

# Homework 1: Probability and Combinatorics

January 22, 2020

**Directions:** Show **all** work for full credit! For this assignment, you can work in groups of 3 and simply turn in one document (a .pdf) to Canvas with the names of all members who contributed. Also, at the bottom of the page, please indicate what proportion of the work was due to each member of the group. (Ex: Tisha - .25, Julie - .50, Frank - .25.)

1. A student has to sell 2 books from a collection of 6 Math, 7 Science, and 4 Data Science books. How many choices are possible if
  - (a) both books are on the same subject?
  - (b) the two books must be on different subjects?
2. This question is to help you better understand “games of chance” so that you can impress your friends with your newly acquired probability and counting knowledge. For those that don’t know the hands of a typical poker game, please see [https://en.wikipedia.org/wiki/List\\_of\\_poker\\_hands](https://en.wikipedia.org/wiki/List_of_poker_hands). If it is assumed that all  $\binom{52}{5}$  poker hands are equally likely, what is the probability of being dealt
  - (a) a flush?
  - (b) one pair?
  - (c) two pairs?
  - (d) three of a kind?
  - (e) four of a kind?
3. Suppose that I choose a letter at random from R E S E R V E and then you choose a letter at random from VERTICAL. What is the probability that we choose the same letter?
4. Balls are randomly removed from an urn initially containing 20 red and 10 blue balls. What is the probability that all of the red balls are removed

before all of the blue ones have been removed? What is the probability that all of the blue balls are removed before all of the red ones have been removed?

5. Independent trials consisting of rolling a pair of fair dice are performed. What is the probability that an outcome of 5 appears before an outcome of 7 when the outcome of a roll is the sum of the dice?
6. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent say that they are Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that he or she is
  - (a) an Independent?
  - (b) a Liberal?
  - (c) a Conservative?
  - (d) What fraction of voters participated in the local election?
7. Suppose that an insurance company classifies people into one of three classes: good risks, average risks, and bad risks. The company's records indicate that the probabilities that good-, average-, and bad-risk persons will be involved in an accident over a 1-year span are, respectively, .05, .15, and .30. If 20 percent of the population is a good risk, 50 percent an average risk, and 30 percent a bad risk, what proportion of people have accidents in a fixed year? If policyholder A had no accidents in 1997, what is the probability that he or she is a good or average risk?
8. A famous restaurant in San Francisco has the following menu:

Suppose that James goes to this restaurant with plans on having one Appetizer, one Entree, and one Dessert.

- (a) My sample space consists of how many outcomes?
- (b) While walking to the restaurant, James realizes that he has cravings for one of the **[fish]** items in the Entree section. As before he plans to have 1 dish from each of the menus. How many possible outcomes are there in the event  $E = \{\text{James picks a fish item from the Entrees menu}\}$ ?

**Appetizers:**

- (1) Wild shrimp and rice soup (2) Local heirloom tomato salad  
(3) Local grass-fed beef tartare (4) Salt and pepper shrimp  
(5) Lantern local pasture-raised pork and chive dumplings

**Entrees**

- (1) Crispy whole NC flounder [fish] (2) Japanese steakhouse-style seared Niman Ranch flat-iron steak (3) Tea and spice smoked White Oak Pastures chicken (4) Steamed wild King salmon [fish] (5) Local tomato and vegetable stew (6) Lion's head meatballs (7) Seafood hotpot [fish]

**Dessert**

- (1) Summer Sidewalk - mango frozen yogurt (2) Yuzu Pudding  
(3) Local peach and riesling sorbet (4) Cherry stone panna cotta

- (c) Suppose that the Chef at this restaurant chose a three course meal for James and that all meal choices are equally likely. Assuming that all of the desserts are vegetarian dishes, what is the probability James will have a vegetarian dinner?
- (d) Suppose that the Chef at this restaurant chose a three course meal for James and that all meal choices are equally likely. What is the probability that the only meat that James will have only one type of meat in his meal: either *beef*, *pork*, *seafood*, *chicken*, or *none*?
- (e) Suppose that the Chef at this restaurant chose a three course meal for James and that all meal choices are equally likely. Given that the Chef chose *Local grass-fed beef tartare* as the appetizer for James, what is the probability that James will have only one type of meat during his meal?
9. Fifteen percent of all email you receive is spam. Your spam filter is 90 percent reliable meaning that it marks 90 percent of spam emails as spam, and marks 90 percent of non-spam mails as not spam. If you see that an email is marked spam by your filter, what is the probability that it is really spam?
10. Consider the grid of points shown below. Suppose that, starting at the point labeled  $A$ , you can go one step up or one step to the right at each move. This procedure is continued until the point labeled  $B$  is reached. How many different paths from  $A$  to  $B$  are possible?

