Darian Irani – irani2

Problem 1 (30 points)**.** Is $0 \le v(t) \le v_0$ an invariant of A? No need to write a complete proof; a two sentence argument would suffice.

① Case 1: $v(t) \le v_0$ because $v_1(0) = v_0$ and car decreases speed /stops if $s(t) = 1$ and maintains current speed if $s(t) = 0$.

Case 2: $v_1(t) \ge 0 \rightarrow$ if $s(t) = 1$ and $v_1(t) < ab$ then $v_1(t+1) = 0$ and if $s(t) = 0$ then car maintains current speed.

These cases imply $0 \le v_1(t) \le v_0$ at all times $\therefore$ $0 \le v_1(t) \le v_0$ is an invariant of A

Problem 2 (30 points)**.** Is $\text{timer}(t) \le v_0/a_b$ an invariant of A? Explain why. Can we use the induction method to prove this invariant? If so, present your proof.

Hint: You may find the usage of other invariants handy in your proof.

② Proof by induction of $\text{timer}(t) \le v_0/a_b$ invariance for automaton A:

Need to prove: $\text{timer}(t+1) \le v_0/a_b$. Induction assumption: $\text{timer}(t) \le v_0/a_b$

Case 1: If $d(t) \le D_{sense}$ and $v_1(t) \ge ab$ then $\text{timer}(t+1) = \text{timer}(t) + 1$ given $\text{timer}(t) + 1 \le v_0/a_b + 1$ $v_1(t)$ decreases by a_b at every time increment. timer will increment v_0/a_b times. When $v_1(t) < a_b$ will not increment.

Case 2: $\text{timer}(t+1) = \text{timer}(t)$ for all other conditions $\text{timer}(t+1) \le v_0/a_b$ is true for induction assumption.

$\therefore$ $\text{timer}(t+1) \le v_0/a_b$ is an invariant of A.

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Problem 3 (40 points)**.** Let us now introduce some delay in the sensing-computation-actuation pipeline, say T_{react}. This could model cognitive delay of a human driver or processing delay in electronics and computers. Assume we have exactly T_{react} seconds delay between the sensing of the pedestrian and the application of the brakes (the **start** of the deceleration). Moreover, let us also introduce acceleration to the vehicle: whenever the vehicle is outside the sensing distance, the vehicle undergoes constant acceleration a_s. Rewrite the new updated model with the sensing delay and vehicle acceleration.

③ SimpleCar $(D_{sense}, V_0, x_1, x_2, a_b)$, $x_{20} > x_{10}$

Initially: $x_1(0) = x_{10}$, $x_2(0) = x_{20}$, $v_1(0) = V_0$, $v_2(0) = 0$

$s(0) = 0$, $timer(0) = 0$, $timer2(0) = 0$

$d(t) = x_2(t + T_{react}) - x_1(t)$

if $d(t) \leq D_{sense}$

$s(t+1) = 1$

if $v_1(t) \geq a_b$

$v_1(t+1) = v_1(t) - a_b$

$timer(t+1) = timer(t) + 1$

$timer2(t+1) = timer2(t)$

else

$v_1(t+1) = 0$

$timer2(t+1) = timer(t)$

$timer2(t+1) = timer2(t)$

else

$s(t+1) = 0$

$v_1(t+1) = v_1(t) + a_s$

$timer(t+1) = timer(t)$

$timer2(t+1) = timer2(t) + 1$

$x_1(t+1) = x_1(t) + v_1(t)$