Lecture 3: Anatomy & Training of Neural Networks

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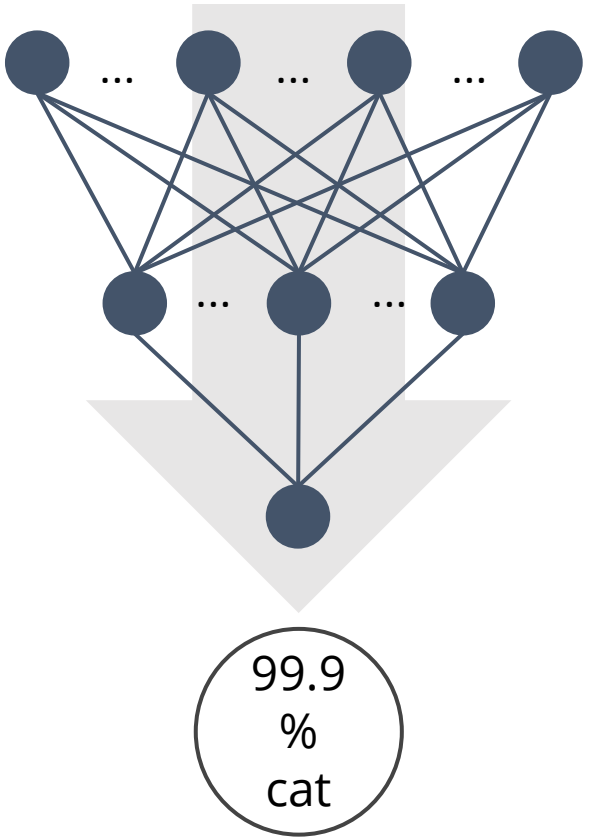
Artificial Neural Networks

1. Machine Learning Algorithm
2. Very simplified parametric models of our brain
3. Networks of basic processing units: neurons
4. Neurons store information which has to be learned (the weights or the state for RNNs)
5. Many types of architectures to solve different tasks
6. They can scale to massive data



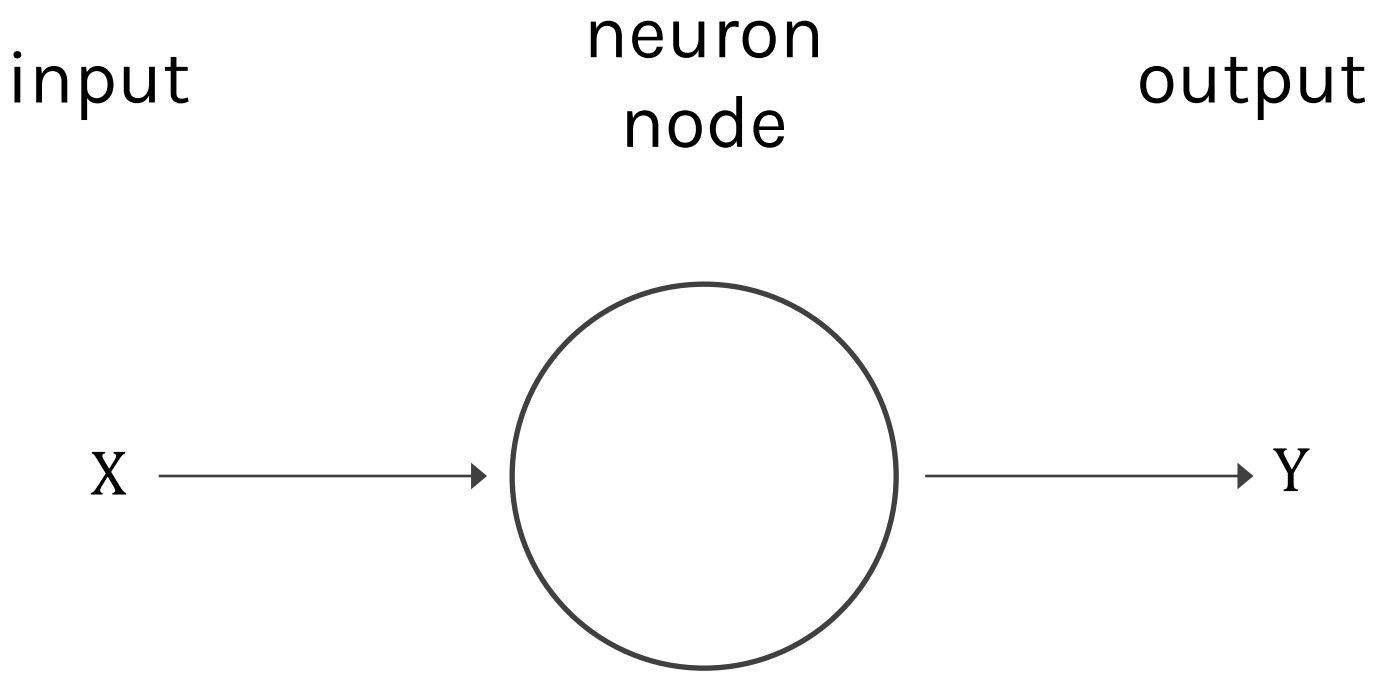
Outline

- Anatomy of a NN
- Design choices
- Learning (Next Monday)
- Wednesday: PyTorch tutorial
- HW1 is out

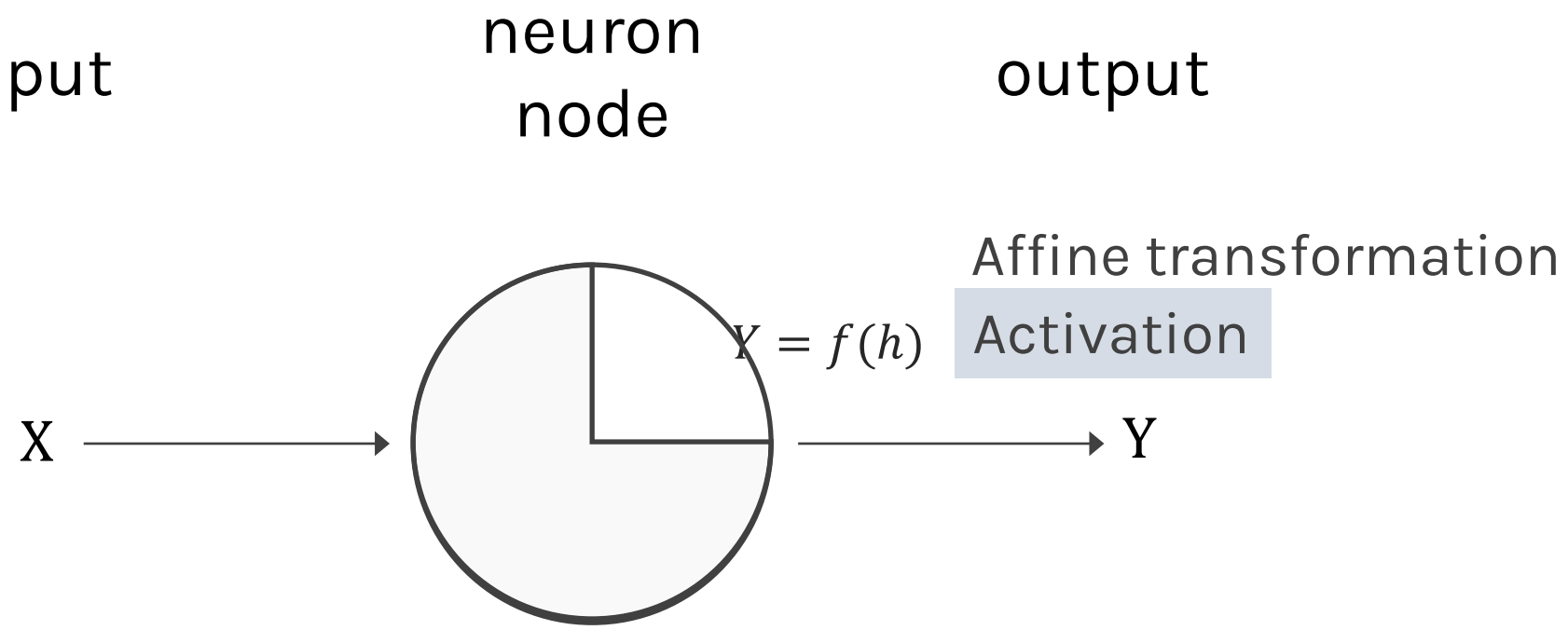
Disclaimer: Slides are heavily inspired by ML and AI courses at CMU and Princeton

Feed Forward Artificial Neural Networks

Anatomy of artificial neural network (ANN)

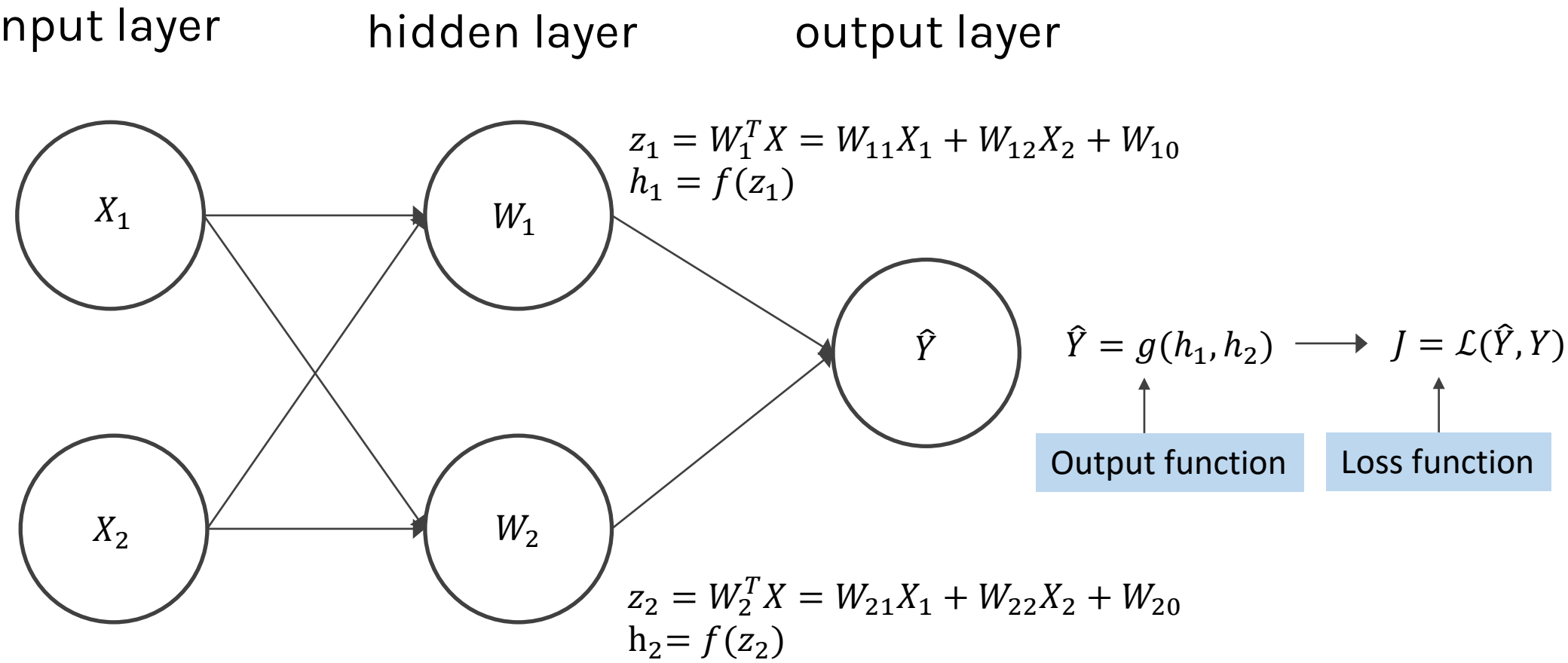


Anatomy of artificial neural network (ANN)



We will talk later about the choice of activation function.

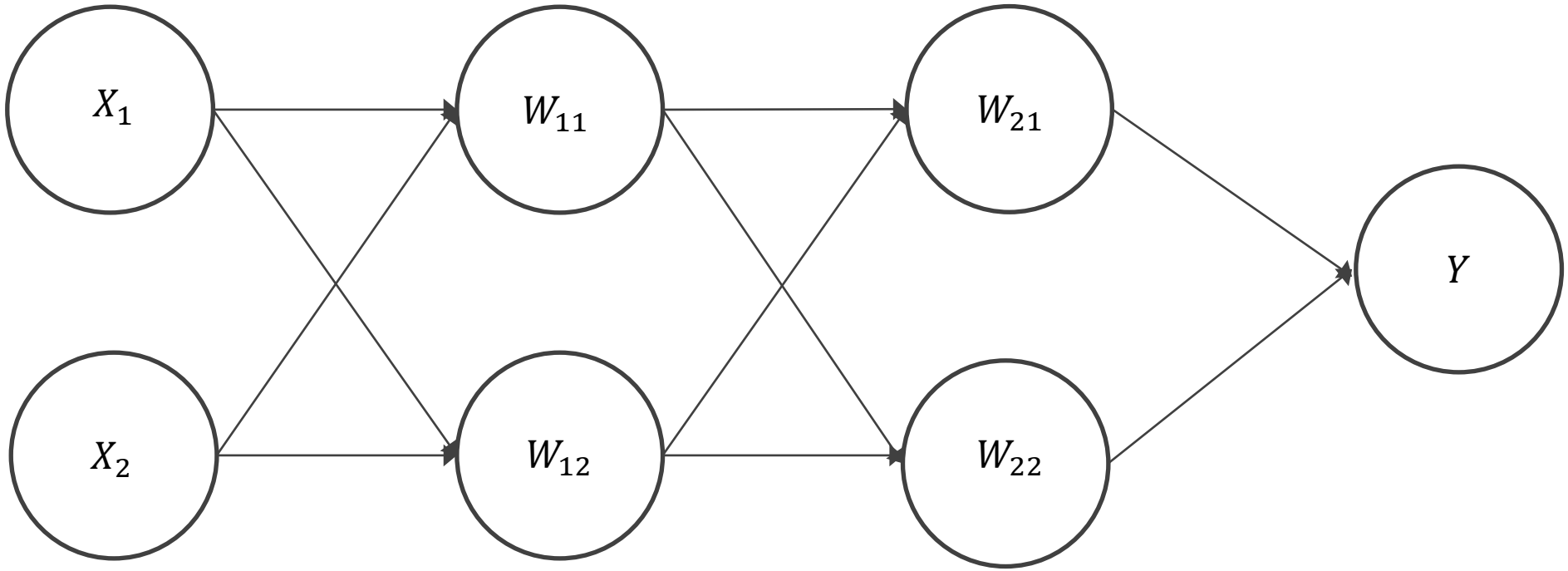
Anatomy of artificial neural network (ANN)



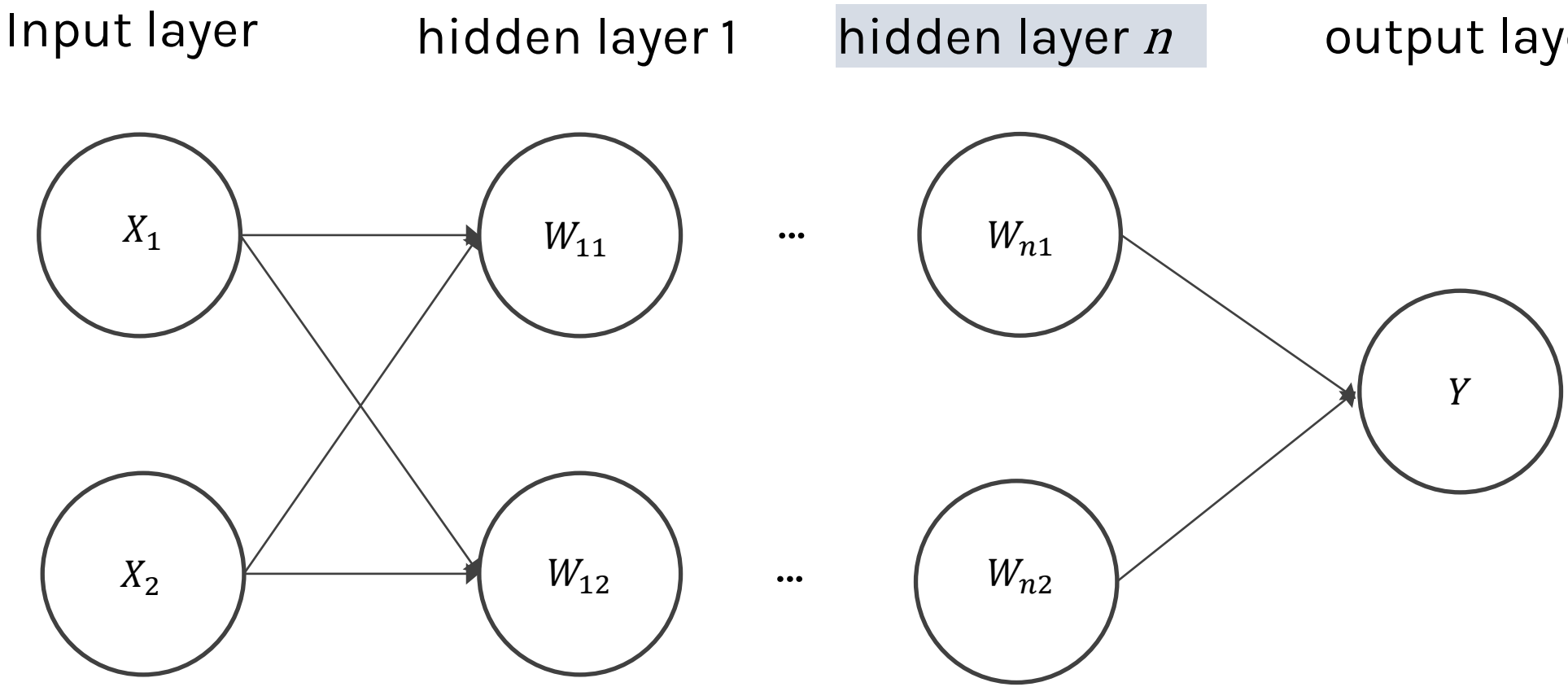
We will talk later about the choice of the output layer and the loss function.

Anatomy of artificial neural network (ANN)

Input layer hidden layer 1 hidden layer 2 output layer



Anatomy of artificial neural network (ANN)



We will talk later about the choice of the number of layers.

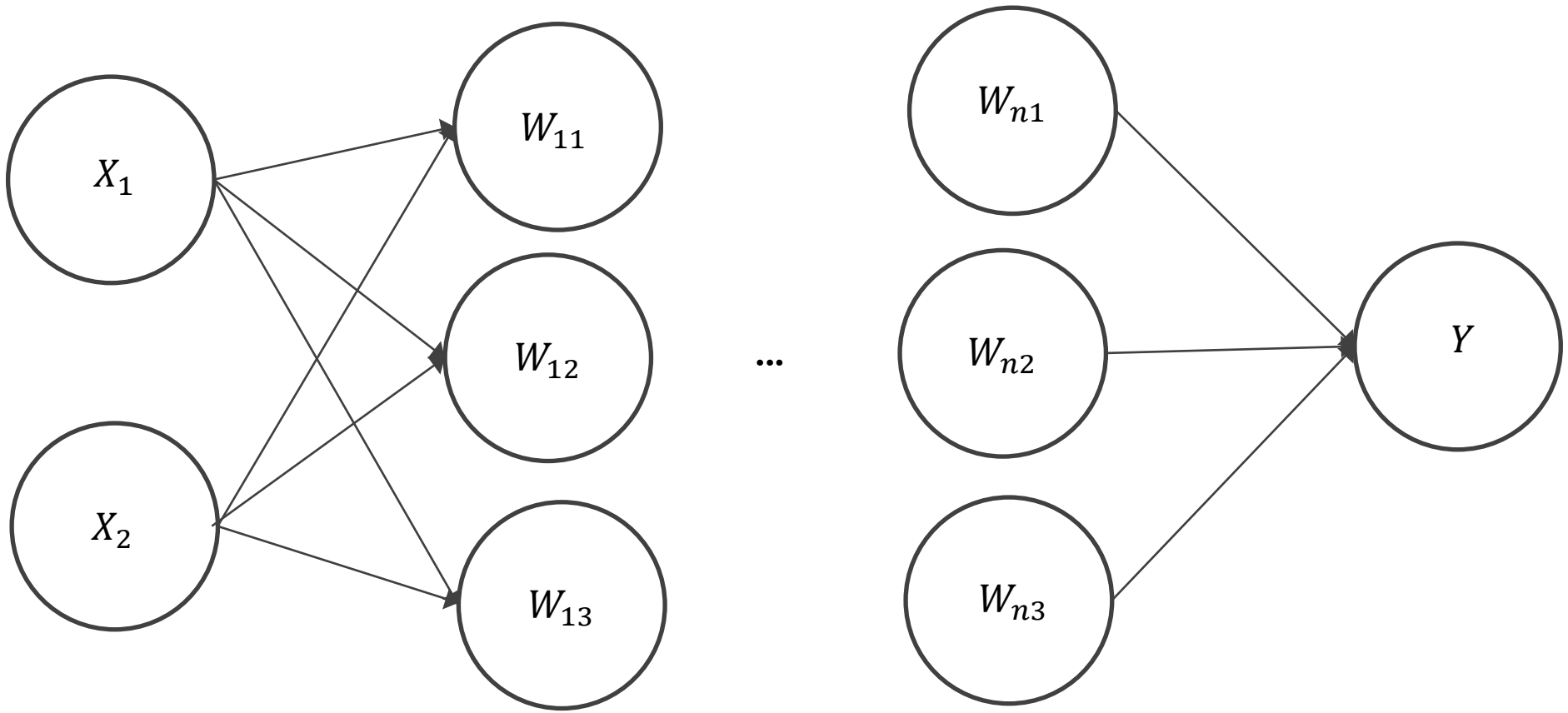
Anatomy of artificial neural network (ANN)

Input layer

hidden layer 1,
3 nodes

hidden layer n
3 nodes

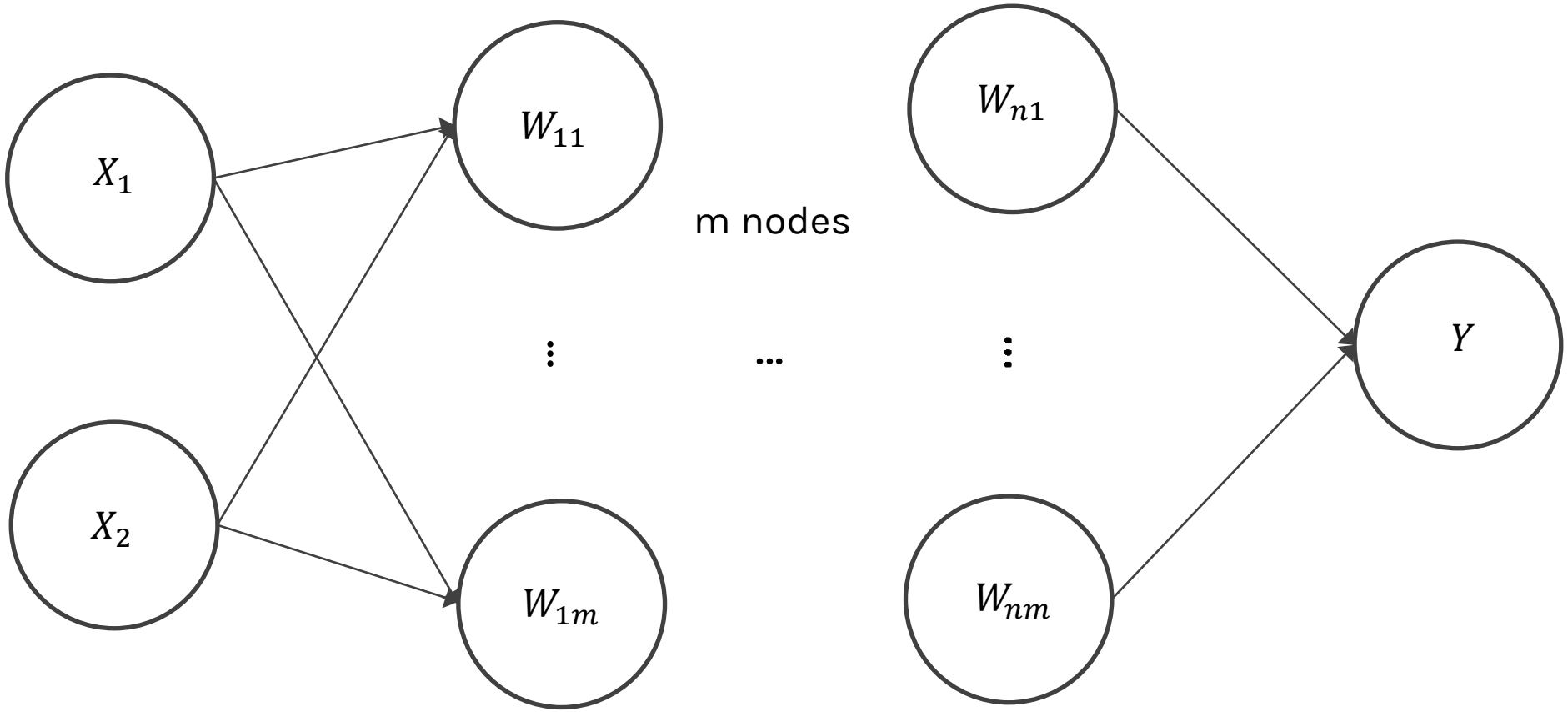
output layer



Anatomy of artificial neural network (ANN)

Input layer hidden layer 1, hidden layer n output layer

m nodes



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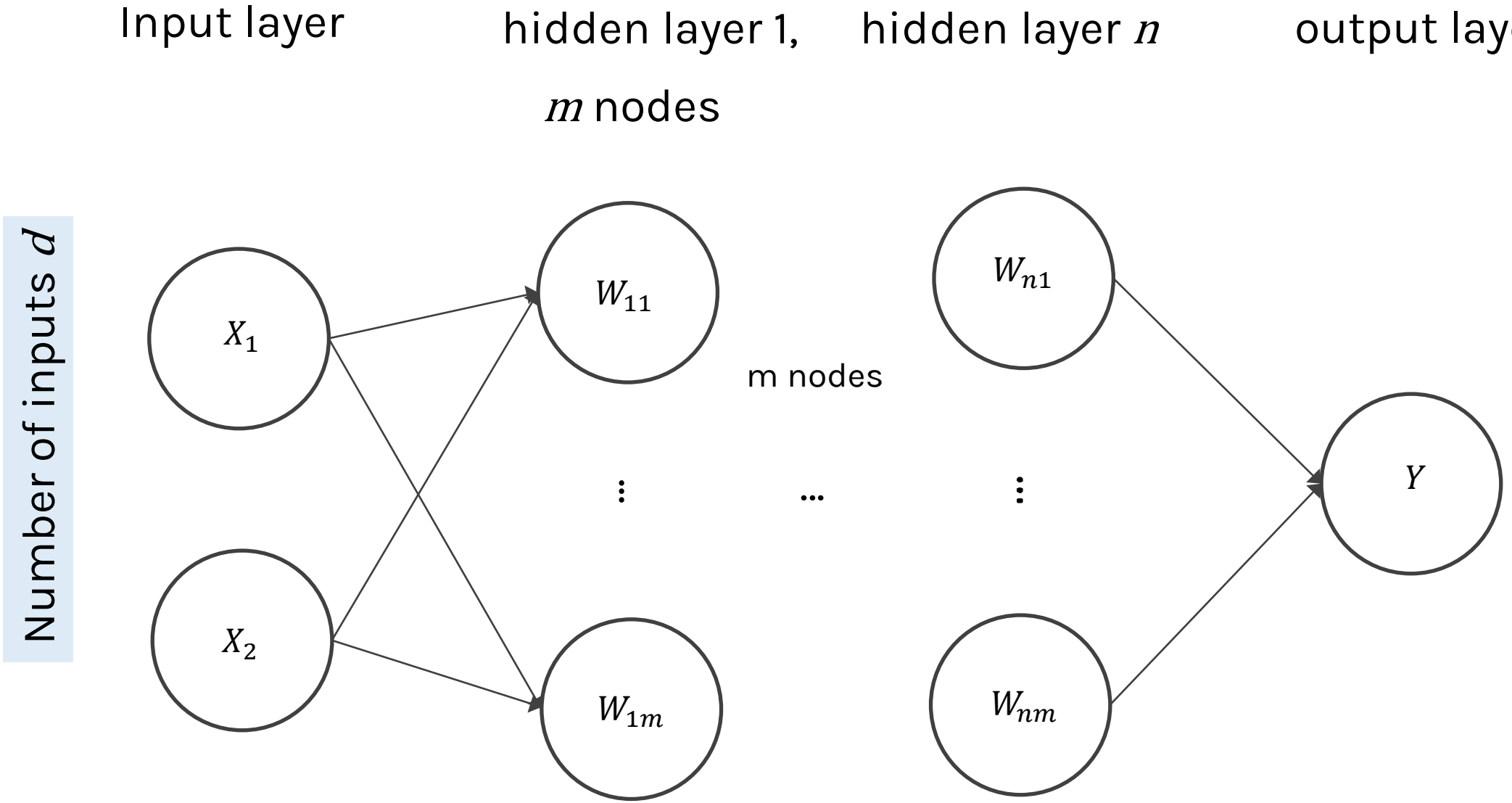
Anatomy of artificial neural network (ANN)



Number of inputs is specified by the data

Beyond Linear Models

- Linear models
  - Can be fit efficiently (via convex optimization)
  - Limited model capacity
- Alternative:

$$f(x) = w^T \phi(x)$$

- Where ϕ is a *non-linear transform*

Traditional ML

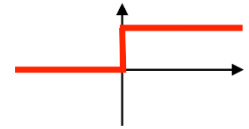
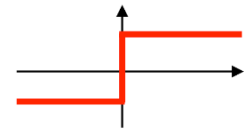
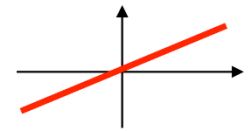
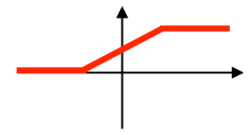
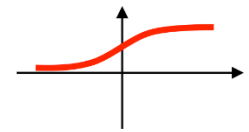
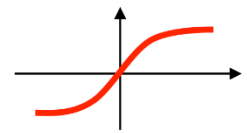
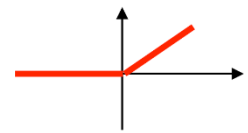
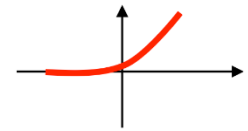
- Manually engineer ϕ
  - Domain specific, enormous human effort
- Generic transform
  - Maps to a higher-dimensional space
  - Kernel methods: e.g. RBF kernels
  - Over fitting: does not generalize well to test set
  - Cannot encode enough prior information

Deep Learning

- Directly learn ϕ

$$f(x;\theta) = W^T \phi(x;\theta)$$

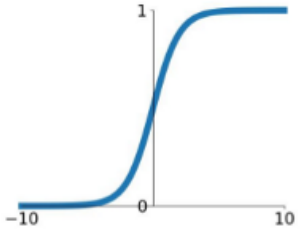
- where θ are parameters of the transform
- ϕ defines hidden layers
- Can encode prior beliefs, generalizes well

Activation function	Equation	Example	1D Graph
Unit step (Heaviside)	$\phi(z) = \begin{cases} 0, & z < 0, \\ 0.5, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Sign (Signum)	$\phi(z) = \begin{cases} -1, & z < 0, \\ 0, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Linear	$\phi(z) = z$	Adaline, linear regression	
Piece-wise linear	$\phi(z) = \begin{cases} 1, & z \geq \frac{1}{2}, \\ z + \frac{1}{2}, & -\frac{1}{2} < z < \frac{1}{2}, \\ 0, & z \leq -\frac{1}{2}, \end{cases}$	Support vector machine	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression, Multi-layer NN	
Hyperbolic tangent	$\phi(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$	Multi-layer Neural Networks	
Rectifier, ReLU (Rectified Linear Unit)	$\phi(z) = max(0, z)$	Multi-layer Neural Networks	
Rectifier, softplus	$\phi(z) = \ln(1 + e^{z})$	Multi-layer Neural Networks	

Modern Activation Function Choices

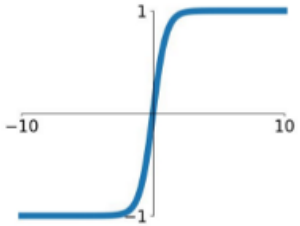
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



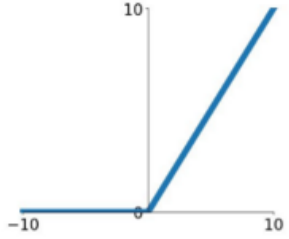
tanh

$$\tanh(x)$$



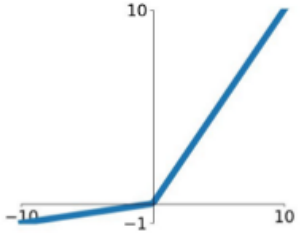
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

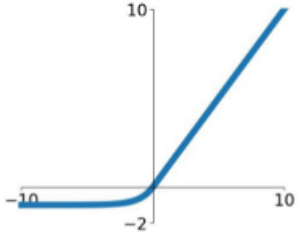


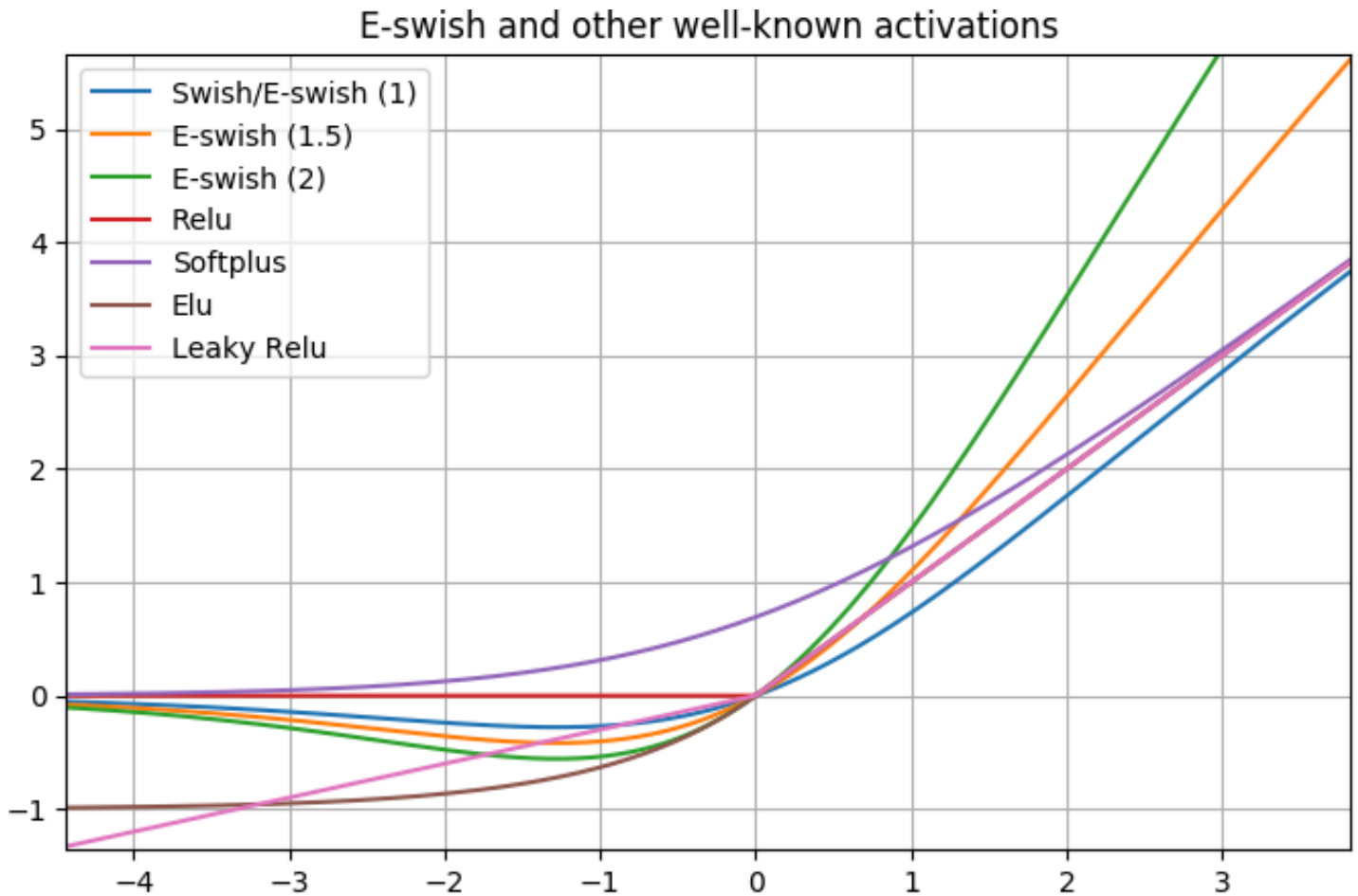
Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$




E-swish and other well-known activations
Swish/E-swish (1)
E-swish (1.5)
E-swish (2)
Relu
Softplus
Elu
Leaky Relu
5
4
3
2
1
0
−1
−4
−3
−2
−1
0
1
2
3

Training and Empirical risk minimization

- We do not know a true distribution of data
- The empirical estimate of the expected error is the average error over the samples

$$R_{emp}(h) = \frac{1}{n}\sum_{i=1}^{n} L(y_i, h(x_i))$$

- This approximation is an unbiased estimate of the expected divergence that we actually want to estimate

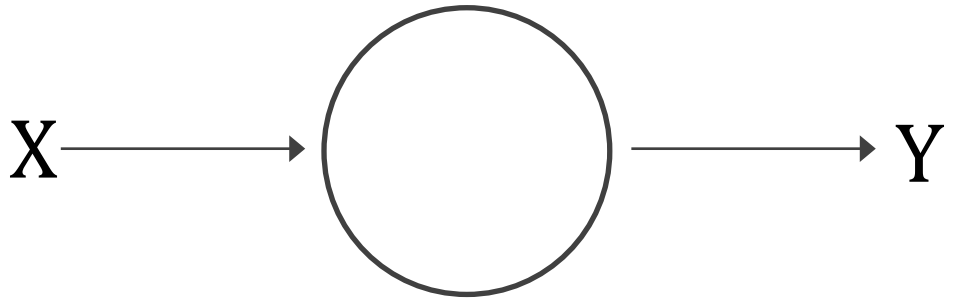
Assumption: minimizing the empirical loss will minimize the true loss

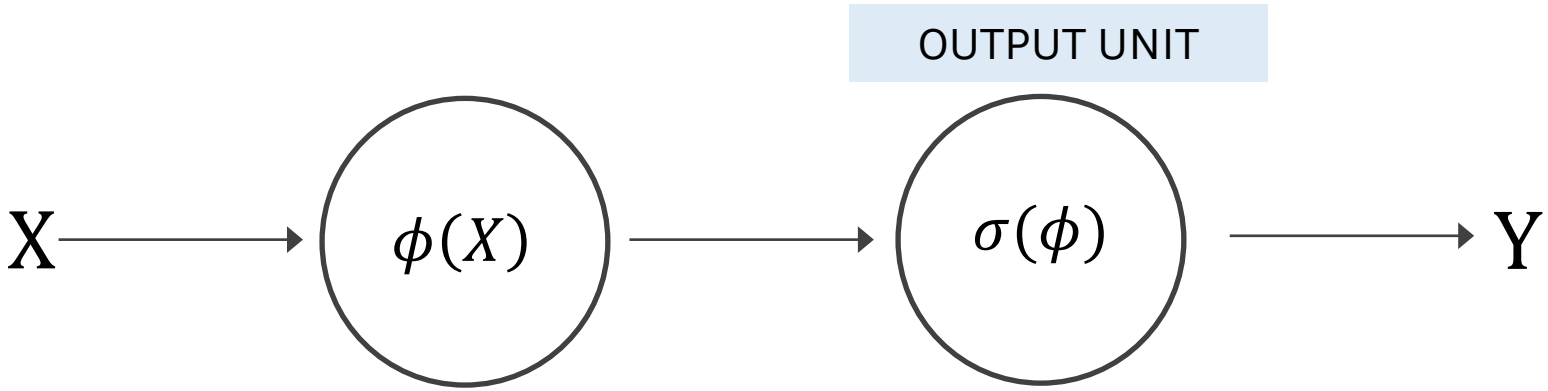
Output Units

Output Type	Output Distribution	Output layer	Cost Function
Binary			

Link function

$$X \Longrightarrow \phi(X) = W^T X \Longrightarrow P(y = 0) = \frac{1}{1 + e^{\phi(X)}}$$





Loss Function

- Cross-entropy between training data and model distribution (i.e. negative log-likelihood)

$$J(W) = -\mathbb{E}_{x,y\sim\hat{p}_{\text{data}}} \log p_{\text{model}}(y|x)$$

- Do not need to design separate loss functions.

  - Gradient of cost function must be large enough

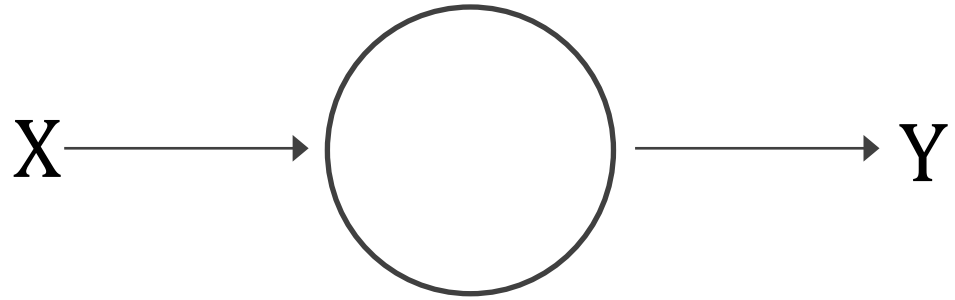
Output Units

Output Type	Output Distribution	Output layer	Cost Function
Binary	Bernoulli	Sigmoid/Softmax	Binary Cross Entropy

Output Units

Output Type	Output Distribution	Output layer	Cost Function
Binary	Bernoulli	Sigmoid/Softmax	Binary Cross Entropy
Discrete			

Link function multi-class problem



OUTPUT UNIT

Output Units

Output Type	Output Distribution	Output layer	Cost Function
Binary	Bernoulli	Sigmoid/Softmax	Binary Cross Entropy
Discrete	Multinoulli	Softmax	Cross Entropy

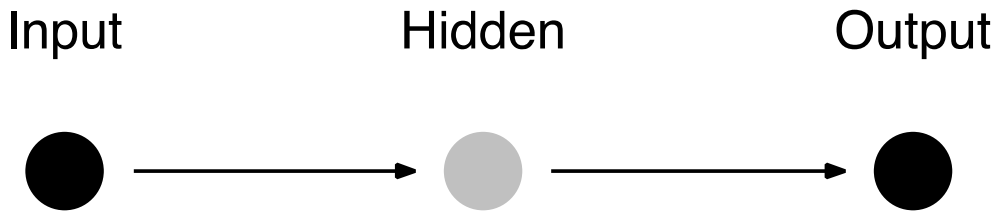
Output Units

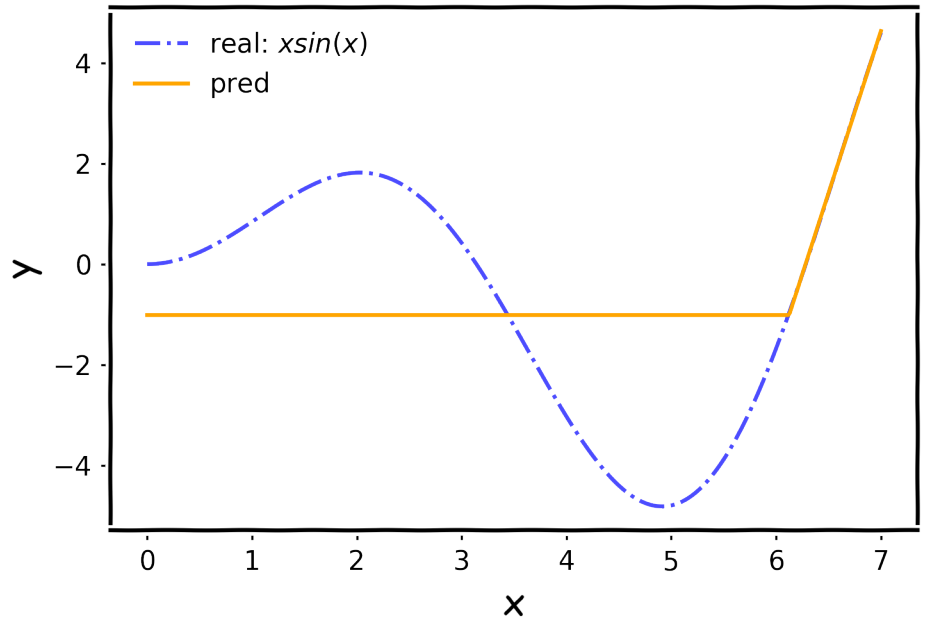
Output Type	Output Distribution	Output layer	Cost Function
Binary	Bernoulli	Sigmoid/Softmax	Binary Cross Entropy
Discrete	Multinoulli	Softmax	Cross Entropy
Continuous	Gaussian	Linear	MSE

Output Units

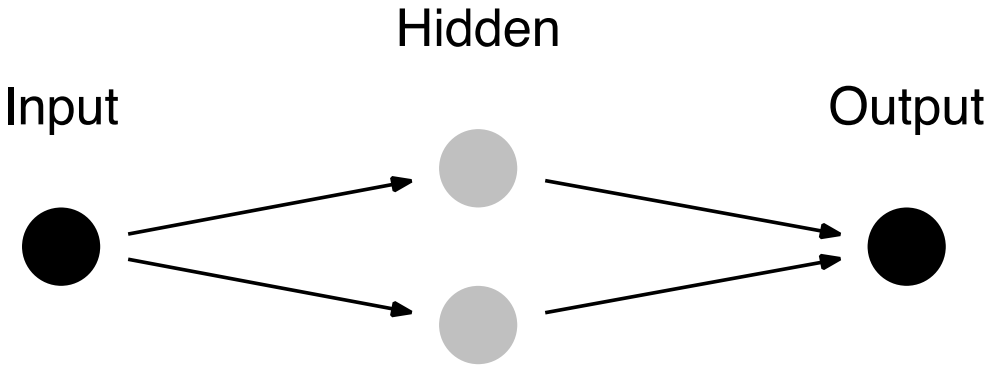
Output Type	Output Distribution	Output layer	Cost Function
Binary	Bernoulli	Sigmoid/Softmax	Binary Cross Entropy
Discrete	Multinoulli	Softmax	Cross Entropy
Continuous	Gaussian	Linear	MSE
Continuous	Arbitrary	-	GANS

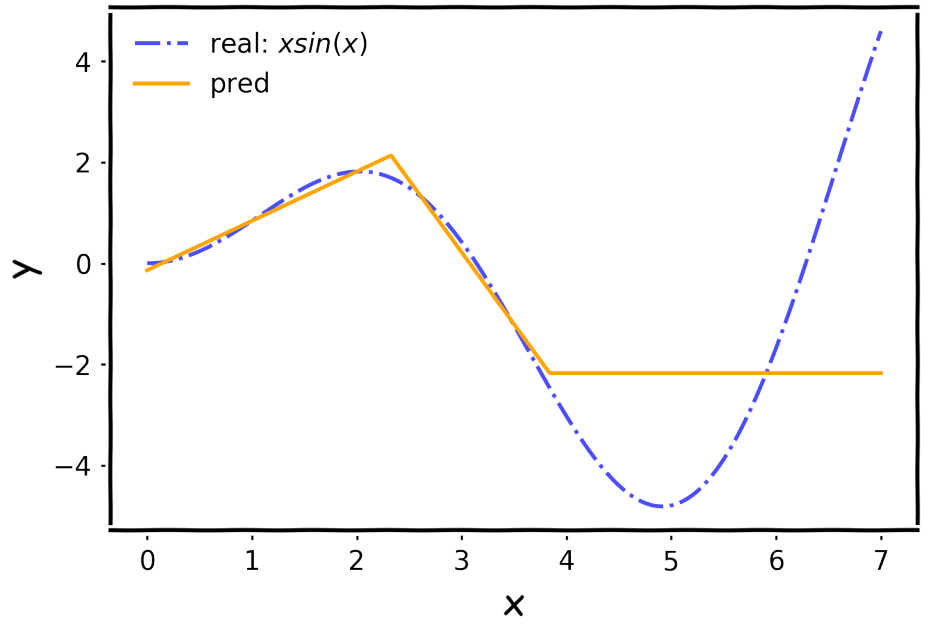
Architecture



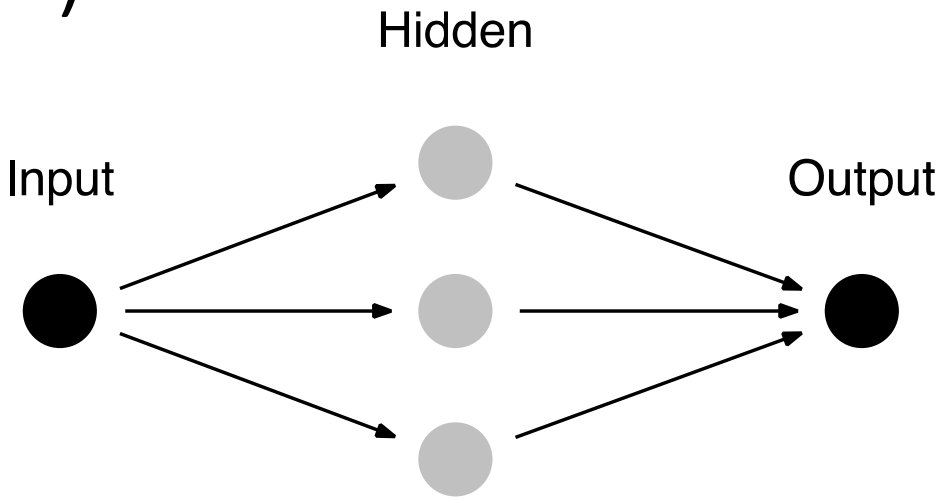


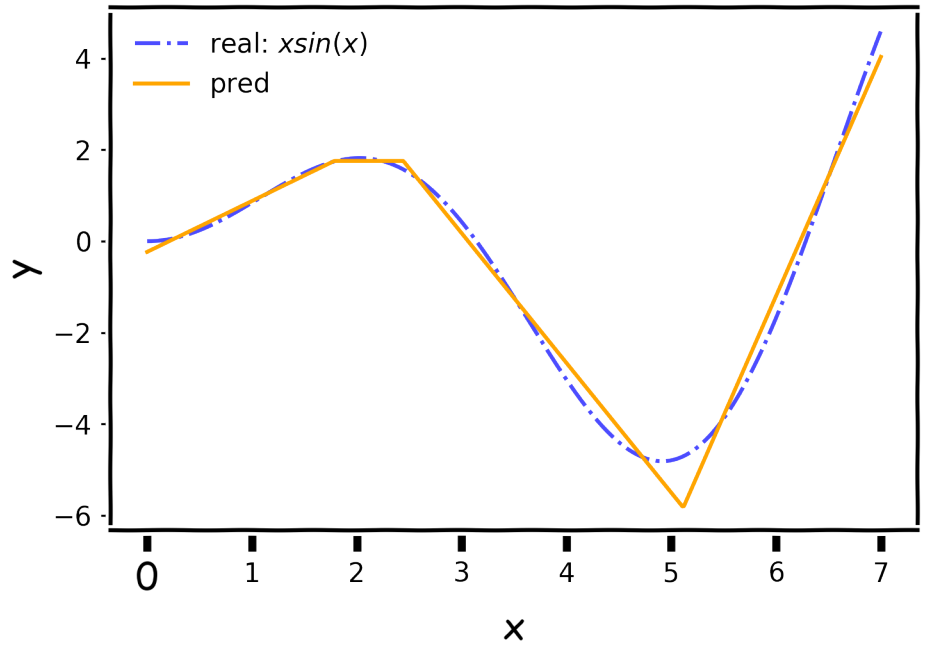
Architecture (cont)



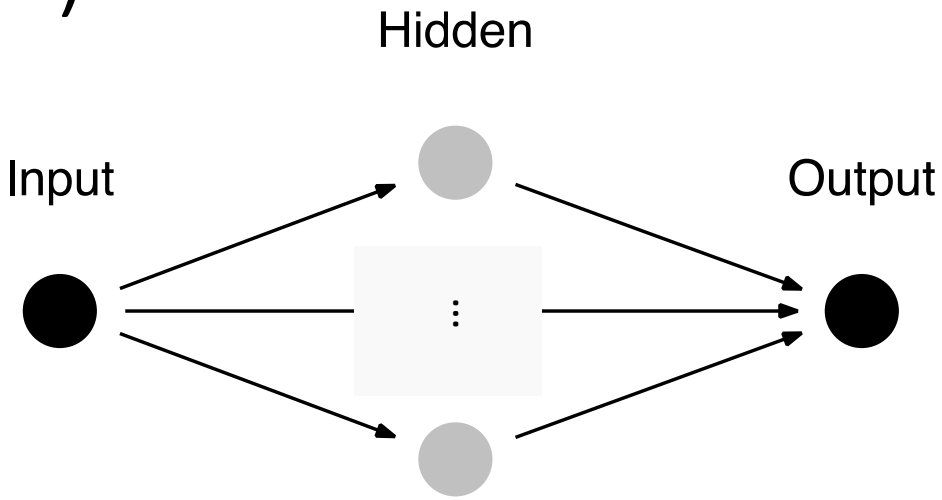


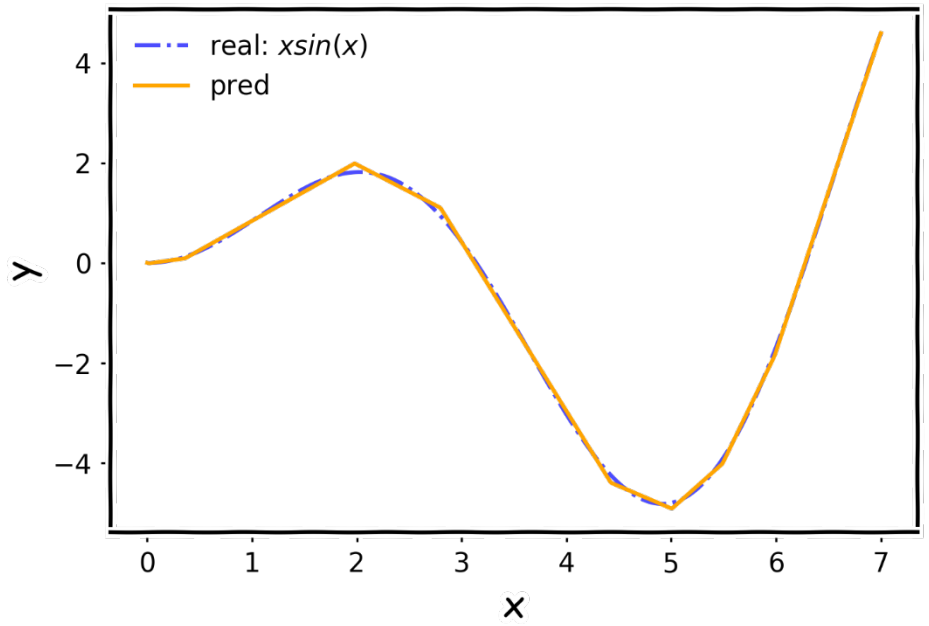
Architecture (cont)



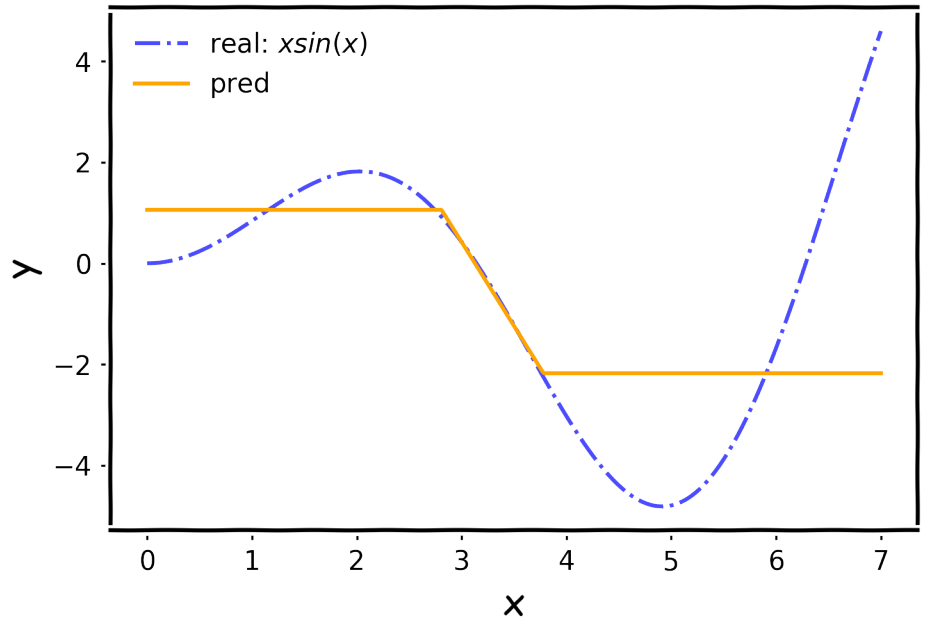


Architecture (cont)

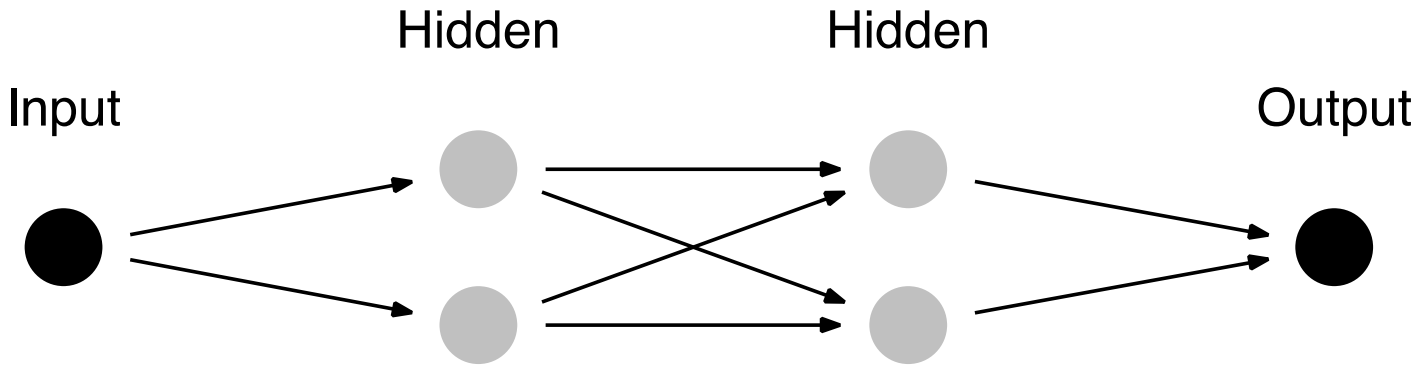


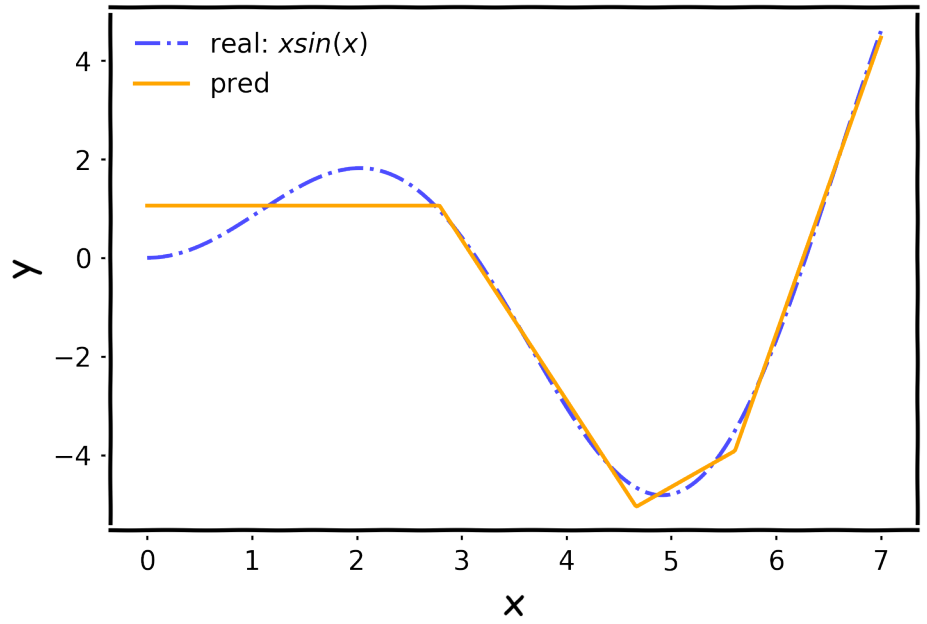


Architecture (cont)

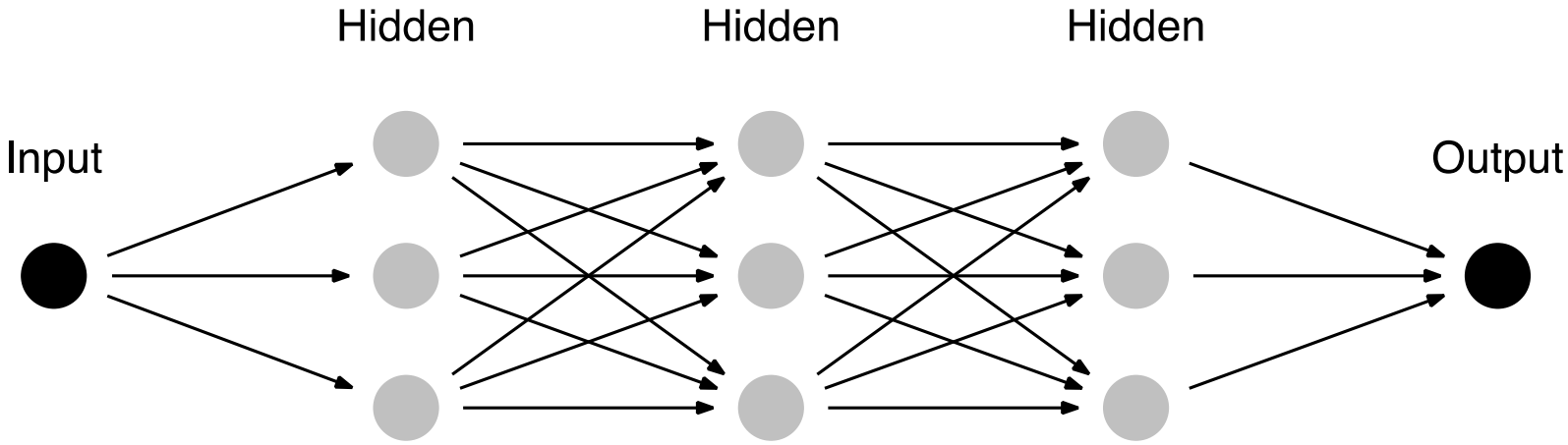


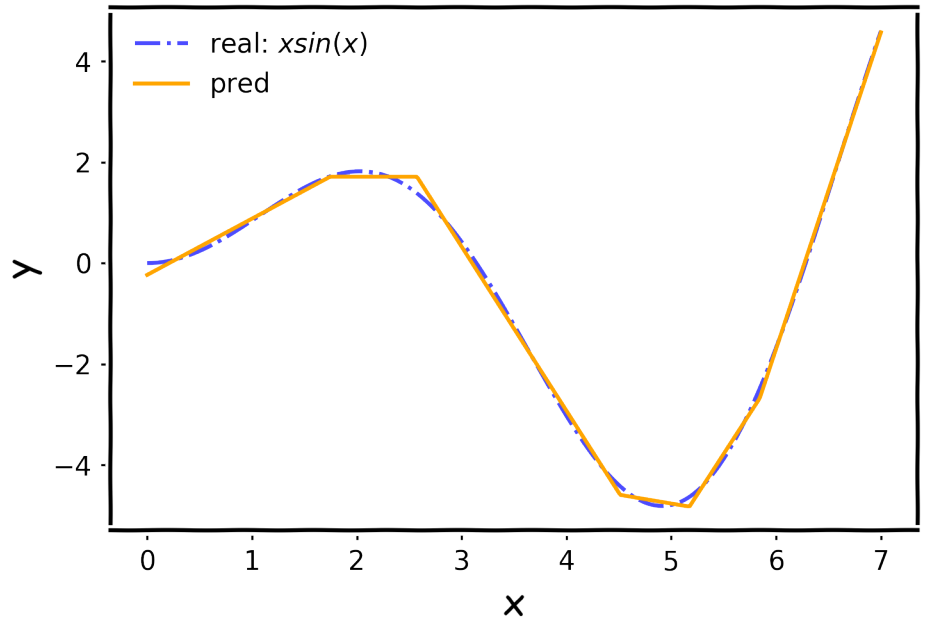
Architecture (cont)





Architecture (cont)

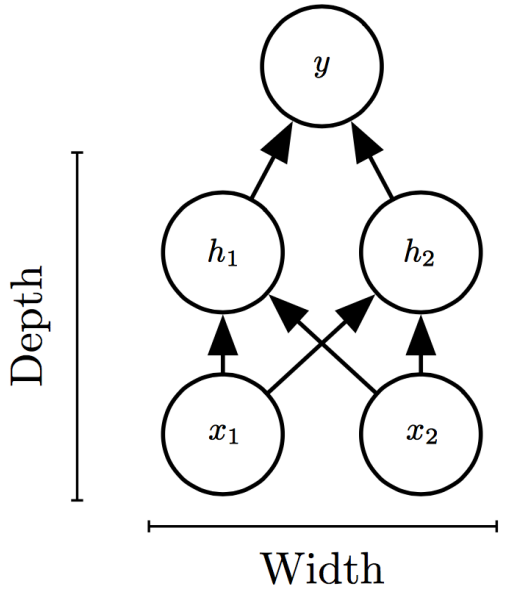




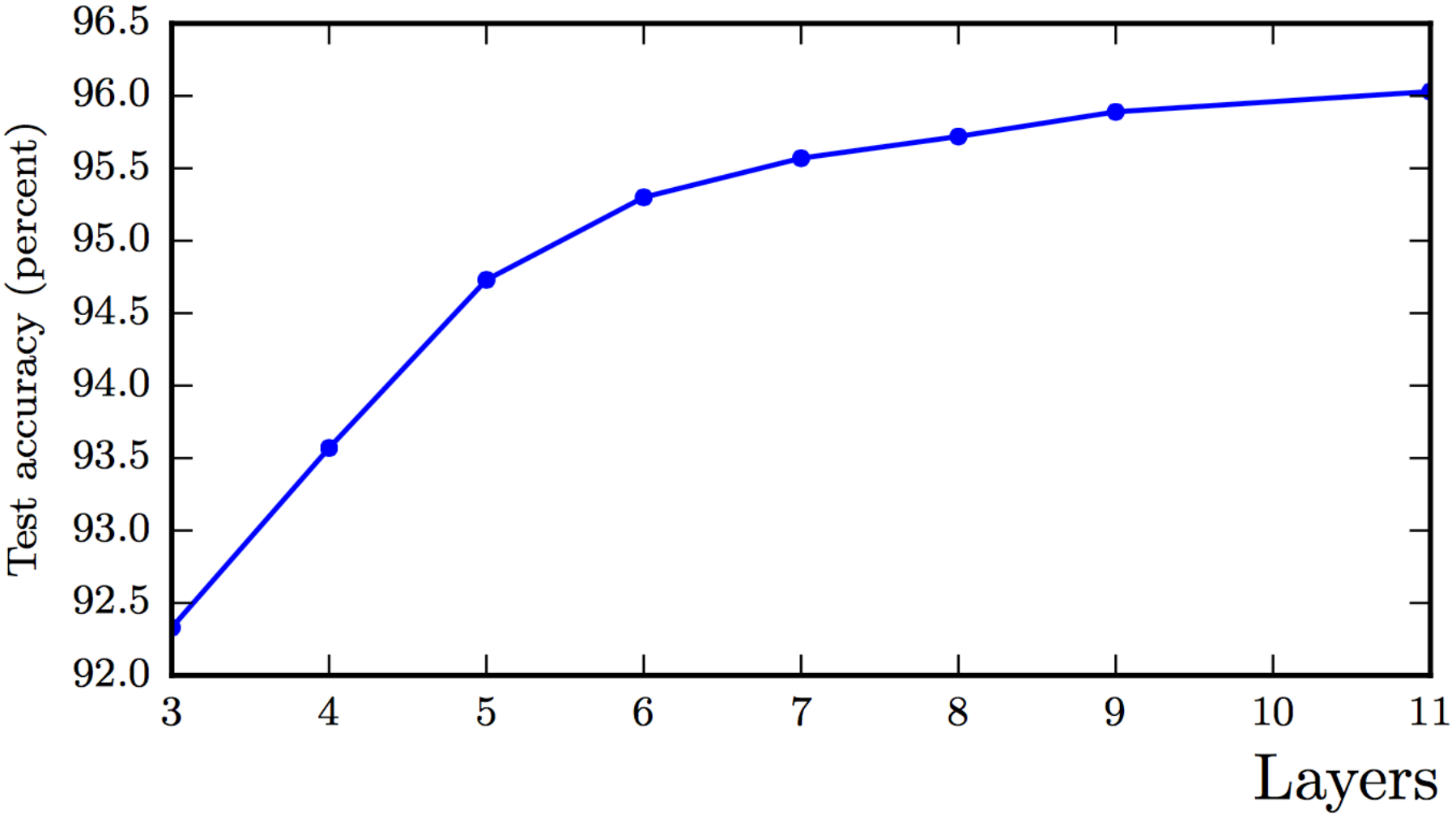
Universal Approximation Theorem

- Think of Neural Network as function approximation.
- $Y = f(x) + \epsilon$
- $Y = \hat{f}(x) + \epsilon$
- NN: $\Longrightarrow \hat{f}(x)$

- **One hidden layer is enough to *represent* an approximation of any function to an arbitrary degree of accuracy**
- So why deeper?
  - Shallow net may need (exponentially) more width
  - Shallow net may overfit more

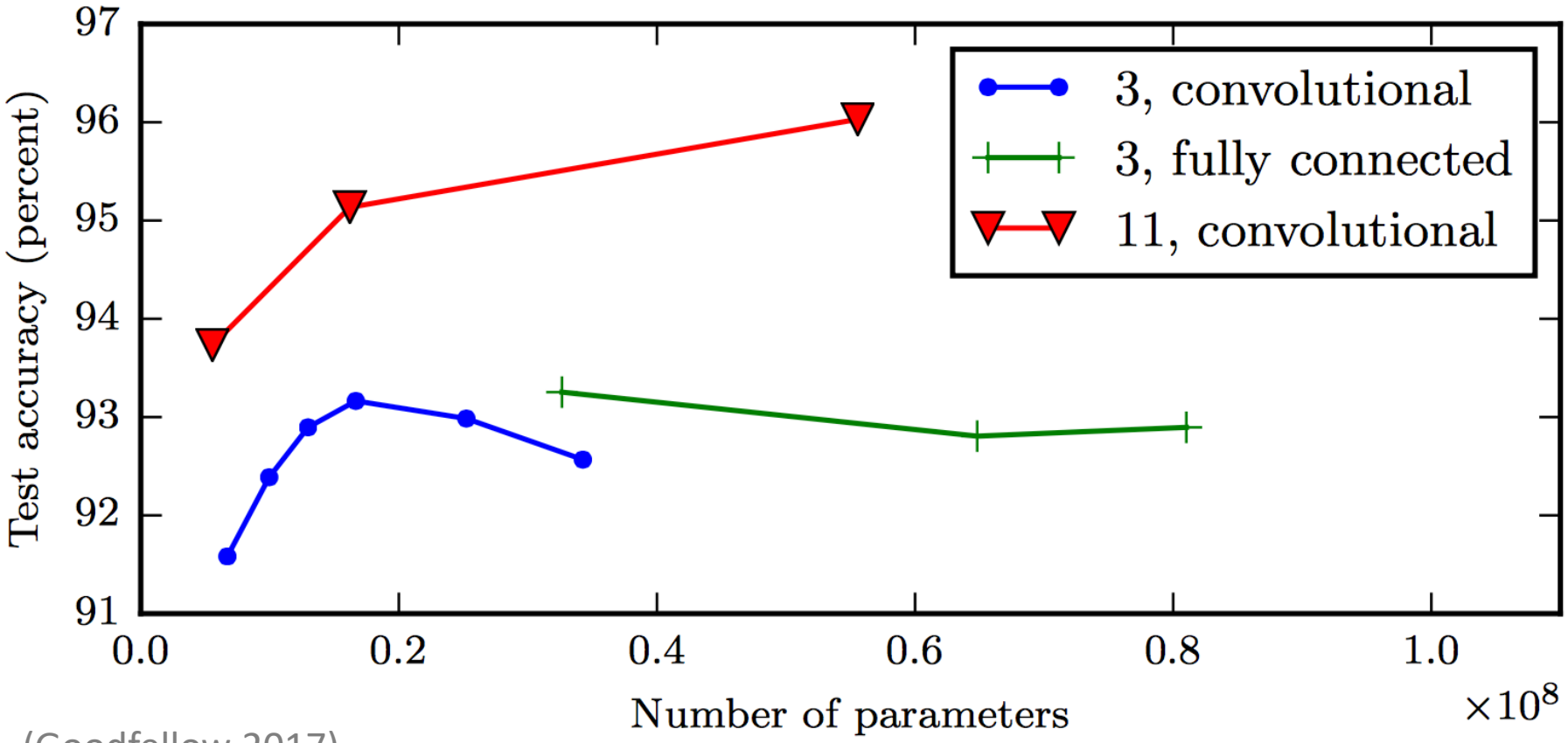


Better Generalization with Depth



(Goodfellow 2017)

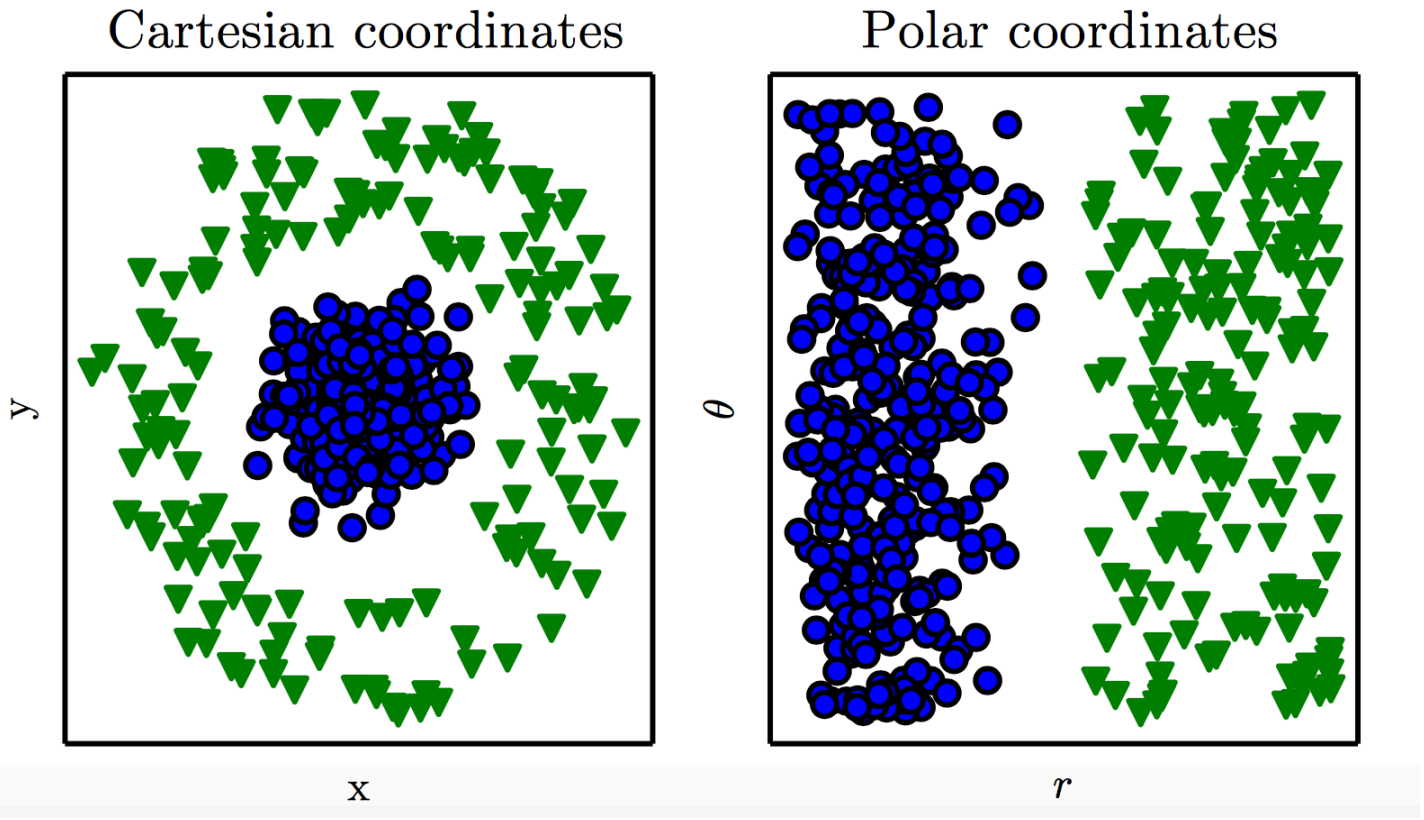
Large, Shallow Nets Overfit More



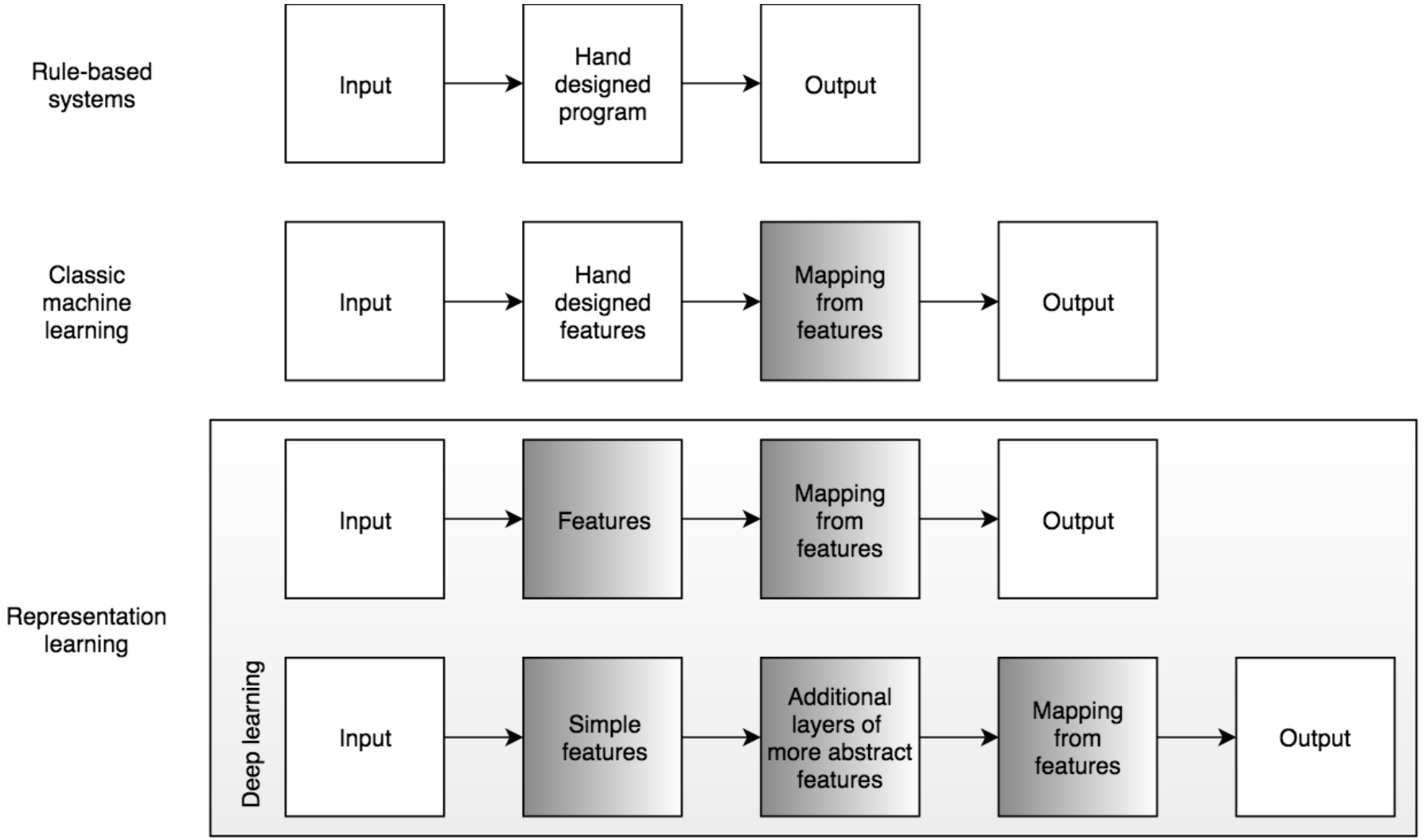
(Goodfellow 2017)

Why layers? Representation

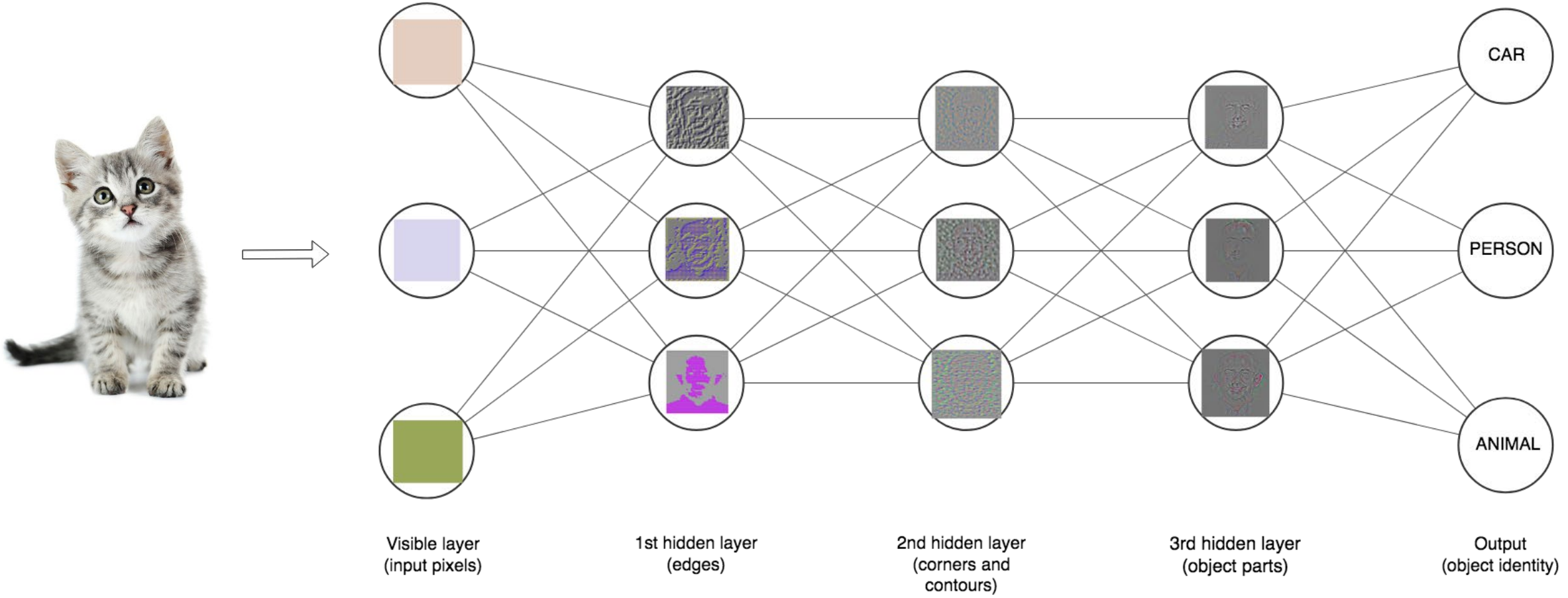
- Representation Matters



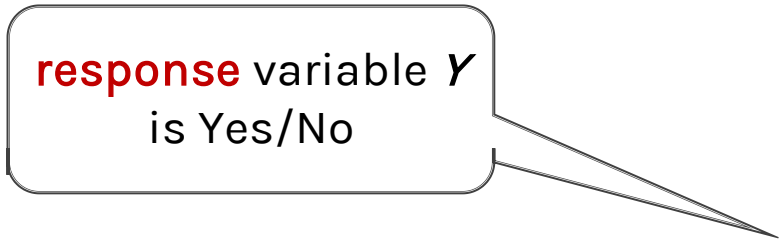
Learning Multiple Components



Depth = Repeated Compositions



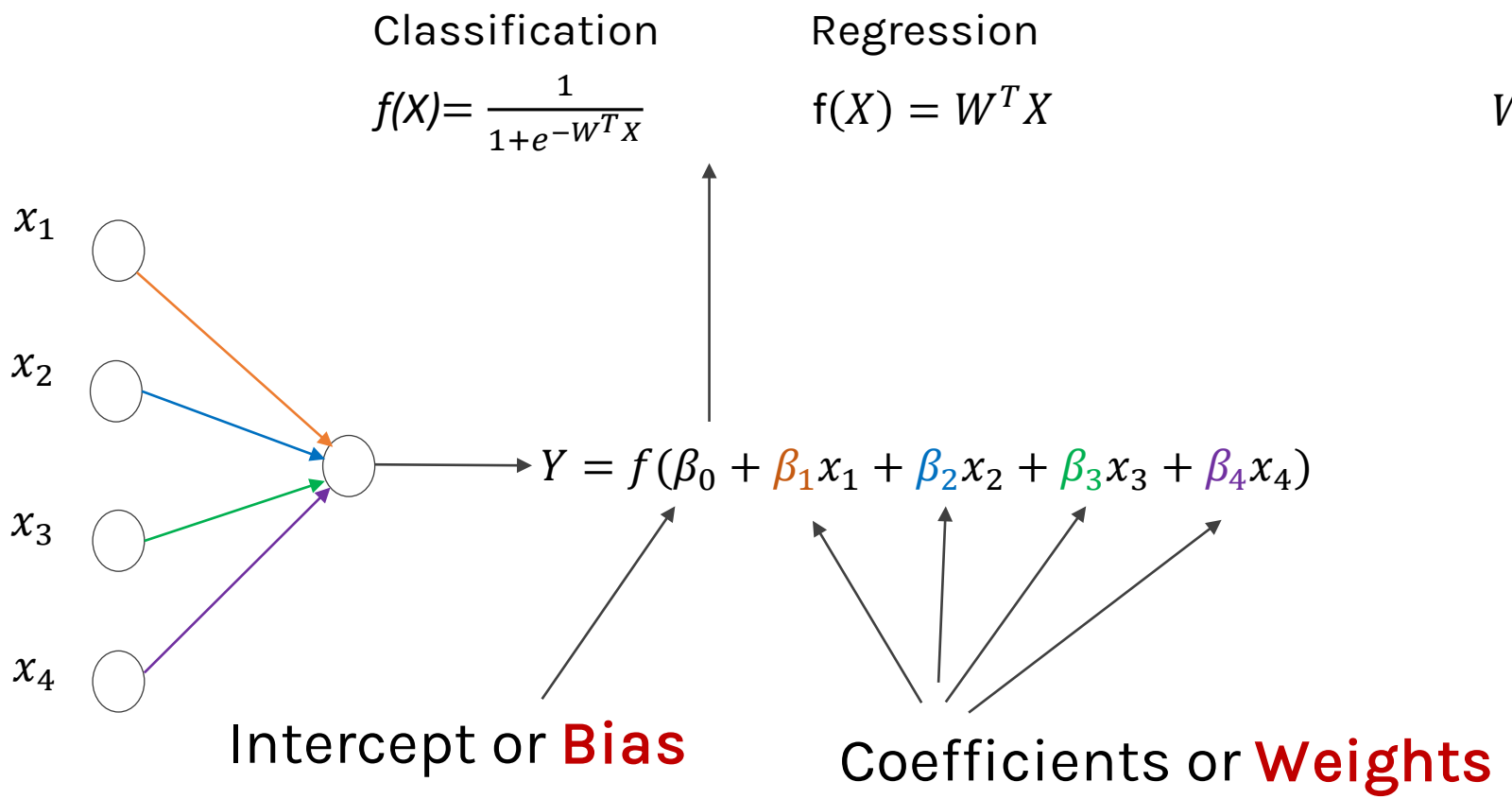
Heart Data



Age	Sex	ChestPain	RestBP	Chol	Fbs	RestECG	MaxHR	ExAng	Oldpeak	Slope	Ca	Thal	AHD
63	1	typical	145	233	1	2	150	0	2.3	3	0.0	fixed	No
67	1	asymptomatic	160	286	0	2	108	1	1.5	2	3.0	normal	Yes
67	1	asymptomatic	120	229	0	2	129	1	2.6	2	2.0	reversable	Yes
37	1	nonanginal	130	250	0	0	187	0	3.5	3	0.0	normal	No
41	0	nontypical	130	204	0	2	172	0	1.4	1	0.0	normal	No

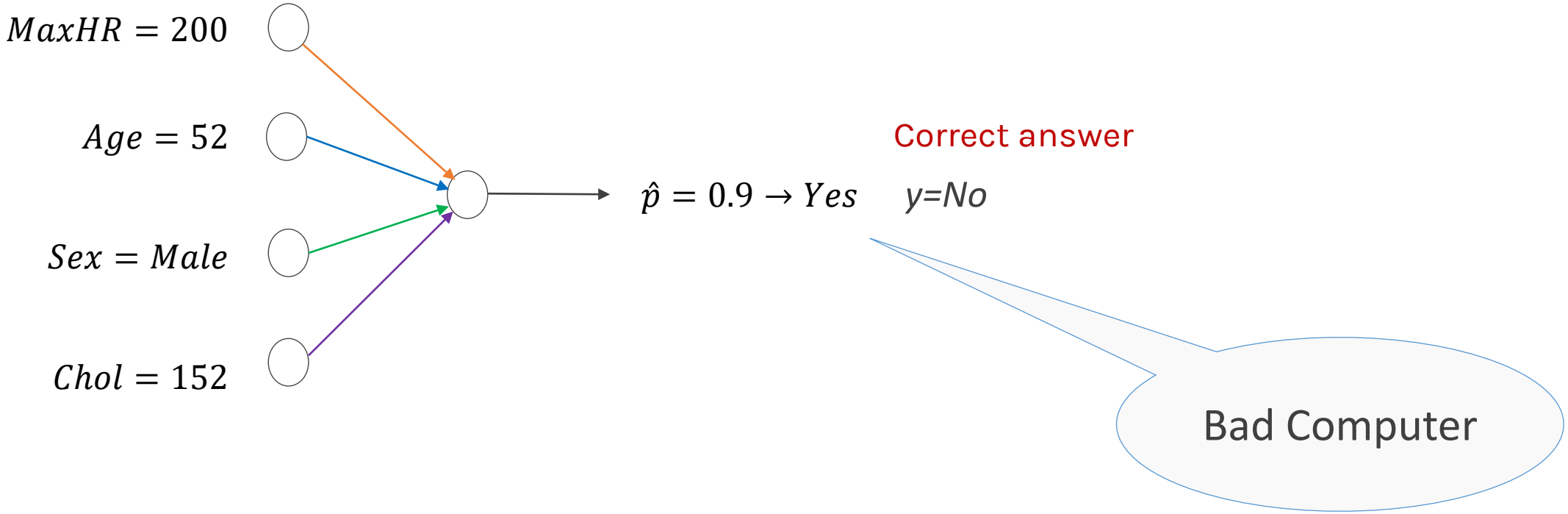
Basic idea of learning

Start with Regression or Logistic Regression



But what is the idea?

Start with all randomly selected weights. Most likely it will perform horribly. For example, in our heart data, the model will be giving us the wrong answer.



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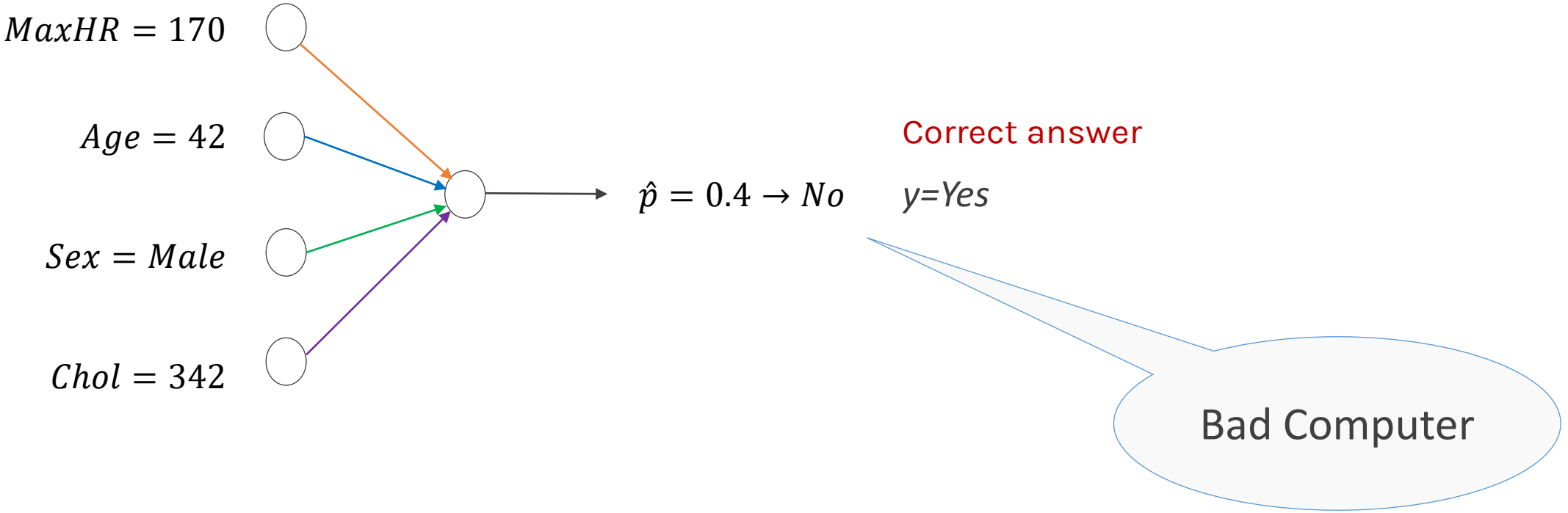
Start with all randomly selected weights. Most likely it will perform horribly. For example, in our heart data, the model will be giving us the wrong answer.



But what is the idea?

- **Loss Function:** Takes all of these results and averages them and tells us how bad or good the computer or those weights are.
- Telling the computer how **bad** or **good** is, does not help.
- You want to tell it how to change those weights so it gets better.

Loss function: $\mathcal{L}(w_0, w_1, w_2, w_3, w_4)$

For now let's only consider one weight, $\mathcal{L}(w_1)$

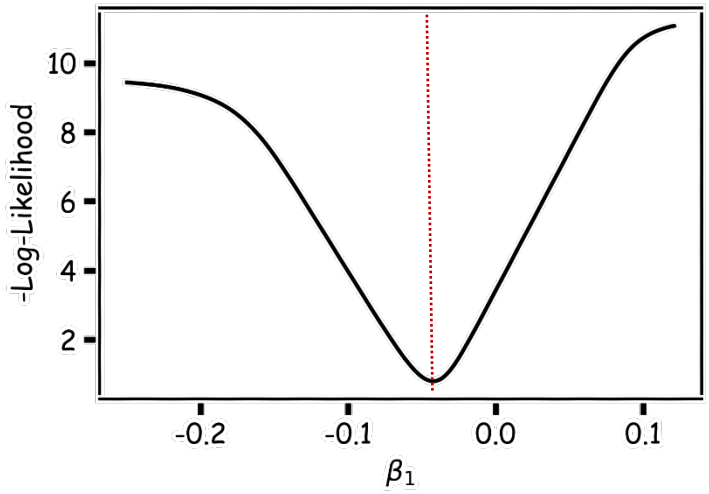
But what is the idea?

Trial and error:

- Change the weights and see the effect.
- This can take long long time especially in NN where we have millions of weights to adjust.

Minimizing the Loss function

Ideally we want to know the value of w_1 that gives the minimul $\mathcal{L}(W)$



To find the optimal point of a function $\mathcal{L}(W)$

$$\frac{d\mathcal{L}(W)}{dW} = 0$$

And find the W that satisfies that equation. Sometimes there is no explicit solution for that.

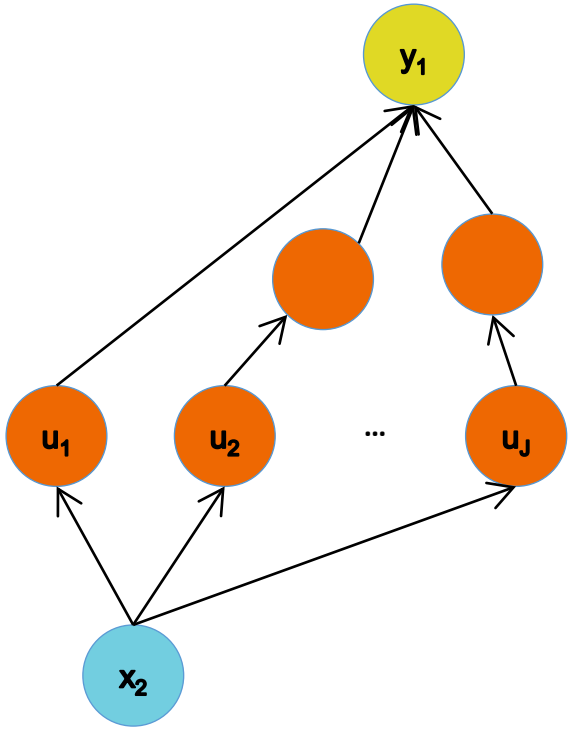
Chain Rule

Given: $\boldsymbol{y} = g(\boldsymbol{u})$ and $\boldsymbol{u} = h(\boldsymbol{x})$.

Chain Rule:

$$\frac{dy_i}{dx_k} = \sum_{j=1}^{J} \frac{dy_i}{du_j} \frac{du_j}{dx_k}, \quad \forall i, k$$

Backpropagation is just repeated application of the **chain rule** from Calculus 101.



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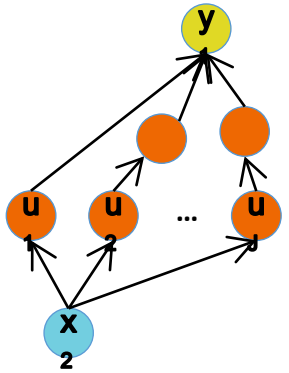


Backpropagation:

1. **Instantiate the computation as a directed acyclic graph**, where each intermediate quantity is a node
2. At each node, store (a) the quantity computed in the forward pass and (b) the **partial derivative** of the goal with respect to that node's intermediate quantity.
3. **Initialize** all partial derivatives to 0.
4. Visit each node in **reverse topological order**. At each node, add its contribution to the partial derivatives of its parents

This algorithm is also called **automatic differentiation in the reverse-mode**

Backpropagation

Simple Example: The goal is to compute $J = \cos(\sin(x^2) + 3x^2)$ on the forward pass and the derivative $\frac{dJ}{dx}$ on the backward pass.

Forward

$J = cos(u)$

$u = u_1 + u_2$

$u_1 = sin(t)$

$u_2 = 3t$

$t = x^2$

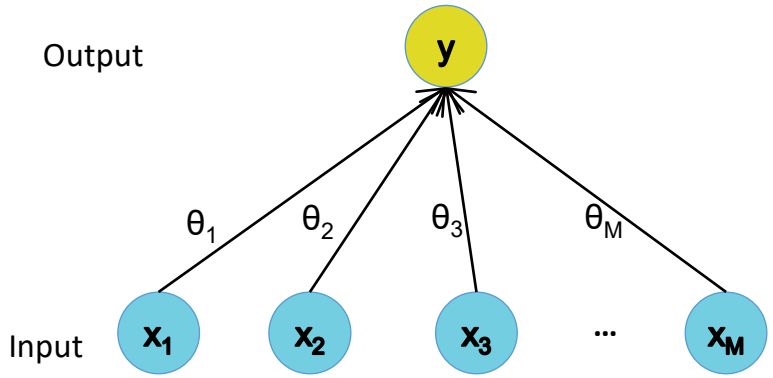
Backpropagation

Simple Example: The goal is to compute $J = \cos(\sin(x^2) + 3x^2)$ on the forward pass and the derivative $\frac{dJ}{dx}$ on the backward pass.

Forward	Backward
$J = cos(u)$	$\frac{dJ}{du}$ += $-sin(u)$
$u = u_1 + u_2$	$\frac{dJ}{du_1}$ += $\frac{dJ}{du}\frac{du}{du_1}$, $\frac{du}{du_1} = 1$ $\quad$ $\frac{dJ}{du_2}$ += $\frac{dJ}{du}\frac{du}{du_2}$, $\frac{du}{du_2} = 1$
$u_1 = sin(t)$	$\frac{dJ}{dt}$ += $\frac{dJ}{du_1}\frac{du_1}{dt}$, $\frac{du_1}{dt} = \cos(t)$
$u_2 = 3t$	$\frac{dJ}{dt}$ += $\frac{dJ}{du_2}\frac{du_2}{dt}$, $\frac{du_2}{dt} = 3$
$t = x^2$	$\frac{dJ}{dx}$ += $\frac{dJ}{dt}\frac{dt}{dx}$, $\frac{dt}{dx} = 2x$

Backpropagation

Case 1: Logistic Regression



Forward

$$J = y^* \log y + (1 - y^*) \log(1 - y)$$

$$y = \frac{1}{1 + \exp(-a)}$$

$$a = \sum_{j=0}^{D} \theta_j x_j$$

Backward

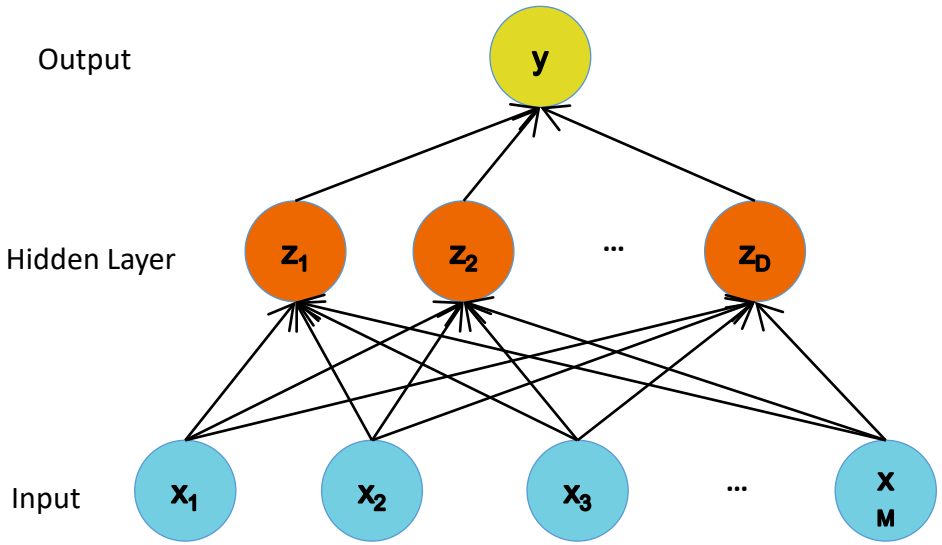
$$\frac{dJ}{dy} = \frac{y^*}{y} + \frac{(1 - y^*)}{y - 1}$$

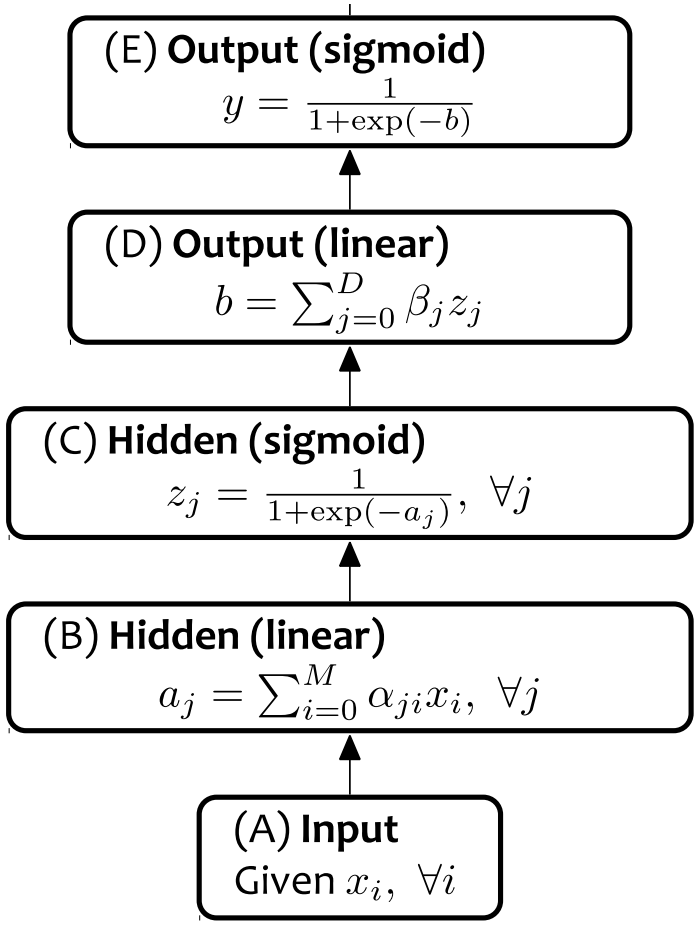
$$\frac{dJ}{da} = \frac{dJ}{dy}\frac{dy}{da}, \frac{dy}{da} = \frac{\exp(-a)}{(\exp(-a) + 1)^2}$$

$$\frac{dJ}{d\theta_j} = \frac{dJ}{da}\frac{da}{d\theta_j}, \frac{da}{d\theta_j} = x_j$$

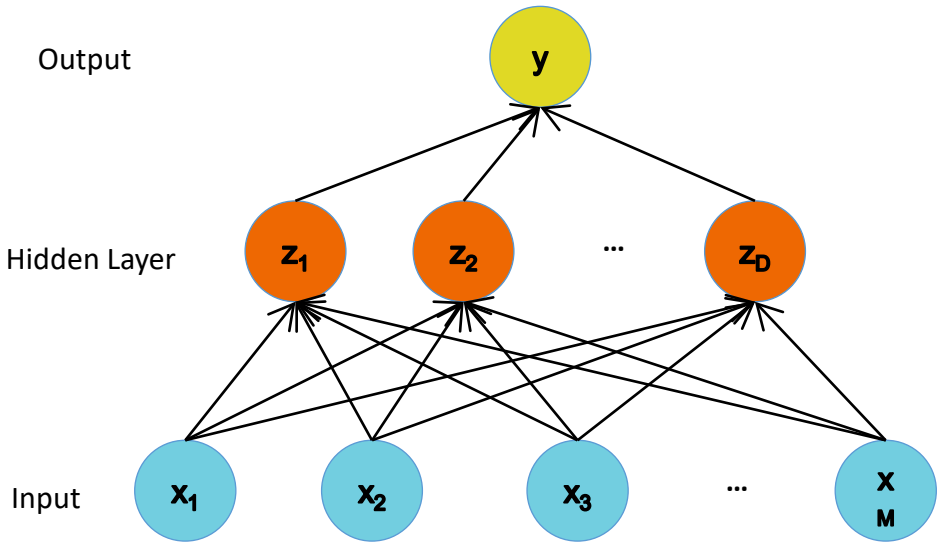
$$\frac{dJ}{dx_j} = \frac{dJ}{da}\frac{da}{dx_j}, \frac{da}{dx_j} = \theta_j$$

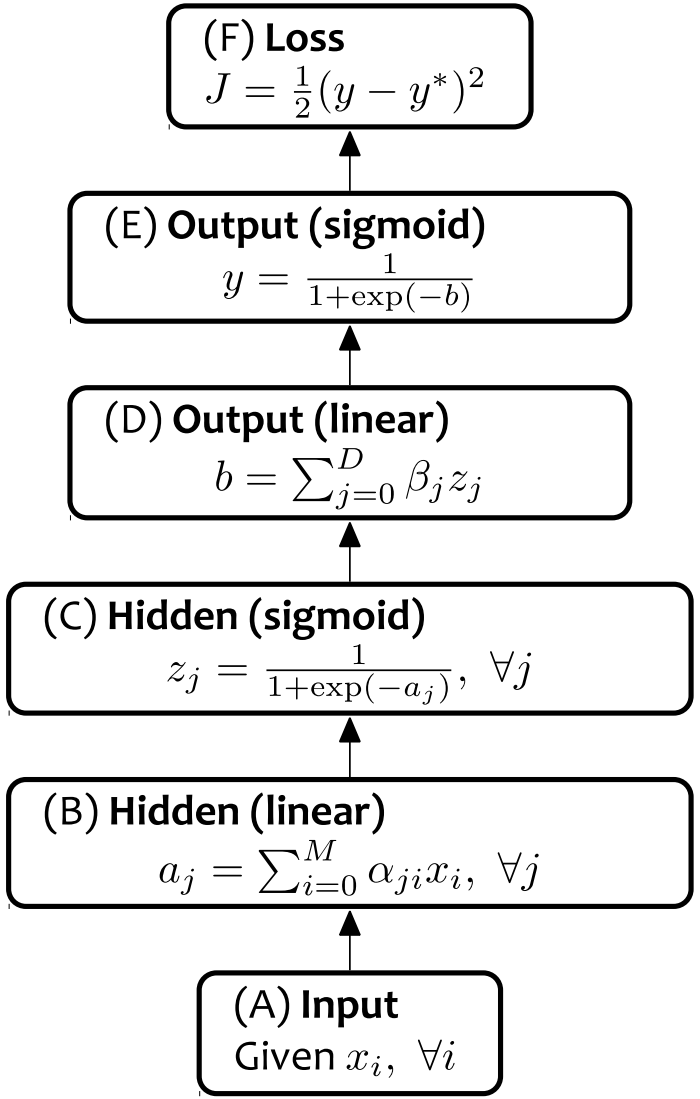
Backpropagation





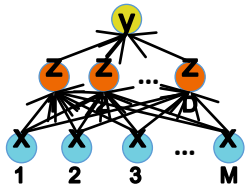
Backpropagation





Backpropagation

Case 2: Neural Network



Forward

$$J = y^* \log y + (1 - y^*) \log(1 - y)$$

$$y = \frac{1}{1 + \exp(-b)}$$

$$b = \sum_{j=0}^{D} \beta_j z_j$$

$$z_j = \frac{1}{1 + \exp(-a_j)}$$

$$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$$

Backward

$$\frac{dJ}{dy} = \frac{y^*}{y} + \frac{(1 - y^*)}{y - 1}$$

$$\frac{dJ}{db} = \frac{dJ}{dy}\frac{dy}{db}, \frac{dy}{db} = \frac{\exp(-b)}{(\exp(-b) + 1)^2}$$

$$\frac{dJ}{d\beta_j} = \frac{dJ}{db}\frac{db}{d\beta_j}, \frac{db}{d\beta_j} = z_j$$

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$$\frac{dJ}{da_j} = \frac{dJ}{dz_j}\frac{dz_j}{da_j}, \frac{dz_j}{da_j} = \frac{\exp(-a_j)}{(\exp(-a_j) + 1)^2}$$

$$\frac{dJ}{d\alpha_{ji}} = \frac{dJ}{da_j}\frac{da_j}{d\alpha_{ji}}, \frac{da_j}{d\alpha_{ji}} = x_i$$

$$\frac{dJ}{dx_i} = \frac{dJ}{da_j}\frac{da_j}{dx_i}, \frac{da_j}{dx_i} = \sum_{j=0}^{D} \alpha_{ji}$$

Chain Rule

Given: $\boldsymbol{y} = g(\boldsymbol{u})$ and $\boldsymbol{u} = h(\boldsymbol{x})$.

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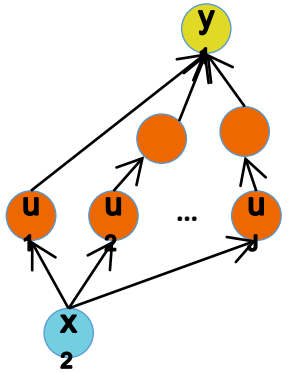

Backpropagation:

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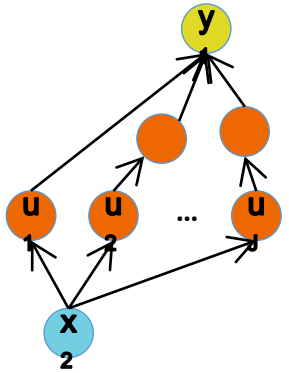

Backpropagation:

1. **Instantiate the computation as a directed acyclic graph**, where each node represents a Tensor.
2. At each node, store (a) the quantity computed in the forward pass and (b) the **partial derivatives** of the goal with respect to that node's Tensor.
3. **Initialize** all partial derivatives to 0.
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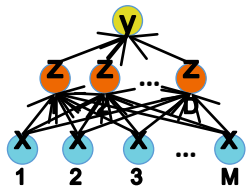
Backpropagation

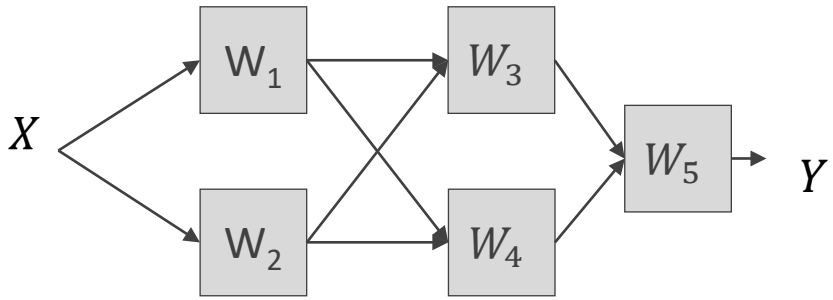
Case 2: Neural Network



	Forward	Backward
Module 5	$J = y^* \log y + (1 - y^*) \log(1 - y)$	$\frac{dJ}{dy} = \frac{y^*}{y} + \frac{(1 - y^*)}{y - 1}$
Module 4	$y = \frac{1}{1 + \exp(-b)}$	$\frac{dJ}{db} = \frac{dJ}{dy}\frac{dy}{db}, \frac{dy}{db} = \frac{\exp(-b)}{(\exp(-b) + 1)^2}$
Module 3	$b = \sum_{j=0}^{D} \beta_j z_j$	$\frac{dJ}{d\beta_j} = \frac{dJ}{db}\frac{db}{d\beta_j}, \frac{db}{d\beta_j} = z_j$ $\frac{dJ}{dz_j} = \frac{dJ}{db}\frac{db}{dz_j}, \frac{db}{dz_j} = \beta_j$
Module 2	$z_j = \frac{1}{1 + \exp(-a_j)}$	$\frac{dJ}{da_j} = \frac{dJ}{dz_j}\frac{dz_j}{da_j}, \frac{dz_j}{da_j} = \frac{\exp(-a_j)}{(\exp(-a_j) + 1)^2}$
Module 1	$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$	$\frac{dJ}{d\alpha_{ji}} = \frac{dJ}{da_j}\frac{da_j}{d\alpha_{ji}}, \frac{da_j}{d\alpha_{ji}} = x_i$ $\frac{dJ}{dx_i} = \frac{dJ}{da_j}\frac{da_j}{dx_i}, \frac{da_j}{dx_i} = \sum_{j=0}^{D} \alpha_{ji}$

Autograd: Auto-differentiation

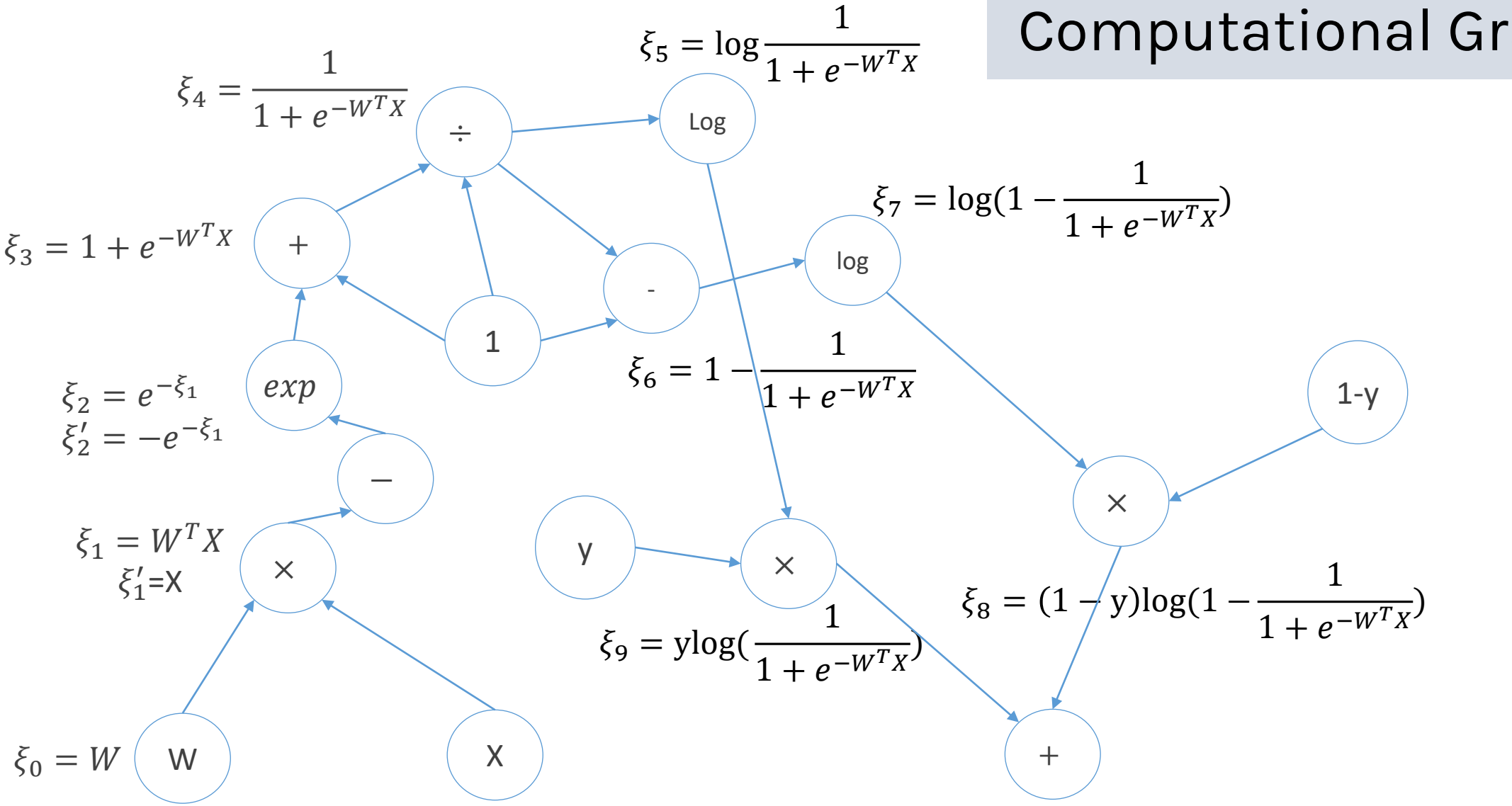
- 1. We specify the network structure



2. We create the computational graph …

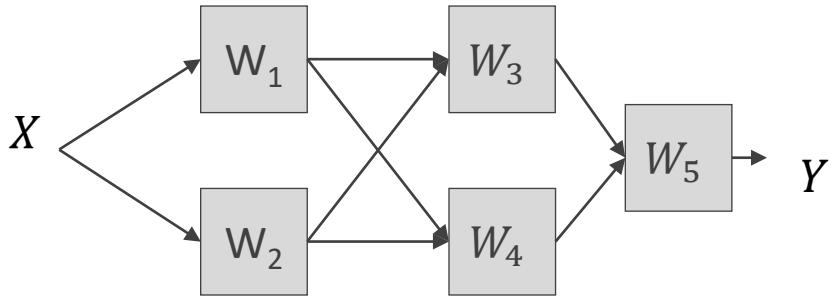
What is computational graph?

Computational Graph



Autograd (cont)

- 1. We specify the network structure



- We create the computational graph.
- At each node of the graph we build two functions: the evaluation of the variable and its partial derivative with respect to the previous variable (as shown in the table few slides back)
- Now we can either go forward or backward depending on the situation. In general, forward is easier to implement and to understand. The difference is clearer when there are multiple nodes per layer.

Forward mode: Evaluate the derivative at:

Variables	derivatives	Value of the variable	Value of the partial derivative	$\frac{d\mathcal{L}}{d\xi_n}$
$\xi_1 = -W^T X$	$\frac{\partial \xi_1}{\partial W} = -X$	-9	-3	-3
$\xi_2 = e^{\xi_1} = e^{-W^T X}$	$\frac{\partial \xi_2}{\partial \xi_1} = e^{\xi_1}$	e^{-9}	e^{-9}	$-3e^{-9}$
$\xi_3 = 1 + \xi_2 = 1 + e^{-W^T X}$	$\frac{\partial \xi_3}{\partial \xi_2} = 1$	$1+e^{-9}$	1	$-3e^{-9}$
$\xi_4 = \frac{1}{\xi_3} = \frac{1}{1 + e^{-W^T X}} = p$	$\frac{\partial \xi_4}{\partial \xi_3} = -\frac{1}{\xi_3^2}$	$\frac{1}{1 + e^{-9}}$	$\left(\frac{1}{1 + e^{-9}}\right)^2$	$-3e^{-9}\left(\frac{1}{1+e^{-9}}\right)^2$
$\xi_5 = \log \xi_4 = \log p = \log \frac{1}{1 + e^{-W^T X}}$	$\frac{\partial \xi_5}{\partial \xi_4} = \frac{1}{\xi_4}$	$\log \frac{1}{1 + e^{-9}}$	$1 + e^{-9}$	$-3e^{-9}\left(\frac{1}{1+e^{-9}}\right)$
$\mathcal{L}_i^A = -y\xi_5$	$\frac{\partial \mathcal{L}}{\partial \xi_5} = -y$	$-\log \frac{1}{1 + e^{-9}}$	-1	$3e^{-9}\left(\frac{1}{1+e^{-9}}\right)$
$\frac{\partial \mathcal{L}_i^A}{\partial W} = \frac{\partial \mathcal{L}_i}{\partial \xi_5}\frac{\partial \xi_5}{\partial \xi_4}\frac{\partial \xi_4}{\partial \xi_3}\frac{\partial \xi_3}{\partial \xi_2}\frac{\partial \xi_2}{\partial \xi_1}\frac{\partial \xi_1}{\partial W}$			-3	0.00037018372

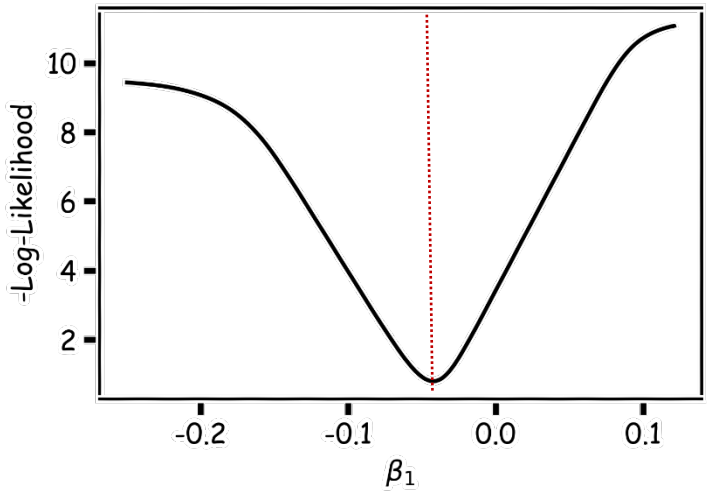
Backward mode: Evaluate the derivative at:

Variables	derivatives	Value of the variable	Value of the partial derivative
$\xi_1 = -W^T X$	$\frac{\partial \xi_1}{\partial W} = -X$	-9	-3
$\xi_2 = e^{\xi_1} = e^{-W^T X}$	$\frac{\partial \xi_2}{\partial \xi_1} = e^{\xi_1}$	e^{-9}	e^{-9}
$\xi_3 = 1 + \xi_2 = 1 + e^{-W^T X}$	$\frac{\partial \xi_3}{\partial \xi_2} = 1$	$1+e^{-9}$	1
$\xi_4 = \frac{1}{\xi_3} = \frac{1}{1 + e^{-W^T X}} = p$	$\frac{\partial \xi_4}{\partial \xi_3} = -\frac{1}{\xi_3^2}$	$\frac{1}{1 + e^{-9}}$	$\left(\frac{1}{1 + e^{-9}}\right)^2$
$\xi_5 = \log \xi_4 = \log p = \log \frac{1}{1 + e^{-W^T X}}$	$\frac{\partial \xi_5}{\partial \xi_4} = \frac{1}{\xi_4}$	$\log \frac{1}{1 + e^{-9}}$	$1 + e^{-9}$
$\mathcal{L}_i^A = -y\xi_5$	$\frac{\partial \mathcal{L}}{\partial \xi_5} = -y$	$-\log \frac{1}{1 + e^{-9}}$	-1
$\frac{\partial \mathcal{L}_i^A}{\partial W} = \frac{\partial \mathcal{L}_i}{\partial \xi_5}\frac{\partial \xi_5}{\partial \xi_4}\frac{\partial \xi_4}{\partial \xi_3}\frac{\partial \xi_3}{\partial \xi_2}\frac{\partial \xi_2}{\partial \xi_1}\frac{\partial \xi_1}{\partial W}$			Type equation here.

Store all these values

Minimizing the Loss function

Ideally we want to know the value of w_1 that gives the minimul $\mathcal{L}(W)$



To find the optimal point of a function $\mathcal{L}(W)$

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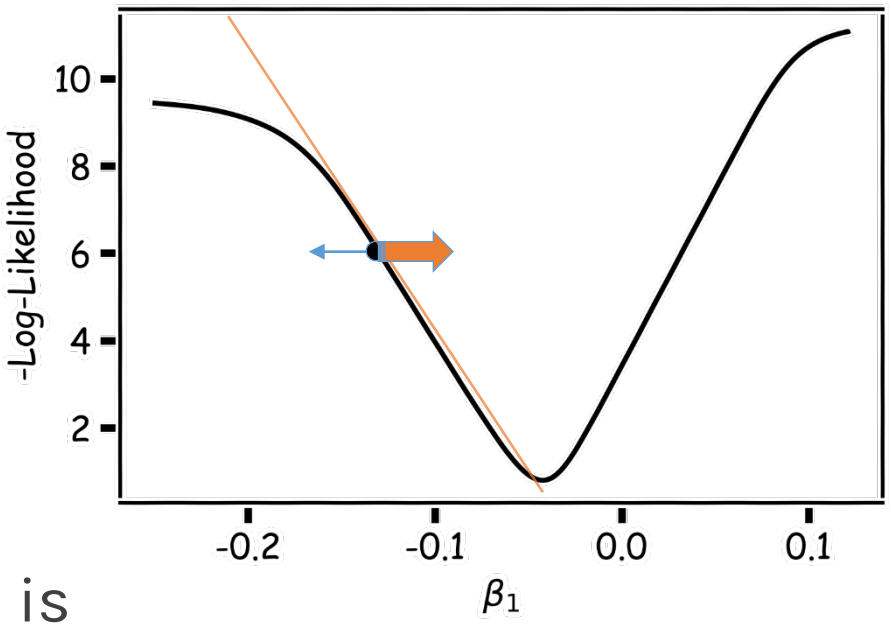
And find the W that satisfies that equation. Sometimes there is no explicit solution for that.

Minimizing the Loss function



A more flexible method is

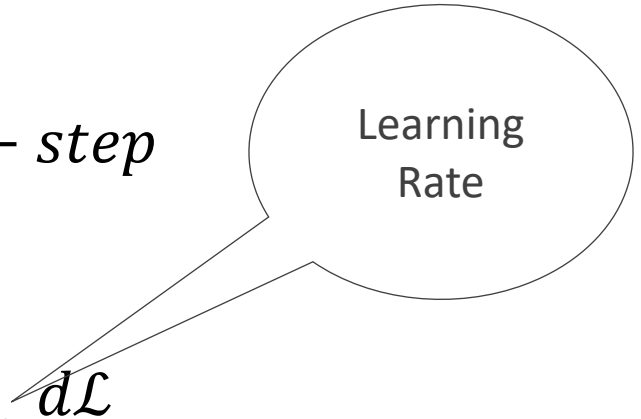
- Start from any point
  - Determine which direction to go to reduce the loss (left or right)
  - Specifically, we can calculate the slope of the function at this point
  - Shift to the right if slope is negative or shift to the left if slope is positive
- Repeat

Let's play the Pavlov's game

- We know that we want to go in the opposite direction of the derivative and we know we want to be making a step proportionally to the derivative.

- Making a step means:

$$w^{new} = w^{old} + step$$



Opposite direction of the derivative means:

$$w^{new} = w^{old} - \lambda \frac{d\mathcal{L}}{dw}$$

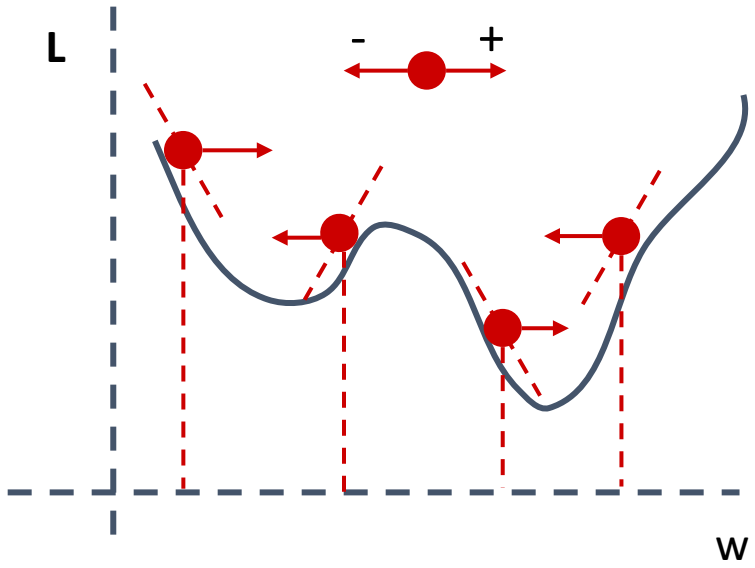
Change to more conventional notation:

$$w^{(i+1)} = w^{(i)} - \lambda \frac{d\mathcal{L}}{dw}$$

Gradient Descent

- Algorithm for optimization of first order to finding a minimum of a function.
- It is an iterative method.
- *L* is decreasing in the direction of the negative derivative.
- The learning rate is controlled by the magnitude of λ.

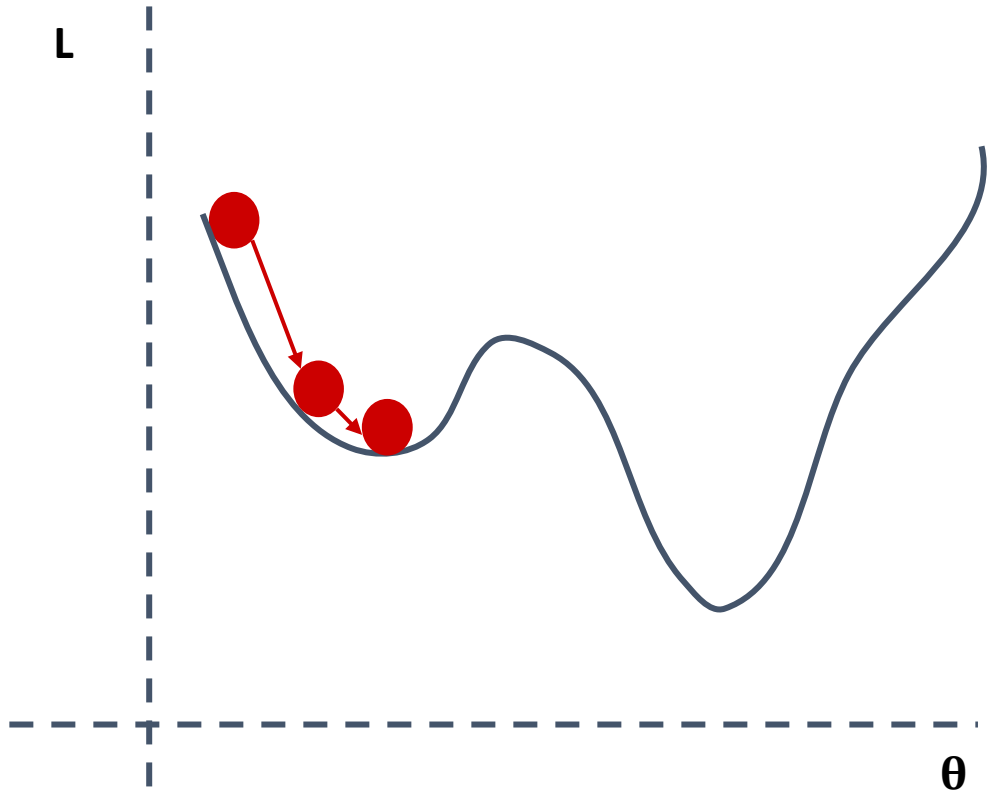
$$w^{(i+1)} = w^{(i)} - \lambda \frac{d\mathcal{L}}{dw}$$



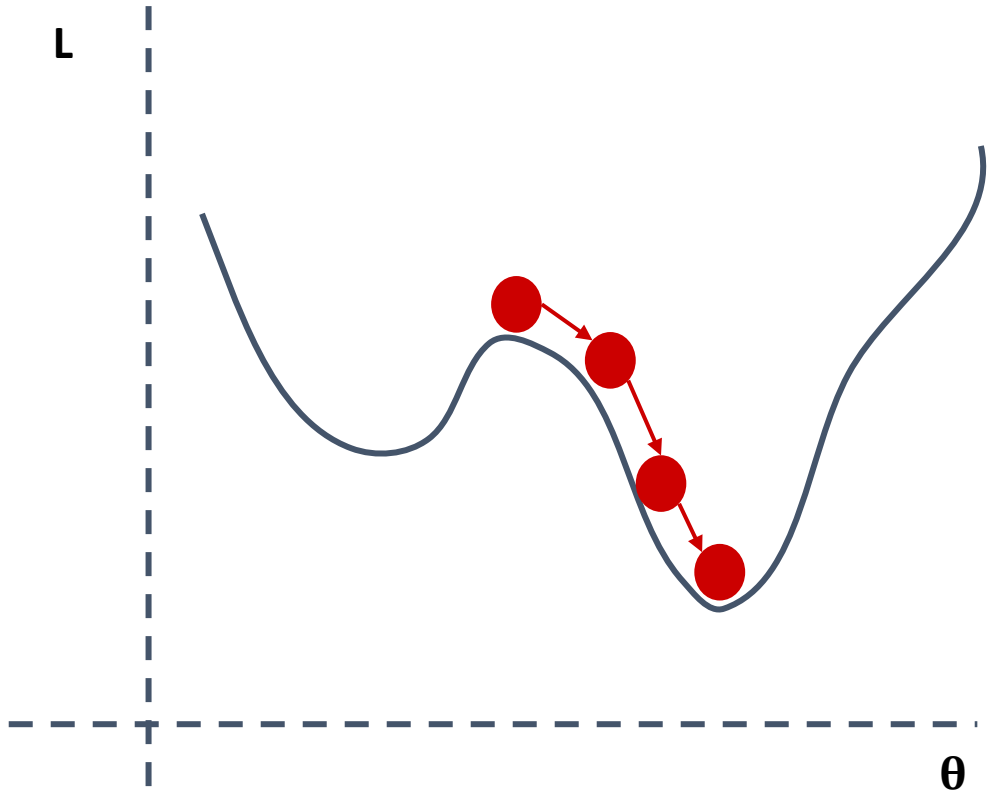
Considerations

- We still need to derive the derivatives.
- We need to know what is the learning rate or how to set it.
- We need to avoid local minima.
- Finally, the full likelihood function includes summing up all individual '*errors'.* Unless you are a statistician, this can be hundreds of thousands of examples.

Local vs Global Minima



Local vs Global Minima



Local vs Global Minima

- No guarantee that we get the global minimum.
- **Question:** What would be a good strategy?

Large data

Batch and Stochastic Gradient Descent

$$\mathcal{L} = -\sum_i [y_i \log p_i + (1 - y_i) \log(1 - p_i)]$$

- Instead of using all the examples for every step, use a subset of them (batch).
- For each iteration *k,* use the following loss function to derive the derivatives:

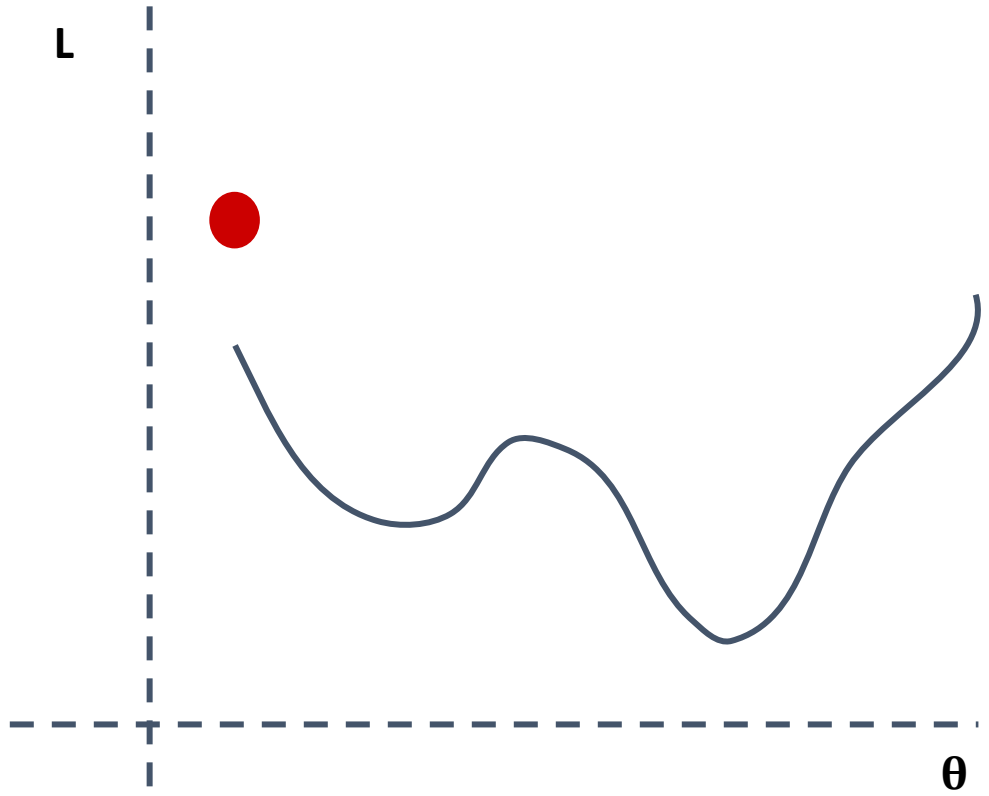
- which is an **approximation** to the full Loss function.

$$\mathcal{L}^k = -\sum_{i \in b^k} [y_i \log p_i + (1 - y_i) \log(1 - p_i)]$$

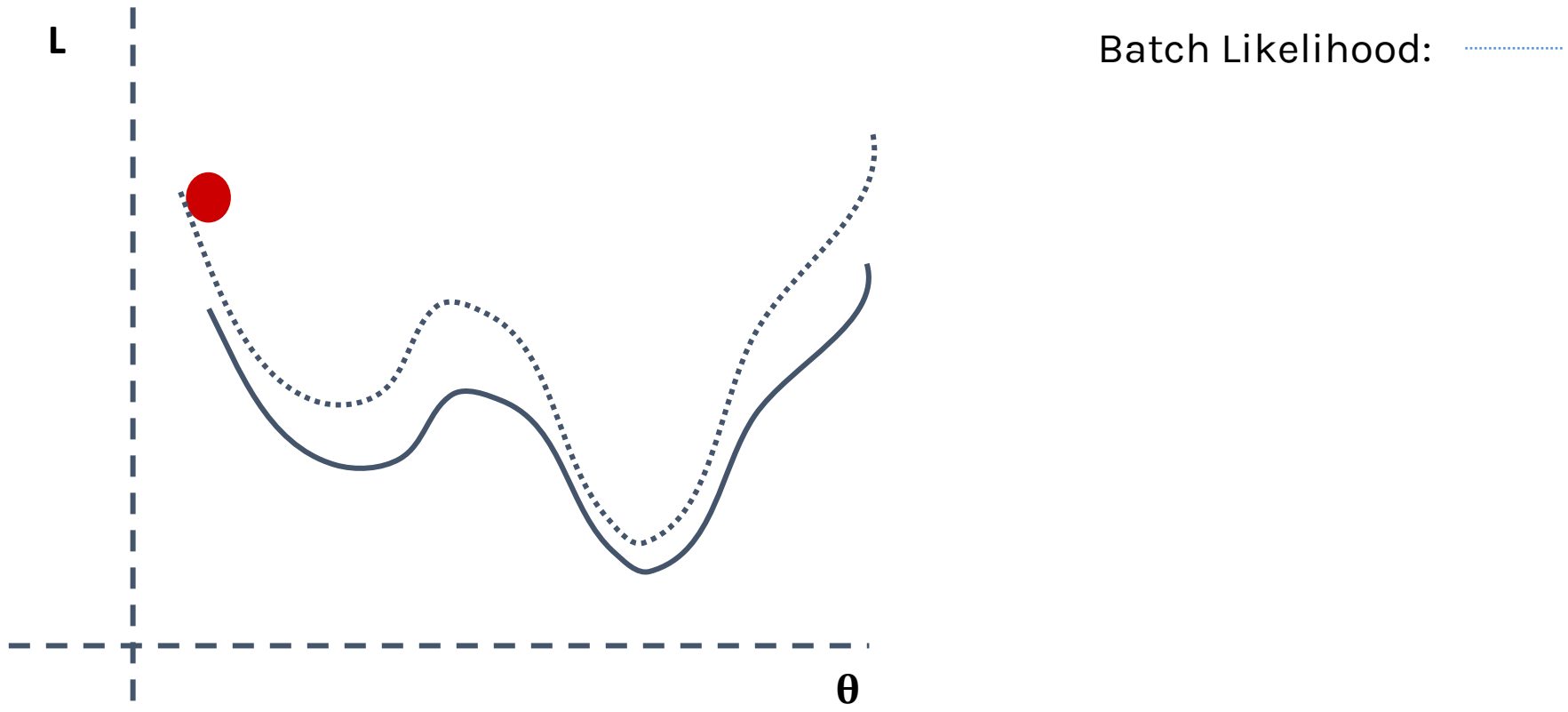
Batch and Stochastic Gradient Descent

Full Likelihood:

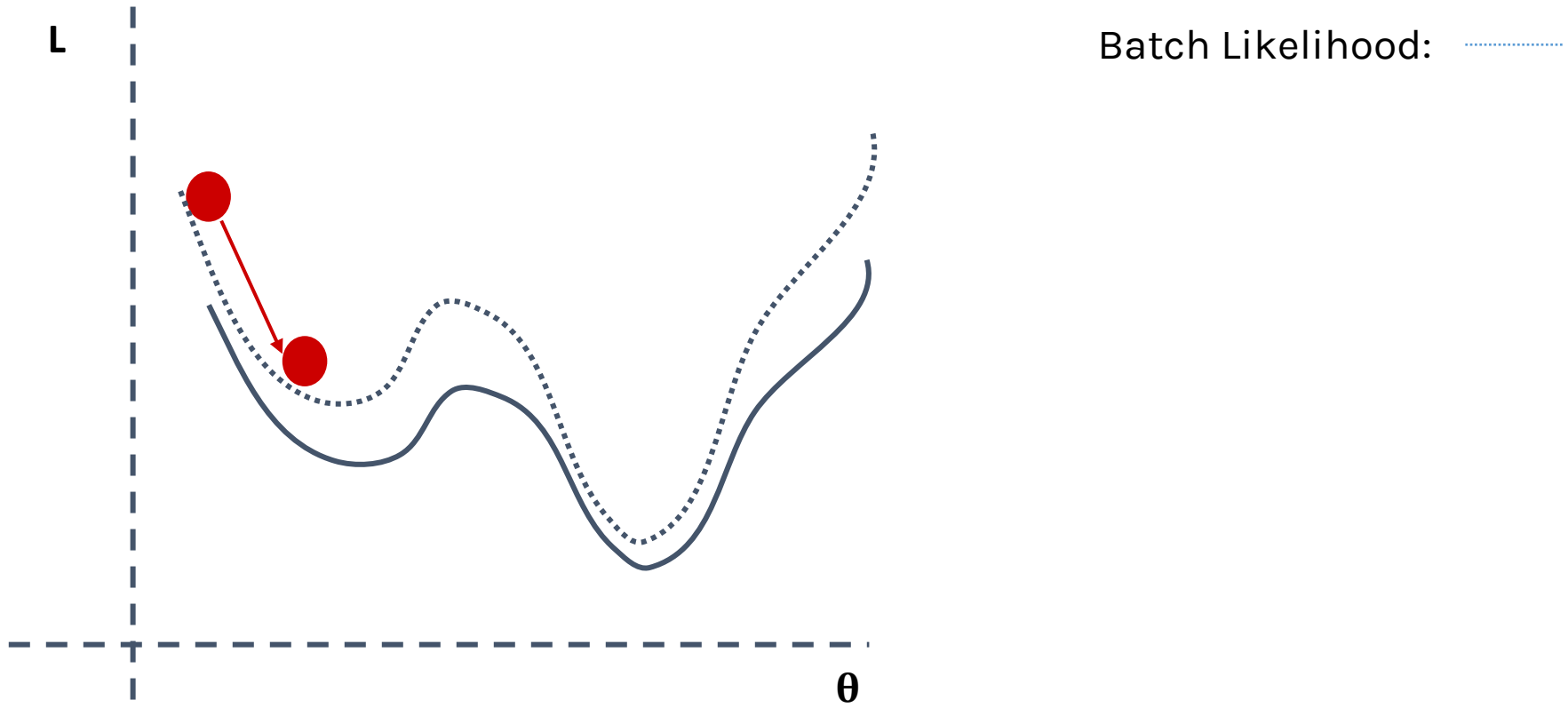
Batch Likelihood:



Batch and Stochastic Gradient Descent



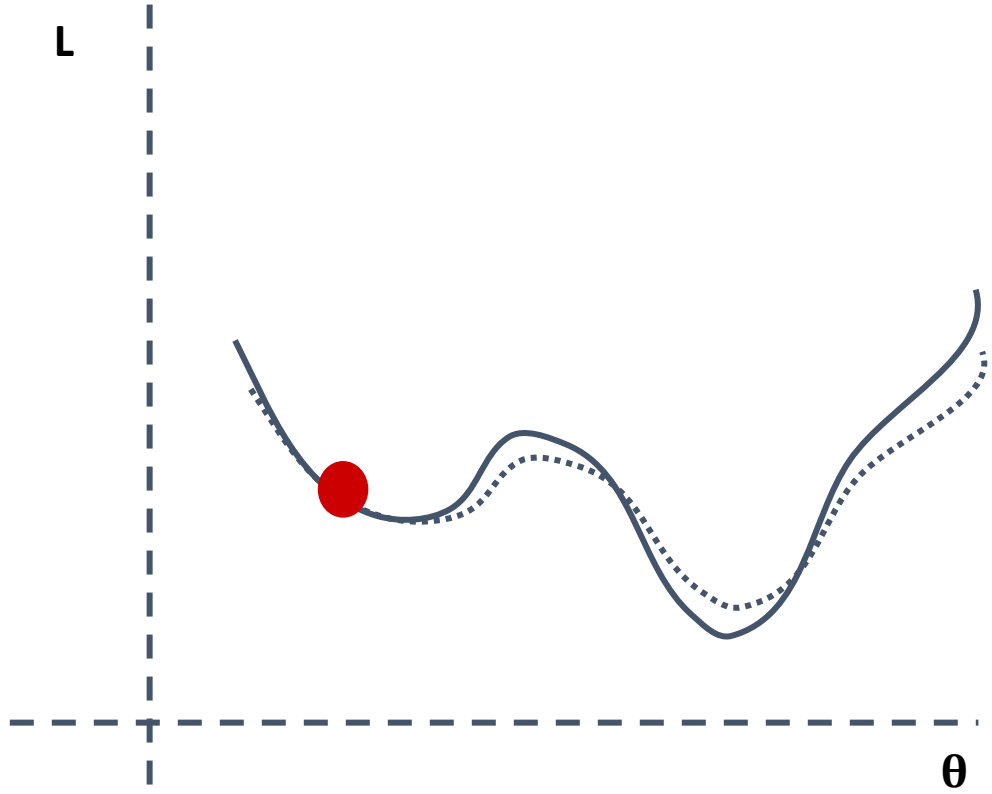
Batch and Stochastic Gradient Descent



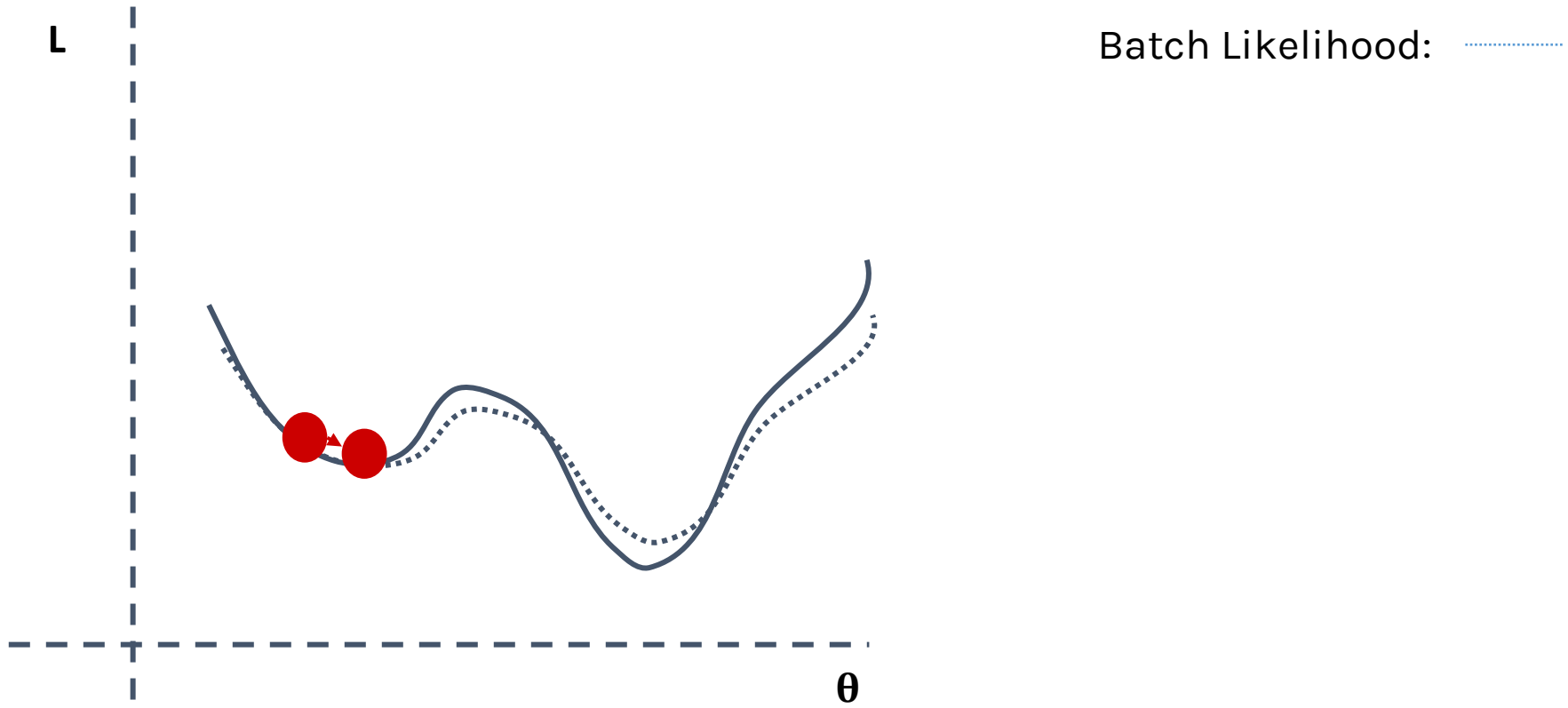
Batch and Stochastic Gradient Descent

Full Likelihood:

Batch Likelihood:



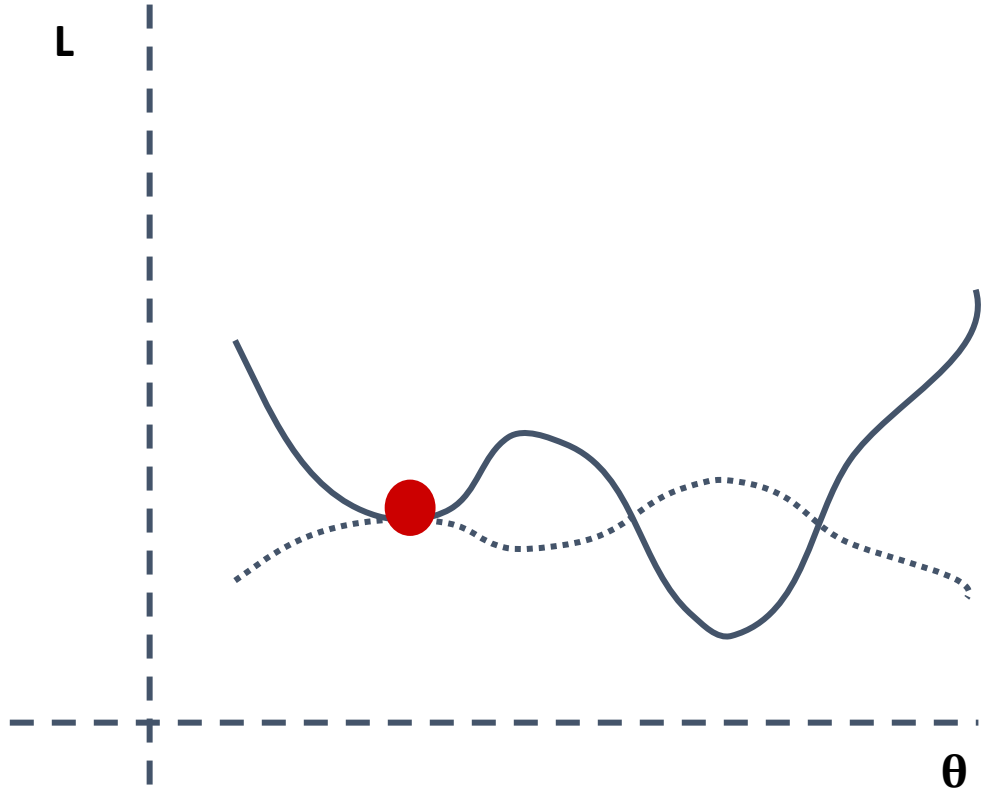
Batch and Stochastic Gradient Descent



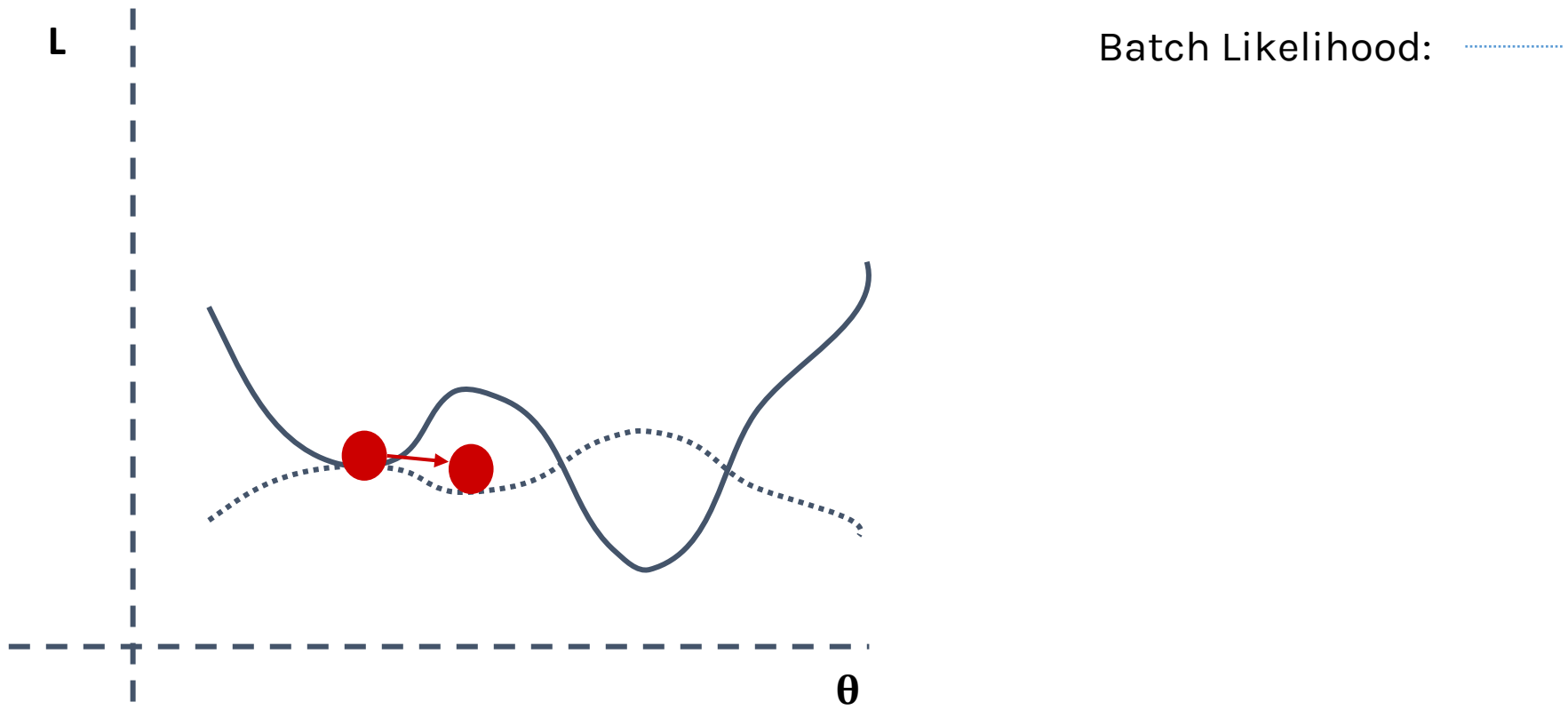
Batch and Stochastic Gradient Descent

Full Likelihood:

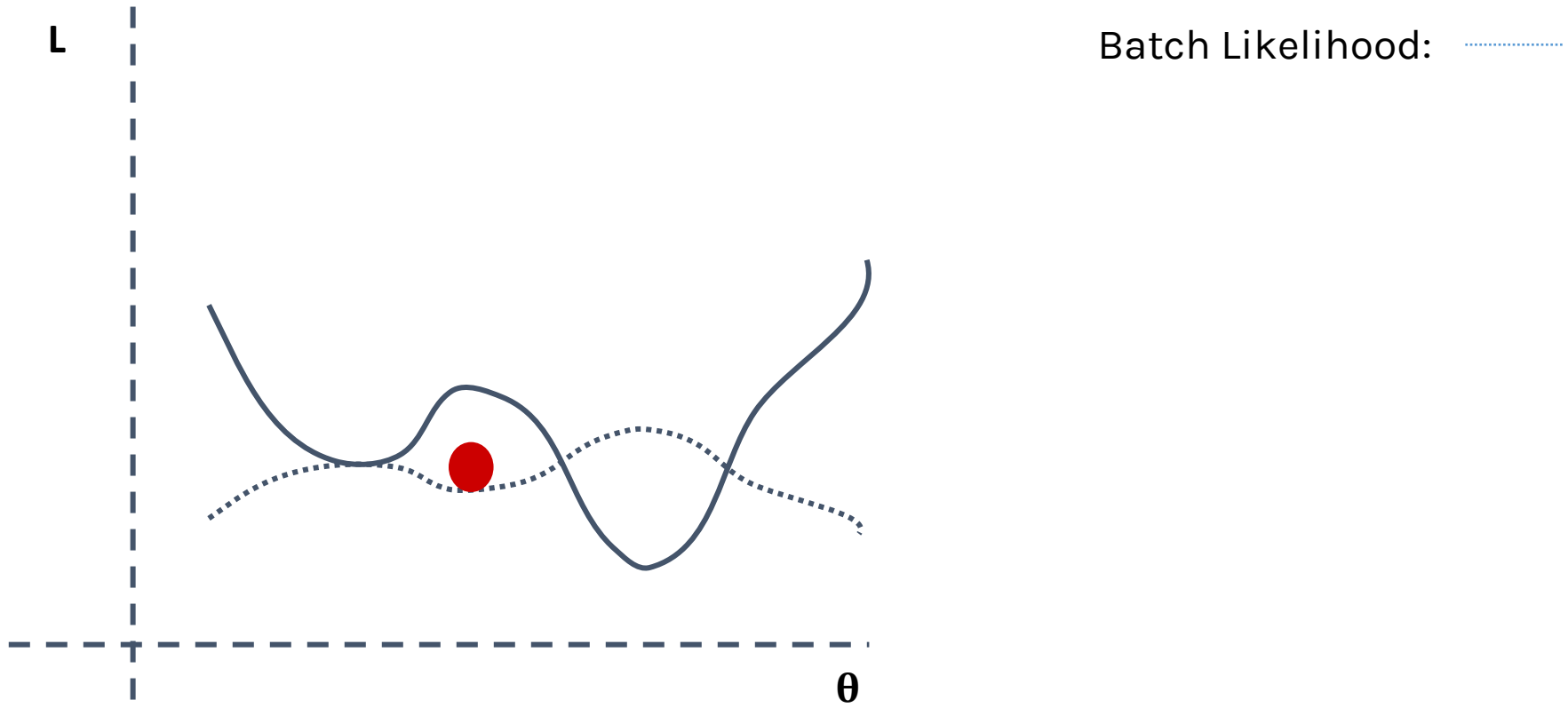
Batch Likelihood:



Batch and Stochastic Gradient Descent



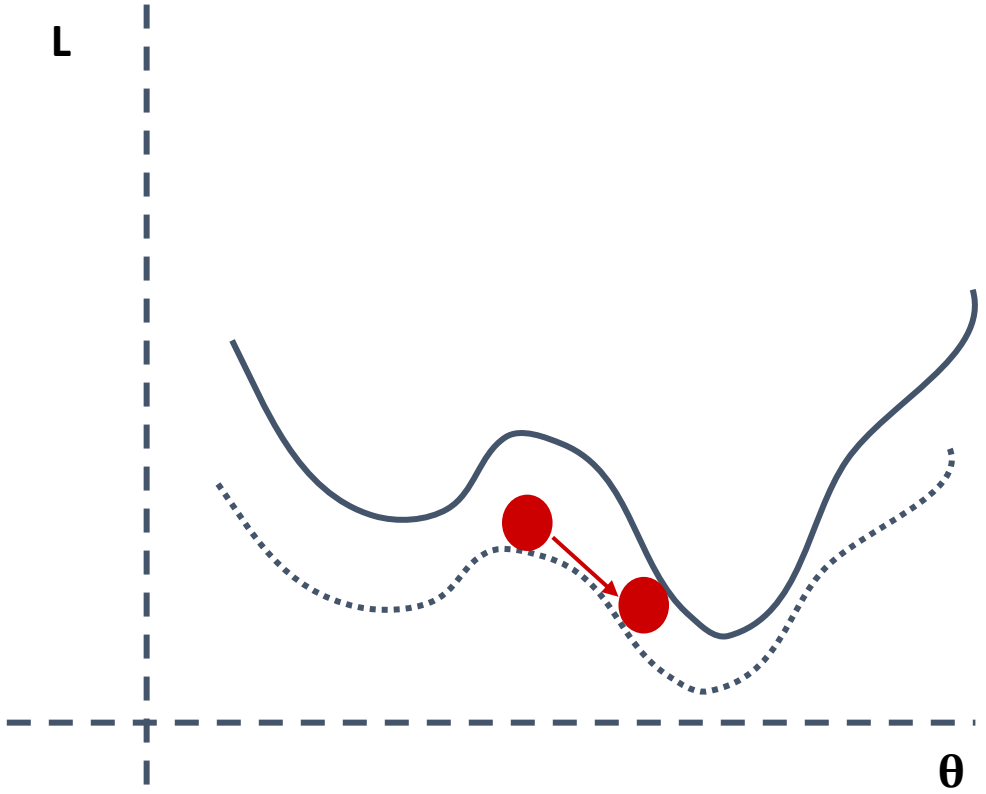
Batch and Stochastic Gradient Descent



Batch and Stochastic Gradient Descent

Full Likelihood:

Batch Likelihood:



Learning Rate

- There are many alternative methods which address how to set or adjust the learning rate, using the derivative or second derivatives and or the momentum. To be discussed in the next lectures on NN.