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CS 441 - HW3: PDFs and Outliers

Complete the sections below. You do not need to fill out the checklist.

Total Points Available	**[ ] / 160**
1. Estimating PDFs	
a. Segmentation with per-channnel PDFs	[ ] / 15
b. Segmentation with clustered value PDFs	[ ] / 15
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2. Robust Estimation	
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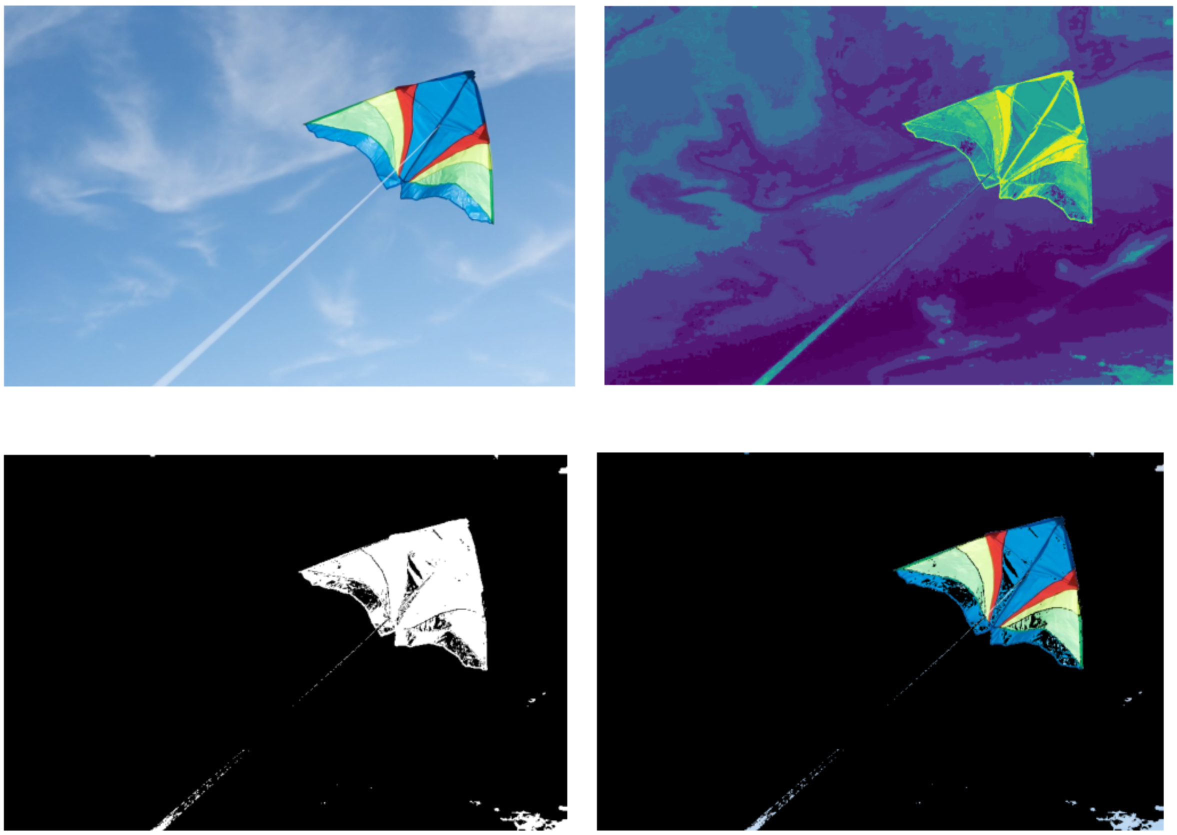
c. Mutual information: discrete pdf	[ ] / 10
d. Mutual information: GMM	[ ] / 10

1. Estimating PDFs

Include the generated images (score map and thresholded RGB) from the display code. List any parameters.

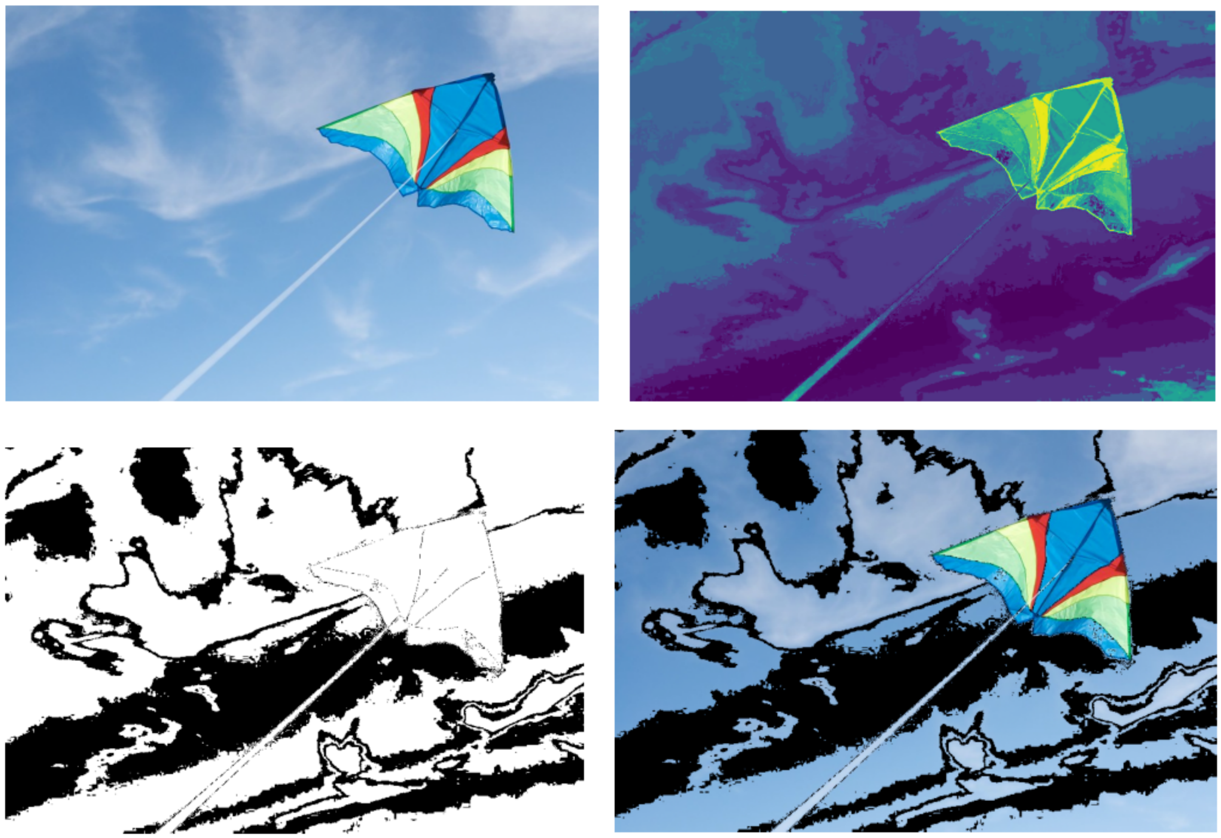
a. Histogram:



Number of bins / discrete values per channel, threshold

Nbins = 32
Threshold = 1

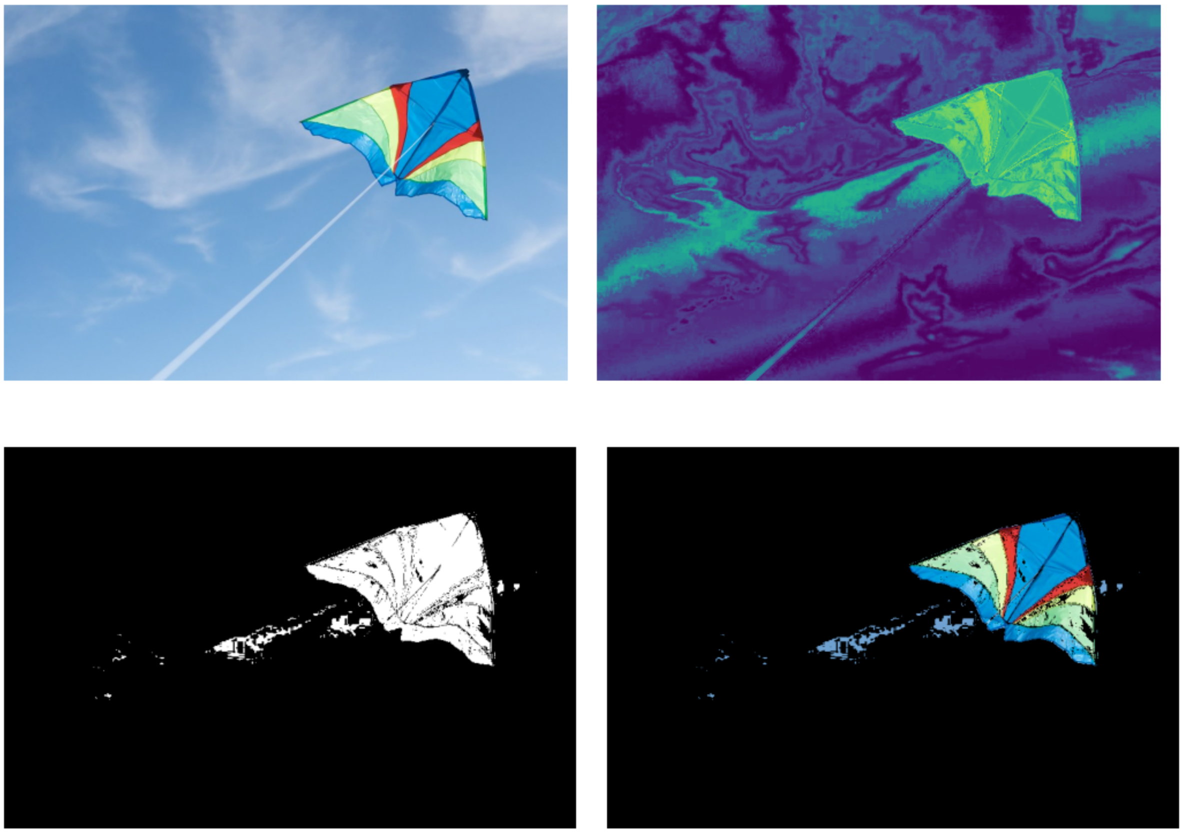
b. Clustering:



Number of clusters, threshold

Clusters= 32
Threshold = 0

c. Gaussian mixture Model:



Number of components, variance model, threshold

Components: 10
Covariance: spherical
Threshold: 2

2. Robust Estimation

Round to nearest whole number.

	a. No noise	b. Percentiles	c. EM
Min	64,694	75,494	64,694
Mean	123,750	113,879	111,984
Std	61,954	15,876	17,966
Max	611,494	159,901	169,008

First five indices of invalid data (based on EM solution, you add last 3)

18	28	49	127	128

3. Stretch Goals

a. Impact of school on salary

Report mean salary overall and for each school

	Average Salary ($)
Overall	123,750
School 0 (UIUC)	122,967
School 1 (MIT)	123,893
School 2 (Cornell)	127,568

Describe your approach to estimate this.

Initial estimates are set based on the standard deviation and mean of other data with valid salaries, the pz value is set to 0.015 as an initial guess using the assumption that UIUC, MIT, and Cornell are the only schools with corrupted data, given that there are 200 entries in the dataset: 15/200 = 0.015. In the E step the probabilities are updated for missing school assignments and then normalized. For each individual, we calculate the probability of belonging to each school based on the current mean and salary estimates and overall SD. In the M step the mean salaries are updated using both the known and estimated assignments.

b. Impact of years of experience on salary

How much are salaries expected to increase with one year of experience?

$ 838

Describe your approach to estimate this.

My approach assumes that ‘p_school_given_salary’ accurately reflects the probability of each salary belonging to each school (calculated is prior code by EM algo). I iterate over each school and filter salary and year data based on highest probability of each data point. I then calculate the salary difference per year of experience for all data point pairs. The mean of the per-year salary differences is the expected salary increase per year of experience, robust to noise due to the application of probabilistic filtering.

c. Mutual information of sex and age, discrete approach

Mutual information (base natural log)

0.693

d. Mutual information of sex and age, GMM approach

Mutual information (base natural log)

Acknowledgments / Attribution

None.

CS441: Applied ML - HW 3

Part 1: Estimating PDFs

In [37]:

```
# initalization code

import numpy as np
from matplotlib import pyplot as plt
import cv2

# read images
datadir = "/Users/darian/Desktop/UIUC/Applied ML/HW3/Code/"
im = cv2.imread(datadir + 'kite.jpg')   # this is the full image
im = cv2.cvtColor(im, cv2.COLOR_BGR2RGB)/255
im = cv2.blur(im, (3, 3))

crop = cv2.imread(datadir + 'kite_crop.jpg')   # this is the cropped image
crop = cv2.cvtColor(crop, cv2.COLOR_BGR2RGB)/255
crop = cv2.blur(crop, (3, 3))

# displays a single image
def display_image(im):
  plt.imshow(im)
  plt.axis('off')
  plt.show()

# displays the image, score map, thresholded score map, and masked image
def display_score(im, score_map, thresh):
  display_image(im)
  display_image(np.reshape(score, (im.shape[:2])))
  plt.imshow(np.reshape(score_map>thresh, (im.shape[0], im.shape[1])), cmap='gray')
  plt.axis('off')
  plt.show()
  display_image(np.tile(np.reshape(score_map>thresh, (im.shape[0], im.shape[1], 1)), (1,
1,3))*im)

print('Whole image')
display_image(im)

print('Foreground')
display_image(crop)
```

```
Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0.
.255] for integers).
```

```
Whole image
```

```
Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).
```

```
Foreground
```

Method 1 (per channel hist)

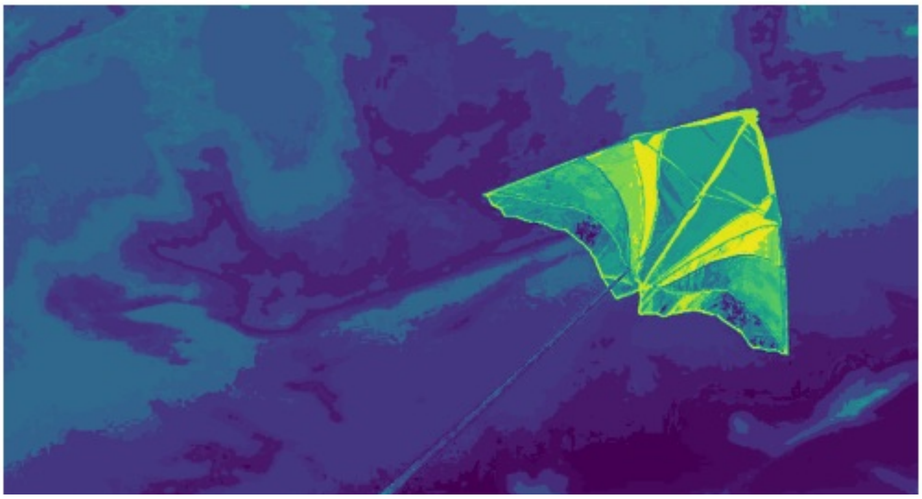
In [142]:

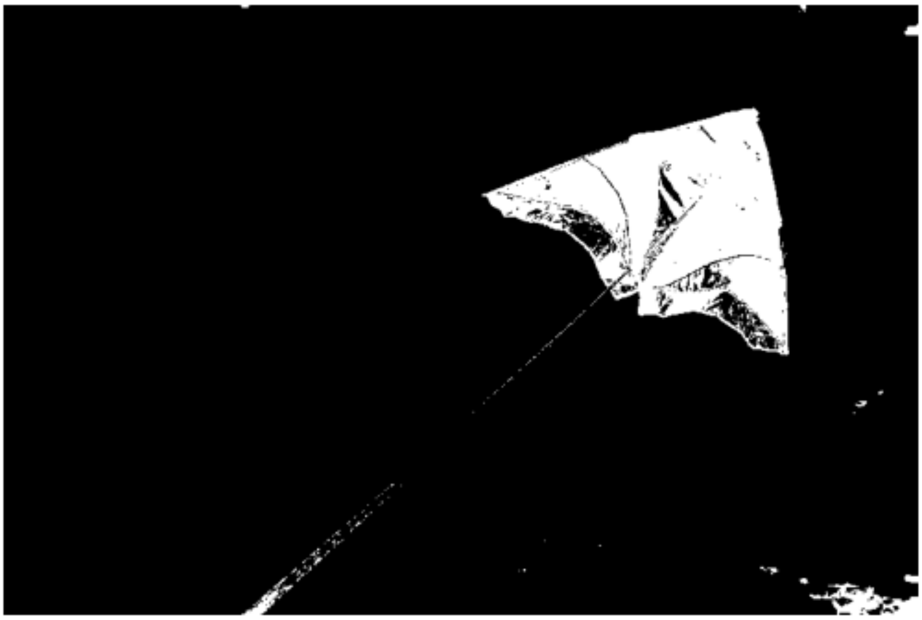
```
import numpy as np
import matplotlib.pyplot as plt


# pdf for 1a
def estimate_discrete_pdf(values, nvalues, prior=1):
    '''
    Estimate P(values=v) for each possible v in (0, nvalues)
    Input:
        values: the values of the data
        nvalues: range of values, such that 0 <= values < nvalues
        prior: initial count used to prevent any value from having zero probability
    Output:
        p[nvalues,]: P(values=v) for each v
    '''

    prob = np.zeros(nvalues) + prior
    # Count occurrences of each value
    for value in values:
        prob[int(value)] += 1
    # Normalize to get probability
    prob /= np.sum(prob)
    return prob

def estimate_channel_pdf(channel_data, bins):
    # Calculate histogram for the channel
    histogram, bin_edges = np.histogram(channel_data, bins=bins, range=(0, 1), density=True)
    bin_centers = (bin_edges[:-1] + bin_edges[1:]) / 2
    return histogram, bin_centers
```

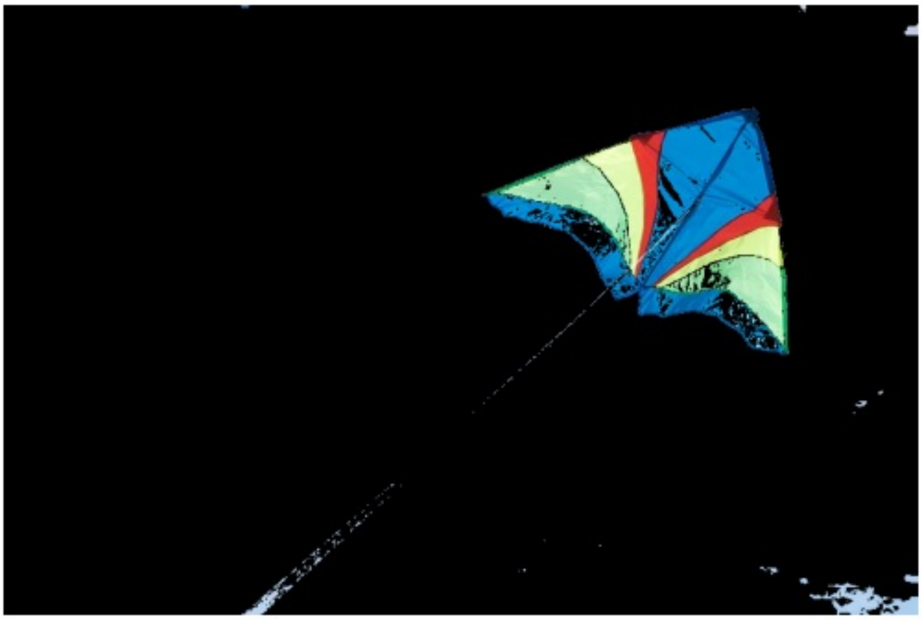
In [153]:

```
import numpy as np
```

```
def compute_score(im, crop, bins):
    # Convert image and crop pixels from 0-255 to 0-(bins-1) range
    im_bins = np.floor(im * (bins - 1)).astype(int)
    crop_bins = np.floor(crop * (bins - 1)).astype(int)

    score_map = np.zeros(im.shape[:2])

    # Estimate PDFs for each color channel and compute score
    for channel in range(3):
        # Estimate PDF for full image and cropped image
        pdf_full = estimate_discrete_pdf(im_bins[:, :, channel].flatten(), bins)
        pdf_crop = estimate_discrete_pdf(crop_bins[:, :, channel].flatten(), bins)

        # Compute log likelihood ratio for each pixel in the channel
        score_channel = np.log((pdf_crop[im_bins[:, :, channel]] + 1e-10) / (pdf_full[im
_bins[:, :, channel]] + 1e-10))

        # Add to the overall score map
        score_map += score_channel.reshape(im.shape[:2])

    return score_map


# Estimate PDFs and compute score per pixel
score = compute_score(im, crop, bins=16)

# Display score map and thresholded images
t = 1
display_score(im=im, score_map=score, thresh=t)
```







Method 2 (Kmeans)

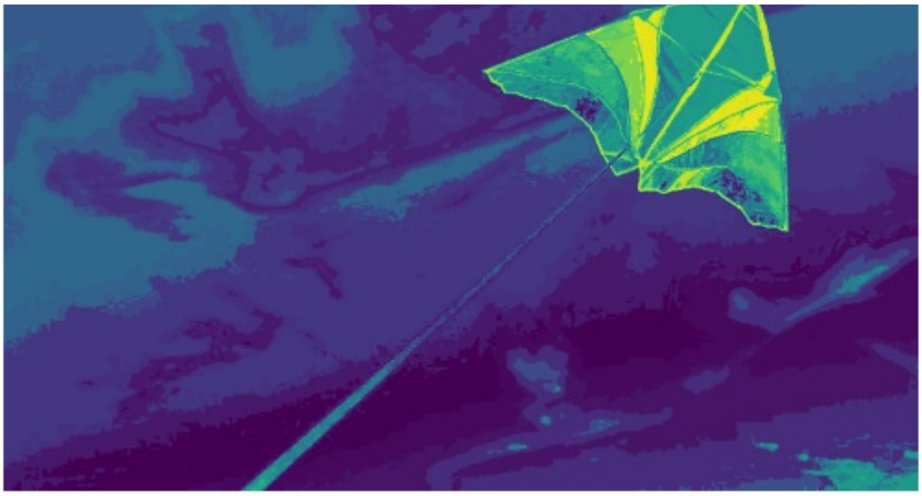
In [ ]:

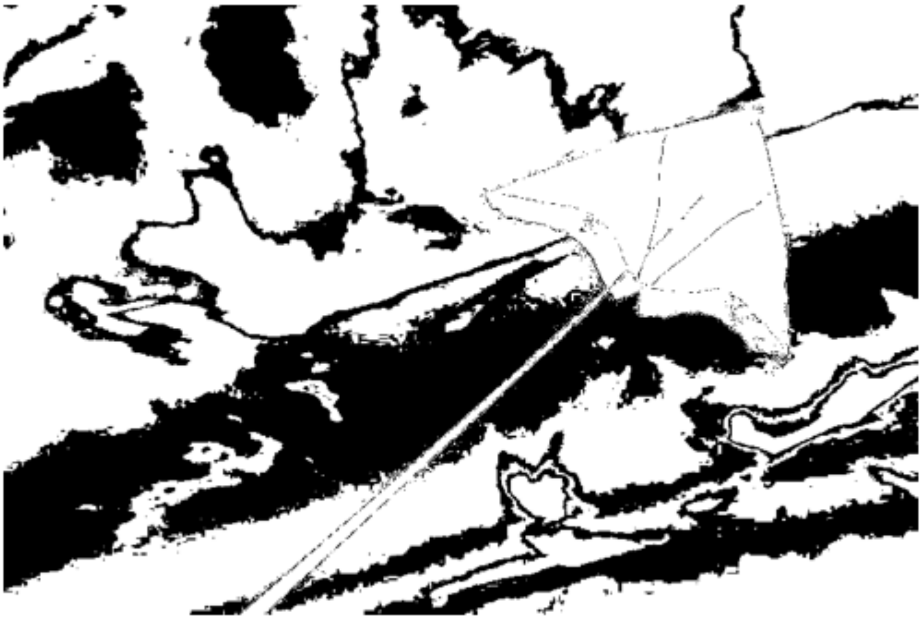
```
# init
!apt install libomp-dev > /dev/null 2>&1
!pip install faiss-cpu > /dev/null 2>&1
import faiss
```

In [175]:

```
# estimate PDFs and compute score per pixel

# TO DO

# pdf for 1b
def estimate_pdf_from_labels(labels, n_clusters):
    # Count the occurrences of each label (cluster)
    counts = np.bincount(labels, minlength=n_clusters)

    # Convert counts to probabilities
    pdf = counts / np.sum(counts)
```

```python
def run_kmeans(data, n_clusters):
    d = data.shape[-1]
    data = data.reshape(-1, d).astype(np.float32)

    # Initialize and train kmeans
    kmeans = faiss.Kmeans(d, n_clusters, niter=20, verbose=False)
    kmeans.train(data)

    # Assign each vector to the nearest cluster center
    D, I = kmeans.index.search(data, 1)
    return kmeans, I.flatten()

# read images
datadir = "/Users/darian/Desktop/UIUC/Applied ML/HW3/Code/"
im = cv2.imread(datadir + 'kite.jpg')
im = cv2.cvtColor(im, cv2.COLOR_BGR2RGB)/255
crop = cv2.imread(datadir + 'kite_crop.jpg')
crop = cv2.cvtColor(crop, cv2.COLOR_BGR2RGB)/255


n_clusters = 32

im_3 = np.reshape(im, (im.shape[0]*im.shape[1], 3))
crop_3 = np.reshape(crop, (crop.shape[0]*crop.shape[1], 3))

# Run FAISS k-means on the image data
kmeans_im, labels_im = run_kmeans(im_3, n_clusters)
kmeans_crop, labels_crop = run_kmeans(crop_3, n_clusters)

# Estimate the PDFs from the k-means clusters
pdf_full = estimate_pdf_from_labels(labels_im, n_clusters)
pdf_crop = estimate_pdf_from_labels(labels_crop, n_clusters)

# Compute the score per pixel
# Use the cluster labels to index into the PDFs
score_map = np.zeros_like(labels_im, dtype=np.float32)
for i in range(n_clusters):
    mask = labels_im == i
    score_map[mask] = np.log((pdf_crop[i] + 1e-10) / (pdf_full[i] + 1e-10))

score_map_reshaped = score_map.reshape(im.shape[0], im.shape[1])

t = -1
display_score(im, score_map_reshaped, t)
```



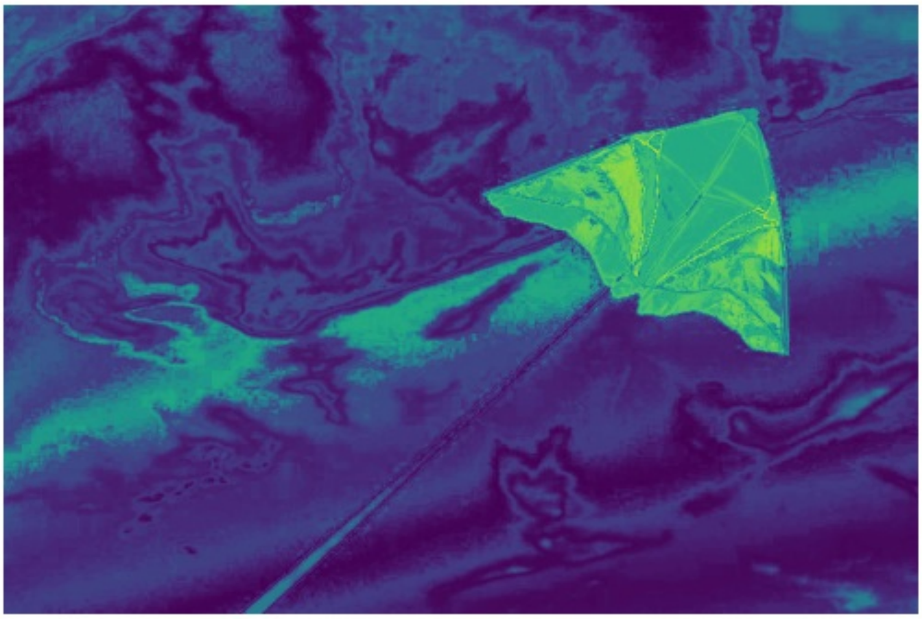


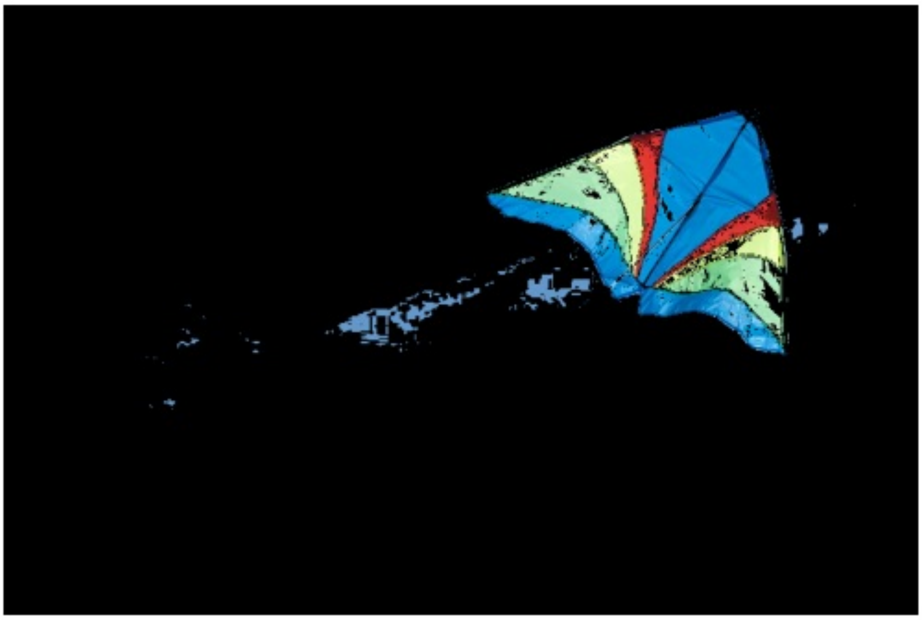
Method 3 (GMM)

In [190]:

```
from sklearn.mixture import GaussianMixture
```

```
gmm_full = GaussianMixture(n_components=10, covariance_type='spherical').fit(im_3)

gmm_crop = GaussianMixture(n_components=10, covariance_type='spherical').fit(crop_3)

# Compute the log probabilities (log PDF)
log_prob_full = gmm_full.score_samples(im_3)
log_prob_crop = gmm_crop.score_samples(im_3)

# Compute the score per pixel as the difference in log probabilities
score = log_prob_crop - log_prob_full

# Reshape score to the original image shape for display
score = score.reshape(im.shape[0], im.shape[1])

t = 2
display_score(im=im, score_map=score, thresh=t)
```





Part 2: Robust Estimation

In [191]:

```
import numpy as np
from matplotlib import pyplot as plt

datadir = "/Users/darian/Desktop/UIUC/Applied ML/HW3/Code/"

# load data
T = np.load(datadir + 'salary.npz')
(salary, years, school) = (T['salary'], T['years'], T['school'])
```

1. No noise

Compute the statistics for the data as a whole

In [86]:

```
# TO DO
import numpy

salary_mu = np.mean(salary)
salary_std = np.std(salary)
salary_min = np.min(salary)
salary_max = np.max(salary)

print('Mean: {}  Std: {}  Min: {}   Max: {}'.format(salary_mu, salary_std, salary_min, s
alary_max))
```

```
Mean: 123749.835  Std: 61953.77348723623  Min: 64694.0   Max: 611494.0
```

In [98]:

```
import numpy as np
from matplotlib import pyplot as plt

pct_5 = np.percentile(salary, 5)
pct_95 = np.percentile(salary, 95)

salary_min = pct_5 - (pct_95 - pct_5) * 0.05 / 0.9
salary_max = pct_95 + (pct_95 - pct_5) * 0.05 / 0.9

salary_range = salary[(salary >= pct_5) & (salary <= pct_95)]
salary_mu = np.mean(salary_range)
salary_std = np.std(salary_range)

print('Mean: {:.2f}  Std: {:.2f}  Min: {:.2f}   Max: {:.2f}'.format(salary_mu, salary_st
d, salary_min, salary_max))
```

```
Mean: 113878.65  Std: 15876.45  Min: 75493.80   Max: 159900.80
```

3. EM

Assume valid data follows a Gaussian distribution, while the fake data has a uniform distribution between the minimum and maximum value of salary.

In [136]:

```
import numpy as np

niter = 20

N = 1
M = len(salary)

salary_mean = np.mean(salary).reshape((1, 1))
salary_std = np.sqrt(np.sum((salary - salary_mean) ** 2) / N/M)

pz = 0.5

for t in range(niter):
    # E-step: Calculate the probability of each salary being valid
    # Assuming Gaussian distribution for valid data and uniform distribution for invalid
data
    prob_valid = pz * (1 / (np.sqrt(2 * np.pi) * salary_std)) * np.exp(-0.5 * ((salary -
salary_mean) / salary_std) ** 2)
    prob_invalid = (1 - pz) * (1 / (salary.max() - salary.min()))
    p_valid_given_salary = prob_valid / (prob_valid + prob_invalid)

    # M-step: Update estimates for mean, standard deviation, and pz
    salary_mean = np.sum(p_valid_given_salary * salary) / np.sum(p_valid_given_salary)
    salary_std = np.sqrt(np.sum(p_valid_given_salary * (salary - salary_mean) ** 2) / np
.sum(p_valid_given_salary))
    pz = np.mean(p_valid_given_salary)


# Calculating min and max salaries based on the valid data range
salary_min = np.min(salary[p_valid_given_salary>0.5])
salary_max = np.max(salary[p_valid_given_salary>0.5])

print('Mean: {:.2f}'.format(salary_mean))
print('Std: {:.2f}'.format(salary_std))
print('Min: {:.2f}'.format(salary_min))
print('Max: {:.2f}'.format(salary_max))

invalid_indices = np.where(p_valid_given_salary < 0.5)[0][:5]
print('Indices of likely invalid salaries:', invalid_indices)
```

```
Indices of likely invalid salaries: [ 18  28  49 127 128]
```

Part 3: Stretch Goals

Include all your code used for any stretch goals in this section. Add headings where appropriate.

3a

In [141]:

```
import numpy as np


num_schools = 3
niter = 20
pz = 0.015  # assuming 3 invalid entries in the dataset of 200 entries

# Initial estimates
salary_means = np.array([np.mean(salary[school == i]) if i in school else np.mean(salary
) for i in range(num_schools)])
salary_std = np.std(salary)

# Placeholder
p_school_given_salary = np.zeros((len(salary), num_schools))

for j in range(niter):
    # E-step: Update probabilities for missing school assignments
    for i in range(num_schools):
        prob_valid = pz * (1 / (np.sqrt(2 * np.pi) * salary_std)) * np.exp(-0.5 * ((sala
ry - salary_means[i]) / salary_std) ** 2)
        p_school_given_salary[:, i] = prob_valid

    # Normalize
    p_school_given_salary /= p_school_given_salary.sum(axis=1, keepdims=True)

    # M-step: Update mean salary for each school
    for i in range(num_schools):
        # Update using both known and estimated assignments
        weights = p_school_given_salary[:, i] + (school == i)
        salary_means[i] = np.sum(weights * salary) / np.sum(weights)

    # Update overall standard deviation based on new means
    weighted_sums = np.sum([p_school_given_salary[:, i] * (salary - salary_means[i])**2
for i in range(num_schools)], axis=0)
    salary_std = np.sqrt(np.sum(weighted_sums) / len(salary))

# Calculate overall mean salary
overall_mean_salary = np.mean(salary)

print("Overall Mean Salary: {:.2f}".format(overall_mean_salary))
for i in range(num_schools):
    print("Mean Salary for School {}: {:.2f}".format(i, salary_means[i]))
```

```
Overall Mean Salary: 123749.84
Mean Salary for School 0: 122966.89
Mean Salary for School 1: 123893.12
Mean Salary for School 2: 127568.09
```

3b

In [196]:

```
import numpy as np
```

```
num_schools = 200
salary_increases = []

# Iterate over each school
for school_id in range(num_schools):
    # Filter data for the current school based on the highest probability from the EM alg
orithm
    school_filter = np.argmax(p_school_given_salary, axis=1) == school_id

    # Extract salary and years for the filtered data
    filtered_salary = salary[school_filter]
    filtered_years = years[school_filter]

    # Calculate differences only within filtered data
    for i in range(len(filtered_salary)-1):
        for j in range(i+1, len(filtered_salary)):
            year_diff = filtered_years[j] - filtered_years[i]
            salary_diff = filtered_salary[j] - filtered_salary[i]

            # Only consider positive year differences to calculate the increase per year
            if year_diff > 0:
                salary_increases.append(salary_diff / year_diff)

# Calculate the weighted average of salary increases: considers all pairwise differences
within the data filtered by school probability
if salary_increases:
    expected_increase_per_year = np.mean(salary_increases)
else:
    expected_increase_per_year = 0

print("Expected increase in salary per year of experience: {:.2f}".format(expected_increa
se_per_year))
```

```
Expected increase in salary per year of experience: 837.63
```

3c

In [197]:

```
import numpy as np
from sklearn.metrics import mutual_info_score

# Example data
data = np.array([
    [59, 2, 32.1, 101, 157, 93.2, 38, 4, 4.8598, 87, 151],
    [48, 1, 21.6, 87, 183, 103.2, 70, 3, 3.8918, 69, 75],
    [72, 2, 30.5, 93, 156, 93.6, 41, 4, 4.6728, 85, 141],
    [24, 1, 25.3, 84, 198, 131.4, 40, 5, 4.8903, 89, 206]
])

age = data[:, 0]
sex = data[:, 1]


mutual_info = mutual_info_score(age, sex)

print(f"The mutual information between AGE and SEX is: {mutual_info}")
```

```
The mutual information between AGE and SEX is: 0.6931471805599452
```

In [ ]:

```
# from https://gist.github.com/jonathanagustin/b67b97ef12c53a8dec27b343dca4abba
# install can take a minute
```

```
#-------------------------------------------------------------------------------#
from google.colab import drive
drive.mount("/content/drive/", force_remount=True)
NOTEBOOK_PATH = f"{NOTEBOOKS_DIR}/{NOTEBOOK_NAME}"
assert os.path.exists(NOTEBOOK_PATH), f"NOTEBOOK NOT FOUND: {NOTEBOOK_PATH}"
!apt install -y texlive-xetex texlive-fonts-recommended texlive-plain-generic > /dev/nul
l 2>&1
!jupyter nbconvert "$NOTEBOOK_PATH" --to pdf > /dev/null 2>&1
NOTEBOOK_PDF = NOTEBOOK_PATH.rsplit('.', 1)[0] + '.pdf'
assert os.path.exists(NOTEBOOK_PDF), f"ERROR MAKING PDF: {NOTEBOOK_PDF}"
print(f"PDF CREATED: {NOTEBOOK_PDF}")
```