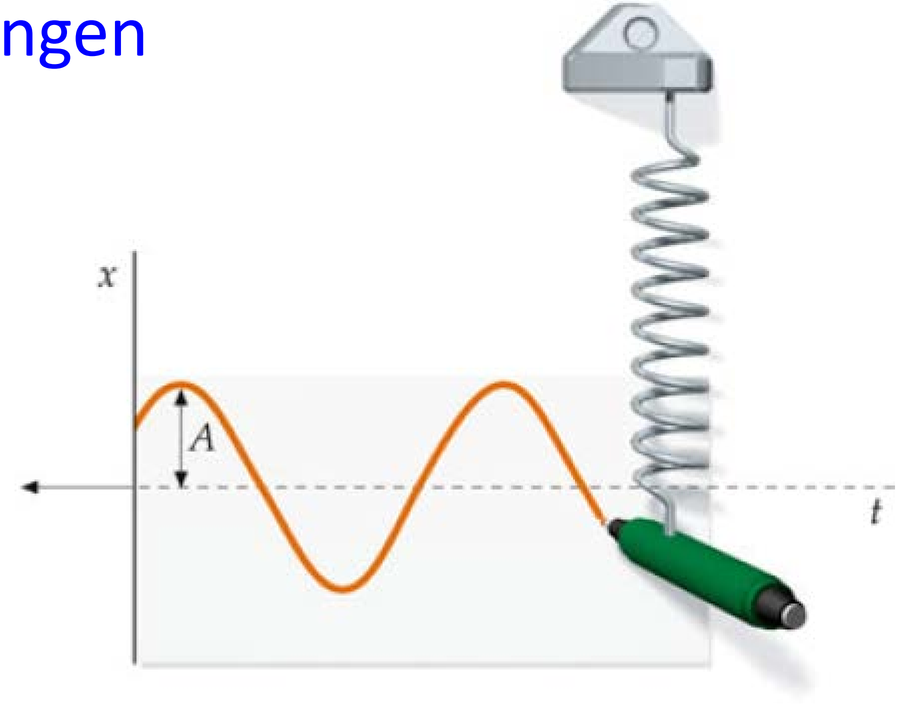
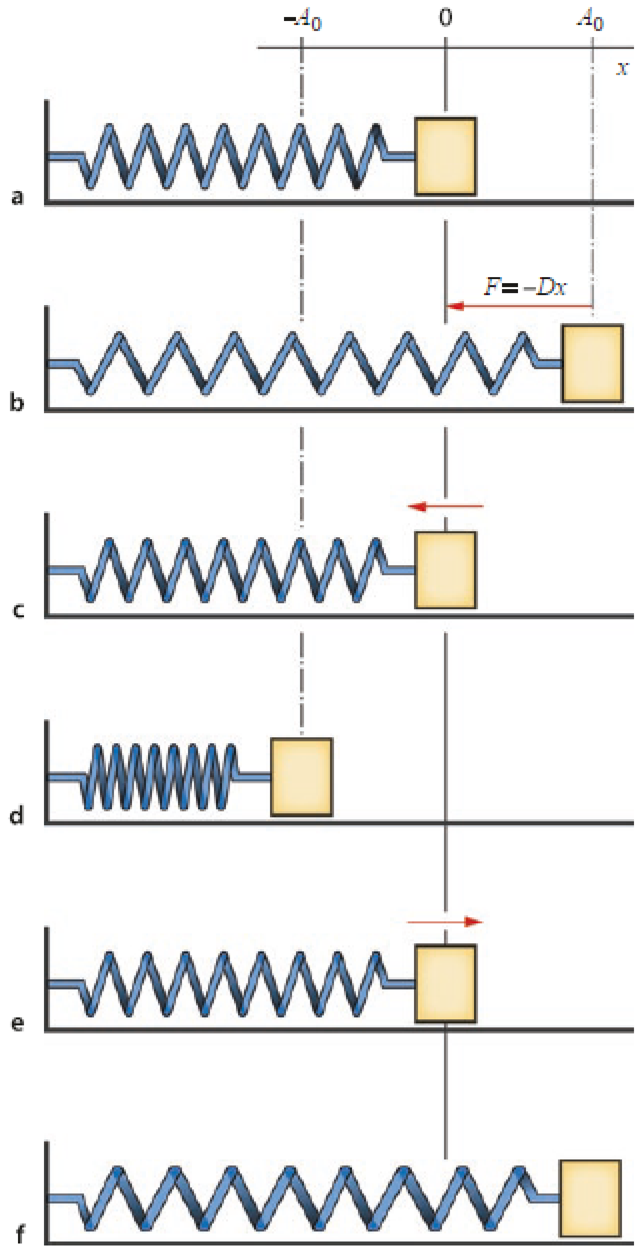
Schwingungen

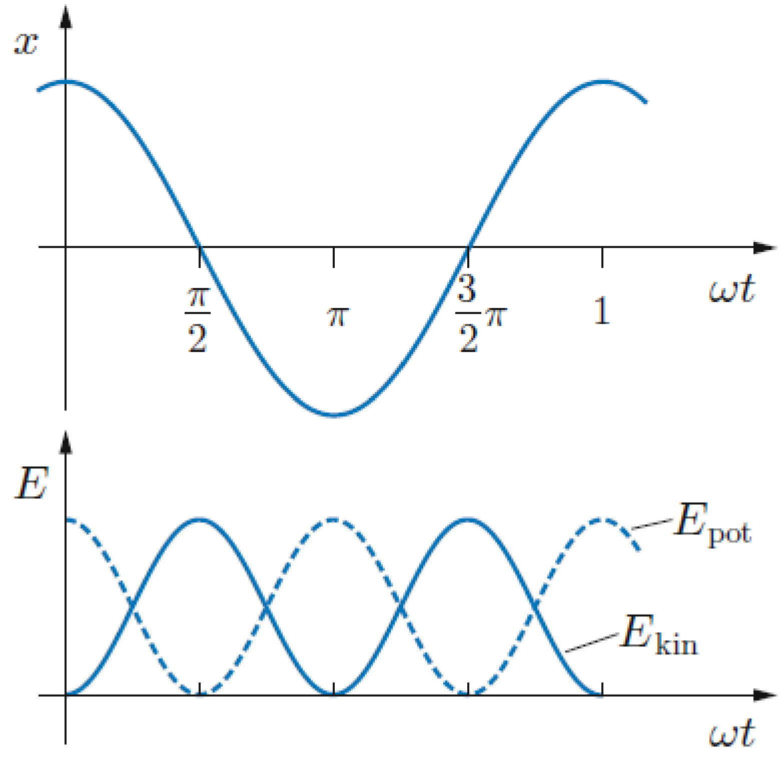
In einem schwingungsfähigen System kann die Energie zwei verschiedene Formen annehmen. Die Schwingung besteht darin, dass eine dem System von außen zugeführte Energie periodisch zwischen diesen möglichen Formen hin und her pendelt.

Schwingungen





-A_0
0
A_0
x
a
$F = -Dx$
b
c
d
e
f



x
$\frac{\pi}{2}$
π
$\frac{3}{2}\pi$
1
ωt
E
E_{pot}
E_{kin}
ωt

Kenngrößen der Harmonischen Schwingung:

- Amplitude = Maximalausschlag $= A$
- Schwingungsdauer T
- Frequenz $f = \frac{1}{T}$
- Kreisfrequenz $\omega = 2\pi \cdot f$

$$\ddot{x} + \omega^2 x = 0$$ Schwingungsgleichung

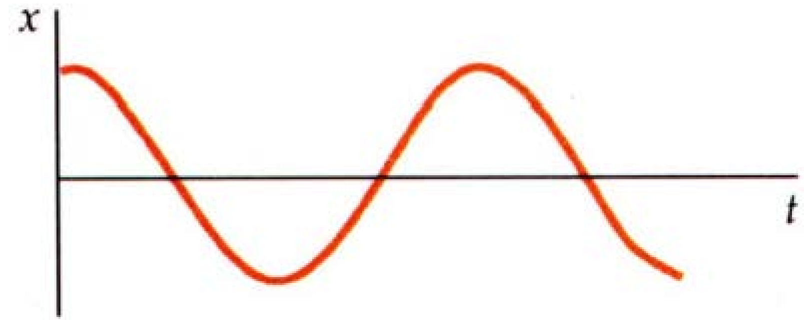
$$x(t) = A \sin(\omega t)$$

$$\dot{x}(t) = A \cdot \omega \cos(\omega t)$$

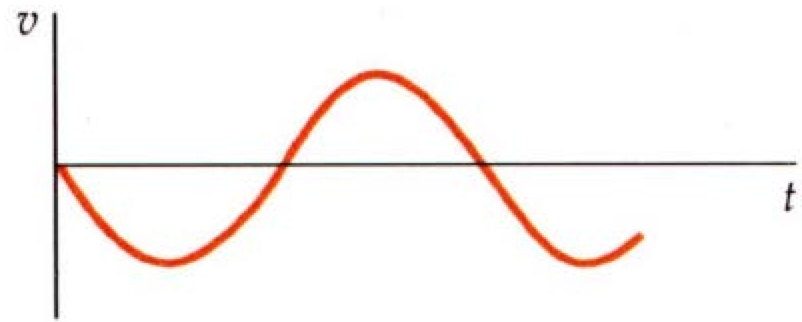
$$\ddot{x}(t) = -A\omega^2 \sin(\omega t)$$

$$\ddot{x} + \omega^2 x = 0$$

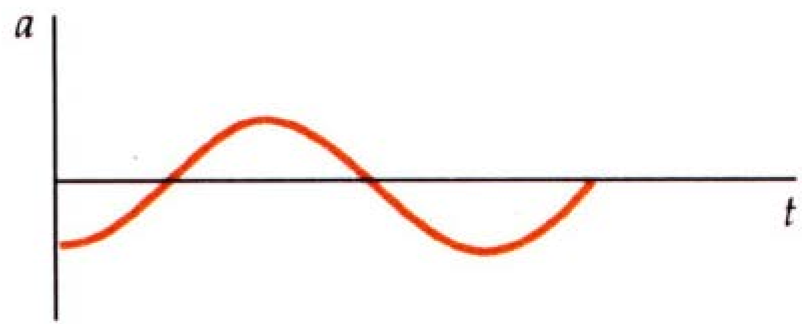
$$x = A\cos(\omega t)$$



$$v = \dot{x} = -A\omega\sin(\omega t)$$



$$a = \dot{v} = \ddot{x} = -A\cos(\omega t)\cdot\omega^2$$



Schwingungen:

Rückstellkräfte

Trägheitskraft

Lineare Schwingungen Rückstellkraft

$\sim$ Auslenkung

Superpositionsprinzip, Linearkombinationen

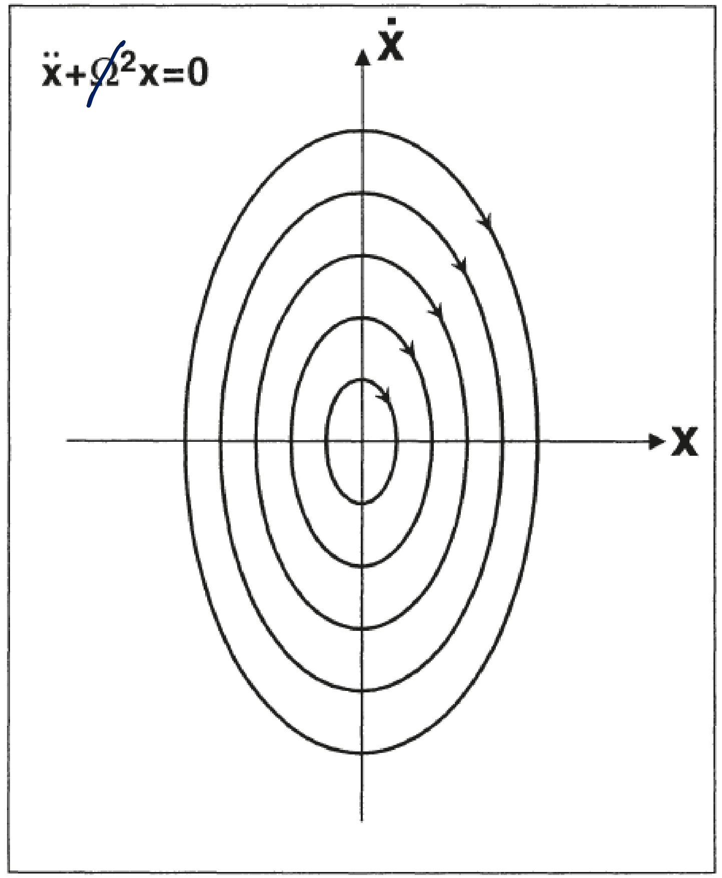
$$\ddot{x} + \omega^2 x = 0$$

Integrieren

Energie : $$\frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega^2 x^2 = \text{konst}$$

Phasendiagramm

$\ddot{x} + w^2 x = 0$



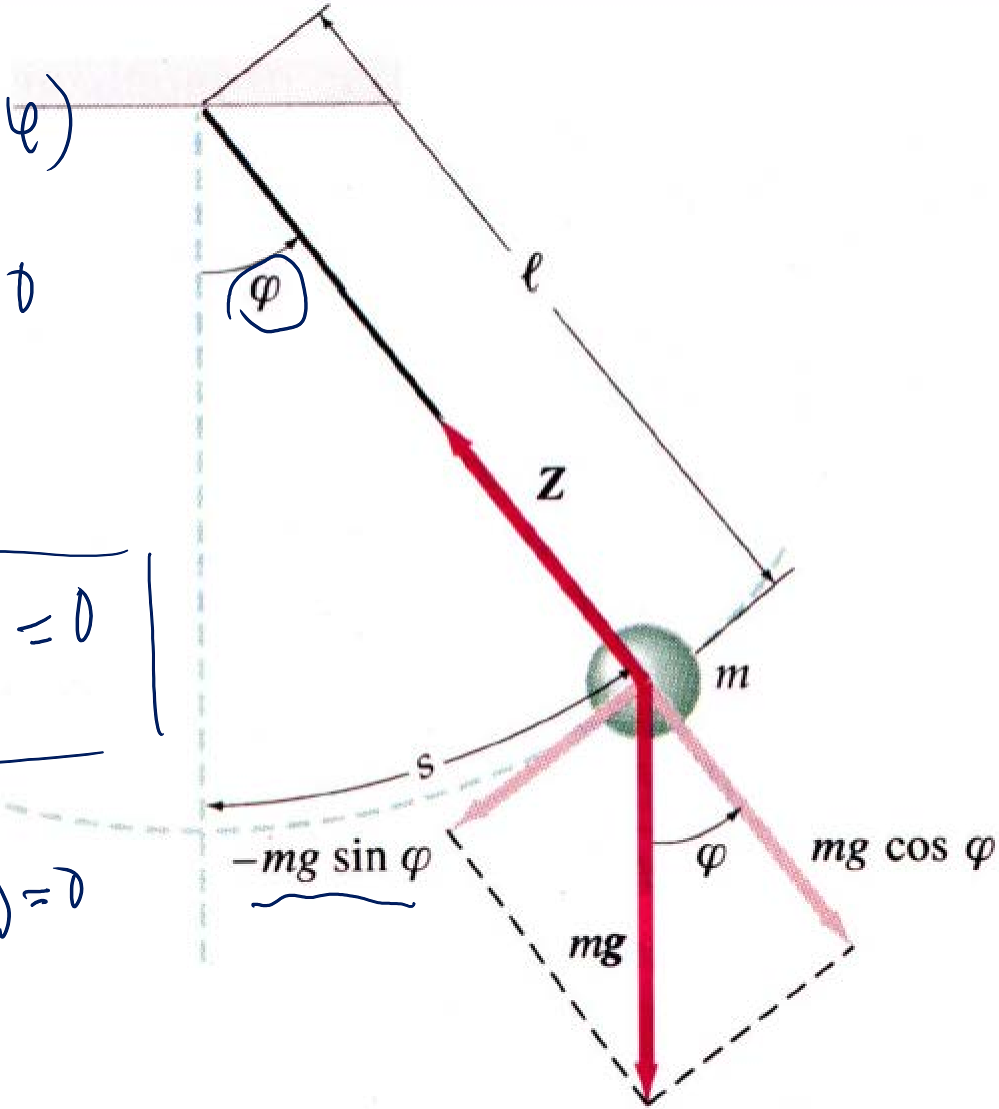
$$\cancel{m}\ddot{x} = -\cancel{m}g\sin(\varphi)$$

$$\ddot{x} + g\sin(\varphi) = 0$$

$$\ddot{x} = \ell\ddot{\varphi}$$

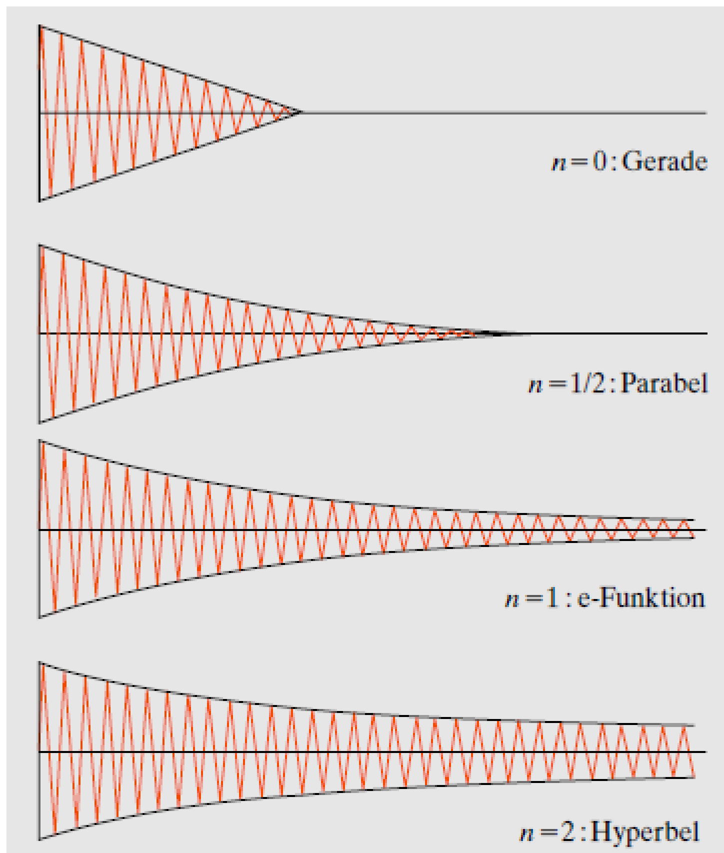
$$\boxed{\ddot{\varphi} + \frac{g}{\ell}\sin(\varphi) = 0}$$

$$\ddot{\varphi} + \omega^2\sin(\varphi) = 0$$



Schwingungen mit Reibung

- $F_R = \mu \cdot N$ Trockene Reibung (z. B. Metall auf Holz, Metall auf Metall,...)
- $F_R = konst \cdot v$ Reibkraft bei laminarer Umströmung (kleine Geschwindigkeiten in Gasen und Flüssigkeiten, z.B. Luftreibung)
- $F_R = konst. \cdot v^2$ Reibungswiderstand in turbulenter Umströmung (es treten Wirbel auf)

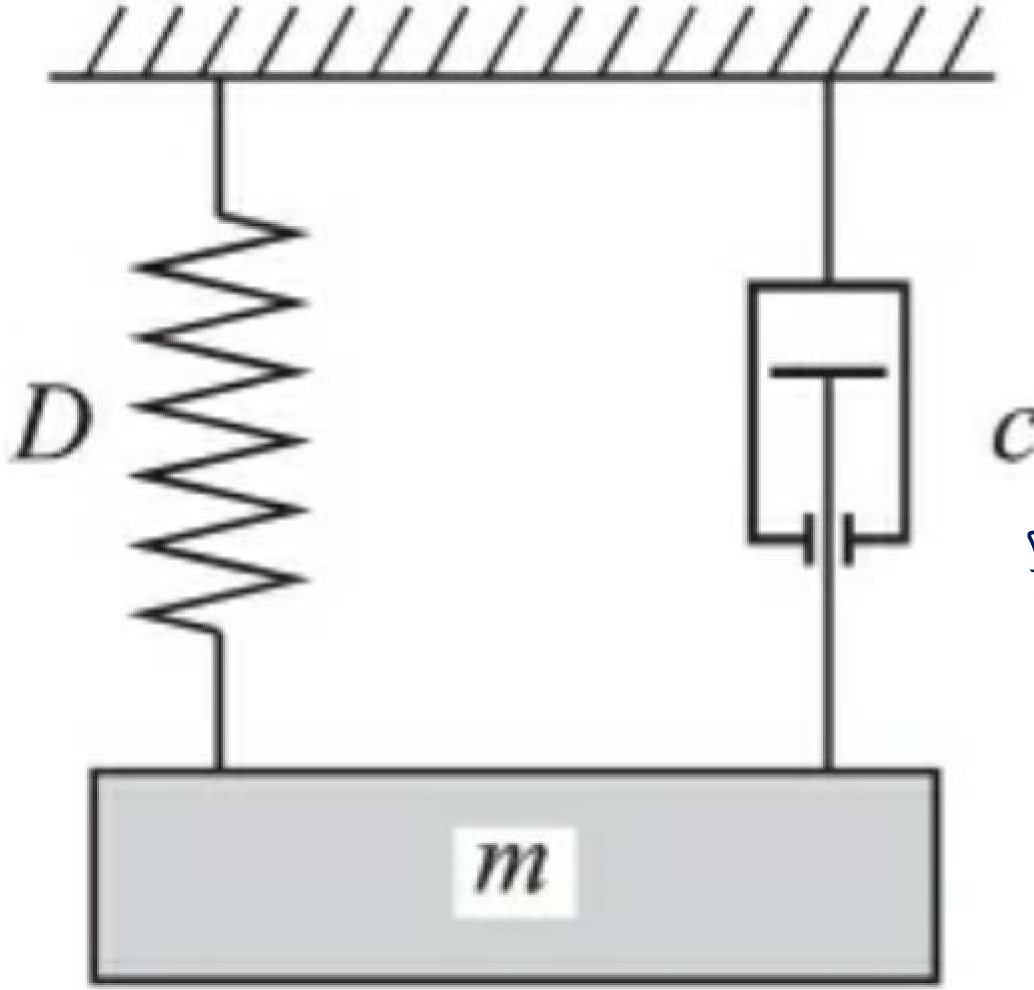


konstant

$\sqrt{v}$

v

v^2



$F \sim v$

$$m\ddot{x} + c\dot{x} + Dx = 0$$

$$\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0$$

$$y_1 = x$$

$$y_2 = \dot{x}$$

$$\dot{y}_2 = \ddot{x} = -2\gamma\dot{x} - \omega^2 x = -2\gamma y_2 - \omega^2 y_1$$

$$\dot{y}_1 = y_2$$

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega^2 & -2\gamma \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\dot{\vec{y}} = A\vec{y} \qquad \longrightarrow$$

$$\dot{\vec{y}} = A\vec{y}$$

$$\vec{y} = e^{At}\,\vec{y}(0)$$

$$e^{At} = I + At + (At)^2/2 + \frac{(At)^3}{3!} + \cdots$$

$$\dot{y} = a y \qquad a \in \mathbb{R}$$

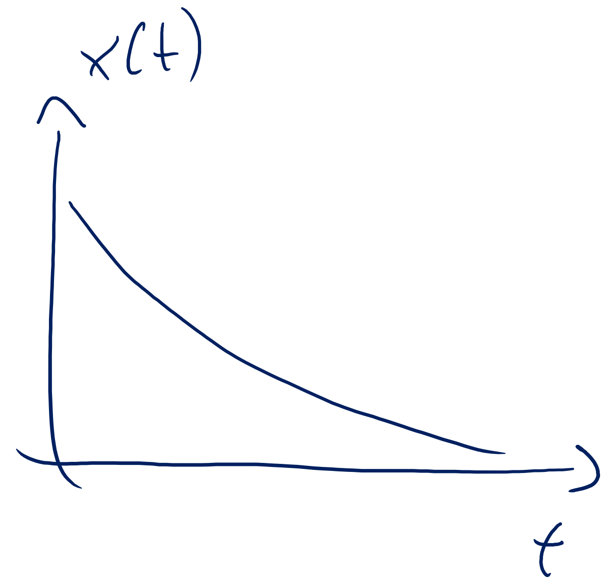
$$e^{at} \cdot y(0)$$

Eigenwerte $\quad A = \begin{pmatrix} 0 & 1 \\ -\omega & -2\gamma \end{pmatrix}$

$$\lambda_{1,2} = -\gamma \pm \sqrt{\gamma^2 - \omega^2}$$

1. Fall $\gamma^2 > \omega^2$

$$x = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$



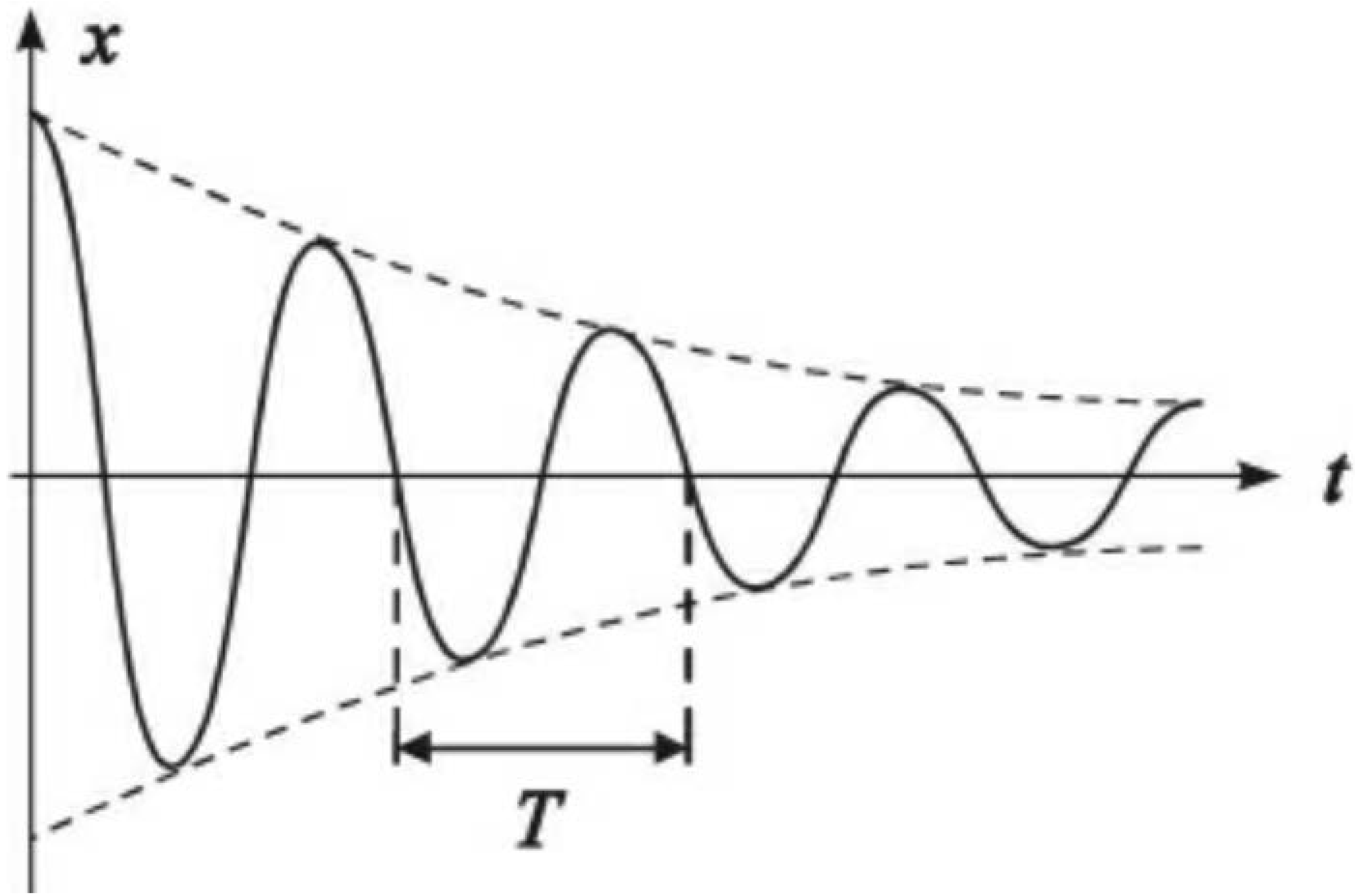
Anfangs- oder Randbed.

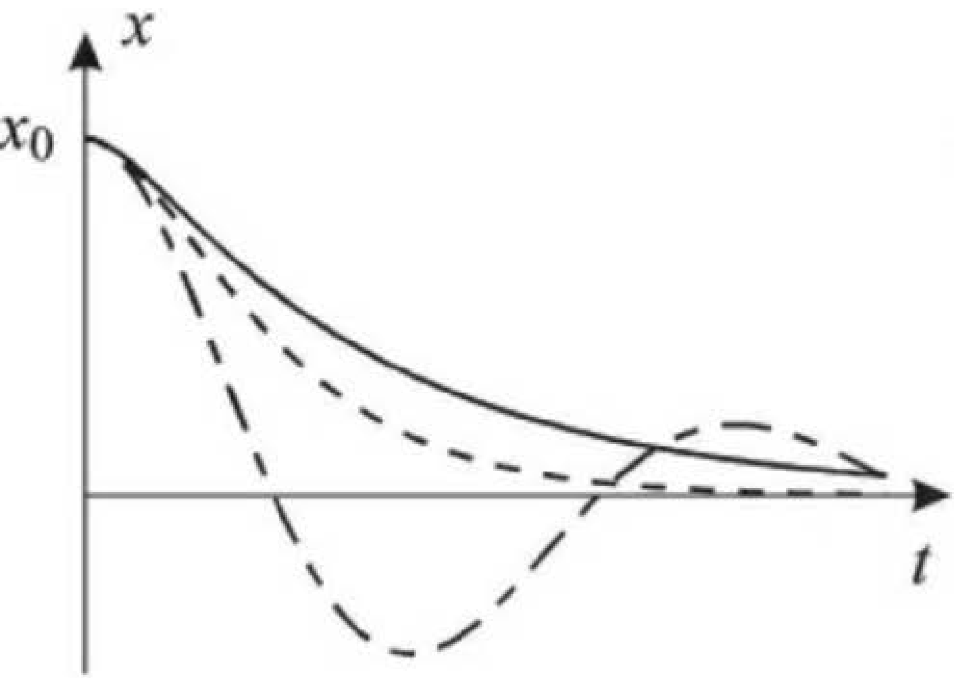
2. Fall $\gamma^2 = \omega^2 \qquad \lambda_{1,2} = -\gamma = \lambda$

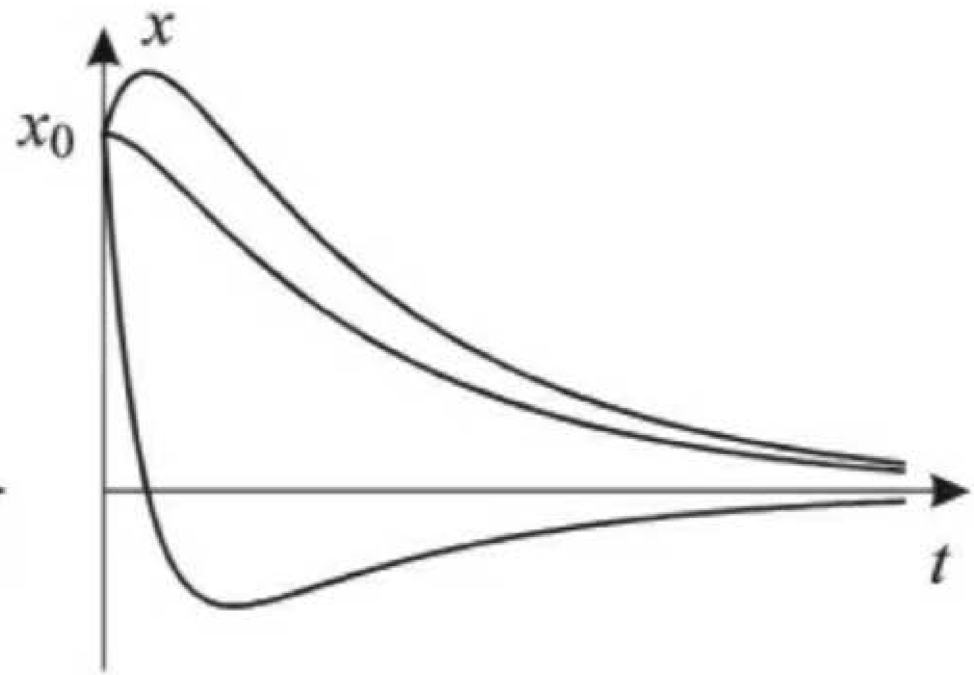
$$x(t) = c_1 e^{\lambda t} + c_2 t e^{\lambda t}$$ aperiodische Grenzfall

3. Fall $\omega^2 > \gamma^2$

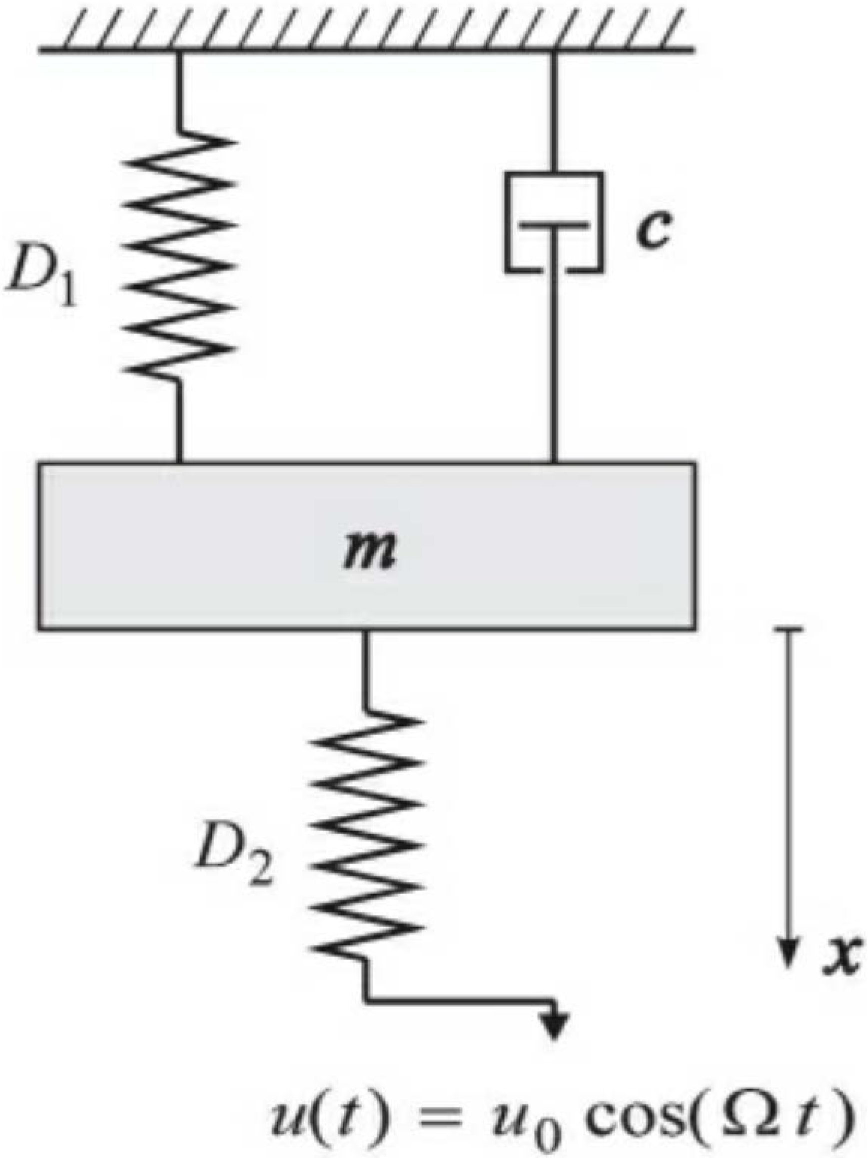
$$x(t) = e^{-\gamma t}\left[A \cdot \cos(\omega t) + B \sin(\omega t)\right]$$


x
t
T


x
x0
t


x
x0
t

Erzwungene Schwingungen



Erzwungene Schwingungen

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f_0 \cos(\omega t)$$

$z = x(t) + i\,y(t) \qquad x(t) = \mathrm{Re}\,(z(t))$

$$\boxed{\ddot{z} + 2\gamma\dot{z} + \omega_0^2 z = f_0\, e^{i\omega t}}$$

$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$

1. Homgene Lösung

$$x_h = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

2. Partikuläre Lösung

$$z = A \cdot e^{i\omega t} \qquad A \in \mathbb{C}$$

$$\ddot{z} + 2\gamma \dot{z} + \omega_0^2 z = f_0 \, e^{i\omega t}$$

$$z = A \, e^{i\omega t} \, , \quad \dot{z} = A i\omega \, e^{i\omega t}$$

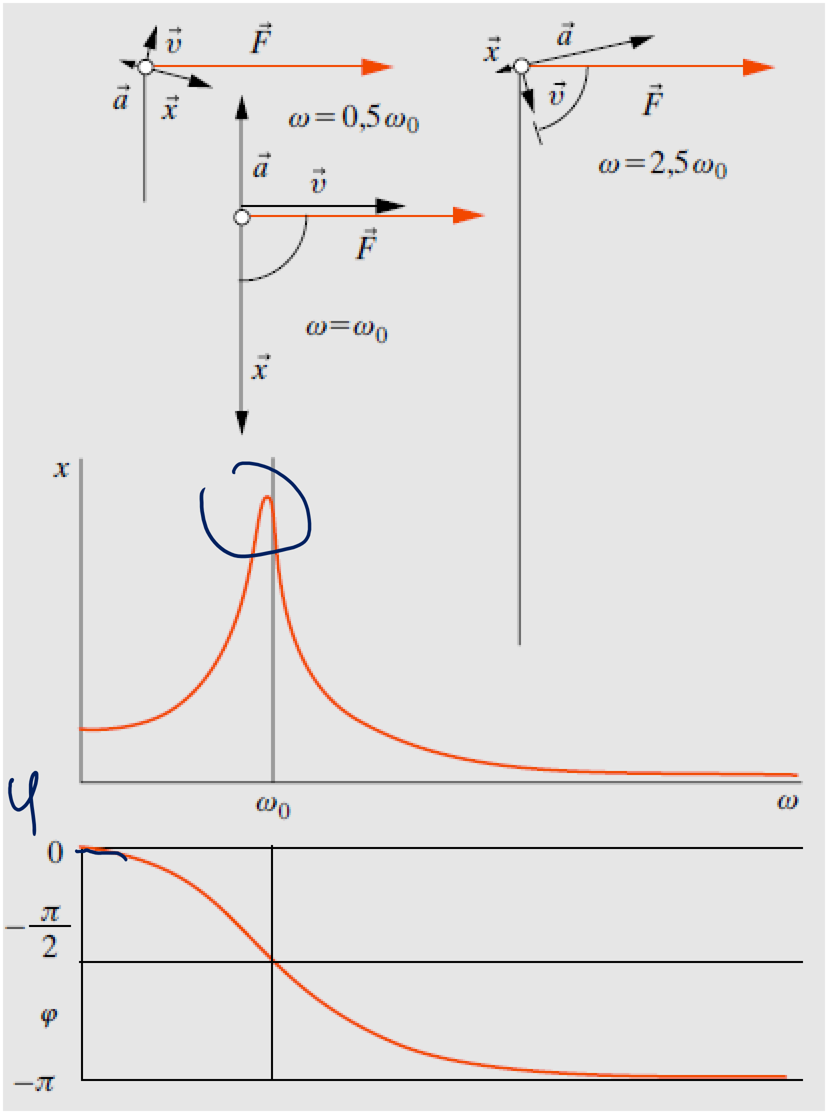
$$\ddot{z} = -A\omega^2 e^{i\omega t}$$

$$-A\omega^2 e^{i\omega t} + 2\gamma A \cdot i\omega e^{i\omega t} + A\omega_0^2 e^{i\omega t} = f_0 \, e^{i\omega t}$$

$$A(\omega_0^2 - \omega^2 + 2\gamma i) = f_0$$

$$A = \frac{f_0}{\omega_0^2 - \omega^2 + 2\gamma i} = |A| \, e^{i\varphi}$$

$$|A| = \frac{f_0}{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2} \qquad \varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$



$\vec{v}$
$\vec{F}$
$\vec{a}$
$\vec{x}$
$\omega = 0{,}5\,\omega_0$
$\vec{a}$
$\vec{v}$
$\vec{F}$
$\omega = \omega_0$
$\vec{x}$
$\vec{x}$
$\vec{a}$
$\vec{v}$
$\vec{F}$
$\omega = 2{,}5\,\omega_0$
x
ω_0
ω
φ
0
$-\frac{\pi}{2}$
φ
$-\pi$

Nichtlineare Schwingungen

$$\ddot{\varphi} + \omega^2 \sin(\varphi) = 0$$

- keine ungestörte Überlagerung
- $x \neq x_h + x_p$
- Neue Phänomene
  - 
  - Oberschwingungen
  - Chaos