3. Queremos una tabulación para aproximar $J_0(x)$ por interpolación cuadrática

error $< \frac{1}{2} \cdot 10^{-6}$

Sea $q(x)$ polinomio cuadrático que interpola a $J_0(x)$ en x_j, x_{j+1}, x_{j+2} $\quad j \in \{1, \dots n-2\}$

Error:

$$J_0 - q(x) = \frac{(x-x_j)(x-x_{j+1})(x-x_{j+2})}{3!} J_0'''(x)$$

$x \in [x_j, x_{j+2}] \Rightarrow$

$$|x - x_j| \leq |x_{j+2} - x_j| = \frac{2}{n}$$

$$|J_0 - q(x)| = \frac{|x-x_j| \cdot |x-x_{j+1}| \cdot |x-x_{j+2}|}{3!} |J_0'''(x)| \leq$$

$$\leq \frac{\left(\frac{2}{n}\right)^3}{3!} |J_0'''(x)| = \frac{8}{n^3} \cdot \frac{1}{6} |J_0'''(x)|$$

$$= \frac{4}{3n^3} |J_0'''(x)|$$

$$J_0''(x) = -\frac{1}{\pi}\int_0^\pi \cos(x \sin(t)) \sin^2(t)\, dt$$

$$J_0'''(x) = -\frac{1}{\pi}\int_0^\pi \frac{\partial}{\partial x} \cos(x \sin(t)) \sin^2(t)\, dt =$$

$$= -\frac{1}{\pi}\int_0^\pi -\sin(x \sin(t)) \cdot \sin^3(t)\, dt =$$

$$= -\frac{1}{\pi}\int_0^\pi \sin(x \sin(t)) \cdot \sin^3(t)\, dt$$

$$|J_0'''(x)| \leq \frac{1}{\pi}\int_0^\pi |\sin(x \sin(t))| \cdot |\sin^3(t)|\, dt \leq \frac{1}{\pi}\int_0^\pi dt = 1$$

$$|J_0 - q(x)| \leq \frac{4}{3n^3} |J_0'''(x)| = \frac{4}{3n^3}$$

Buscamos n tal que

$$\frac{4}{3n^3} < \frac{1}{2} 10^{-6}$$

$$\frac{1}{n^3} < \frac{3}{8} 10^{-6}$$

$$n^3 > \frac{8}{3} \cdot 10^6$$

$$\sqrt[3]{n^3} > \sqrt[3]{\frac{8}{3}} \cdot \sqrt[3]{10^6}$$

$$n > \frac{2}{\sqrt[3]{3}} \cdot 10^2$$