Task 8 Khaetskaya Daria

k) $U_c(0)=U_c(0_+)=U_c(0_-)=\frac{E}{2R}R=\frac{E}{2}$, $U_c(\infty)=E$

$i_L(0)=\frac{E}{2R}$

$U_c(t)=U_{cв}+U_{пр.}=E+A_1e^{p_1t}+A_2e^{p_2t}$

Характ. ур-е: $j\omega L+R+\frac{1}{j\omega C}=0$

$p^2LC+pRC+1=0$

Колебательный процесс:
∃ пара комплексно-сопр. корня
$\Rightarrow D<0$

$$p_{1,2}=\frac{-RC\pm\sqrt{R^2C^2-4LC}}{2LC}=\frac{-6\,2\cdot10^{-5}\pm\sqrt{36\cdot4\cdot10^{-10}-4\cdot2\cdot10^{-5}\cdot10^{-4}}}{2\cdot2\cdot10^{-5}\cdot10^{-4}}=\frac{-12\pm8}{4\cdot10^{-4}}=\left[\begin{array}{l}p_1=-5\cdot10^{4}\\ p_2=10^{4}\end{array}\right.$$

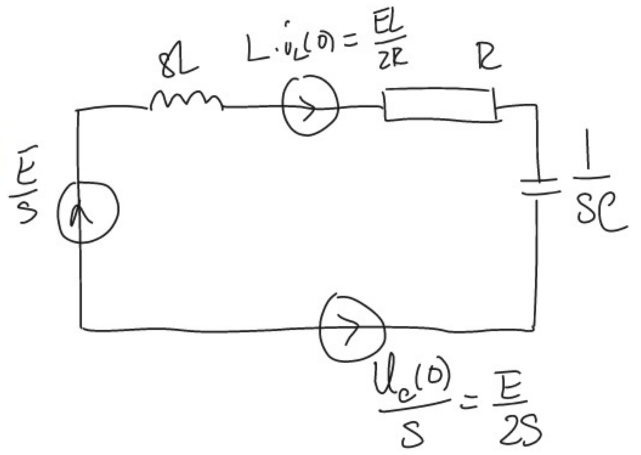
$$\begin{cases}U_c(0)=E+A+B=\frac{E}{2}\\ U_c'(0)=\frac{I_c(0)}{C}=\frac{E}{2RC}=p_1A+p_2\end{cases}\Rightarrow\begin{cases}A=\frac{E/2\left(1/RC+p_2\right)}{p_1-p_2}\\ B=-\frac{E}{2}\left(1+\frac{\left(\frac{1}{RC}+p_2\right)}{p_1-p_2}\right)\end{cases}$$

$$U_c(t)=E+\frac{E/2\left(\frac{1}{RC}+p_1\right)}{p_1-p_2}e^{p_1t}-\frac{E}{2}\left(1+\frac{\left(1/RC+p_2\right)}{p_1-p_2}\right)e^{p_2t}=$$

$$=E+\frac{E}{48}e^{-5\cdot10^4t}-\frac{25}{48}Ee^{-10^4t}$$

$$i(t)=\frac{dU_c}{dt}\cdot C=E\left(-\frac{5\cdot10^4}{48}e^{-5\cdot10^4t}+\frac{5}{48}e^{-10^4t}\right)$$

Оп.)



$$I(s)=\frac{\frac{E}{s}-\frac{E}{2S}-\frac{EL}{2R}}{R+sL+\frac{1}{sC}}=$$

$$= E\left(\underbrace{\frac{C/2}{s^2LC+sRC+1}}_{\frac{C}{2LC(s+10^4)(s+5\cdot 10^4)}} + \underbrace{\frac{L/2R}{R+sL+\frac{1}{sC}}}_{\frac{L/2R}{sRC+s^2LC+1}}\right) \ominus$$

Input interpretation

solve $2\times 10^{-9}s^2 + 12\times 10^{-5}s + 1 = 0$ for s

Results

$s = -50000$

$s = -10000$

$$\frac{L/2R}{sRC+s^2LC+1} = \frac{LC}{2R}\left(\frac{s}{s^2LC+sRC+1}\right) = \frac{LC}{2R}\left(\frac{s}{LC(s+10^4)(s+5\cdot 10^4)}\right) =$$

$$= \frac{1}{2R}\left(\frac{A}{s+10^4} + \frac{B}{s+5\cdot 10^4}\right) = \left[\text{Partial fraction expansion: } \frac{5}{4(s+50000)} - \frac{1}{4(s+10000)}\right] = \frac{1}{2R}\left(\frac{5}{4(s+5\cdot 10^4)} - \frac{1}{4(s+10^4)}\right)$$

$$\ominus\; E\left(\frac{s}{2LC(s+10^4)(s+5\cdot 10^4)} + \frac{1}{2R}\left(\frac{5}{4(s+5\cdot 10^4)} - \frac{1}{4(s+10^4)}\right)\right) \risingdotseq$$

$$\risingdotseq E\left[\frac{1}{8}\left(e^{-10^4t} - e^{-5\cdot 10^4 t}\right) + \frac{1}{48}\left(5e^{-10^4t} - e^{-5\cdot 10^4 t}\right)\right] =$$

$$i(t) = E\left(\frac{5}{48}e^{-10^4t} - \frac{1}{48}e^{-5\cdot 10^4 t}\right)$$

$$U_C(t) = \frac{1}{C}\int i_C(t)\,dt = \frac{E}{C}\left[\frac{5}{48}\int e^{-10^4t}dt - \frac{1}{48}\int e^{-5\cdot 10^4 t}dt\right] =$$

$$= E\left(\frac{1}{48}e^{-5\cdot 10^4 t} - \frac{25}{48}e^{-10^4t}\right) + C \underset{(*)}{=} E\left(\frac{1}{48}e^{-5\cdot 10^4 t} - \frac{25}{48}e^{-10^4t} + 1\right)$$

$$(*)\; U_C'(0) = \frac{E}{2} \Rightarrow -\frac{1}{2}E + C = \frac{E}{2} \Rightarrow C = E$$