;a2l a22] P = poly(A) Eig = eig(A) [V,D] = eig(A); V1 = V(:,1); V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) %    Cl + 1.5C2 = 160 % -2Cl + C2 = 0 J = [V22,V11] b = [160;0]; C = inv(J)*b %    Y1 = 80 e^(-0.124)t + 80 e^(-0.516)t %    Y2 = 130.4 e^(-0.124)t - 130.4 e^(-0.516)t  8.2.4 Run 4 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -all(Y1)+ al2(Y2)</pre>	<pre>8.2.5 Run 5 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -all(Y1)+ al2(Y2) b=6; c=500; al2=b/c d=16; f=50; al1=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2) d=16; f=50; a21=d/f g=10; h=500; i= 6; a22= -i/h-g/h</pre>

A =[a11 a12;a21 a22]	
P = poly(A)	8.2.7 Run 7
Eig = eig(A)	% Tank 1
[V,D] = eig(A);	% Y'= Inflow/Min - Outflow/Min
V1 = V(:,1);	% Y'= -a11(Y1)+ a12(Y2)
V11 = V1/V(1,1)	b=6;
V2 = V(:,2);	c=20;
V22 = V2/V(1,2)	a12=b/c
% C1 + 1.5C2 = 160	d=16;
% -2C1 + C2 = 0	
J = [V22, V11]	f=100;
b = [160;0];	all=-d/f
C = inv(J)*b	% Tank 2
% Y1 = $6.5153$ e <sup>(-0.3328)</sup> t + $153.487$	% Y'= Inflow/Min - Outflow/Min
e^(-0.0192)t	% Y' = a21(Y1) + a22(Y2)
% $Y2 = -6.9316 e^{(-0.3328)t} + 3846.92$	d=16;
e^(-0.0192)t	f=100;
e (-0.0192)t	a21=d/f
	g=10;
8.2.6 Run 6	h=20;
% Tank 1	i= 6;
% Y'= Inflow/Min - Outflow/Min	a22 = -i/h-g/h
% $Y' = -a11(Y1) + a12(Y2)$	_
	A =[a11 a12;a21 a22]
b=6;	P = poly(A)
c=10;	Eig = eig(A)
a12=b/c	
d=16;	[V,D] = eig(A);
f=100;	V1 = V(:,1);
all=-d/f	V11 = V1/V(1,1)
% Tank 2	V2 = V(:,2);
% Y'= Inflow/Min - Outflow/Min	V22 = V2/V(1,2)
% Y' = a21(Y1) + a22(Y2)	% C1 + 1.5C2 = 160
d=16;	% -2C1 + C2 = 0
f=100;	J = [V22, V11]
a21=d/f	b = [160;0];
g=10;	C = inv(J)*b
h=10;	$%$ Y1 = 13.98 e^(-0.0922)t + 146.01
i= 6;	e^(-0.8678)t
a22 = -i/h - g/h	$%$ Y2 = 3.1579 e^(-0.0922)t - 344.49
	e^(-0.8678)t
A =[all al2;a21 a22]	
P = poly(A)	
Eig = eig(A)	8.2.8 Run 8
[V,D] = eig(A);	% Tank 1
V1 = V(:,1);	% Y'= Inflow/Min - Outflow/Min
V11 = V1/V(1,1)	% Y' = -a11(Y1) + a12(Y2)
V2 = V(:,2);	b=6;
V22 = V2/V(1,2)	c=50;
% C1 + 1.5C2 = 160	a12=b/c
% -2C1 + C2 = 0	d=16;
J = [V22, V11]	f=100;
b = [160;0];	a11=-d/f
C = inv(J)*b	% Tank 2
$% Y1 = 6.5153 e^{(-0.0962)t} + 153.4847$	% Y'= Inflow/Min - Outflow/Min
e^(-1.6638)t	% Y'= a21(Y1)+ a22(Y2)
$%$ Y2 = 0.6932 e^(-0.0962)t - 384.69	d=16;
e^(-1.6638)t	f=100;

```
a21=d/f
                                                b = [160;0];
g=10;
                                                C = inv(J)*b
                                                % Y1 = 40 e^{(-0.2)t} + 120 e^{(-0.04)t}
h=50;
                                                Y2 = -53.32 e^{(-0.2)t} + 480 e^{(-0.2)t}
i = 6;
a22 = -i/h-g/h
                                                0.04)t
A = [a11 \ a12; a21 \ a22]
P = poly(A)
                                                8.2.10 Run 10
Eig = eig(A)
                                                % Tank 1
[V,D] = eig(A);
                                                % Y'= Inflow/Min - Outflow/Min
V1 = V(:,1);
                                                % Y' = -a11(Y1) + a12(Y2)
V11 = V1/V(1,1)
                                                b=6;
V2 = V(:,2);
                                                c = 500;
V22 = V2/V(1,2)
                                                a12=b/c
% C1 + 1.5C2 = 160
                                                d=16;
% -2C1 + C2 = 0
                                                f=100;
J = [V22, V11]
                                                a11=-d/f
b = [160;0];
                                                % Tank 2
C = inv(J)*b
                                                % Y'= Inflow/Min - Outflow/Min
% Y1 = 40 e^(-0.08)t + 120 e^(-0.40)t
                                                % Y' = a21(Y1) + a22(Y2)
% Y2 = 26.68 e^(-0.08)t - 240 e^(-
                                                d=16;
0.40)t
                                                f = 100;
                                                a21=d/f
                                                q = 10;
                                                h=500;
8.2.9 Run 9
                                                i= 6;
% Tank 1
                                                a22 = -i/h-g/h
% Y'= Inflow/Min - Outflow/Min
% Y' = -a11(Y1) + a12(Y2)
b=6;
                                                A = [all \ al2; a21 \ a22]
c = 200;
                                                P = poly(A)
a12=b/c
                                                Eig = eig(A)
d=16;
                                                [V,D] = eig(A);
f=100;
                                                V1 = V(:,1);
a11=-d/f
                                                V11 = V1/V(1,1)
% Tank 2
                                                V2 = V(:,2);
% Y'= Inflow/Min - Outflow/Min
                                                V22 = V2/V(1,2)
% Y' = a21(Y1) + a22(Y2)
                                               % C1 + 1.5C2 = 160
d=16;
                                                % -2C1 + C2 = 0
f=100;
                                                J = [V22, V11]
a21=d/f
                                                b = [160;0];
q=10;
                                                C = inv(J)*b
h=200;
                                                % Y1 = 13.98 e^{(-0.1736)t} + 120 e^{(-0.1736)t}
i= 6;
                                                0.0184)t
a22 = -i/h-g/h
                                                % Y2 = -15.80 e^{(-0.1736)t} + 1722.46
                                                e^{(-0.0184)}t
A = [all \ al2; a21 \ a22]
P = poly(A)
Eig = eig(A)
                                                8.2.11 Run 11
[V,D] = eig(A);
                                                % Tank 1
V1 = V(:,1);
                                                % Y'= Inflow/Min - Outflow/Min
V11 = V1/V(1,1)
                                                % Y' = -a11(Y1) + a12(Y2)
V2 = V(:,2);
                                                b=6;
V22 = V2/V(1,2)
                                                c = 10;
% C1 + 1.5C2 = 160
                                                a12=b/c
% -2C1 + C2 = 0
                                                d=16;
J = [V22, V11]
```

```
f=150;
                                                V1 = V(:,1);
a11=-d/f
                                                V11 = V1/V(1,1)
% Tank 2
                                                V2 = V(:,2);
                                               V22 = V2/V(1,2)
% Y'= Inflow/Min - Outflow/Min
                                               % C1 + 1.5C2 = 160
% Y' = a21(Y1) + a22(Y2)
d=16;
                                               % -2C1 + C2 = 0
f=150;
                                               J = [V22, V11]
a21=d/f
                                               b = [160;0];
                                                C = inv(J)*b
q=10;
h=10;
                                                % Y1 = 8.9071 e^{(-0.0632)t} + 151.0929
i = 6;
                                               e^{(-0.8434)}t
a22 = -i/h-g/h
                                               % Y2 = 1.291 e^{(-0.0632)t} - 371.0690
                                                e^{(-0.8434)}t
A = [a11 \ a12; a21 \ a22]
P = poly(A)
                                                8.2.13 Run 13
Eig = eig(A)
                                                % Tank 1
[V,D] = eig(A);
                                                % Y'= Inflow/Min - Outflow/Min
V1 = V(:,1);
                                                % Y' = -a11(Y1) + a12(Y2)
V11 = V1/V(1,1)
                                                b=6;
V2 = V(:,2);
                                                c = 50;
V22 = V2/V(1,2)
                                                a12=b/c
% C1 + 1.5C2 = 160
                                                d=16;
% -2C1 + C2 = 0
                                               f=150;
J = [V22, V11]
                                                a11=-d/f
b = [160;0];
                                                % Tank 2
C = inv(J)*b
                                               % Y'= Inflow/Min - Outflow/Min
% Y1 = 4.23 e^{(-0.065)t} + 153.76 e^{(-0.065)t}
                                               % Y' = a21(Y1) + a22(Y2)
Y2 = 0.294 e^{(-0.065)t} - 393.40
                                                f = 150;
e^{(-1.64)}t
                                                a21=d/f
                                                g = 10;
                                                h=50;
8.2.12 Run 12
                                                i = 6;
% Tank 1
                                                a22 = -i/h-g/h
% Y'= Inflow/Min - Outflow/Min
% Y' = -a11(Y1) + a12(Y2)
                                                A = [all \ al2; a21 \ a22]
b=6;
                                                P = poly(A)
c = 20;
                                                Eig = eig(A)
a12=b/c
                                               [V,D] = eig(A);
d=16;
                                                V1 = V(:,1);
f=150;
                                                V11 = V1/V(1,1)
a11=-d/f
                                                V2 = V(:,2);
% Tank 2
                                               V22 = V2/V(1,2)
% Y'= Inflow/Min - Outflow/Min
                                               % C1 + 1.5C2 = 160
% Y' = a21(Y1) + a22(Y2)
                                               % -2C1 + C2 = 0
d=16;
                                               J = [V22, V11]
f=150;
                                               b = [160;0];
a21=d/f
                                               C = inv(J)*b
q=10;
                                                % Y1 = 25.1205 e^{(-0.0578)t} +
h=20;
                                               134.8795 e^(-0.3688)t
i= 6;
                                               % Y2 = 10.221 e^(-0.0578)t - 294.67
a22 = -i/h-g/h
                                                e^{(-0.3688)}t
A = [a11 \ a12; a21 \ a22]
P = poly(A)
```

Eig = eig(A)[V,D] = eig(A);

8.2.14 Run 14 % Tank 1	g=10; h=500;
% Y'= Inflow/Min - Outflow/Min	i= 6; a22= -i/h-g/h
% Y'= -a11(Y1)+ a12(Y2) b=6;	a221/11-9/11
c=200;	A =[a11 a12;a21 a22]
al2=b/c	P = poly(A)
d=16;	Eig = eig(A)
f=150;	[V,D] = eig(A);
all=-d/f	V1 = V(:,1);
% Tank 2	V11 = V1/V(1,1)
<pre>% Y'= Inflow/Min - Outflow/Min % Y'= 021(Y1) + 022(Y2)</pre>	V2 = V(:,2); V22 = V2/V(1,2)
% Y'= a21(Y1)+ a22(Y2) d=16;	% C1 + 1.5C2 = 160
f=150;	% -2C1 + C2 = 0
a21=d/f	J = [V22,V11]
g=10;	b = [160;0];
h=200;	C = inv(J)*b
i= 6;	$% Y1 = 22.2404 e^{(-0.1210)t} +$
a22 = -i/h - g/h	137.7596 e^(-0.0176)t
	$Y2 = -26.641 e^{(-0.1210)t} + 1022.2$
A =[a11 a12;a21 a22]	e^(-0.0176)t
P = poly(A)	
Eig = eig(A)	
[V,D] = eig(A);	8.2.16 Run 16
V1 = V(:,1); V11 = V1/V(1,1)	% Tank 1
V2 = V(:,2);	% Y'= Inflow/Min - Outflow/Min
V22 = V2/V(1,2)	% Y'= -all(Y1)+ al2(Y2) b=6;
% C1 + 1.5C2 = 160	c=10;
% -2C1 + C2 = 0	a12=b/c
J = [V22, V11]	d=16;
b = [160;0];	f=200;
C = inv(J)*b	a11=-d/f
$%$ Y1 = 61.6467 e^(-0.1515)t + 98.3533	% Tank 2
$e^{(-0.0352)t}$ % Y2 = -92.026 $e^{(-0.1515)t} + 234.248$	% Y'= Inflow/Min - Outflow/Min
e^(-0.0352)t	% Y'= a21(Y1)+ a22(Y2)
C ( 0.0332/C	d=16; f=200;
	a21=d/f
8.2.15 Run 15	g=10;
% Tank 1	h=10;
% Y'= Inflow/Min - Outflow/Min	i= 6;
% Y'= -all(Y1)+ al2(Y2)	a22 = -i/h-g/h
b=6;	
c=500;	A =[a11 a12;a21 a22]
al2=b/c	P = poly(A)
d=16;	Eig = eig(A)
f=150;	[V,D] = eig(A);
all=-d/f	V1 = V(:,1);
% Tank 2 % Y'= Inflow/Min - Outflow/Min	V11 = V1/V(1,1) V2 = V(:,2);
% Y'= inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2)	V2 = V(.,2), V22 = V2/V(1,2)
d=16;	% C1 + 1.5C2 = 160
f=150;	% -2C1 + C2 = 0
a21=d/f	J = [V22, V11]

```
f=200;
b = [160;0];
C = inv(J)*b
                                               a11=-d/f
% Y1 = 3.1303 e^(-0.0491)t + 156.8693
                                               % Tank 2
                                                % Y'= Inflow/Min - Outflow/Min
e^{(-1.6309)}t
% Y2 = 0.1615 e^(-0.0491)t - 405.4924
                                               % Y' = a21(Y1) + a22(Y2)
e^{(-1.6309)}t
                                                d=16;
                                                f=200;
                                                a21=d/f
                                                q=10;
8.2.17 Run 17
                                               h=50;
% Tank 1
                                               i= 6;
% Y'= Inflow/Min - Outflow/Min
                                               a22 = -i/h-g/h
% Y' = -a11(Y1) + a12(Y2)
b=6;
                                               A = [all \ al2; a21 \ a22]
c = 20;
                                                P = poly(A)
a12=b/c
                                               Eig = eig(A)
d=16;
                                               [V,D] = eig(A);
f = 200;
                                               V1 = V(:,1);
a11=-d/f
                                               V11 = V1/V(1,1)
% Tank 2
                                               V2 = V(:,2);
% Y'= Inflow/Min - Outflow/Min
                                               V22 = V2/V(1,2)
% Y' = a21(Y1) + a22(Y2)
                                               % C1 + 1.5C2 = 160
d=16;
                                               % -2C1 + C2 = 0
f = 200;
                                               J = [V22, V11]
a21=d/f
                                               b = [160;0];
q=10;
                                               C = inv(J)*b
h=20;
                                               % Y1 = 18.0323 e^{(-0.0451)t} +
i = 6;
                                               141.9677 e^(-0.3549)t
a22 = -i/h-g/h
                                                % Y2 = 5.2473 e^{(-0.-0451)}t - 325.24
                                               e^{(-0.3549)}t
A = [a11 \ a12; a21 \ a22]
P = poly(A)
Eig = eig(A)
                                                8.2.19 Run 19
[V,D] = eig(A);
V1 = V(:,1);
                                                % Tank 1
V11 = V1/V(1,1)
                                               % Y'= Inflow/Min - Outflow/Min
                                               % Y' = -a11(Y1) + a12(Y2)
V2 = V(:,2);
V22 = V2/V(1,2)
                                               b=6;
% C1 + 1.5C2 = 160
                                               c = 200;
% -2C1 + C2 = 0
                                               a12=b/c
J = [V22, V11]
                                               d=16;
b = [160;0];
                                               f=200;
C = inv(J)*b
                                               a11=-d/f
% Y1 = 6.1557 e^(-0.0481)t + 153.4847
                                               % Tank 2
e^{(-0.8319)}t
                                               % Y'= Inflow/Min - Outflow/Min
% Y2 = 0.6932 e^(-0.0481)t - 384.69
                                               % Y' = a21(Y1) + a22(Y2)
e^{(-0.8319)}t
                                               d=16;
                                               f=200;
                                                a21=d/f
8.2.18 Run 18
                                               q=10;
% Tank 1
                                               h=200;
% Y'= Inflow/Min - Outflow/Min
                                               i= 6;
% Y' = -a11(Y1) + a12(Y2)
                                               a22 = -i/h - q/h
b=6;
c = 50;
                                               A = [all \ al2; a21 \ a22]
a12=b/c
                                                P = poly(A)
d=16;
                                                Eig = eig(A)
```

<pre>[V,D] = eig(A); V1 = V(:,1); V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) %    C1 + 1.5C2 = 160 % -2C1 + C2 = 0 J = [V22,V11] b = [160;0]; C = inv(J)*b %    Y1 = 80 e^(-0.0310)t + 80 e^(-0.1290)t %    Y2 = 130.64 e^(-0.0310)t - 130.64 e^(-0.1290)t</pre>	<pre>8.2.21 Run 21 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -a11(Y1) + a12(Y2) b=6; c=10; a12=b/c d=16; f=250; a11=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1) + a22(Y2) d=16; f=250;</pre>
<pre>8.2.20 Run 20 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -all(Y1)+ al2(Y2) b=6; c=500; al2=b/c d=16; f=200; al1=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2) d=16; f=200; a21=d/f g=10; h=500; i= 6; a22= -i/h-g/h</pre> A =[al1 al2;a21 a22] P = poly(A) Eig = eig(A) [V Dl = eig(A)]	a21=d/f g=10; h=10; i= 6; a22= -i/h-g/h  A =[a11 a12;a21 a22] P = poly(A) Eig = eig(A) [V,D] = eig(A); V1 = V(:,1); V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) % C1 + 1.5C2 = 160 % -2C1 + C2 = 0 J = [V22,V11] b = [160;0]; C = inv(J)*b % Y1 = 2.4835 e^(-0.0394)t + 157.5156 e^(-1.6246)t % Y2 = 0.1018 e^(-0.0394)t - 409.7 e^(-1.6246)t
[V,D] = eig(A); V1 = V(:,1); V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) % C1 + 1.5C2 = 160 % -2C1 + C2 = 0 J = [V22,V11] b = [160;0]; C = inv(J)*b % Y1 = 31.0102 e^(-0.0952)t + 128.98 e^(-0.0168)t % Y2 = -39.258 e^(-0.0952)t + 679.26 e^(-0.0168)t	<pre>8.2.22 Run 22 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -a11(Y1)+ a12(Y2) b=6; c=20; a12=b/c d=16; f=250; a11=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2) d=16; f=250; a21=d/f</pre>

```
q=10;
                                               b = [160;0];
h=20;
                                               C = inv(J)*b
                                                % Y1 = 13.98 e^(-0.0369)t + 146.0110
i = 6;
a22 = -i/h-g/h
                                               e^{(-0.3471)t}
                                               % Y2 = 3.1594 e^(-0.0369)t + 344.498
                                               e^{(-0.3471)}t
A = [a11 \ a12; a21 \ a22]
P = poly(A)
Eig = eig(A)
                                               8.2.24 Run 24
[V,D] = eig(A);
                                               % Tank 1
V1 = V(:,1);
                                               % Y'= Inflow/Min - Outflow/Min
V11 = V1/V(1,1)
                                               % Y' = -a11(Y1) + a12(Y2)
V2 = V(:,2);
                                               b=6i
V22 = V2/V(1,2)
                                               c = 200;
% C1 + 1.5C2 = 160
                                               a12=b/c
% -2C1 + C2 = 0
                                               d=16;
J = [V22, V11]
                                               f=250;
b = [160;0];
                                               a11=-d/f
C = inv(J)*b
                                               % Tank 2
% Y1 = 5.1315 e^(-0.0388)t + 154.8586
                                               % Y'= Inflow/Min - Outflow/Min
e^{(-0.8252)t}
                                               % Y' = a21(Y1) + a22(Y2)
% Y2 = 0.4315 e^(-0.0388)t - 392.96
                                               d=16;
e^{(-0.8252)t}
                                               f=250;
                                               a21=d/f
                                               q = 10;
                                               h=200;
8.2.23 Run 23
                                               i= 6;
% Tank 1
                                               a22 = -i/h-g/h
% Y'= Inflow/Min - Outflow/Min
% Y' = -a11(Y1) + a12(Y2)
                                               A = [a11 \ a12; a21 \ a22]
b=6;
                                               P = poly(A)
c = 50;
a12=b/c
                                               Eig = eig(A)
                                               [V,D] = eig(A);
d=16;
                                               V1 = V(:,1);
f=250;
                                               V11 = V1/V(1,1)
a11=-d/f
                                               V2 = V(:,2);
% Tank 2
                                               V22 = V2/V(1,2)
% Y'= Inflow/Min - Outflow/Min
                                               % C1 + 1.5C2 = 160
% Y' = a21(Y1) + a22(Y2)
d=16;
                                               % -2C1 + C2 = 0
                                               J = [V22, V11]
f=250;
                                               b = [160;0];
a21=d/f
                                               C = inv(J)*b
q=10;
                                               % Y1 = 65.6316 e^{(-0.0275)t} + 94.3684
h=50;
                                               e^{(-0.1165)}t
i= 6;
                                               % Y2 = 79.9458 e^{(-0.0275)t} + 165.276
a22 = -i/h-g/h
                                               e^{(-0.1165)}t
A = [all \ al2; a21 \ a22]
P = poly(A)
Eig = eig(A)
                                               8.2.25 Run 25
[V,D] = eig(A);
                                                % Tank 1
V1 = V(:,1);
                                               % Y'= Inflow/Min - Outflow/Min
V11 = V1/V(1,1)
                                               % Y' = -a11(Y1) + a12(Y2)
V2 = V(:,2);
V22 = V2/V(1,2)
                                               c=500;
% C1 + 1.5C2 = 160
                                               a12=b/c
% -2C1 + C2 = 0
                                               d=16;
J = [V22, V11]
                                               f = 250;
```

```
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y' = a21(Y1) + a22(Y2)
d=16;
f=250;
a21=d/f
q = 10;
h=500;
i = 6;
a22 = -i/h - g/h
A = [a11 \ a12; a21 \ a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
% C1 + 1.5C2 = 160
% -2C1 + C2 = 0
J = [V22, V11]
b = [160;0];
C = inv(J)*b
% Y1 = 40 e^(-0.080)t + 120 e^(-
0.0160)t
% Y2 = -53.332 e^(-0.080)t + 480 e^(-
0.0160)t
```

## 8.3 Part 3 Matlab Code (Flow Changes)

## 8.3.1 Run 1

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y' = -a11(Y1) + a12(Y2)
b=1;
c = 100;
a12=b/c
d=1;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y' = a21(Y1) + a22(Y2)
d=1;
f=200;
a21=d/f
q=10;
h=100;
i = 1;
a22 = -i/h-g/h
A = [all \ al2; a21 \ a22]
P = poly(A)
```

```
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
% C1 + 1.5C2 = 160
% -2C1 + C2 = 0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
% Y1 = 0.7159 e^(-0.0045)t + 159.284
e^(-0.1103)t
% Y2 = 0.0339 e^(-0.0045)t - 1680.03
e^(-0.1103)t
```

% Y'= Inflow/Min - Outflow/Min

# **8.3.2 Run 2** % Tank 1

```
% Y' = -a11(Y1) + a12(Y2)
b=2;
c=100;
 a12=b/c
d=1;
f=200;
a11=-d/f
% Tank 2
 % Y'= Inflow/Min - Outflow/Min
 % Y' = a21(Y1) + a22(Y2)
 d=1;
 f=200;
 a21=d/f
q = 10;
h=100;
i=2;
a22 = -i/h-g/h
 A = [all \ al2; a21 \ a22]
 P = poly(A)
 Eig = eig(A)
 [V,D] = eig(A);
 V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
% C1 + 1.5C2 = 160
 % -2C1 + C2 = 0
 J = [V22, V11]
b = [160;0];
C = inv(J)*b
% Y1 = 1.1831 e^{(-0.0041)t} + 158.81
e^{(-0.1209)}t
% Y2 = 0.0511 e^{(-0.0041)t} - 920 e^{(-0.0041)t}
0.1209)t
```

8.3.3 Run 3	g=10;
% Tank 1	h=100;
% Y'= Inflow/Min - Outflow/Min	i= 10;
% Y' = -a11(Y1) + a12(Y2)	a22= -i/h-g/h
b=5;	
c=100;	A =[a11 a12;a21 a22]
a12=b/c	P = poly(A)
d=1;	<pre>Eig = eig(A)</pre>
f=200;	[V,D] = eig(A);
all=-d/f	V1 = V(:,1);
% Tank 2	V11 = V1/V(1,1)
% Y'= Inflow/Min - Outflow/Min	V2 = V(:,2);
% Y' = a21(Y1) + a22(Y2)	V22 = V2/V(1,2)
d=1;	% C1 + 1.5C2 = 160
f=200;	% -2C1 + C2 = 0
a21=d/f	J = [V22, V11]
g=10;	b = [160;0];
h=100;	C = inv(J)*b
i= 5;	$%$ Y1 = 2.0244 e^(-0.0025)t + 157.97
a22 = -i/h - g/h	e^(-0.2025)t
	$Y2 = 0.0512 e^{(-0.0025)t} - 312.049$
A =[all al2;a21 a22]	e^(-0.2025)t
P = poly(A)	
Eig = eig(A)	
[V,D] = eig(A);	0.2 5 D 5
V1 = V(:,1);	8.3.5 Run 5
V1 = V(1,1) V11 = V1/V(1,1)	% Tank 1
V2 = V(:,2);	% Y'= Inflow/Min - Outflow/Min
V2 = V(.,2), V22 = V2/V(1,2)	% Y' = -a11(Y1) + a12(Y2)
	b=100;
% C1 + 1.5C2 = 160 % 2G1 + G2 = 0	c=100;
% -2C1 + C2 = 0	a12=b/c
J = [V22, V11]	d=1;
b = [160;0];	f=200;
C = inv(J)*b	all=-d/f
$% Y1 = 1.8377 e^{(-0.0033)t} + 158.16$	% Tank 2
e^(-0.1517)t	% Y'= Inflow/Min - Outflow/Min
$%$ Y2 = 0.0626 e^(-0.0033)t - 464.06	% Y' = a21(Y1) + a22(Y2)
e^(-0.1517)t	d=1;
	f=200;
	a21=d/f
8.3.4 Run 4	g=10;
% Tank 1	h=100;
% Y'= Inflow/Min - Outflow/Min	i= 100;
% Y'= -a11(Y1)+ a12(Y2)	a22= -i/h-g/h
b=10;	
c=100;	A =[a11 a12;a21 a22]
a12=b/c	P = poly(A)
d=1;	Eig = eig(A)
f=200;	[V,D] = eig(A);
all=-d/f	V1 = V(:,1);
% Tank 2	$V1 - V(\cdot, 1)$ , V11 = V1/V(1, 1)
% Y'= Inflow/Min - Outflow/Min	V11 - V1/V(1,1) V2 = V(:,2);
% Y'= a21(Y1)+ a22(Y2)	V22 = V2/V(1,2) % C1 + 1.5C2 = 160
d=1; f=200:	
f=200;	
a21=d/f	J = [V22, V11]

b = [160;0];	f=200;
C = inv(J)*b	all=-d/f
$% Y1 = 0.6590 e^{(-0.0005)t} + 159.34$	% Tank 2
e^(-1.1045)t	% Y'= Inflow/Min - Outflow/Min
$% Y2 = 0.00296 e^{(-00005)t - 175.19}$	% Y' = a21(Y1) + a22(Y2)
e^(-1.1045)t	d=2;
	f=200;
	a21=d/f
0.2.6 D	g=10;
8.3.6 Run 6	h=100;
% Tank 1	i= 2;
% Y'= Inflow/Min - Outflow/Min	a22 = -i/h - g/h
% Y'= -a11(Y1)+ a12(Y2)	
b=1;	A =[a11 a12;a21 a22]
c=100;	P = poly(A)
a12=b/c	Eig = eig(A)
d=2;	[V,D] = eig(A);
f=200;	V1 = V(:,1);
a11=-d/f	V11 = V1/V(1,1)
% Tank 2	V2 = V(:,2);
% Y'= Inflow/Min - Outflow/Min	V22 = V2/V(1,2)
% Y' = a21(Y1) + a22(Y2)	% C1 + 1.5C2 = 160
d=2; f=200;	% -2C1 + C2 = 0
	J = [V22, V11]
a21=d/f	b = [160;0];
g=10; h=100;	C = inv(J)*b
i= 1;	% $Y1 = 2.5203 e^{(-0.0082)t} + 157.47$
	e^(-0.1218)t
a22 = -i/h - g/h	% $Y2 = 0.2253 e^{(-0.0082)t} - 880.23$
- [ 44	e^(-0.1218)t
A =[all al2;a21 a22]	
P = poly(A)	
Eig = eig(A)	0.2.0 D 0
[V,D] = eig(A);	8.3.8 Run 8
V1 = V(:,1);	% Tank 1
V11 = V1/V(1,1)	% Y'= Inflow/Min - Outflow/Min
V2 = V(:,2);	% Y' = -a11(Y1) + a12(Y2)
V22 = V2/V(1,2)	b=5;
% C1 + 1.5C2 = 160	c=100;
% -2C1 + C2 = 0	a12=b/c
J = [V22, V11]	d=2;
b = [160;0];	f=200;
C = inv(J)*b	all=-d/f
$% Y1 = 158.44 e^{(-0.1110)t} + 1.5535$	% Tank 2
e^(-0.0090)t	% Y'= Inflow/Min - Outflow/Min
$%$ Y2 = -1600.08 e^(-0.1110)t + 0.1537	% Y'= a21(Y1)+ a22(Y2)
e^(-0.0090)t	d=2; f=200;
	a21=d/f
8.3.7 Run 7	g=10; h=100:
% Tank 1	h=100;
% Y'= Inflow/Min - Outflow/Min	i= 5; a22= -i/h-g/h
% Y' = -a11(Y1) + a12(Y2)	a221/11-9/11
b=2;	7 [ 11 10 01 00]
c=100;	A =[a11 a12;a21 a22]
al2=b/c	P = poly(A)
d=2;	Eig = eig(A)

<pre>[V,D] = eig(A); V1 = V(:,1); V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) %    C1 + 1.5C2 = 160 % -2C1 + C2 = 0 J = [V22,V11] b = [160;0]; C = inv(J)*b %    Y1 = 3.7937 e^((-0.0065)t + 156.20 e^((-0.1535)t %    Y2 = 0.2644 e^((-0.0065)t - 448.24 e^((-0.1535)t</pre>	8.3.10 Run 10 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -all(Y1)+ al2(Y2) b=100; c=100; al2=b/c d=2; f=200; al1=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2) d=2; f=200; a21=d/f
<pre>8.3.9 Run 9 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -all(Y1) + al2(Y2) b=10; c=100; al2-b/c</pre>	g=10; h=100; i= 100; a22= -i/h-g/h A =[a11 a12;a21 a22]
<pre>a12=b/c d=2; f=200; a11=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2) d=2; f=200; a21=d/f a-10:</pre>	<pre>P = poly(A) Eig = eig(A) [V,D] = eig(A); V1 = V(:,1); V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) % C1 + 1.5C2 = 160 % -2C1 + C2 = 0 J = [V22,V11] b = [160;0];</pre>
g=10; h=100; i= 10; a22= -i/h-g/h A =[a11 a12;a21 a22] P = poly(A)	C = $inv(J)*b$ % Y1 = 1.3136 e^(-0.0009)t + 158.68 e^(-1.1091)t % Y2 = 0.01195 e^(-0.0009)t - 174.4 e^(-1.1091)t
Eig = eig(A) [V,D] = eig(A); V1 = V(:,1); V11 = V1/V(1,1) V2 = V(:,2); V22 = V2/V(1,2) % C1 + 1.5C2 = 160 % -2C1 + C2 = 0 J = [V22,V11] b = [160;0]; C = inv(J)*b % Y1 = 4.094 e^(-0.0049)t + 153.905 e^(-0.2051)t % Y2 = 0.2094 e^(-0.0049)t - 300.29 80 e^(-0.2051)t	8.3.11 Run 11 % Tank 1 % Y'= Inflow/Min - Outflow/Min % Y'= -a11(Y1)+ a12(Y2) b=1; c=100; a12=b/c d=5; f=200; a11=-d/f % Tank 2 % Y'= Inflow/Min - Outflow/Min % Y'= a21(Y1)+ a22(Y2) d=5; f=200; a21=d/f

```
q=10;
                                               b = [160;0];
h=100;
                                               C = inv(J)*b
                                                % Y1 = 7.619 e^{(-0.02)t} + 152.381
i= 1;
a22 = -i/h-g/h
                                               e^{(-0.1250)}t
                                               % Y2 = 1.904 e^{(-0.02)t} - 761.9 e^{(-0.02)t}
                                                0.1250)t
A = [a11 \ a12; a21 \ a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
                                               8.3.13 Run 13
V1 = V(:,1);
                                                % Tank 1
V11 = V1/V(1,1)
                                               % Y'= Inflow/Min - Outflow/Min
V2 = V(:,2);
                                               % Y' = -a11(Y1) + a12(Y2)
V22 = V2/V(1,2)
                                               b=5;
% C1 + 1.5C2 = 160
                                               c=100;
% -2C1 + C2 = 0
                                               a12=b/c
J = [V22, V11]
                                               d=5;
b = [160;0];
                                               f=200;
C = inv(J)*b
                                               a11=-d/f
% Y1 = 5.0208 e^(-0.0222)t + 154.97
                                               % Tank 2
e^{(-0.1128)}t
                                               % Y'= Inflow/Min - Outflow/Min
% Y2 = 1.4289 e^(-0.0222)t - 1361.43
                                               % Y' = a21(Y1) + a22(Y2)
e^{(-0.1128)t}
                                               d=5;
                                               f=200;
                                                a21=d/f
8.3.12 Run 12
                                               q = 10;
% Tank 1
                                               h=100;
% Y'= Inflow/Min - Outflow/Min
                                               i = 5;
                                               a22 = -i/h - q/h
% Y' = -a11(Y1) + a12(Y2)
b=2;
c = 100;
                                               A = [a11 \ a12; a21 \ a22]
a12=b/c
                                               P = poly(A)
d=5;
                                               Eig = eig(A)
f=200;
                                               [V,D] = eig(A);
                                               V1 = V(:,1);
a11=-d/f
% Tank 2
                                               V11 = V1/V(1,1)
% Y'= Inflow/Min - Outflow/Min
                                               V2 = V(:,2);
% Y' = a21(Y1) + a22(Y2)
                                               V22 = V2/V(1,2)
d=5;
                                               % C1 + 1.5C2 = 160
f = 200;
                                               % -2C1 + C2 = 0
a21=d/f
                                               J = [V22, V11]
                                               b = [160;0];
q=10;
h=100;
                                               C = inv(J)*b
i = 2i
                                               % Y1 = 10.368 e^{(-0.0157)t} + 149.631
a22 = -i/h-g/h
                                               e^{(-0.1593)}t
                                               % Y2 = 1.9294 e^(-0.0157)t - 401.92
                                               e^{(-0.1593)t}
A = [a11 \ a12; a21 \ a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
                                                8.3.14 Run 14
V1 = V(:,1);
                                                % Tank 1
V11 = V1/V(1,1)
                                               % Y'= Inflow/Min - Outflow/Min
V2 = V(:,2);
                                               % Y' = -a11(Y1) + a12(Y2)
V22 = V2/V(1,2)
                                               b=10;
% C1 + 1.5C2 = 160
                                               c=100;
% -2C1 + C2 = 0
                                               a12=b/c
J = [V22, V11]
                                                d=5;
```

```
f=200;
                                               [V,D] = eig(A);
a11=-d/f
                                               V1 = V(:,1);
% Tank 2
                                               V11 = V1/V(1,1)
                                               V2 = V(:,2);
% Y'= Inflow/Min - Outflow/Min
                                               V22 = V2/V(1,2)
% Y' = a21(Y1) + a22(Y2)
d=5;
                                               % C1 + 1.5C2 = 160
f=200;
                                               % -2C1 + C2 = 0
a21=d/f
                                               J = [V22, V11]
q = 10;
                                               b = [160;0];
h=100;
                                               C = inv(J)*b
i = 10;
                                               % Y1 = 3.2518 e^{(-0.0022)t} + 156.74
a22 = -i/h-g/h
                                              e^(-1.1228)t
                                               % Y2 = 0.0741 e^{(-0.0022)t - 172.07}
                                               e^{(-1.1228)t}
A = [a11 \ a12; a21 \ a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
                                               8.3.16 Run 16
V1 = V(:,1);
                                               % Tank 1
V11 = V1/V(1,1)
                                               % Y'= Inflow/Min - Outflow/Min
V2 = V(:,2);
                                               % Y' = -a11(Y1) + a12(Y2)
V22 = V2/V(1,2)
                                              b=1;
% C1 + 1.5C2 = 160
                                               c=100;
% -2C1 + C2 = 0
                                               a12=b/c
J = [V22, V11]
                                               d=10;
b = [160;0];
                                               f=200;
C = inv(J)*b
                                              a11=-d/f
% Y1 = 10.54 e^(-0.0117)t + 149.45
                                              % Tank 2
e^{(-0.2133)t}
                                               % Y'= Inflow/Min - Outflow/Min
% Y2 = 1.3937 e^(-0.0117)t - 281.38
                                               % Y' = a21(Y1) + a22(Y2)
e^{(-0.2133)t}
                                               d=10;
                                               f = 200;
                                               a21=d/f
8.3.15 Run 15
                                               g=10;
                                               h=100;
% Tank 1
                                               i = 1;
% Y'= Inflow/Min - Outflow/Min
                                               a22 = -i/h-g/h
% Y' = -a11(Y1) + a12(Y2)
b=100;
c=100;
                                               A = [a11 \ a12; a21 \ a22]
a12=b/c
                                               P = poly(A)
d=5;
                                               Eig = eig(A)
                                               [V,D] = eig(A);
f=200;
a11=-d/f
                                               V1 = V(:,1);
% Tank 2
                                               V11 = V1/V(1,1)
% Y'= Inflow/Min - Outflow/Min
                                               V2 = V(:,2);
% Y' = a21(Y1) + a22(Y2)
                                              V22 = V2/V(1,2)
d=5;
                                               % C1 + 1.5C2 = 160
f=200;
                                               % -2C1 + C2 = 0
a21=d/f
                                               J = [V22, V11]
q=10;
                                               b = [160;0];
h=100;
                                               C = inv(J)*b
i = 100;
                                              % Y1 = 15.857 e^{(-0.0126)t} + 144.14
a22 = -i/h-g/h
                                              e^{(-0.1174)}t
                                               % Y2 = 11.761 e^(-0.0126)t - 971.74
                                               e^{(-0.1174)}t
A = [al1 \ al2; a21 \ a22]
P = poly(A)
```

Eig = eig(A)

8.3.17 Run 17	g=10;
% Tank 1	h=100;
% Y'= Inflow/Min - Outflow/Min	i= 5;
% Y' = -a11(Y1) + a12(Y2)	a22 = -i/h - g/h
b=2;	
c=100;	A =[a11 a12;a21 a22]
al2=b/c	P = poly(A)
d=10;	Eig = eig(A)
f=200;	[V,D] = eig(A);
all=-d/f	V1 = V(:,1);
% Tank 2	V11 = V1/V(1,1)
% Y'= Inflow/Min - Outflow/Min	V2 = V(:,2);
% Y' = a21(Y1) + a22(Y2)	V22 = V2/V(1,2)
d=10;	% C1 + 1.5C2 = 160
f=200;	% -2C1 + C2 = 0
a21=d/f	J = [V22, V11]
g=10;	b = [160;0];
h=100;	C = inv(J)*b
i= 2;	% Y1 = $136.56 e^{(-0.1707)t} + 23.431$
a22 = -i/h - q/h	e^(-0.0293)t
422 1/11 9/11	% $Y2 = -329.68 e^{(-0.1707)t} + 9.705$
7 -[-11 -10:-01 -00]	e^(-0.0293)t
A =[a11 a12;a21 a22]	C ( 0.0255/C
P = poly(A)	
Eig = eig(A)	0.5.10.7
[V,D] = eig(A);	8.3.19 Run 19
V1 = V(:,1);	% Tank 1
V11 = V1/V(1,1)	% Y'= Inflow/Min - Outflow/Min
V2 = V(:,2);	% Y' = -a11(Y1) + a12(Y2)
V22 = V2/V(1,2)	b=10;
% C1 + 1.5C2 = 160	c=100;
% -2C1 + C2 = 0	a12=b/c
J = [V22, V11]	d=10;
b = [160;0];	f=200;
C = inv(J)*b	a11=-d/f
$% Y1 = 20.64 e^{(-0.0328)t} + 139.359$	% Tank 2
e^(-0.1322)t	% Y'= Inflow/Min - Outflow/Min
$%$ Y2 = 12.559 e^(-0.0328)t - 572.53	% Y'= a21(Y1)+ a22(Y2)
e^(-0.1322)t	d=10;
	f=200;
	a21=d/f
8.3.18 Run 18	g=10;
% Tank 1	h=100;
• - •	i= 10;
<pre>% Y'= Inflow/Min - Outflow/Min</pre>	a22 = -i/h - g/h
% Y' = -a11(Y1) + a12(Y2)	422- 1/11 g/11
b=5;	- [ 11 10 01 00]
c=100;	A =[all al2;a21 a22]
al2=b/c	P = poly(A)
d=10;	Eig = eig(A)
f=200;	[V,D] = eig(A);
all=-d/f	V1 = V(:,1);
% Tank 2	V11 = V1/V(1,1)
% Y'= Inflow/Min - Outflow/Min	V2 = V(:,2);
% Y' = a21(Y1) + a22(Y2)	V22 = V2/V(1,2)
d=10;	% C1 + 1.5C2 = 160
f=200;	% -2C1 + C2 = 0
a21=d/f	J = [V22, V11]

```
f=200;
b = [160;0];
C = inv(J)*b
                                               a11=-d/f
% Y1 = 21.791 e^(-0.0219)t + 138.208
                                               % Tank 2
                                                % Y'= Inflow/Min - Outflow/Min
e^{(-0.2281)}t
Y2 = 6.119 e^{(-0.0219)t} - 246.12
                                               % Y' = a21(Y1) + a22(Y2)
e^{(-0.2281)t}
                                                d=100;
                                                f=200;
                                                a21=d/f
                                                q=10;
8.3.20 Run 20
                                               h=100;
% Tank 1
                                               i = 1;
% Y'= Inflow/Min - Outflow/Min
                                               a22 = -i/h-g/h
% Y' = -a11(Y1) + a12(Y2)
b=100;
                                               A = [all \ al2; a21 \ a22]
c=100;
                                                P = poly(A)
a12=b/c
                                               Eig = eig(A)
d=10;
                                               [V,D] = eig(A);
f=200;
                                               V1 = V(:,1);
a11=-d/f
                                               V11 = V1/V(1,1)
% Tank 2
                                               V2 = V(:,2);
% Y'= Inflow/Min - Outflow/Min
                                               V22 = V2/V(1,2)
% Y' = a21(Y1) + a22(Y2)
                                               % C1 + 1.5C2 = 160
d=10;
                                               % -2C1 + C2 = 0
f = 200;
                                               J = [V22, V11]
a21=d/f
                                               b = [160;0];
q=10;
                                               C = inv(J)*b
h=100;
                                               % Y1 = 4.7920 e^{(-0.5124)t} + 153.208
i = 100;
                                               e^{(-0.0976)}t
a22 = -i/h - g/h
                                                % Y2 = -5.954 e^{(-0.5124)t} + 6165.47
                                               e^{(-0.0976)}t
A = [al1 \ al2; a21 \ a22]
P = poly(A)
Eig = eig(A)
                                                8.3.22 Run 22
[V,D] = eig(A);
V1 = V(:,1);
                                                % Tank 1
V11 = V1/V(1,1)
                                                % Y'= Inflow/Min - Outflow/Min
V2 = V(:,2);
                                               % Y' = -a11(Y1) + a12(Y2)
V22 = V2/V(1,2)
                                               b=2i
% C1 + 1.5C2 = 160
                                               c=100;
% -2C1 + C2 = 0
                                               a12=b/c
J = [V22, V11]
                                               d=100;
b = [160;0];
                                               f=200;
C = inv(J)*b
                                               a11=-d/f
% Y1 = 6.3979 e^(-0.0044)t + 153.6
                                               % Tank 2
e^{(-1.1456)}t
                                               % Y'= Inflow/Min - Outflow/Min
% Y2 = 0.2917 e^(-0.0044)t - 168.28
                                               % Y' = a21(Y1) + a22(Y2)
e^{(-1.1456)}t
                                               d=100;
                                                f=200;
                                                a21=d/f
                                               q=10;
8.3.21 Run 21
                                               h=100;
% Tank 1
                                               i = 2;
% Y'= Inflow/Min - Outflow/Min
                                               a22 = -i/h-g/h
% Y' = -a11(Y1) + a12(Y2)
b=1;
                                               A = [a11 \ a12; a21 \ a22]
c=100;
                                                P = poly(A)
a12=b/c
                                                Eig = eig(A)
d=100;
```

```
[V,D] = eig(A);
                                               8.3.24 Run 24
V1 = V(:,1);
                                               % Tank 1
V11 = V1/V(1,1)
                                               % Y'= Inflow/Min - Outflow/Min
V2 = V(:,2);
                                               % Y' = -a11(Y1) + a12(Y2)
V22 = V2/V(1,2)
                                              b=10;
% C1 + 1.5C2 = 160
                                               c=100;
% -2C1 + C2 = 0
                                               a12=b/c
J = [V22, V11]
                                               d=100;
b = [160;0];
                                              f=200;
C = inv(J)*b
                                              all=-d/f
% Y1 = 9.2063 e^{(-0.5247)t} + 150.79
                                              % Tank 2
e^{(-0.0953)t}
                                              % Y'= Inflow/Min - Outflow/Min
% Y2 = -7.9708 e^(-0.5247)t + 3051.31
                                              % Y' = a21(Y1) + a22(Y2)
e^{(-0.0953)t}
                                               d=100;
                                               f=200;
                                               a21=d/f
8.3.23 Run 23
                                               q = 10;
                                              h=100;
% Tank 1
                                              i = 10;
% Y'= Inflow/Min - Outflow/Min
                                               a22 = -i/h-g/h
% Y' = -a11(Y1) + a12(Y2)
b=5;
                                               A = [a11 \ a12; a21 \ a22]
c=100;
a12=b/c
                                               P = poly(A)
d=100;
                                               Eig = eig(A)
                                               [V,D] = eig(A);
f = 200;
                                               V1 = V(:,1);
a11=-d/f
                                               V11 = V1/V(1,1)
% Tank 2
% Y'= Inflow/Min - Outflow/Min
                                               V2 = V(:,2);
% Y' = a21(Y1) + a22(Y2)
                                               V22 = V2/V(1,2)
                                               % C1 + 1.5C2 = 160
d=100;
                                               % -2C1 + C2 = 0
f = 200;
                                               J = [V22, V11]
a21=d/f
                                               b = [160;0];
g = 10;
                                              C = inv(J)*b
h=100;
                                              % Y1 = 35.43 e^(-0.6193)t + 124.56
i = 5;
                                              e^{(-0.0807)t}
a22 = -i/h-g/h
                                               Y2 = -42.25 e^{(-0.6193)t} + 522.23
                                               e^{(-0.0807)}t
A = [a11 \ a12; a21 \ a22]
P = poly(A)
Eig = eig(A)
                                               8.3.25 Run 25
[V,D] = eig(A);
                                               % Tank 1
V1 = V(:,1);
                                              % Y'= Inflow/Min - Outflow/Min
V11 = V1/V(1,1)
                                              % Y' = -a11(Y1) + a12(Y2)
V2 = V(:,2);
                                               b=100;
V22 = V2/V(1,2)
                                               c=100;
% C1 + 1.5C2 = 160
                                               a12=b/c
% -2C1 + C2 = 0
                                               d=100;
J = [V22, V11]
                                              f=200;
b = [160;0];
                                              a11=-d/f
C = inv(J)*b
                                              % Tank 2
% Y1 = 20.64 e^{(-0.5608)t} + 139.35
                                             % Y'= Inflow/Min - Outflow/Min
e^{(-0.0892)}t
                                              % Y' = a21(Y1) + a22(Y2)
% Y2 = -25.11 e^(-0.5608)t + 1145.03
                                               d=100;
e^{(-0.0892)t}
                                               f=200;
                                               a21=d/f
                                               g = 10;
```

```
h=100;
i = 100;
a22 = -i/h-g/h
A = [a11 \ a12; a21 \ a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
% C1 + 1.5C2 = 160
% -2C1 + C2 = 0
J = [V22, V11]
b = [160;0];
C = inv(J)*b
% Y1 = 48.754 e^(-0.0319)t + 111.24
e^{(-1.5681)t}
% Y2 = 22.824 e^(-0.0319)t - 118.81
e^{(-1.5681)t}
```

## 8.4 Part 1 Matlab Code (Including Plots)

#### 8.4.1 Run 1

```
clear
clear all
clc
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y' = -a11(Y1) + a12(Y2)
b=6;
c=100;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y' = a21(Y1) + a22(Y2)
d=16;
f = 200;
a21=d/f
q=10;
h=100;
i= 6;
a22 = -i/h-g/h
A = [al1 \ al2; a21 \ a22]
P = poly(A)
roots(P)
% Eigenvalues
Eigenvalues = eig(A)
```

```
[V,D] = eig(A);
 V1 = V(:,1);
 V11 = V1/V(1,1);
 V2 = V(:,2);
 V22 = V2/V(1,2);
 Vectors = table(V22,V11)
 % Displaying Eigenvalues and
 Eigenvectors in a table
K = table (Eigenvalues, V22, V11)
% C1 + 1.5C2 = 160
% -2C1 + C2 = 0
 J = [V22, V11]
 b = [160;0];
 % Solving for Constants
C = inv(J)*b
% Y1 = 40 e^(-0.2)t + 120 e^(-0.04)t
% Y2 = -80 e^(-0.2)t + 80 e^(-0.04)t
 % To plot y(t) we must model y(t) =
 C1(y1)+C2(y2)
 % Where C are the constants C1 and C2
 t = [0 \ 27.5 \ 50 \ 100]';
 y = [160 \ 40.1080 \ 16.2420 \ 2.1979]';
 T = (0:27.5:50:100)';
 % Modeling y(t) = C1(y1)+C2(y2)=40
 e^{(-0.2)t} + 120 e^{(-0.04)t}
 Y1 = [\exp(-0.2*T) \exp(-0.04*T)]*C;
 \% % % Solving for T and Y
 % Modeling y(t) = C1(y1)+C2(y2) = -80
 e^{(-0.2)t} + 80 e^{(-0.04)t}
 z = [-80 80]
 Y2 = [exp(-0.2*T) exp(-0.04*T)]*z
 % Displaying Tables for T,Y1 and T2,Y2
 B = table(T, Y1)
 w = table(T, Y2)
 plot(T,Y1,'-',t,y,'o',T,Y2,'r')
 title('y(t) vs. Time (Minutes)')
 xlabel('Time (Minutes)')
 ylabel('y(t)')
 grid
```

% Eigenvectors

## 8.5 Part 2 Matlab Code (Including Plots)

## 8.5.1 Run 1

```
clear
clear all
clc
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -all(Y1)+ al2(Y2)
```

```
b=6;
                                                plot(T,Y1,'-',t,y,'o',T,Y2,'r')
c = 20;
                                                title('y(t) vs. Time (Minutes)')
a12=b/c
                                                xlabel('Time (Minutes)')
d=16;
                                                ylabel('y(t)')
f = 50;
                                                grid
a11=-d/f
% Tank 2
                                                8.6 Part 3 Matlab Code (Including Plots)
% Y'= Inflow/Min - Outflow/Min
% Y' = a21(Y1) + a22(Y2)
                                                8.6.1 Run 1
d=16;
f = 50;
a21=d/f
                                                % Tank 1
q=10;
                                                % Y'= Inflow/Min - Outflow/Min
h=20;
                                                % Y' = -a11(Y1) + a12(Y2)
i = 6;
                                                b=1;
a22 = -i/h-g/h
                                                c=100;
                                                a12=b/c
A = [a11 \ a12; a21 \ a22]
                                                d=1;
P = poly(A)
                                                f=200;
Eig = eig(A)
                                                a11=-d/f
[V,D] = eig(A);
                                                % Tank 2
V1 = V(:,1);
                                                % Y'= Inflow/Min - Outflow/Min
V11 = V1/V(1,1)
                                                % Y' = a21(Y1) + a22(Y2)
V2 = V(:,2);
                                                d=1;
V22 = V2/V(1,2)
                                                f = 200;
% C1 + 1.5C2 = 160
                                                a21=d/f
% -2C1 + C2 = 0
                                                q = 10;
J = [V22, V11]
                                                h=100;
b = [160;0];
                                                i = 1;
C = inv(J)*b
                                                a22 = -i/h-g/h
% Y1 = 31.01 e^{(-0.1681)t} + 128.98
e^{(-0.9519)t}
                                                A = [al1 \ al2; a21 \ a22]
% Y2 = 15.70 e^(-0.1681)t - 271.70
                                                P = poly(A)
e^{(-0.9519)}t
                                                Eig = eig(A)
                                                [V,D] = eig(A);
% To plot y(t) we must model y(t) =
                                                V1 = V(:,1);
C1(y1)+C2(y2)
                                                V11 = V1/V(1,1)
% Where C are the constants C1 and C2
                                                V2 = V(:,2);
t = [0 27.5 50 100]';
                                                V22 = V2/V(1,2)
y = [160 \quad 0.33142 \quad 0.0069384 \quad 1.5524e -
                                                % C1 + 1.5C2 = 160
06]';
                                                % -2C1 + C2 = 0
T = (0:27.5:50:100)';
                                                J = [V22, V11]
% Modeling y(t) = C1(y1)+C2(y2)=
                                                b = [160;0];
31.01 e^(-0.1681)t + 128.98 e^(-
                                                C = inv(J)*b
0.9519)t
                                                % Y1 = 0.7159 e^{(-0.0045)t} + 159.284
Y1 = [\exp(-0.1681*T) \exp(-
                                                e^{(-0.1103)t}
0.9519*T)]*C;
                                                % Y2 = 0.0339 e^(-0.0045)t - 1680.03
% % % Solving for T and Y
                                                e^{(-0.1103)}t
% Modeling y(t) = C1(y1)+C2(y2)=
15.70 e^{(-0.1681)}t - 271.70 e^{(-0.1681)}t
0.9519)t
                                                % To plot y(t) we must model y(t) =
z = [15.70 -271.70]
                                                C1(y1)+C2(y2)
Y2 = [\exp(-0.1681*T) \exp(-0.9519*T)]*z
                                                % Where C are the constants C1 and C2
% Displaying Tables
                                                t = [0 \ 27.5 \ 50 \ 100]';
B = table(T, Y1)
                                                y = [160 \ 8.7399 \ 1.2129 \ 0.45906]';
w = table(T, Y2);
                                                T = (0:27.5:50:100)';
```

```
Modeling y(t) = C1(y1)+C2(y2)= 40
e^{(-0.2)t} + 120 e^{(-0.04)t}
Y1 = [exp(-0.0045*T) exp(-
0.1103*T)]*C;
% % % Solving for T and Y
Modeling y(t) = C1(y1)+C2(y2) = -80
e^{(-0.2)t} + 80 e^{(-0.04)t}
z = [0.0339 -1680.03]
Y2 = [\exp(-0.0045*T) \exp(-0.1103*T)]*z
% Displaying Tables
B = table(T, Y1)
w = table(T, Y2)
plot(T,Y1,'-',t,y,'o',T,Y2,'r')
xlabel('Time (Mins)')
ylabel(' y(t)')
grid
```