Broscoteanu Daria-Mihaela

Tema 3

6. $X \sim Pois(\lambda)$
$Y \sim Pois(\mu)$ $\Rightarrow X+Y \sim Pois(\lambda+\mu)$

$$P(X=k \mid X+Y=m) = \frac{P((X=k)\cap(X+Y=m))}{P(X+Y=m)}$$

$$P(X=k \mid X+Y=m) = \frac{P(X=k, X+Y=m)}{P(X+Y=m)}$$

$$P(X=k \mid X+Y=m) = \frac{P(X=k, Y=m-k)}{P(X+Y=m)}$$

$$P(X=k \mid X+Y=m) = \frac{P(X=k)\,P(Y=m-k)}{P(X+Y=m)}$$

$X \sim Pois(\lambda) \Rightarrow P(X=k) \simeq e^{-\lambda}\cdot\frac{\lambda^k}{k!}$

$Y \sim Pois(\mu) \Rightarrow P(y=m-k) \simeq e^{-\mu}\cdot\frac{\mu^{m-k}}{(m-k)!}$

$X+Y \sim Pois(\lambda+\mu) \Rightarrow P(X+Y=m) \simeq e^{-(\lambda+\mu)}\cdot\frac{(\lambda+\mu)^m}{m!}$

$$P(X=k \mid X+Y=m) = \frac{\cancel{e^{-\lambda}}\cdot\frac{\lambda^k}{k!}\cdot\cancel{e^{-\mu}}\cdot\frac{\mu^{m-k}}{(m-k)!}}{\cancel{e^{-(\lambda+\mu)}}\cdot\frac{(\lambda+\mu)^m}{m!}} = \frac{\lambda^k\cdot\mu^{m-k}\cdot m!}{k!\,(m-k)!\,(\lambda+\mu)^m}$$

$$= \frac{m!}{k!\,(m-k)!}\cdot\frac{\lambda^k}{(\lambda+\mu)^k}\cdot\frac{\mu^{m-k}}{(\lambda+\mu)^{m-k}} = C_m^k\cdot\left(\frac{\lambda}{\lambda+\mu}\right)^k\cdot\left(\frac{\mu}{\lambda+\mu}\right)^{m-k}$$

$$= \binom{m}{k}\left(\frac{\lambda}{\lambda+\mu}\right)^k\cdot\left(\frac{\mu}{\lambda+\mu}\right)^{m-k}$$

7.

Fie C_K - evenimentul în care sacul conține K bile albe, $0 \le K \le 10$ (la final, după cele 10 aruncări, deci obținem K de heads și $10-K$ de tails)

$$P(\text{să dea de } K \text{ ori head}) = \left(\frac{1}{2}\right)^K$$

$$P(\text{să dea de } 10-K \text{ ori tails}) = \left(\frac{1}{2}\right)^{10-K}$$

$$P(C_K) = C_{10}^K \left(\frac{1}{2}\right)^K \cdot \left(\frac{1}{2}\right)^{10-K} = C_{10}^K \cdot \left(\frac{1}{2}\right)^{10}$$

Fie E - evenimentul în care obținem în 10 extrageri, 10 bile albe

$$P(E|C_K) = \left(\frac{K}{10}\right)^{10}$$

$$P(\text{să fie doar bile albe}) = P(C_{10}|E)$$

$$P(C_{10}|E) = \frac{P(C_{10}) \cdot P(E|C_{10})}{P(E)} = \frac{P(C_{10}) \cdot P(E|C_{10})}{\sum_{K=0}^{10} P(C_K) \cdot P(E|C_K)}$$

$$P(C_{10}|E) = \frac{C_{10}^{10} \cdot \left(\frac{1}{2}\right)^{10} \cdot \left(\frac{1}{2}\right)^0 \cdot \left(\frac{10}{10}\right)^{10}}{\sum_{K=0}^{10} C_{10}^K \cdot \left(\frac{1}{2}\right)^{10} \cdot \left(\frac{K}{10}\right)^{10}} = \frac{\left(\frac{1}{2}\right)^{10}}{\left(\frac{1}{2}\right)^{10} \cdot \sum_{K=0}^{10} C_{10}^K \cdot \left(\frac{K}{10}\right)^{10}}$$

$$= \frac{1}{\frac{1}{10^{10}} \sum_{K=0}^{10} C_{10}^K \cdot K^{10}} = \frac{10^{10}}{\sum_{K=0}^{10} C_{10}^K \cdot K^{10}}$$

$$\sum_{K=0}^{10} C_{10}^K \cdot K^{10} = 2^7 \cdot 10 \cdot 111\,304\,237$$

$$\Rightarrow P(C_{10}|E) = \frac{\cancel{10} \cdot \cancel{2^7} \cdot 5^7 \cdot 10^2}{\cancel{2^7} \cdot \cancel{10} \cdot 111\,304\,237} = 0.0702$$

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8.

$$P(N=k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$$

H = obținut din N selectând doar prob. p.

$$P(H=k \mid N=t) = C_t^k \; p^k \cdot (1-p)^{t-k}$$

$$P(H=k) = \sum_{t=k}^{\infty} P(H=k \mid N=t)\, P(N=t)$$

$$= \sum_{t=k}^{\infty} C_t^k \cdot p^k \cdot (1-p)^{t-k} \cdot \frac{\lambda^t \cdot e^{-\lambda}}{t!}$$

$$\overset{t=m+k}{=} \sum_{m=0}^{\infty} C_{m+k}^k \cdot p^k \, (1-p)^m \cdot \frac{\lambda^{m+k} \cdot e^{-\lambda}}{(m+k)!}$$

$$= \sum_{m=0}^{\infty} \frac{\cancel{(m+k)!}}{k!\, m!} \, p^k \cdot (1-p)^m \cdot \frac{\lambda^k \cdot \lambda^m \cdot e^{-\lambda}}{\cancel{(m+k)!}}$$

$$= \sum_{m=0}^{\infty} \frac{\lambda^k \cdot p^k \cdot e^{-\lambda}}{k!} \cdot \frac{(1-p)^m \cdot \lambda^m}{m!}$$

$$= \frac{(\lambda p)^k \cdot e^{-\lambda}}{k!} \sum_{m=0}^{\infty} \frac{((1-p)\lambda)^m}{m!}$$

$$= \frac{(\lambda p)^k \cdot e^{-\lambda}}{k!} \cdot e^{(1-p)\lambda}$$

$$= \frac{(\lambda p)^k}{k!} \cdot e^{-p\lambda} .$$

$\hookrightarrow$ distribuție poisson de parametru $p\lambda$