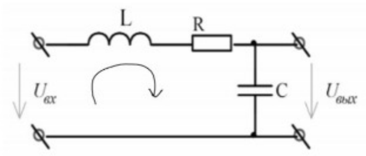
Task 10 Khaetskaya Daria

(10) 

$$U_{вх}(t) = L\cdot\frac{di}{dt} + Ri + U_{вых}$$

Выполним преобр. Лапласа, учитывая что $I(s) = \dfrac{U_{вых}(s)}{\frac{1}{sC}}$

$$U_{вх} = LsI(s) + RI(s) + U_{вых}(s) = (LCs^2 + RCs + 1)\,U_{вых}(s)$$

Передаточная ф-я для звена:

$$W(s) = \frac{U_{вых}(s)}{U_{вх}(s)} = \frac{1}{s^2LC + RCs + 1} = \frac{1/LC}{(s-s_1)(s-s_2)}, \quad s_{1,2} = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L} - \frac{1}{LC}}$$

Переходная характеристика:

$$h(t) = \mathcal{L}^{-1}\left\{\frac{1/LC}{(s-s_1)(s-s_2)s}\right\} = \frac{1(t)}{LC}\left(\frac{1}{s_1(s_1-s_2)}e^{s_1t} + \frac{1}{s_2(s_2-s_1)}e^{s_2t} + \frac{1}{s_1s_2}\right)$$

$t^n e^{\lambda t},\ n\in N$	$\dfrac{n!}{(p-\lambda)^{n+1}}$

Временная хар-ка:

$$w(t) = \frac{dh(t)}{dt} = \frac{1(t)}{LC}\cdot\frac{1}{s_1-s_2}\left(e^{s_1t} - e^{s_2t}\right)$$

Реакция цепи на единичный импульс:

1) Через передаточную ф-цию:

$$U_{вх}(t) = U_m\left(1(t) - 1(t-t_1)\right) \doteqdot U_m\frac{1-e^{-st_1}}{s}$$

- $$U_{вых}(t) = \mathcal{L}^{-1}\left\{W(s)U_{вх}(s)\right\} = \mathcal{L}^{-1}\left\{\frac{1/LC}{(s-s_1)(s-s_2)}\cdot U_m\frac{1-e^{-st_1}}{s}\right\}$$

$$= U_m\left(\mathcal{L}^{-1}\left\{\frac{1/LC}{s(s-s_1)(s-s_2)}\right\} - \mathcal{L}^{-1}\left\{\frac{1/LC\cdot e^{-st_1}}{s(s-s_1)(s-s_2)}\right\}\right) =$$

$$= U_m\left(h(t) - h(t-t_1)\right)$$

2) Интеграл наложения

- $U_{вых}(t) = \int_0^t U_{вх}(\tau)\, w(t-\tau)\, d\tau = U_m \int_0^t \left[1(t) - 1(\tau - t_1) \right] w(t-\tau)\, d\tau =$

$$= U_m \Big(\int_0^t \overset{1(t)>0\ \forall t>0}{} w(t-\tau)\, d\tau - \int_0^{t_1} \underbrace{1(\tau - t_1)}_{=0\ (\tau<t_1)} w(t-\tau)\, d\tau - \int_{t_1}^t \underbrace{1(\tau-t_1)}_{=1\ (\tau>t_1)} w(t-\tau)\, d\tau \Big) =$$

$$= U_m \Big(\int_0^t w(t-\tau)\, d\tau - \int_{t_1}^t w(t-\tau)\, d\tau \Big) = U_m \int_0^{t_1} w(t-\tau)\, d\tau =$$

$$= U_m \big(h(t) - h(t-t_1) \big)$$

3) Интеграл Дюамеля

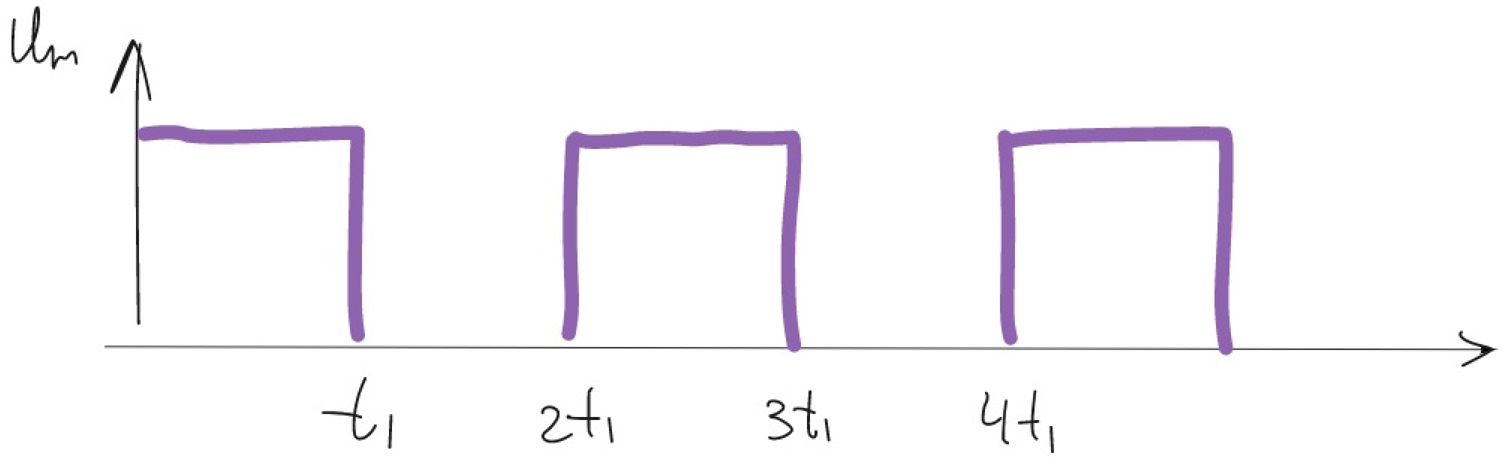
- $U_{вых}(t) = U_{вх}(0) \cdot h(t) + \int_0^t U'_{вх}(\tau)\, h(t-\tau)\, d\tau =$

$$= U_m \cdot h(t) + U_m \int_0^t \big(\delta(\tau) - \delta(\tau - t_1) \big)\, h(t-\tau)\, d\tau =$$

$$= U_m \Big[h(t) + \int_0^t \delta(\tau)\, \underbrace{h(t-\tau)}_{=0} d\tau - \int_0^t \delta(\tau - t_1)\, h(t-\tau)\, d\tau \Big] =$$

$$= U_m \big(h(t) - h(t-t_1) \big)$$

Последовательность импульсов



$$U_{вх}(t)=\sum_{n=0}^{\infty} U_m[1(t-2n\cdot t_1)-1(t-(2k+1)t_1)]$$

$$U_{вых}(t)=\sum_{n=0}^{\infty} U_m[h(t-2nt_1)-h(t-(2k+1)t_1)]$$

Реакция цепи на гармонический сигнал

$$U_{вх}(t)=U_m\cdot\sin(\omega t+\varphi_0)=U_m\frac{e^{j(\omega t+\varphi_0)}-e^{-j(\omega t+\varphi_0)}}{2i}$$

$$U_{вых}(t)=\int_0^t U_{вх}(\tau)w(t-\tau)d\tau=\frac{U_m}{LC(s_1-s_2)\cdot 2i}\Big[\int_0^t e^{j(\omega\tau+\varphi_0)}\left(e^{s_1(t-\tau)}-e^{s_2(t-\tau)}\right)d\tau-$$

$$-\int_0^t e^{-j(\omega\tau+\varphi_0)}\left(e^{s_1(t-\tau)}-e^{s_2(t-\tau)}\right)d\tau\Big]=\frac{U_m}{LC(s_1-s_2)\cdot 2i}\Big[e^{j\varphi_0}\int_0^t e^{j\tau\omega}\Big(\underbrace{e^{s_1(t-\tau)}}_{e^{j\tau\omega+s_1t-s_1\tau}=e^{s_1t}e^{\tau(j\omega-s_1)}}-e^{s_2(t-\tau)}\Big)d\tau-$$

$$-e^{-j\varphi_0}\int_0^t e^{-j\tau\omega}\Big(\underbrace{e^{s_1(t-\tau)}}_{e^{-j\omega\tau+s_1t-s_1\tau}=e^{s_1t}e^{-\tau(j\omega+s_1)}}-e^{s_2(t-\tau)}\Big)d\tau\Big]=\frac{U_m}{LC(s_1-s_2)2i}\Big[e^{j\varphi_0}\Big(e^{s_1t}\frac{e^{\tau(j\omega-s_1)}}{j\omega-s_1}\Big|_0^t-e^{s_2}\frac{e^{\tau(j\omega-s_2)}}{j\omega-s_2}\Big|_0^t\Big)-$$

$$-e^{j\varphi_0}\Big(e^{s_1t}\frac{e^{-\tau(j\omega+s_1)}}{-s_1-j\omega}\Big|_0^t-e^{s_2}\frac{e^{-\tau(j\omega+s_2)}}{-s_2-j\omega}\Big|_0^t\Big)\Big]=\frac{U_m}{LC(s_1-s_2)\cdot 2i}\Big[e^{j\varphi_0}\Big(\frac{e^{s_1t}}{j\omega-s_1}\left(e^{t(j\omega-s_1)}-1\right)-\frac{e^{s_2t}}{j\omega-s_2}\left(e^{t(j\omega-s_2)}-1\right)\Big)-$$

$$-e^{-j\varphi_0}\Big(\frac{e^{s_2}}{s_2+j\omega}\left(e^{-t(j\omega+s_2)}-1\right)-\frac{e^{s_1t}}{s_1+j\omega}\left(e^{-t(j\omega+s_1)}-1\right)\Big)\Big]$$

$$K(j\omega) = \frac{1}{1 + j\omega CR - \omega^2 LC} = \frac{1}{\sqrt{(1-\omega^2 LC)^2 - (\omega RC)^2}} e^{-j \operatorname{arctg}\left(\frac{\omega CR}{1-\omega^2 LC}\right)}$$

$$\sin \omega_0 t = \frac{\omega_0}{S^2 + \omega_0^2}$$

$$U_{вых}(s) = U_1(s) W(s) = \frac{U_m \omega_0}{s^2 + \omega_0^2} \cdot \frac{1}{s^2 LC + sCR + 1} \doteqdot K(\omega_0) \sin(\omega_0 t + \varphi(\omega_0)) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

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$$A_{1,2} = \frac{F_1(S_k)}{F_2'(S_k)} = \frac{U_m \omega_0}{4 S_k^3 LC + 3 S_k^2 RC + 2 S_k (1 + \omega_0^2 LC) + RC\omega_0^2}$$

$$S_{1,2} = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L} - \frac{1}{LC}}$$

$$U_{вых}(t) = \frac{1}{\sqrt{(1-\omega_0^2 LC)^2 - (\omega_0 RC)^2}} \cdot \sin\left(\omega_0 t - \operatorname{arctg}\left(\frac{\omega CR}{1-\omega^2 LC}\right)\right) +$$

$$+ \frac{U_m \omega_0}{4 S_1^3 LC + 3 S_1^2 RC + 2 S_1 (1 + \omega_0^2 LC) + RC\omega_0^2} e^{s_1 t} +$$

$$+ \frac{U_m \omega_0}{4 S_2^3 LC + 3 S_2^2 RC + 2 S_2 (1 + \omega_0^2 LC) + RC\omega_0^2}$$

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