California State University, Los Angeles

ME 409 – Mechanical Engineering Analysis

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1. Abstract

The purpose of this study is to analyze a non-homogenous system. To better understand this, the approach for calculating our results will be by building a mathematical model with a programming aid such as Matlab. This mathematical model is used towards the non-homogenous system which in this case is known as a mixture problem with two separate tanks. The method used in this study for calculating is known as the Method of Undetermined Coefficients. The findings reveal different equations or solutions when using the method of Undetermined Coefficients when changing the values of Tank 1 and Tank 2. Furthermore, a total of 50 combinations were made to fully understand the methods used and determining the amounts of salt for both Tank 1 and Tank2.

Nomenclatures:

Greek Symbol	Meaning	Unit
λ	Scalar (Real or Complex Number)	–

2. Introduction

One of the main applications of differential equations in general is known as Modeling. Modeling is the process of writing a differential equation to describe a physical situation that is occurring. Most of the differential equations that are used in industry for engineers are there because somebody, at some point modeled a situation and came up with the differential equation that we are using. There are times when the modeling problem is a non-homogenous system, enabling the undetermined coefficients method to be used when calculating the differential equation.

2.1 Overview and Purpose

In the modeling system problem that we will be solving starts with a substance that is being dissolved in a liquid. This liquid entering the tank may or may not contain more of the substance that is being dissolved in it. The liquid leaving the tank will contain the substance dissolved in it. We know that if *Q(t)* gives the amount of the substance that is being dissolved in the liquid in the tank at any time *t* we want to develop a differential equation that when it is solved, it will give us an expression for *Q(t)*.

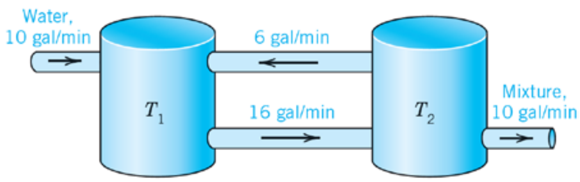
The main assumption that we are making in this problem is that the concentration of the substance in the liquid is uniform throughout the tank. Taking into consideration that there are two tanks that are being interconnected with liquid while potentially entering both and with an exit.

Our team has made the following assumptions for this situation involving two tanks. First of all, the inflow and outflow from each tank are equal, or in a simpler detail the volume in each tank is constant. Next, the concentration of contaminate in each tank will be the same at each point in the tank. Also, the concentration of contaminate in the outflow of the tank 1 will be the same as the concentration in tank 1, while the concentration of contaminate in the outflow from tank 2 will be the same as the concentration in tank 2. In addition, the outflow from tank 1 will be exiting tank 1 and reaching tank 2. The total liquid will be exiting the system completely.

As mentioned before, our team is dealing with a system containing two tanks. The first tank initially contains 200 gallons of water with 160 lb of salt within being dissolved. The second tank initially contains 100 gallons of pure water while the liquid is pumped throughout the system and the mixtures are kept uniform while being stirred. Our team proposed the idea for calculating the solution with the method of undetermined coefficients for this non-homogenous system.

2.2 Statement of the Problem

1.) Tank $T1$ in Fig. 101 initially contains 200 gal of water in which 160 lb of salt are dissolved. Tank $T2$ initially contains 100 gal of pure water. Liquid is pumped through the system as indicated, and the mixtures are kept uniform by stirring. Find the amounts of salt $y1(t)$ and $y2(t)$ in $T1$ and $T2$, respectively.



2.) Repeat the same problem for which T1 contains: 50, 100, 150, 200, 250 gal of water and T2 contains 10, 20, 50, 200, 500 gal of water. (25 combinations)

3.) Repeat the same problem if the flow from tank T1 to T2 changes from 1,2,5,10,20 gal/min and T2 to T1 1,2,5,10,100 gal/min. (25 combinations)

3. Theory and Principles

In this study, most of our linear systems will consist of two linear Ordinary Differential Equations (ODEs) in two unknown functions $y_1(t), y_2(t)$,

$$y_1' = a_{11}\, y_1 + a_{12}\, y_2$$

$$y_2' = a_{21}\, y_1 + a_{22}\, y_2$$

In addition, a linear system of n first-order Ordinary Differential Equations (ODEs) in n unknown functions is of the form

$$y_1' = a_{11}\, y_1 + a_{12}\, y_2$$

$$y_2' = a_{21}\, y_1 + a_{22}\, y_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$y_n' = a_{n1}\, y_1 + a_{n2}\, y_2 + a_{nn}\, y_n$$

3.1 Matrices

The (constant or variable) coefficients form a **2x2 matrix** $\boldsymbol{A}$, that are,

an array

$$A = [a_{jk}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

In addition, the coefficients in the equation form an ***n*** **matrix**

$$A = [a_{jk}] \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & & a \\ a_{n1} & K_{n2} & \dots & a_{nn} \end{bmatrix}$$

The $a_{11}, a_{12}, \cdots$ are called **entries**, the horizontal lines **rows**, and the vertical lines **columns**. Thus, in the equation, the first row is $[\, a_{11}\; a_{12}]$, the second row is $[\, a_{21}\; a_{22}]$ and the first and second columns are

$$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} \text{ and } \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}.$$

In the "***double subscript notation***" for various entries, the first subscript is that one that denotes the *row* and the second the *column* in which the entry stands. Similarly in the previous equation. The **main diagonal** is the diagonal $a_{11}\; a_{12} \cdots a_{nn}$, hence $a_{11}\; a_{12}$.

We shall need only **square matrices**, that is, matrices with the same number of rows and columns, as in the previous equations in this project.

3.2 Vectors

A **column vector** $\boldsymbol{x}$ with n **components** such as these $x_1 \cdots, x_n$ is of the form

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \qquad \text{thus if } n = 2, \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Similarly, a **row vector v** is of the following form below,

$$\mathbf{v} = [v_1 \quad \cdots \quad v_n], \;\; \text{thus if } n = 2, \qquad v = [v_1 \quad v_2]$$

3.3 Equality

Now we move on to calculations with matrices and vectors. Two $n\ x\ n$ matrices will be *equal* if and only if corresponding entries are equal as well. For example, let's take this for instance, Thus for $= 2$, let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \; and \;\; B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Then $A = B$ if and only if

$$a_{11} = b_{11}, \quad a_{12} = b_{12}$$

$$a_{21} = b_{21}, \quad a_{22} = b_{22}$$

Now, two column vectors (or two row vectors) are *equal* if and only if they both have n components and corresponding components are equal as well. For example, let

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad and \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$then\, v = x \quad if\ and\ only\ if \quad v_1 = x_1\,, v_2 = x_2$$

3.4 Addition

This is performed by adding the corresponding entries (or components). The matrices must both be $n\ x\ n$, and vectors must both have the same number of components. For example, for $= 2$,

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}, \qquad \mathbf{v} + \mathbf{x} = \begin{bmatrix} v_1 + x_1 \\ v_2 + x_2 \end{bmatrix}$$

3.5 Matrix Multiplication.

The product $\boldsymbol{C} = \boldsymbol{AB}$ **(**in this order) of two $n\ x\ n$ matrices $A = [a_{jk}]$ and $B = [b_{jk}]$ **is the $\boldsymbol{n\ x\ n}$ matrix** $C = [c_{jk}]$ with entries the following;

$$c_{jk} = \sum_{m=1}^{n} a_{jm}\, b_{mk}$$

$$j = 1, \cdots, n \quad \text{and} \quad k = 1, \cdots, n$$

Let us keep caution with the following note; Matrix multiplication is **not commutative $\mathbf{AB} \neq \mathbf{BA}$.**

Furthermore, a multiplication of an $\boldsymbol{n\ x\ n}$ matrix **A** by a vector **x** with n components is defined by the same rule: $v = Ax$ is the vector with the n components

$$v_j = \sum_{m=1}^{n} a_{jm}\, x_m \qquad\qquad j = 1, \cdots, n$$

3.6 Differentiation

Now we will move on to Systems of ODEs as Vector Equations. The *derivative* of a matrix (or vector) with variable entries (or components) is obtained by differentiating each entry (or component). By using the matrix multiplication and differentiation, we can now write the following;

$$y' = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \mathbf{Ay} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Similarly, the $\boldsymbol{n\ x\ n}$ matrix **A** and a column vector y with n components, namely, $y' = \mathbf{Ay}$ The vector equation is equivalent to two equations for the components, and these are precisely the two ODEs in the first equation.

3.7 Inverse Matrix

The $\boldsymbol{n\ x\ n}$ unit matrix I is the $\boldsymbol{n\ x\ n}$ matrix with main diagonal 1, 1,…, 1 and all of the other entries that are zero. If, for a given $\boldsymbol{n\ x\ n}$ matrix A, there is a $\boldsymbol{n\ x\ n}$ matrix B such that AB = BA = I, then A is called nonsingular and B is called the Inverse of A and is denoted by A^{-1} so the following is true;

$$AA^{-1} = A^{-1}A = I$$

The inverse will exist if the determinant det A of A is not zero. If A has not inverse, then we will call it a singular. For example is n = 2, then

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} a_{11} & -a_{12} \\ -a_{21} & a_{22} \end{bmatrix}$$

Where the determinant of A is

det A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ = $a_{11}a_{22} - a_{12}a_{21}$

3.8 Eigenvalues, Eigenvectors

This next process is quite important and useful. The Eigenvalues and eigenvectors will be very important during the process of calculating our solution.

Let $A = [a_{jk}]$ be an $\boldsymbol{n\ x\ n}$ matrix. Consider the following equation;

$$\mathbf{Ax} = \boldsymbol{\lambda}\mathbf{x}$$

where λ is a scalar (a real or complex number) to be determined and **x** is a vector to be determined. Now, for every λ, a solution is $= 0$. A scalar λ such that it holds for some vector $\mathrm{x} \neq 0$ is called an **eigenvalue** of **A**, and this vector is called an **eigenvector** of **A** corresponding to this eigenvalue λ.

$$(\mathbf{A} - \boldsymbol{\lambda}\mathbf{I})\mathbf{x} = \mathbf{0}$$

These are n linear algebraic equations in the n unknowns $x_1 \cdots, x_n$ (the components of **x**). For these equations to have a solution x $\neq 0$, the determinant of the coefficient matrix $\mathbf{A} - \boldsymbol{\lambda}\mathbf{I}$ must be zero. For example, we only need this only for $= 2$. Then the following is

$$\begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

In components,

$$(a_{11} - \lambda)x_1 + a_{12}x_2 = 0$$

$$a_{21}x_1 + (a_{22} - \lambda)x_2 = 0$$

Now, $\mathbf{A} - \boldsymbol{\lambda}\mathbf{I}$ is singular if and only if its determinant det $(\boldsymbol{A} - \boldsymbol{\lambda}\boldsymbol{I})$, called the **characteristic determinant** of **A** (also for general *n*), is zero. This will give us the following;

$$\det(\boldsymbol{A} - \boldsymbol{\lambda}\boldsymbol{I}) = \begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix}$$

$$= (a_{11} - \lambda)(a_{22} - \lambda) - a_{12}\, a_{21}$$

$$= \lambda^2(a_{11} + a_{22})\lambda + a_{11}\, a_{22} - a_{12}\, a_{21} = 0$$

This quadratic equation in λ is called the **characteristic equation** of **A**. Its solutions are the eigenvalues λ_1 and λ_2 of **A**. First determine these. Then use with $\lambda = \lambda_1$ to determine an eigenvector $\mathbf{x}^{(1)}$of **A** corresponding to λ_1 . Finally use with $\lambda = \lambda_2$ to find an eigenvector $\mathbf{x}^{(2)}$ of **A** corresponding to λ_2 . Note that if **x** is an eigenvector of **A**, so is kx with any $k \neq 0$.

3.9 Nonhomogeneous Linear Systems of ODEs

We will now move on to the situation where the linear system of the Ordinary Differential Equation is Non-homogenous. Here we will use the method for solving nonhomogeneous linear systems of ODEs;

$$y' = \mathbf{Ay} + \mathbf{g}$$

where the vector $g(t)$ is not identically zero. We assume $g(t)$ and the entries of the n x n matrix $\mathrm{A}(t)$ to be continuous on some interval $J$of the t-axis. From a general solution $y^{(h)}(t)$ of the homogeneous system $y' = \mathbf{Ay}$ on J and a **particular solution** $y^{(p)}(t)$ of $y' = \mathbf{Ay}$.

$$y = y^{(h)}(t) + y^{(p)}(t)$$

$\boldsymbol{y}$ is called a **general solution** of (1) on J because it includes every solution of $y' = \mathbf{Ay}$ on J.

3.10 Method of Undetermined Coefficients

Now for the actual use of the method. Just as for a single ODE, this method is suitable only if the entries of **A** are constants and the components of **g** are constants, positive integer powers of t, exponential functions, or cosines and sines. For instance, in such a case, a particular solution $y^{(p)}$ is assumed in a form similar to **g**; for instance, $y^{(p)} = \mathbf{u} + \mathbf{v}t + \mathbf{w}t^2$ if **g** has components quadratic in t, with **u**, **v**, **w** to be determined by substitution into $y' = \mathbf{Ay} + \mathbf{g}$.

4. Data Analysis

4.1. Analysis of Calculation and Tables

4.1.1 Sample Calculation using Method of Undetermined Coefficients for Original Problem

The following calculation demonstrates a sample calculation or the original problem in finding the equations to determine the amount of salt in Tank 1 and Tank 2. This was done by hand before using the aid of Matlab to see if the solution would match. Verification was very crucial for our team. Here is our sample calculation:

Given:

$T_1 = 200\ gal$, $160\ lb\ salt$, Inflow= $6\ gal/\mathrm{min}$, $\mathrm{Outflow} = 16\ gal/min$

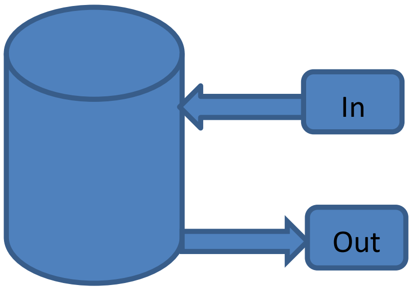
$T_2 = 100\ gal$, $pure\ water$, Inflow= $16\ gal/\mathrm{min}$, $\mathrm{Outflow} = 10\ gal/min$

Required:

$$y_1(t)\ and\ y_2(t)\ in\ T_1\ and\ T_2$$

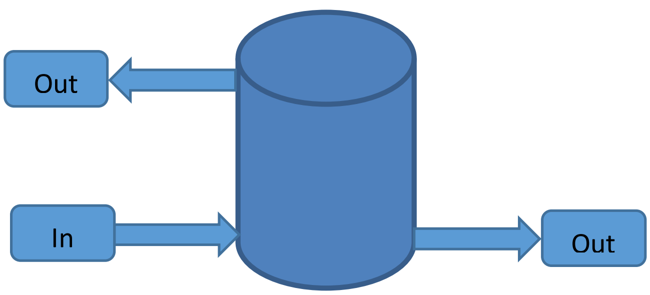
Solution:

Free Body Diagram (FBD):



$$y_1' = \frac{Inflow}{min} - \frac{Outflow}{min} = \frac{6}{100}y_2 - \frac{16}{200}y_1$$
$$= -0.08y_1 + 0.06\,y_2$$

Free Body Diagram (FBD):



$$y_2' = \frac{Inflow}{min} - \frac{Outflow}{min} = \frac{16}{200}y_2 - \frac{6}{200}y_1 - \frac{10}{200}y_1$$
$$= -0.08y_1 + 0.06\,y_2$$

$$y' = \mathbf{Ay}$$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} -0.08 & 0.06 \\ 0.08 & -0.16 \end{bmatrix}$$

$$(\mathbf{A} - \boldsymbol{\lambda}\mathbf{I}) = \mathbf{0}$$

$$\det(\boldsymbol{A} - \boldsymbol{\lambda I}) = \begin{bmatrix} -0.08-\lambda & 0.06 \\ 0.08 & -0.16-\lambda \end{bmatrix}$$

Eigenvalues

$$(-0.08-\lambda)(-0.16-\lambda) - (0.08)(0.06)$$
$$= 0.128 + 0.08\lambda + 0.16\lambda + \lambda^2$$
$$- 0.048$$

$$\lambda^2 + 0.24\lambda + 0.008$$

$$\lambda = \frac{(-0.24) \pm \sqrt{(0.24)^2 - 4(1)(0.008)}}{2(1)}$$
$$= \frac{(-0.24) \pm \sqrt{0.0256}}{2(1)}$$
$$= \frac{(-0.24) \pm 0.16}{2(1)}$$

$$\lambda = -0.2 \;and\; \lambda = -0.4$$

Eigenvectors for $\lambda = -0.2$

$$\begin{bmatrix} -0.08-(-0.2) & 0.06 \\ 0.08 & -0.16-(-0.2) \end{bmatrix}$$

$$\begin{bmatrix} -0.08 & 0.06 \\ 0.08 & -0.16 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -0.08 & 0.06 & 0 \\ 0.08 & -0.16 & 0 \end{array}\right] -> \begin{bmatrix} 0.12 & 0.06 \\ 0.0696 & 0.0448 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$0.12x_1 + 0.06x_2 = 0$$

$$\frac{0.12}{0.12}x_1 = \frac{0.06}{0.12}x_2$$

$$x_1 = -0.5\,x_2 \qquad x_1 = 1$$

$$\frac{1}{-0.5} = \frac{-0.5}{-0.5}x_2 \qquad x_2 = -2$$

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix} \; for\; \lambda = -0.2$$

Eigenvectors for $\lambda = -0.04$

$$\begin{bmatrix} -0.08-(-0.04) & 0.06 \\ 0.08 & -0.16-(-0.04) \end{bmatrix}$$

$$\begin{bmatrix} -0.04 & 0.06 \\ 0.08 & -0.12 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -0.04 & 0.06 & 0 \\ 0.08 & -0.12 & 0 \end{array}\right] -> \begin{bmatrix} -0.0635 & -0.1152 \\ 0.08 & -0.12 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$0.8x_1 - 0.12x_2 = 0$$

$$\frac{0.08}{0.08}x_1 = \frac{0.12}{0.08}x_2$$

$$x_1 = 1.5\,x_2 \qquad x_2 = 1$$

$$x_1 = 1.5(1) \qquad x_1 = 1.5$$

$$\begin{bmatrix} 1.5 \\ 1 \end{bmatrix} \; for\; \lambda = -0.04$$

$$y = c_1 x^{(1)} e^{\lambda_1 t} + c_2 x^{(2)} e^{\lambda_2 t}$$

$$y = c_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} e^{\lambda_2 t}$$

$$y = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-0.2t} + c_2 \begin{bmatrix} 1.5 \\ 1 \end{bmatrix} e^{-0.04t}$$

$$\begin{bmatrix} 160 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$$

$$160 = c_1 + 1.5c_2$$

$$0 = -2c_1 + c_2$$

$$c_2 = 2c_1$$

$$160 = c_1 + 1.5(2c_1)$$

$$\frac{160}{4} = \frac{4}{4} c_1$$

$$c_1 = 40$$

$$0 = -2c_1 + c_2$$

$$0 = -2(40) + c_2$$

$$c_2 = 80$$

$$y_1 = 40e^{-0.2t} + 120e^{-0.04t}$$

$$y_2 = -80e^{-0.2t} + 80e^{-0.04t}$$

4.1.2 Sample Table Data for Part 1

Eigenvalues	**Vector V11**	**Vector V22**	**Constants for y1**	**Constants for y2**
-0.04	1	1	40	-80
-0.2	-2	0.666666667	120	80
T(min)	y1		T(min)	y2
0	0		0	160
1	11.36469489		1	148.0439628
2	20.22370403		2	137.5867634
3	27.04870405		3	128.3829178
4	32.22518599		4	120.2304132
5	36.06810495		5	112.962868
6	38.83469193		6	106.4431118
7	40.7349422		7	100.5579275
8	41.94020153		8	95.21374517
9	42.59019503		9	90.33311466
10	42.79878102		10	85.85181685
11	42.65866102		11	81.71649686
12	42.24523508		12	77.88272515
13	41.61975758		13	74.31340888
14	40.8319201		14	70.97749017
15	39.92196542		15	67.84887907
16	38.92241761		16	64.90557904
17	37.85949779		17	62.12896988
18	36.75428268		18	59.50321961
19	35.62365241		19	57.01480212
20	34.48106602		20	54.65210125
21	33.33719573		21	52.40508588
22	32.20044574		22	50.265043
23	31.07737643		23	48.22435836
24	29.97305111		24	46.2763362
25	28.89131953		25	44.41505082
26	27.8350494		26	42.63522441
27	26.80631558		27	40.93212632
28	25.80655447		28	39.3014899

29	24.83669009		29	37.7394439
30	23.89723678		30	36.24245552
31	22.98838298		31	34.80728338
32	22.11005945		32	33.43093835
33	21.26199471		33	32.11065096
34	20.44376014		34	30.84384424
35	19.65480656		35	29.62811095
36	18.89449383		36	28.46119447
37	18.16211485		37	27.34097272
38	17.45691484		38	26.26544449
39	16.7781069		39	25.23271794
40	16.12488443		40	24.24100066
41	15.4964311		41	23.28859122
42	14.8919287		42	22.37387182
43	14.31056337		43	21.49530198
44	13.75153046		44	20.65141298
45	13.21403827		45	19.84080298
46	12.69731094		46	19.06213271
47	12.20059054		47	18.31412165
48	11.72313867		48	17.59554461
49	11.26423755		49	16.90522857
50	10.82319066		50	16.24204999
51	10.39932324		51	15.60493212
52	9.991982377		52	14.99284276
53	9.600537		53	14.40479206

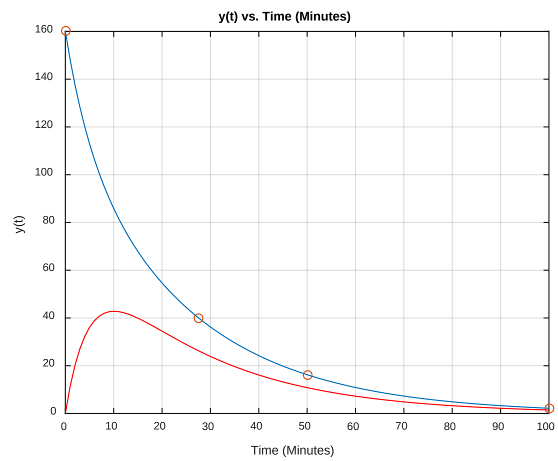
54	9.224377723		54	13.8398305
55	8.862916533		55	13.29704707
56	8.515586415		56	12.77556749
57	8.181840898		57	12.27455263
58	7.861153561		58	11.79319692
59	7.553017491		59	11.33072697
60	7.256944726		60	10.88640016
61	6.972465681		61	10.45950339
62	6.69912856		62	10.04935181
63	6.436498779		63	9.655287691
64	6.184158374		64	9.276679284
65	5.941705431		65	8.912919799
66	5.708753516		66	8.563426371
67	5.484931121		67	8.227639104
68	5.269881115		68	7.905020151
69	5.063260218		69	7.595052828
70	4.864738488		70	7.297240776
71	4.673998813		71	7.011107148
72	4.490736436		72	6.736193836
73	4.314658476		73	6.47206073
74	4.145483483		74	6.218285006
75	3.982940997		75	5.97446044
76	3.826771123		76	5.740196757
77	3.676724128		77	5.515119
78	3.532560043		78	5.298866926

79	3.39404829		79	5.091094428
80	3.260967315		80	4.891468979
81	3.133104237		81	4.699671097
82	3.01025451		82	4.515393834
83	2.892221599		83	4.338342281
84	2.77881667		84	4.168233096
85	2.669858285		85	4.004794051
86	2.565172115		86	3.847763595
87	2.464590663		87	3.696890434
88	2.367952996		88	3.551933129
89	2.275104489		89	3.41265971
90	2.185896577		90	3.278847303
91	2.10018652		91	3.150281775
92	2.01783717		92	3.026757389
93	1.938716759		93	2.908076476
94	1.862698683		94	2.794049119
95	1.7896613		95	2.684492847
96	1.719487741		96	2.579232345
97	1.652065714		97	2.478099172
98	1.587287334		98	2.380931492
99	1.525048942		99	2.287573816
100	1.465250946		100	2.197876749

4.2 Data Analysis Plot Graphs

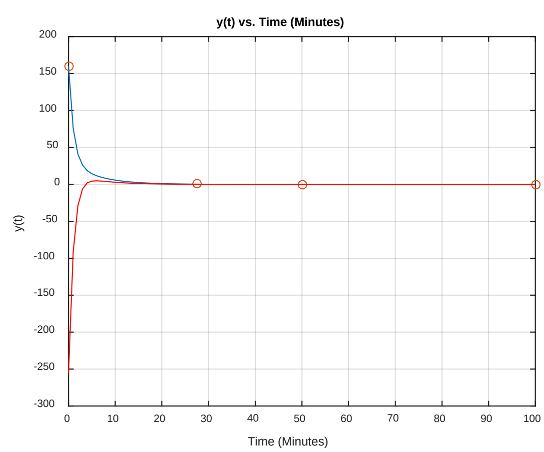
The following analysis was made with the aid of Matlab programming. The following graphs demonstrate the mathematical model for the differential equation. The differential equation was then plotted to demonstrate the function over time. The team began with graphing a plot for the Part 1. Following up with the modeling system, the graphs were then made for Part 2 and Part 3 to demonstrate the change of gallons of water and change of flow of the water, respectively. The following graphs show the complete run for Part A since it was only 1 run. Next, the graphs for Part 2 and Part 3 only show for the first 5 runs, respectively. This was done to save time and demonstrate how the function will be changing over time.

4.2.1 Graph for Part 1

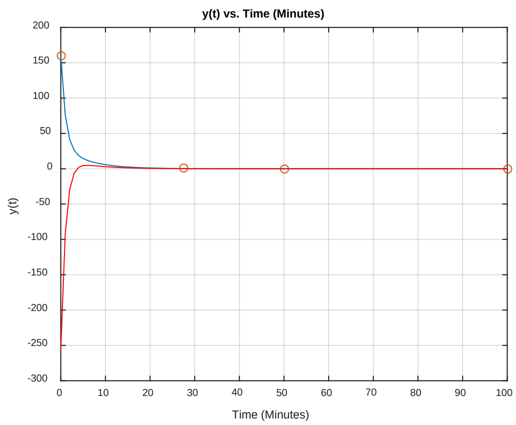


Graph 1: Part 1 1st Run

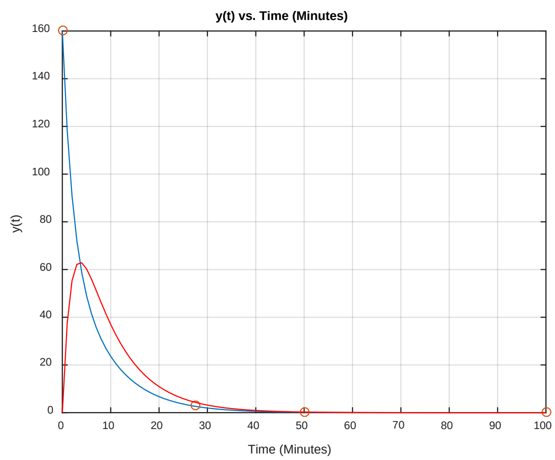
4.2.2 Graphs for Part 2



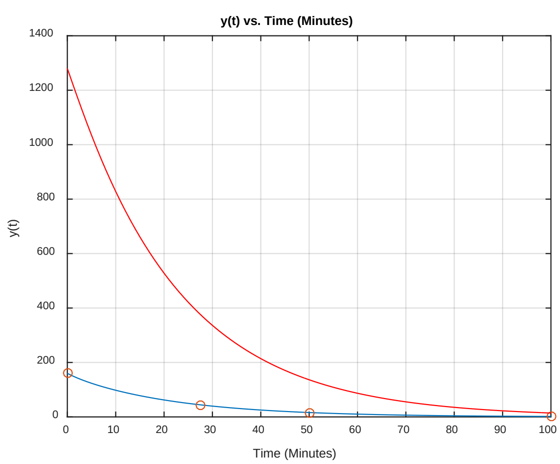
Graph 2: Part 2 1st Run



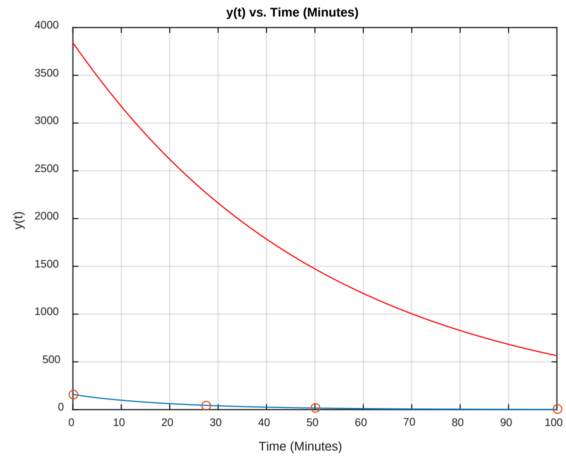
Graph 3: Part 2 2nd Run



Graph 4: Part 2 3rd Run

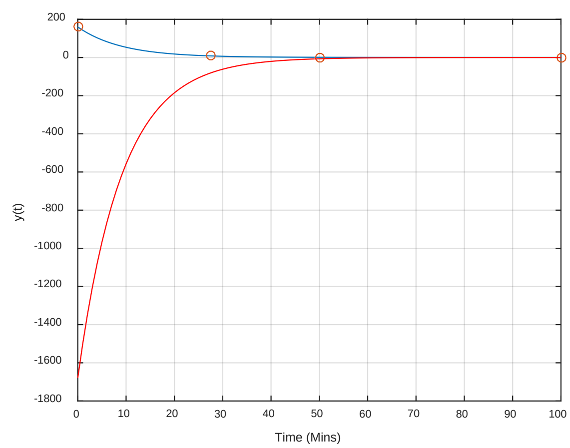


Graph 5: Part 2 4th Run

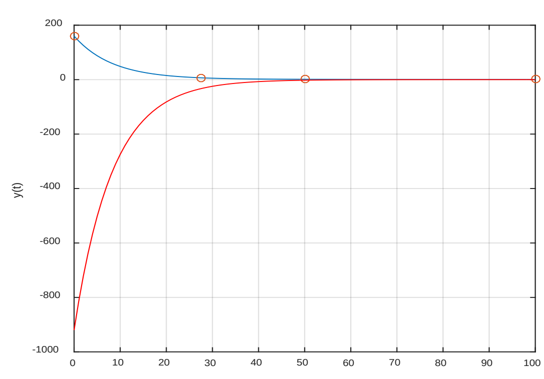


Graph 6: Part 2 5th Run

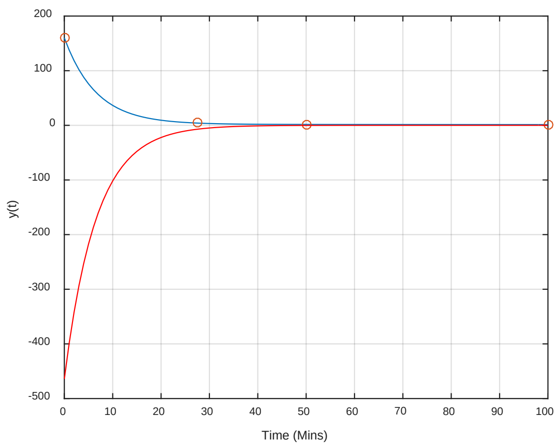
4.2.3 Graphs for Part 3



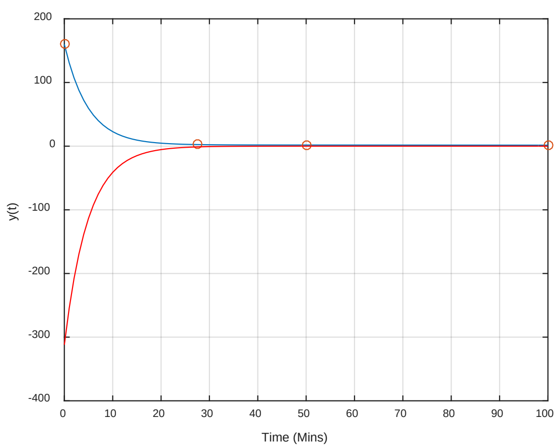
Graph 7: Part 3 1st Run



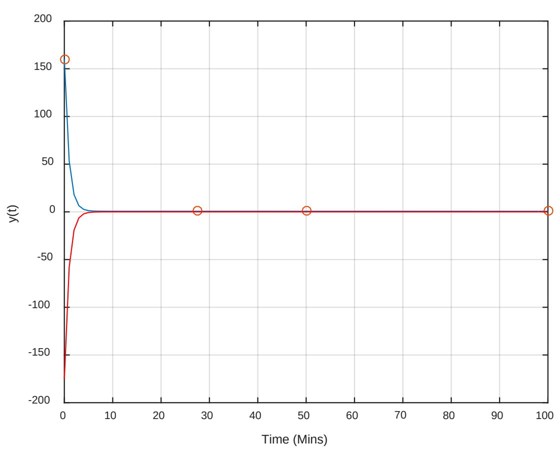
Graph 8: Part 3 2nd Run



Graph 9: Part 3 3rd Run



Graph 10: Part 3 4th Run



Graph 11: Part 3 5th Run

5. Results and Discussion

Our team managed to compute a differential equation model for the non-homogeneous system involving two tanks. The mixture problem was an ideal situation for our understanding of modeling systems. The eigenvectors and eigenvalues were shown to be quite different when solving each mathematical model. For Part 1, this was basic due to the fact that we had everything given in terms of what we needed to solve the non-homogenous system. For Part 2, we had to change the gallons for the tank1 and tank 2. When this was applied, the values were prompted to change. This concept was also seen in Part 3, when the flow of water was changed for tank 1 and tank 2. When the flow was changed as well, the values given to us were quite different as well. Each graph shows a differential equation for tank 1 and tank 2. The graphs show the differences between times for each tank and also it displays differences of each. At some point, both differential equations will intersect at a neutral point in which shows the mixture of liquid and a substance (salt) combine into one and thus revealing a mixture flow throughout the function. Overall, the graphs give us an understanding where the mixture is occurring and conclude this understanding of non-homogenous system. The results give us a great ideal of information to see how this non-homogenous system behaves as a mathematical model in a differential equation.

6. Conclusion

This cumulative study on modeling a differential equation for a non-homogeneous system involving a mixture of two tanks was a great opportunity to broaden our understanding. This situation gave us a broad idea of what modeling differential equation is like and how to show a functional mathematical model with the aid of programming such as Matlab. The values show different outcomes when the differential equation for the modeling system were created. This project allowed us to see how the differential equations behave when plotted and helps us view what is occurring over time. The overall task was completed for the first part with a great understanding. The team then proceeded to repeat the same problem when T1 contained different values such as 50, 100, 150, 200, 250 gal of water and while T2 contained 10, 20, 50, 200, 500 gal of water. This allowed us to see different situations for our modeling systems. Once this was completed, the team repeated the same problem when the flow from tank T1 to T2 changes with different values such as 1,2,5,10,20 gal/min and while T2 to T1 with the following values 1, 2,5,10,100 gal/min. We did a total of 25 combinations for both the second and third parts of this project giving us a total of 50 combinations. Overall, the study of modeling a non-homogenous system was enlightening and can be concurrently applied practically in research and design within the engineering industry.

7. References

[1] Kreyszig, Erwin. "Systems of ODEs. Phase Plane. Qualitative Methods." Advanced Engineering Mathematics. 10th ed. Hoboken, New Jersey: John Wiley & Sons Inc., 2011. 124-165. Print.

[2] "Differential Equations - Modeling." *Differential Equations - Modeling*. Paul's Online Math Notes, n.d. Web. 04 June 2016. <http://tutorial.math.lamar.edu/Classes/DE/SystemsModeling.aspx>.

[3] "Differential Equations - Undetermined Coefficients." *Differential Equations - Undetermined Coefficients*. Paul's Online Math Notes, n.d. Web. 04 June 2016. <http://tutorial.math.lamar.edu/Classes/DE/UndeterminedCoefficients.aspx>.

[4] "Differential Equations - Modeling with First Order DE's." *Differential Equations - Modeling with First Order DE's*. Paul's Online Math Notes, n.d. Web. 04 June 2016. <http://tutorial.math.lamar.edu/Classes/DE/Modeling.aspx>.

8. Appendix

8.1 Part 1 Matlab Code

8.1.1 Run 1

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=100;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=200;
a21=d/f
g=10;
h=100;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 40 e^(-0.04)t + 120 e^(-0.2)t
%  Y2 = 26.68 e^(-0.04)t - 240 e^(-
0.2)t
```

8.2 Part 2 Matlab Code

8.2.1 Run 1

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=10;
a12=b/c
d=16;
f=50;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=50;
a21=d/f
g=10;
h=10;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 13.98 e^(-0.1844)t + 146.01
e^(-1.7356)t
%  Y2 = 3.1615 e^(-0.1844)t - 344.49
e^(-1.7356)t
```

8.2.2 Run 2

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=20;
a12=b/c
d=16;
f=50;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=50;
a21=d/f
g=10;
h=20;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
```

```
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 31.01 e^(-0.1681)t + 128.98
e^(-0.9519)t
%  Y2 = 15.70 e^(-0.1681)t – 271.70
e^(-0.9519)t
```

8.2.3 Run 3

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=50;
a12=b/c
d=16;
f=50;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=50;
a21=d/f
g=10;
h=50;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 80 e^(-0.124)t + 80 e^(-
0.516)t
%  Y2 = 130.4 e^(-0.124)t - 130.4 e^(-
0.516)t
```

8.2.4 Run 4

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=200;
a12=b/c
d=16;
f=50;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=50;
a21=d/f
g=10;
h=200;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 18.0323 e^(-0.3549)t +
141.9677 e^(-0.0451)t
%  Y2 = -20.98 e^(-0.3549)t + 1300.99
e^(-0.0451)t
```

8.2.5 Run 5

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=500;
a12=b/c
d=16;
f=50;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=50;
a21=d/f
g=10;
h=500;
i= 6;
a22= -i/h-g/h
```

```
A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 6.5153 e^(-0.3328)t + 153.487
e^(-0.0192)t
%  Y2 = -6.9316 e^(-0.3328)t + 3846.92
e^(-0.0192)t
```

8.2.6 Run 6

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=10;
a12=b/c
d=16;
f=100;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=100;
a21=d/f
g=10;
h=10;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 6.5153 e^(-0.0962)t + 153.4847
e^(-1.6638)t
%  Y2 = 0.6932 e^(-0.0962)t - 384.69
e^(-1.6638)t
```

8.2.7 Run 7

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=20;
a12=b/c
d=16;
f=100;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=100;
a21=d/f
g=10;
h=20;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 13.98 e^(-0.0922)t + 146.01
e^(-0.8678)t
%  Y2 = 3.1579 e^(-0.0922)t - 344.49
e^(-0.8678)t
```

8.2.8 Run 8

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=50;
a12=b/c
d=16;
f=100;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=100;
```

```
a21=d/f
g=10;
h=50;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 40 e^(-0.08)t + 120 e^(-0.40)t
%  Y2 = 26.68 e^(-0.08)t - 240 e^(-
0.40)t
```

8.2.9 Run 9

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=200;
a12=b/c
d=16;
f=100;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=100;
a21=d/f
g=10;
h=200;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 40 e^(-0.2)t + 120 e^(-0.04)t
%  Y2 = -53.32 e^(-0.2)t + 480 e^(-
0.04)t
```

8.2.10 Run 10

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=500;
a12=b/c
d=16;
f=100;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=100;
a21=d/f
g=10;
h=500;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 13.98 e^(-0.1736)t + 120 e^(-
0.0184)t
%  Y2 = -15.80 e^(-0.1736)t + 1722.46
e^(-0.0184)t
```

8.2.11 Run 11

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=10;
a12=b/c
d=16;
```

```
f=150;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=150;
a21=d/f
g=10;
h=10;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 4.23 e^(-0.065)t + 153.76 e^(-
1.64)t
%  Y2 = 0.294 e^(-0.065)t - 393.40
e^(-1.64)t
```

8.2.12 Run 12

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=20;
a12=b/c
d=16;
f=150;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=150;
a21=d/f
g=10;
h=20;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 8.9071 e^(-0.0632)t + 151.0929
e^(-0.8434)t
%  Y2 = 1.291 e^(-0.0632)t - 371.0690
e^(-0.8434)t
```

8.2.13 Run 13

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=50;
a12=b/c
d=16;
f=150;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=150;
a21=d/f
g=10;
h=50;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 25.1205 e^(-0.0578)t +
134.8795 e^(-0.3688)t
%  Y2 = 10.221 e^(-0.0578)t – 294.67
e^(-0.3688)t
```

8.2.14 Run 14

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=200;
a12=b/c
d=16;
f=150;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=150;
a21=d/f
g=10;
h=200;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 61.6467 e^(-0.1515)t + 98.3533
e^(-0.0352)t
%  Y2 = -92.026 e^(-0.1515)t + 234.248
e^(-0.0352)t
```

8.2.15 Run 15

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=500;
a12=b/c
d=16;
f=150;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=150;
a21=d/f
g=10;
h=500;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 22.2404 e^(-0.1210)t +
137.7596 e^(-0.0176)t
%  Y2 = -26.641 e^(-0.1210)t + 1022.2
e^(-0.0176)t
```

8.2.16 Run 16

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=10;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=200;
a21=d/f
g=10;
h=10;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
```

```
b = [160;0];
C = inv(J)*b
%  Y1 = 3.1303 e^(-0.0491)t + 156.8693
e^(-1.6309)t
%  Y2 = 0.1615 e^(-0.0491)t – 405.4924
e^(-1.6309)t
```

8.2.17 Run 17

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=20;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=200;
a21=d/f
g=10;
h=20;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 6.1557 e^(-0.0481)t + 153.4847
e^(-0.8319)t
%  Y2 = 0.6932 e^(-0.0481)t - 384.69
e^(-0.8319)t
```

8.2.18 Run 18

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=50;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=200;
a21=d/f
g=10;
h=50;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 18.0323 e^(-0.0451)t +
141.9677 e^(-0.3549)t
%  Y2 = 5.2473 e^(-0.-0451)t - 325.24
e^(-0.3549)t
```

8.2.19 Run 19

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=200;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=200;
a21=d/f
g=10;
h=200;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
```

```
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 80 e^(-0.0310)t + 80 e^(-
0.1290)t
%  Y2 = 130.64 e^(-0.0310)t - 130.64
e^(-0.1290)t
```

8.2.20 Run 20

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=500;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=200;
a21=d/f
g=10;
h=500;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 31.0102 e^(-0.0952)t + 128.98
e^(-0.0168)t
%  Y2 = -39.258 e^(-0.0952)t + 679.26
e^(-0.0168)t
```

8.2.21 Run 21

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=10;
a12=b/c
d=16;
f=250;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=250;
a21=d/f
g=10;
h=10;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 2.4835 e^(-0.0394)t + 157.5156
e^(-1.6246)t
%  Y2 = 0.1018 e^(-0.0394)t – 409.7
e^(-1.6246)t
```

8.2.22 Run 22

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=20;
a12=b/c
d=16;
f=250;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=250;
a21=d/f
```

```
g=10;
h=20;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 5.1315 e^(-0.0388)t + 154.8586
e^(-0.8252)t
%  Y2 = 0.4315 e^(-0.0388)t – 392.96
e^(-0.8252)t
```

8.2.23 Run 23

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=50;
a12=b/c
d=16;
f=250;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=250;
a21=d/f
g=10;
h=50;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 13.98 e^(-0.0369)t + 146.0110
e^(-0.3471)t
%  Y2 = 3.1594 e^(-0.0369)t + 344.498
e^(-0.3471)t
```

8.2.24 Run 24

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=200;
a12=b/c
d=16;
f=250;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=250;
a21=d/f
g=10;
h=200;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 65.6316 e^(-0.0275)t + 94.3684
e^(-0.1165)t
%  Y2 = 79.9458 e^(-0.0275)t + 165.276
e^(-0.1165)t
```

8.2.25 Run 25

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=500;
a12=b/c
d=16;
f=250;
```

```
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=250;
a21=d/f
g=10;
h=500;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 40 e^(-0.080)t + 120 e^(-
0.0160)t
%  Y2 = -53.332 e^(-0.080)t + 480 e^(-
0.0160)t
```

8.3 Part 3 Matlab Code (Flow Changes)

8.3.1 Run 1

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=1;
c=100;
a12=b/c
d=1;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=1;
f=200;
a21=d/f
g=10;
h=100;
i= 1;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 0.7159 e^(-0.0045)t + 159.284
e^(-0.1103)t
%  Y2 = 0.0339 e^(-0.0045)t - 1680.03
e^(-0.1103)t
```

8.3.2 Run 2

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=2;
c=100;
a12=b/c
d=1;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=1;
f=200;
a21=d/f
g=10;
h=100;
i= 2;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 1.1831 e^(-0.0041)t + 158.81
e^(-0.1209)t
%  Y2 = 0.0511 e^(-0.0041)t - 920 e^(-
0.1209)t
```

8.3.3 Run 3

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=5;
c=100;
a12=b/c
d=1;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=1;
f=200;
a21=d/f
g=10;
h=100;
i= 5;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 1.8377 e^(-0.0033)t + 158.16
e^(-0.1517)t
%  Y2 = 0.0626 e^(-0.0033)t - 464.06
e^(-0.1517)t
```

8.3.4 Run 4

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=10;
c=100;
a12=b/c
d=1;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=1;
f=200;
a21=d/f
g=10;
h=100;
i= 10;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 2.0244 e^(-0.0025)t + 157.97
e^(-0.2025)t
%  Y2 = 0.0512 e^(-0.0025)t - 312.049
e^(-0.2025)t
```

8.3.5 Run 5

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=100;
c=100;
a12=b/c
d=1;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=1;
f=200;
a21=d/f
g=10;
h=100;
i= 100;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
```

```
b = [160;0];
C = inv(J)*b
%  Y1 = 0.6590 e^(-0.0005)t + 159.34
e^(-1.1045)t
%  Y2 = 0.00296 e^(-0.-0005)t - 175.19
e^(-1.1045)t
```

8.3.6 Run 6

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=1;
c=100;
a12=b/c
d=2;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=2;
f=200;
a21=d/f
g=10;
h=100;
i= 1;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 158.44 e^(-0.1110)t + 1.5535
e^(-0.0090)t
%  Y2 = -1600.08 e^(-0.1110)t + 0.1537
e^(-0.0090)t
```

8.3.7 Run 7

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=2;
c=100;
a12=b/c
d=2;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=2;
f=200;
a21=d/f
g=10;
h=100;
i= 2;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 2.5203 e^(-0.0082)t + 157.47
e^(-0.1218)t
%  Y2 = 0.2253 e^(-0.0082)t - 880.23
e^(-0.1218)t
```

8.3.8 Run 8

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=5;
c=100;
a12=b/c
d=2;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=2;
f=200;
a21=d/f
g=10;
h=100;
i= 5;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
```

```
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 3.7937 e^(-0.0065)t + 156.20
e^(-0.1535)t
%  Y2 = 0.2644 e^(-0.0065)t – 448.24
e^(-0.1535)t
```

8.3.9 Run 9

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=10;
c=100;
a12=b/c
d=2;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=2;
f=200;
a21=d/f
g=10;
h=100;
i= 10;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 4.094 e^(-0.0049)t + 153.905
e^(-0.2051)t
%  Y2 = 0.2094 e^(-0.0049)t - 300.29
80 e^(-0.2051)t
```

8.3.10 Run 10

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=100;
c=100;
a12=b/c
d=2;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=2;
f=200;
a21=d/f
g=10;
h=100;
i= 100;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 1.3136 e^(-0.0009)t + 158.68
e^(-1.1091)t
%  Y2 = 0.01195 e^(-0.0009)t - 174.4
e^(-1.1091)t
```

8.3.11 Run 11

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=1;
c=100;
a12=b/c
d=5;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=5;
f=200;
a21=d/f
```

```
g=10;
h=100;
i= 1;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 5.0208 e^(-0.0222)t + 154.97
e^(-0.1128)t
%  Y2 = 1.4289 e^(-0.0222)t - 1361.43
e^(-0.1128)t
```

8.3.12 Run 12

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=2;
c=100;
a12=b/c
d=5;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=5;
f=200;
a21=d/f
g=10;
h=100;
i= 2;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 7.619 e^(-0.02)t + 152.381
e^(-0.1250)t
%  Y2 = 1.904 e^(-0.02)t - 761.9 e^(-
0.1250)t
```

8.3.13 Run 13

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=5;
c=100;
a12=b/c
d=5;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=5;
f=200;
a21=d/f
g=10;
h=100;
i= 5;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 10.368 e^(-0.0157)t + 149.631
e^(-0.1593)t
%  Y2 = 1.9294 e^(-0.0157)t - 401.92
e^(-0.1593)t
```

8.3.14 Run 14

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=10;
c=100;
a12=b/c
d=5;
```

```
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=5;
f=200;
a21=d/f
g=10;
h=100;
i= 10;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 10.54 e^(-0.0117)t + 149.45
e^(-0.2133)t
%  Y2 = 1.3937 e^(-0.0117)t - 281.38
e^(-0.2133)t
```

8.3.15 Run 15

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=100;
c=100;
a12=b/c
d=5;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=5;
f=200;
a21=d/f
g=10;
h=100;
i= 100;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 3.2518 e^(-0.0022)t + 156.74
e^(-1.1228)t
%  Y2 = 0.0741 e^(-0.0022)t – 172.07
e^(-1.1228)t
```

8.3.16 Run 16

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=1;
c=100;
a12=b/c
d=10;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=10;
f=200;
a21=d/f
g=10;
h=100;
i= 1;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 15.857 e^(-0.0126)t + 144.14
e^(-0.1174)t
%  Y2 = 11.761 e^(-0.0126)t - 971.74
e^(-0.1174)t
```

8.3.17 Run 17

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=2;
c=100;
a12=b/c
d=10;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=10;
f=200;
a21=d/f
g=10;
h=100;
i= 2;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 20.64 e^(-0.0328)t + 139.359 e^(-0.1322)t
%  Y2 = 12.559 e^(-0.0328)t – 572.53 e^(-0.1322)t
```

8.3.18 Run 18

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=5;
c=100;
a12=b/c
d=10;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=10;
f=200;
a21=d/f
g=10;
h=100;
i= 5;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 136.56 e^(-0.1707)t + 23.431 e^(-0.0293)t
%  Y2 = -329.68 e^(-0.1707)t + 9.705 e^(-0.0293)t
```

8.3.19 Run 19

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=10;
c=100;
a12=b/c
d=10;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=10;
f=200;
a21=d/f
g=10;
h=100;
i= 10;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
```

```
b = [160;0];
C = inv(J)*b
%  Y1 = 21.791 e^(-0.0219)t + 138.208
e^(-0.2281)t
%  Y2 = 6.119 e^(-0.0219)t - 246.12
e^(-0.2281)t
```

8.3.20 Run 20

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=100;
c=100;
a12=b/c
d=10;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=10;
f=200;
a21=d/f
g=10;
h=100;
i= 100;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 6.3979 e^(-0.0044)t + 153.6
e^(-1.1456)t
%  Y2 = 0.2917 e^(-0.0044)t – 168.28
e^(-1.1456)t
```

8.3.21 Run 21

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=1;
c=100;
a12=b/c
d=100;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=100;
f=200;
a21=d/f
g=10;
h=100;
i= 1;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 4.7920 e^(-0.5124)t + 153.208
e^(-0.0976)t
%  Y2 = -5.954 e^(-0.5124)t + 6165.47
e^(-0.0976)t
```

8.3.22 Run 22

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=2;
c=100;
a12=b/c
d=100;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=100;
f=200;
a21=d/f
g=10;
h=100;
i= 2;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
```

```
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 9.2063 e^(-0.5247)t + 150.79
e^(-0.0953)t
%  Y2 = -7.9708 e^(-0.5247)t + 3051.31
e^(-0.0953)t
```

8.3.23 Run 23

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=5;
c=100;
a12=b/c
d=100;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=100;
f=200;
a21=d/f
g=10;
h=100;
i= 5;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 20.64 e^(-0.5608)t + 139.35
e^(-0.0892)t
%  Y2 = -25.11 e^(-0.5608)t + 1145.03
e^(-0.0892)t
```

8.3.24 Run 24

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=10;
c=100;
a12=b/c
d=100;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=100;
f=200;
a21=d/f
g=10;
h=100;
i= 10;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 35.43 e^(-0.6193)t + 124.56
e^(-0.0807)t
%  Y2 = -42.25 e^(-0.6193)t + 522.23
e^(-0.0807)t
```

8.3.25 Run 25

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=100;
c=100;
a12=b/c
d=100;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=100;
f=200;
a21=d/f
g=10;
```

```
h=100;
i= 100;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 48.754 e^(-0.0319)t + 111.24
e^(-1.5681)t
%  Y2 = 22.824 e^(-0.0319)t - 118.81
e^(-1.5681)t
```

8.4 Part 1 Matlab Code (Including Plots)

8.4.1 Run 1

```
clear
clear all
clc

% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=6;
c=100;
a12=b/c
d=16;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=200;
a21=d/f
g=10;
h=100;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
roots(P)
% Eigenvalues
Eigenvalues = eig(A)
% Eigenvectors
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1);
V2 = V(:,2);
V22 = V2/V(1,2);

Vectors = table(V22,V11)
% Displaying Eigenvalues and
Eigenvectors in a table
K = table (Eigenvalues,V22,V11)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
% Solving for Constants
C = inv(J)*b
%  Y1 = 40 e^(-0.2)t + 120 e^(-0.04)t
%  Y2 = -80 e^(-0.2)t + 80 e^(-0.04)t

% To plot y(t) we must model y(t) =
C1(y1)+C2(y2)
% Where C are the constants C1 and C2
t = [0 27.5 50 100]';
y = [160 40.1080 16.2420 2.1979]';
T = (0:27.5:50:100)';
%  Modeling y(t) = C1(y1)+C2(y2)= 40
e^(-0.2)t + 120 e^(-0.04)t
Y1 = [exp(-0.2*T) exp(-0.04*T)]*C;
% % % Solving for T and Y
%  Modeling y(t) = C1(y1)+C2(y2)= -80
e^(-0.2)t + 80 e^(-0.04)t
z =[-80 80]'
Y2 = [exp(-0.2*T) exp(-0.04*T)]*z
% Displaying Tables for T,Y1 and T2,Y2
B = table(T,Y1)
w = table(T,Y2)
plot(T,Y1,'-',t,y,'o',T,Y2,'r')
title('y(t) vs. Time (Minutes)')
xlabel('Time (Minutes)')
ylabel(' y(t)')
grid
```

8.5 Part 2 Matlab Code (Including Plots)

8.5.1 Run 1

```
clear
clear all
clc

% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
```

```
b=6;
c=20;
a12=b/c
d=16;
f=50;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=16;
f=50;
a21=d/f
g=10;
h=20;
i= 6;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 31.01 e^(-0.1681)t + 128.98
e^(-0.9519)t
%  Y2 = 15.70 e^(-0.1681)t – 271.70
e^(-0.9519)t

% To plot y(t) we must model y(t) =
C1(y1)+C2(y2)
% Where C are the constants C1 and C2
t = [0 27.5 50 100]';
y = [160  0.33142 0.0069384 1.5524e-
06]';
T = (0:27.5:50:100)';
%  Modeling y(t) = C1(y1)+C2(y2)=
31.01 e^(-0.1681)t + 128.98 e^(-
0.9519)t
Y1 = [exp(-0.1681*T) exp(-
0.9519*T)]*C;
% % % Solving for T and Y
%  Modeling y(t) = C1(y1)+C2(y2)=
15.70 e^(-0.1681)t – 271.70 e^(-
0.9519)t
z =[15.70 -271.70]'
Y2 = [exp(-0.1681*T) exp(-0.9519*T)]*z
% Displaying Tables
B = table(T,Y1)
w = table(T,Y2);
plot(T,Y1,'-',t,y,'o',T,Y2,'r')
title('y(t) vs. Time (Minutes)')
xlabel('Time (Minutes)')
ylabel(' y(t)')
grid
```

8.6 Part 3 Matlab Code (Including Plots)

8.6.1 Run 1

```
% Tank 1
% Y'= Inflow/Min - Outflow/Min
% Y'= -a11(Y1)+ a12(Y2)
b=1;
c=100;
a12=b/c
d=1;
f=200;
a11=-d/f
% Tank 2
% Y'= Inflow/Min - Outflow/Min
% Y'= a21(Y1)+ a22(Y2)
d=1;
f=200;
a21=d/f
g=10;
h=100;
i= 1;
a22= -i/h-g/h

A =[a11 a12;a21 a22]
P = poly(A)
Eig = eig(A)
[V,D] = eig(A);
V1 = V(:,1);
V11 = V1/V(1,1)
V2 = V(:,2);
V22 = V2/V(1,2)
%  C1 + 1.5C2 = 160
% -2C1 + C2   =  0
J = [V22,V11]
b = [160;0];
C = inv(J)*b
%  Y1 = 0.7159 e^(-0.0045)t + 159.284
e^(-0.1103)t
%  Y2 = 0.0339 e^(-0.0045)t - 1680.03
e^(-0.1103)t


% To plot y(t) we must model y(t) =
C1(y1)+C2(y2)
% Where C are the constants C1 and C2
t = [0 27.5 50 100]';
y = [160 8.7399 1.2129 0.45906]';
T = (0:27.5:50:100)';
```

```
%  Modeling y(t) = C1(y1)+C2(y2)= 40
e^(-0.2)t + 120 e^(-0.04)t
Y1 = [exp(-0.0045*T) exp(-
0.1103*T)]*C;
% % % Solving for T and Y
%  Modeling y(t) = C1(y1)+C2(y2)= -80
e^(-0.2)t + 80 e^(-0.04)t
z =[0.0339 -1680.03]'
Y2 = [exp(-0.0045*T) exp(-0.1103*T)]*z
% Displaying Tables
B = table(T,Y1)
w = table(T,Y2)
plot(T,Y1,'-',t,y,'o',T,Y2,'r')
xlabel('Time (Mins)')
ylabel(' y(t)')
grid
```