The Fibonacci Sequence

1 Introduction

The Fibonacci sequence is a series of numbers where each number is the sum of the two preceding ones, starting from 0 and 1. It is defined recursively as follows:

$$F_n = F_{n-1} + F_{n-2} \tag{1}$$

where $F_0 = 0$ and $F_1 = 1$.

2 The Sequence

The first few terms of the Fibonacci sequence are
    0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

3 Mathematical Properties

The Fibonacci sequence has many interesting mathematical properties. Here are a few:

- **The Ratio of Consecutive Fibonacci Numbers:** The ratio of consecutive Fibonacci numbers approaches the golden ratio, denoted by ϕ:

$$\lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \phi = \frac{1 + \sqrt{5}}{2} \approx 1.618 \tag{2}$$

- **Binet's Formula:** The n-th Fibonacci number can be calculated directly using Binet's formula:
$$F_n = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}} \tag{3}$$

4 Applications

The Fibonacci sequence appears in various fields, including:

- **Mathematics:** Number theory, combinatorics.

- **Nature:** The arrangement of leaves on a stem, the spirals in a sunflower head, the branching of trees.

- **Computer Science:** Algorithms, data structures.

5 Example Code (Python)

Here's a simple Python code to generate the Fibonacci sequence:

```python
def fibonacci(n):
    a, b = 0, 1
    for _ in range(n):
        print(a)
        a, b = b, a + b

fibonacci(10)
```