- **1.2 THE BASIC PRINCIPLE OF COUNTING**

When you are asked to count the number of ways a group of consequent events leads to a specific outcome, then the first thing you have to do:

- counting all possible ways that lead to that outcome
- recall the basic principle of counting

Definitely, you can use the first approach, but it will probably take too much time. Instead, you can use the basic principle of counting, which is all about counting different ways to achieve a desired outcome. Formal definition is:

Consider two experiments that are to be taken. If the first experiment results in n ways and the second experiment results in m ways, then the number of outcomes after performing both experiments is n*m.

EXAMPLE:

Consider cities A, B and C. The number of roads from A to B is 10 and from B to C is 5. In how many ways a person can reach C from A?

Possible answers: 15; 50

Using the basic principle of counting, the problem is trivial: 10*5 = 50

The same applies to a group of more than 2 events, which is called the generalized principle of counting. If we had to calculate the number of ways from A to m (the number of ways from A to A1 is n1, from A1 to A2 is n2, etc.), how can we achieve that?

Possible answers:

Multiplying n1, n2 up to n[m]; Adding the values

- **PERMUTATIONS and COMBINATIONS**

1.Is order a priority?

2.Yes: Can the objects repeat?

Yes: Use allocations without replacement
No: Use permutations(allocations with replacement)

No: Do you have to group the objects?

Yes: Use multinomial coefficients

No: Use combinations

EXAMPLE:

There are 10 people and we have to choose the director and the secretary. In how many ways can we do that?

1. Order is indeed a priority, so we have to use permutations

The order is the most important difference between the permutations and the combinations. If the order if significant-go for permutations and combinations, otherwise.

2. No, the director and the secretary cannot be a one person

The example, where objects can repeat, is the binary number of length n, where in each place there are two options to place 0 or 1.

Therefore, in the end we have to use permutations, which are also called allocations with replacement: $A^k n = n!/(n-k)!$, which results in $10!/8! = 90$ ways.

- **A little bit of theory**

As was mentioned before, the difference between permutations and combinations is whether the order of the objects matters or not.

But even if we already decided which one we need, we have to take into consideration whether the objects can repeat (like digits in the number), etc.

Multinomial coefficients are computed when we have indistinguishable objects, and the order is not important. The common example is the order of letters in a word, some letters of which are the same.

Allocations with replacements mean that the objects can repeat, i.e. every object can be replaced with any other, and allocations without replacements mean that we cannot do that.