Rule Induction and Reasoning in Knowledge Graphs

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ODSC 2022

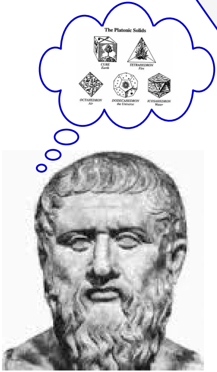
BOSCH
Invented for life

What is Knowledge?

Plato: *“Knowledge is justified true belief”*

350 BC

Personal



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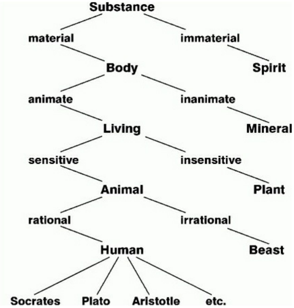

Tree of Knowledge

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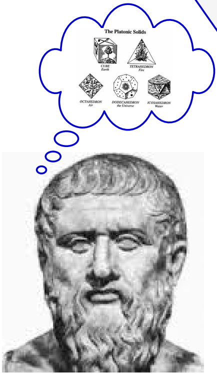

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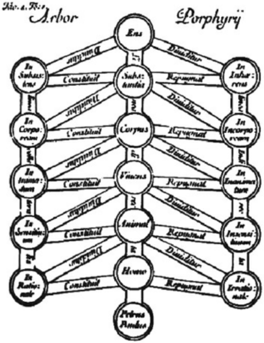


Knowledge Graphs as Digital Knowledge

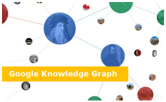
"Digital knowledge is semantically enriched machine processable data"

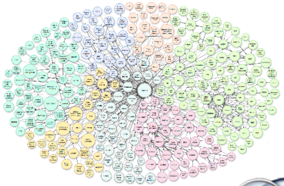
2022

Personal

External

yago
select knowledge





WIKIDATA

NELL

Knowledge Graph

Industrial KGs

Thousands of companies are developing their own KGs, not only for search and indexing but advanced reasoning tasks on top of machine learning

Semantic Web Search

Google | winner of Australian Open 2022

Rafael Nadal

Spanish tennis player

Rafael Nadal Parera is a Spanish professional tennis player. He is ranked world No. 5 in singles by the Association of Tennis Professionals; he has been ranked world No. 1 for 209 weeks and finished as the year-end No. 1 five times. Wikipedia

Born: June 3, 1986 (age 35 years), Manacor, Spain

Grand slams won (singles): 21

Height: 1.85 m

Spouse: Maria Francisca Perello (m. 2019)

Books: Rafa

Parents: Sebastián Nadal, Ana María Parera

Nicknames: El Nino, King of clay, Rafa, Rafi, Spain's Raging Bull

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Google $\exists X\ winnerOf(X, AustralianOpen)$

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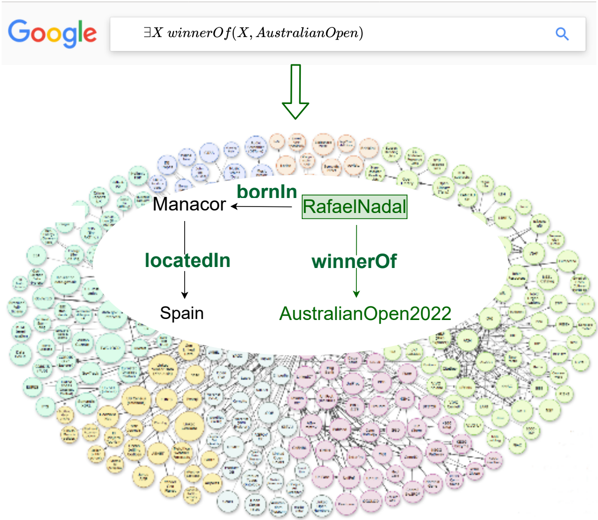


Semantic Web Search

Google

living place of Rafael Nadal

All Images News Videos Shopping More Tools

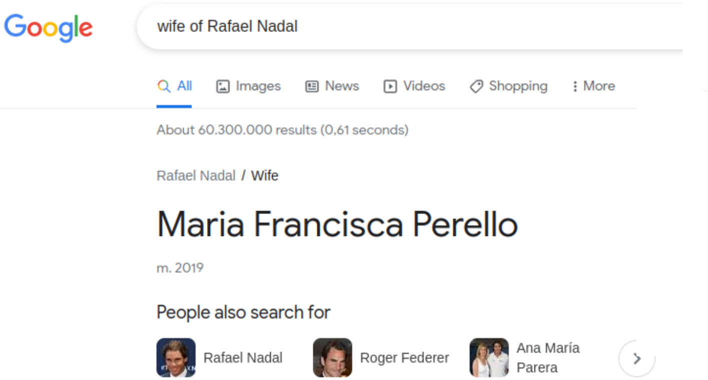
About 23.300.000 results (0,68 seconds)

Manacor, Mallorca, Spain

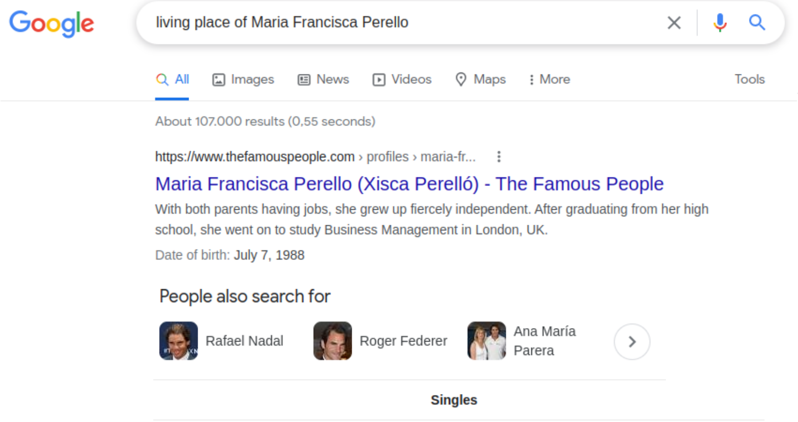
Rafael Nadal

Full name	Rafael Nadal Parera
Country (sports)	Spain
Residence	**Manacor, Mallorca, Spain**
Born	3 June 1986 Manacor, Mallorca, Spain
Singles	

Semantic Web Search



Incompleteness of KGs



Human Reasoning

$\mathit{livesIn}(Y, Z) \leftarrow \mathit{marriedTo}(X, Y), \mathit{livesIn}(X, Z)$	*Married people live together*
$\mathit{marriedTo}(\mathit{rafael}, \mathit{mariaFrancisca})$	*Rafael is married to Maria Francisca*
$\mathit{livesIn}(\mathit{rafael}, \mathit{manacor})$	*Rafael lives in Manacor*
________________________	________________________

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Rafael is married to Maria Francisca

$$\mathit{livesIn}(\mathit{rafael}, \mathit{manacor})$$

Rafael lives in Manacor

$$\mathit{livesIn}(\mathit{mariaFrancisca}, \mathit{manacor})$$

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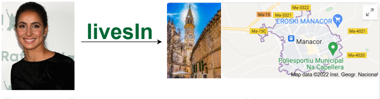

Maria Franciso Borello

Manacor

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$livesIn(mariaFrancisca, manacor)$	*Maria Francisca lives in Manacor*

Maria Franciso Borello

livesIn →

Manacor

But where can a machine get such rules from?

Applications of Rule Learning

- Fact prediction
- Fact checking
- Data cleaning
- Finding trends in KGs
- ...

Horn Rules

Rule: $\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \ldots, b_m.}_{\text{body}}$

Informal semantics: If $b_1, \ldots, b_m$ are true, then a must be true.

Logic program: Set of rules

Example: ground rule

% If Nadal is married to Maria and lives in M., then Maria lives there too
$livesIn(maria, manacor) \leftarrow isMarried(nadal, maria), livesIn(nadal, manacor)$

Horn Rules

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Logic program: Set of rules

Example: non-ground rule

% Married people live together
$livesIn(Y, Z) \leftarrow isMarried(X, Y), livesIn(X, Z)$

Rules wih Negation

Rule: $\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \ldots, b_m, \textit{not}\ b_{m+1}, \ldots, \textit{not}\ b_n.}_{\text{body}}$

Informal semantics: If $b_1, \ldots, b_m$ are true and none of $b_{m+1}, \ldots, b_n$ is known, then a must be true.

Default reasoning: Facts not known to be true are assumed to be false

Example: rule with negation

% Two married live together unless one is a researcher
$livesIn(Y, Z) \leftarrow isMarried(X, Y), livesIn(X, Z), not\ \ researcher(Y)$

Reasoning with Incomplete Information

Default Reasoning

Assume normal state of affairs, unless there is evidence to the contrary

By default married people live together.

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Choose between several explanations that explain an observation

John and Mary live together. They must be married.

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Generalize a number of similar observations into a hypothesis

Given many examples of spouses living together generalize this knowledge.

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History of Inductive Learning

- AI & Machine Learning 1960s-70s:
  Banerji, Plotkin, Vere, Michalski, ...
- AI & Machine Learning 1980s:
  Shapiro, Sammut, Muggleton, ...
- Inductive Logic Programming (ILP) 1990s:
  Muggleton, Quinlan, De Raedt, ...
- Statistical Relational Learning 2000s:
  Getoor, Koller, Domingos, Sato, ...
- Neuro-symbolic AI 2015 -...
  Hitzler, De Raedt, Leskovec, Tannenbaum, ...

Learning from Examples

Inductive Learning from Examples [Muggleton, 1991]

Given:

- $E^+ = \{fatherOf(john, mary), fatherOf(david, steve)\}$
- $E^- = \{fatherOf(kathy, ellen), fatherOf(john, steve)\}$
- $T = \{parentOf(john, mary), male(john),$
  $parentOf(david, steeve), male(david),$
  $parentOf(kathy, ellen), female(kathy)\}$
- Language bias: Horn rules with 2 body atoms

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Possible hypothesis:

- $Hyp: \; fatherOf(X, Y) \leftarrow parentOf(X, Y), male(X)$

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  - $livesIn(nadal, manacor)$? $livesIn(nadal, spain)$

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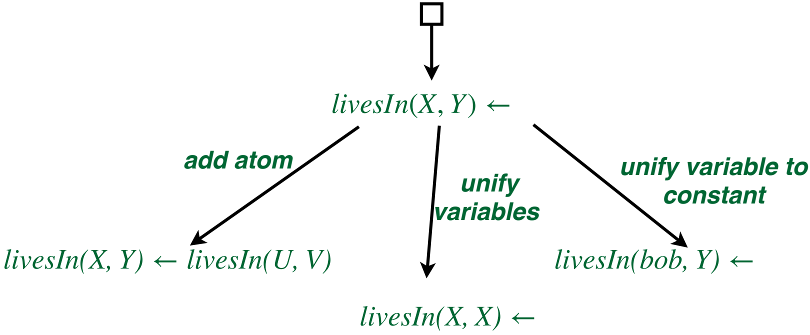
Common Techniques in ILP

- Clause refinement [Shapiro, 1991]: e.g., MIS, FOIL, etc.
  - Explore clause search space from general to specific or vice versa to find a hypothesis that covers all examples.



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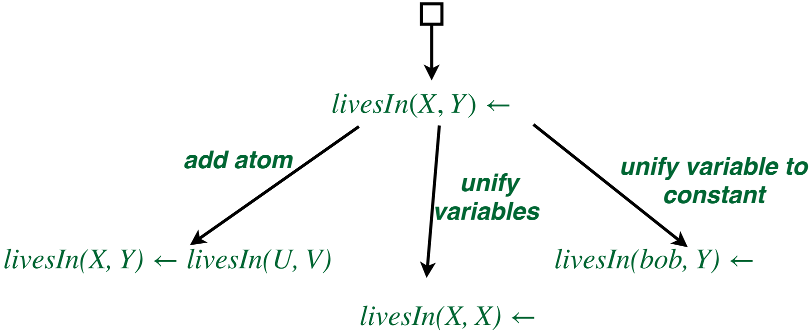

- Inverse entailment [Muggleton, 1995]: e.g., Progol, etc.
  - Properties of deduction to make hypothesis search space finite

Zoo of Other ILP Tasks

ILP tasks can be classified along several dimensions:

- type of the data source, e.g., positive/negative examples, interpretations, answer sets [Law *et al.*, 2015]
- type of the output knowledge, e.g., rules, ontologies [Lehmann, 2009]
- the way the data is given as input, e.g., all at once, incrementally [Katzouris *et al.*, 2015]
- availability of an oracle, e.g., human in the loop
- quality of the data source, e.g., noisy [Evans and Grefenstette, 2018]
- data (in)completeness, e.g., complete, incomplete, partially complete
- background knowledge, e.g., ontology [d'Amato *et al.*, 2016], hybrid theories [Lisi, 2010]

Challenges of Rule Induction from KGs

Open World Assumption: negative facts cannot be easily derived

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Maybe R. Nadal is a researcher and A. Einstein was a dancer?

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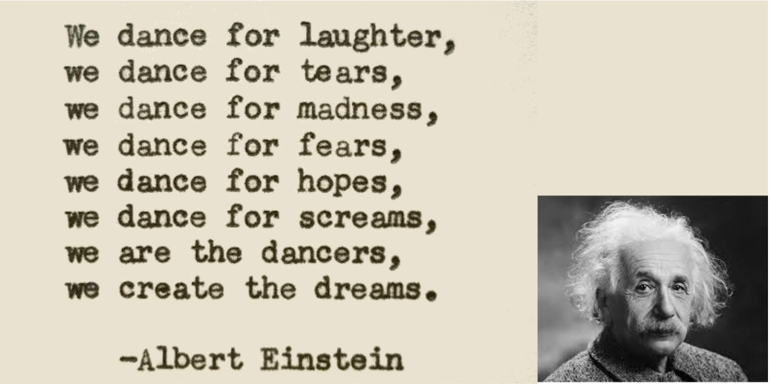


Challenges of Rule Induction from KGs

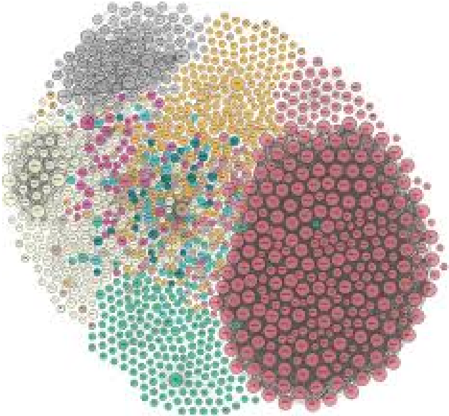
Data bias: KGs are extracted from text, which typically mentions only popular entities and interesting facts about them.

"Man bites dog phenomenon"[1]

[1] https://en.wikipedia.org/wiki/Man_bites_dog_(journalism)

Challenges of Rule Induction from KGs

Huge size: Modern KGs contain billions of facts
E.g., Google KG stores 70 billion facts



Challenges of Rule Induction from KGs

World knowledge is complex, none of its "models" is perfect

Exploratory Data Analysis

Question:

How can we still learn rules from KGs, which do not perfectly fit the data, but reflect interesting correlations that can predict sufficiently many correct facts?

Answer:

Relational association rule mining! Roots in classical datamining.

Association Rules

- Classical data mining task: Given a transaction database, find out products (called itemsets) that are frequently bought together and form recommendation rules.

Transaction 1	🍎 🍺 🍚 🍗
Transaction 2	🍎 🍺 🍚
Transaction 3	🍎 🍺
Transaction 4	🍎 🍐
Transaction 5	🍼 🍺 🍚 🍗
Transaction 6	🍼 🍺 🍚
Transaction 7	🍼 🍺
Transaction 8	🍼 🍐

Out of 4 people who bought apples, 3 also bought beer.

Some Rule Measures

Support, confidence, lift

Support [🍎] = 4

Transaction 1	🍎 🍺 🍚 🍗
Transaction 2	🍎 🍺 🍚
Transaction 3	🍎 🍺
Transaction 4	🍎 🍐
Transaction 5	🍼 🍺 🍚 🍗
Transaction 6	🍼 🍺 🍚
Transaction 7	🍼 🍺
Transaction 8	🍼 🍐

Some Rule Measures

Support, confidence, lift

Support [🍎] = 4

$$\text{Confidence}\ \{🍎 \rightarrow 🍺\} = \frac{\text{Support}\ \{🍎, 🍺\}}{\text{Support}\ \{🍎\}}$$

Transaction 1	🍎 🍺 🍚 🍗
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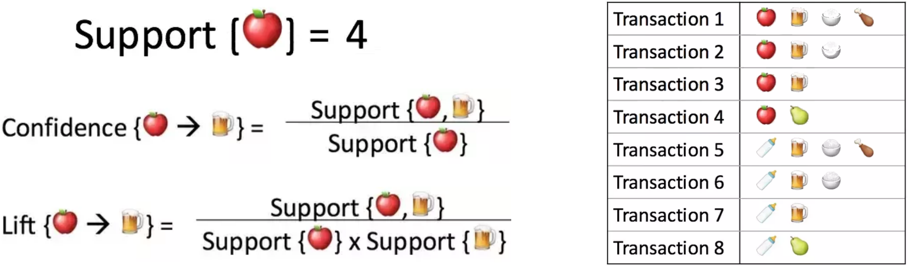
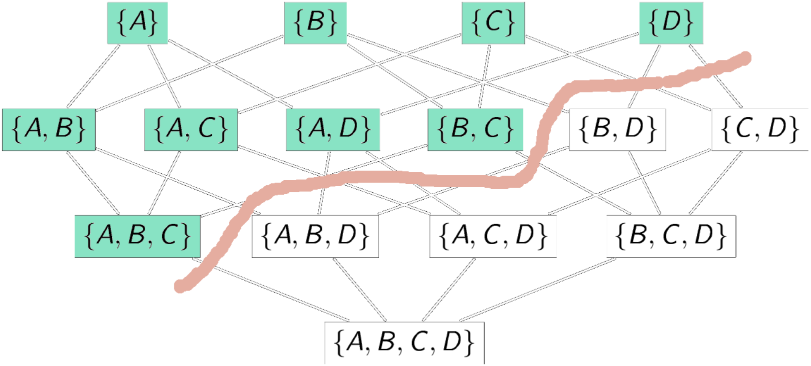


Frequent Itemset Mining

- A=apple, B=beer... Frequent patterns are in green.
- Monotonicity: any superset of an infrequent pattern is infrequent
  At the heart of frequent itemset mining algorithm



How to Apply this to Relational Data?

- DOWNGRADING DATA: Can we change the representation from richer representations to simpler ones? (So we can use systems working with simpler representations)

- UPGRADING SYSTEMS: Can we develop systems that work with richer representations (starting from systems for simpler representations)?

Downgrading the Data

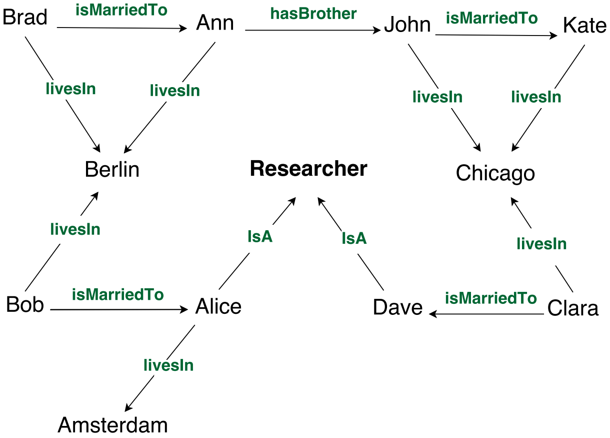
- **Propositionalization** [Krogel *et al.*, 2003]: transform a KG into a transaction database

	bornInUS	*livesInUS*	*isMarriedToSinger*	*researcher*	*sportsman*
p1	✓	✓			✓
p2	✓	✓		✓	
p3	✓	✓			
p4	✓	✓			
p5	✓		✓		
p6	✓		✓		✓
p7	✓			✓	
p8	✓	✓			

Upgrading the Systems

- Start from existing system for simpler representation
- Extend it for use with richer representation (while trying to keep the original system as a special case)

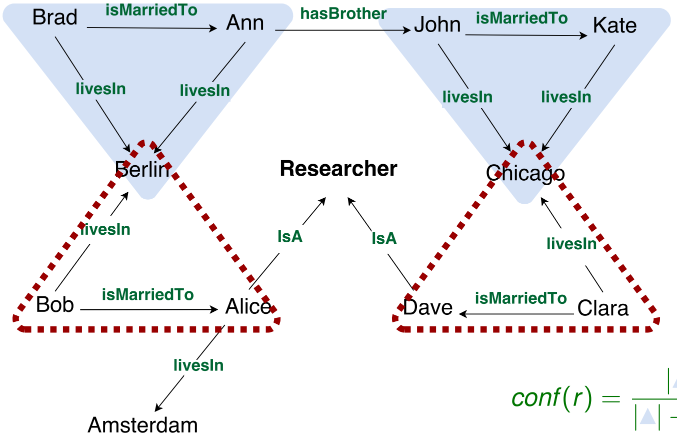
Relational Association Rule Learning



Relational Association Rule Learning

Confidence, e.g., WARMER [Goethals and den Bussche, 2002], AnyBurl [Meilicke *et al.*, 2019]

Closed World Assumption (CWA): Whatever is missing is false



$$conf(r) = \frac{|\blacktriangle|}{|\blacktriangle| + |\triangle|} = \frac{2}{4}$$

$r: \ livesIn(X, Y) \leftarrow isMarriedTo(Z, X), livesIn(Z, Y)$

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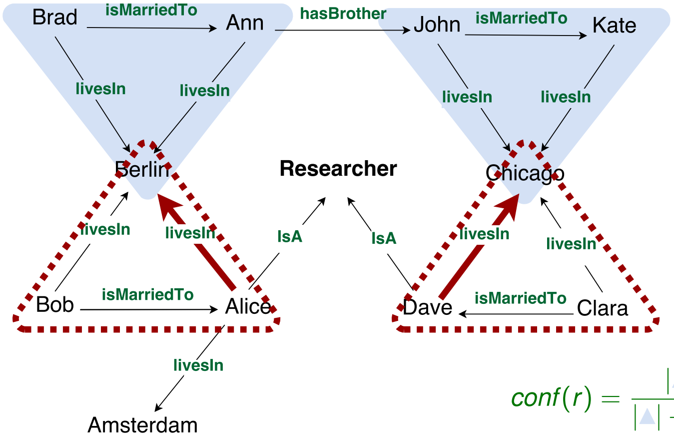

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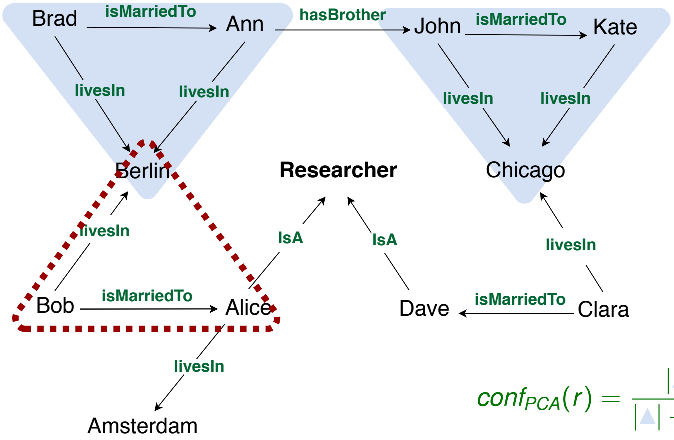
Relational Association Rule Learning

PCA confidence AMIE [Galarraga *et al.*, 2015]

Partial CA: Since Alice has a living place already, all others are incorrect.

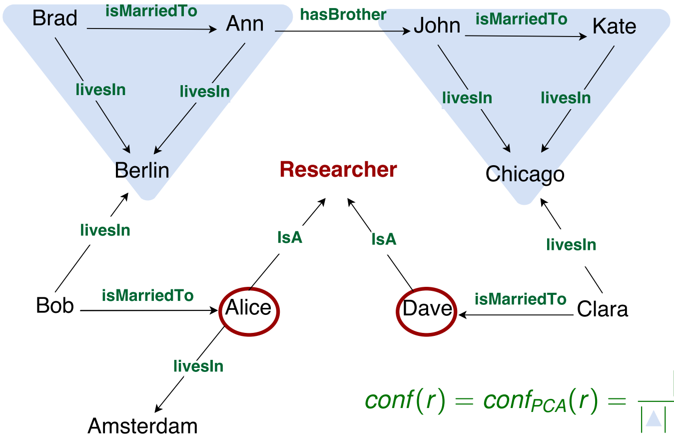


$$conf_{PCA}(r) = \frac{|\blacktriangle|}{|\blacktriangle| + |\triangle|} = \frac{2}{3}$$

$r: \ livesIn(X, Y) \leftarrow isMarriedTo(Z, X), livesIn(Z, Y)$

Relational Association Rule Learning

Exception-enriched rules: **Open World Assumption** is a challenge!



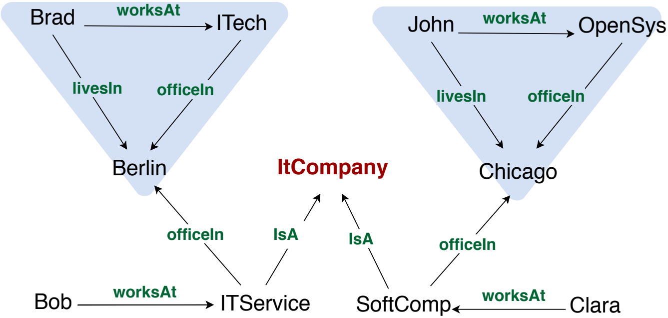
$$conf(r) = conf_{PCA}(r) = \frac{|\blacktriangle|}{|\blacktriangle| + |\triangle|} = 1$$

$$r: \ livesIn(X, Y) \leftarrow isMarriedTo(Z, X), livesIn(Z, Y), not \ \ isA(X, researcher)$$

M. Gad-Elrab, D. Stepanova, J. Urbani, G. Weikum. Exception-enriched Rule Learning from Knowledge Graphs. ISWC2016
D. Tran, D. Stepanova, M. Gad-Elrab, F. Lisi, G. Weikum. Towards Nonmonotonic Relational Learning from KGs. ILP2016
`https://github.com/htran010589/nonmonotonic-rule-mining.git`

Absurd Rules due to Data Incompleteness

Problem: rules learned from highly incomplete KGs might be absurd..

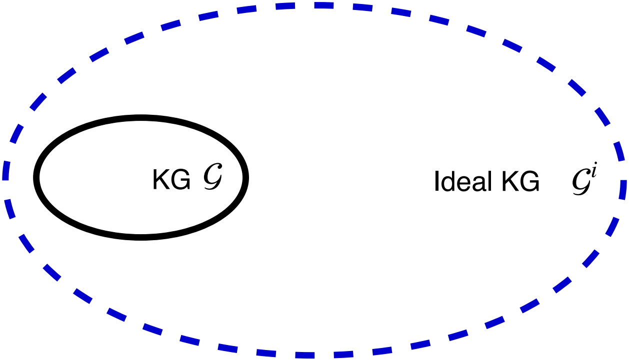


$$conf(r) = conf_{PCA}(r) = 1$$

$$livesIn(X, Y) \leftarrow worksAt(X, Z), officeIn(Z, Y), not\ isA(Z, itCompany)$$

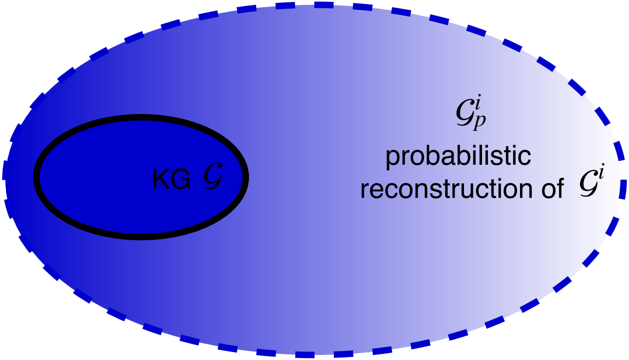
Ideal KG

$\mu(r, \mathcal{G}^i)$: measure quality of the rule r on $\mathcal{G}^i$, but $\mathcal{G}^i$ is unknown



Probabilistic Reconstruction of Ideal KG

$\mu(r, \mathcal{G}_p^i)$: measure quality of r on $\mathcal{G}_p^i$



Hybrid Rule Measure

$$\mu(r, \mathcal{G}_p^i) = (1 - \lambda) \times \mu_1(r, \mathcal{G}) + \lambda \times \mu_2(r, \mathcal{G}_p^i)$$

- **$\lambda \in$ [0..1] : weighting factor**
- **μ_1 : descriptive quality** of rule r over the available KG $\mathcal{G}$
  - confidence
  - PCA confidence
- **μ_2 : predictive quality** of r relying on $\mathcal{G}_p^i$ (probabilistic reconstruction of the ideal KG $\mathcal{G}^i$)

Knowledge Graph Embeddings

- Map entities $\mathcal{E}$ and relations $\mathcal{R}$ to some vector space
  - $emb_E : \mathcal{E} \rightarrow \mathbb{R}^{d_E}$ and $emb_R : \mathcal{R} \rightarrow \mathbb{R}^{d_R}$

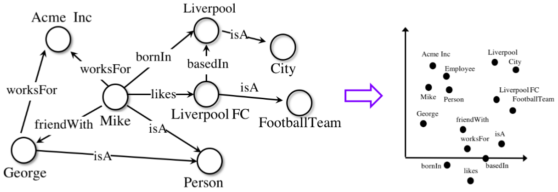


Figure from Ampligraph documentation

Knowledge Graph Embeddings

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  - $emb_E : \mathcal{E} \rightarrow \mathbb{R}^{d_E}$ and $emb_R : \mathcal{R} \rightarrow \mathbb{R}^{d_R}$
- KG facts are scored using a dedicated scoring function:
  $score(h, r, t) = s(emb_E(h), emb_R(r), emb_E(t))$

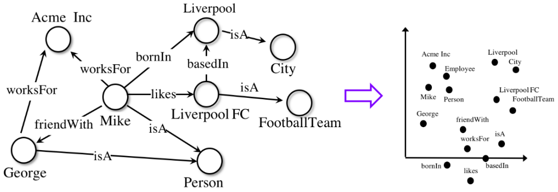

Figure from Ampligraph documentation

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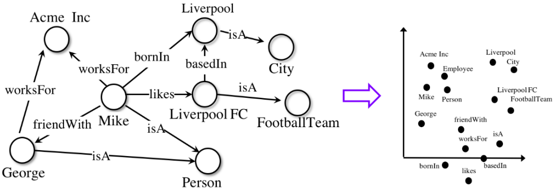
- E.g., TransE [Bordes *et al.*, 2013]: $score(h, r, t) = -\|\mathbf{h} + \mathbf{r} - \mathbf{t}\|$
  Text-enhanced variations exist, e.g., SSP [Xiao *et al.*, 2017]



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- E.g., TransE [Bordes *et al.*, 2013]: $score(h, r, t) = -\|\mathbf{h} + \mathbf{r} - \mathbf{t}\|$ Text-enhanced variations exist, e.g., SSP [Xiao *et al.*, 2017]
- Ranking interpretation: If $score(h_1, r_1, t_1) > score(h_2, r_2, t_2)$, then (h_1, r_1, t_1) is more likely to hold than (h_2, r_2, t_2)

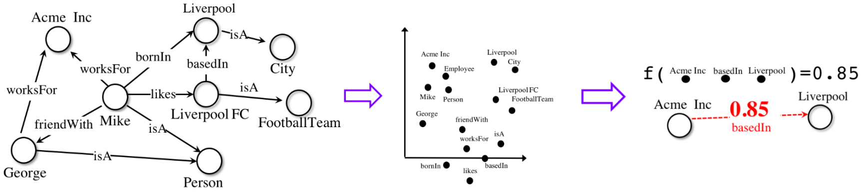
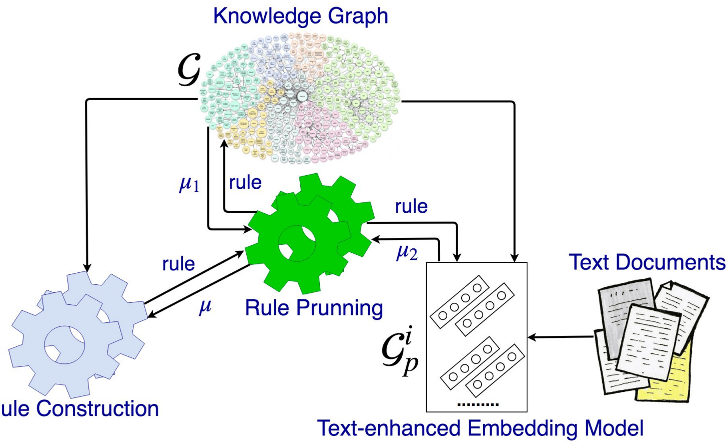


Figure from Ampligraph documentation

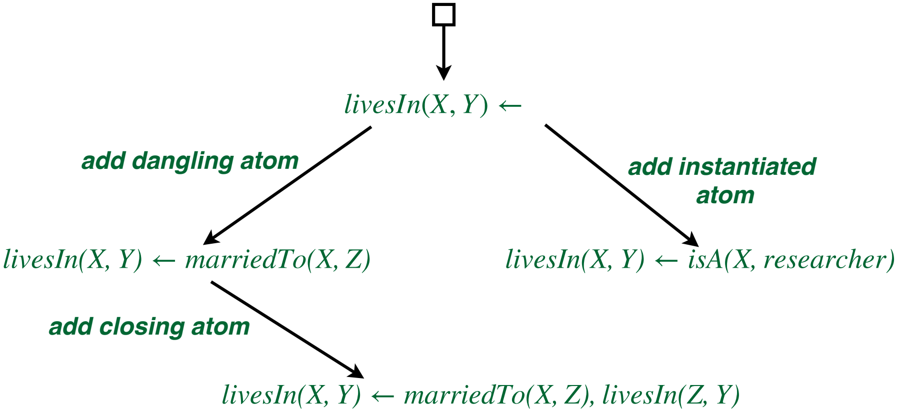
Embedding-based Rule Learning



V. Thinh Ho, D. Stepanova, M. Gad-Elrab, E. Kharlamov, G. Weikum. Rule Learning from KGs Guided by Embeddings. ISWC2018
`https://github.com/hovinhthinh/RuLES`

Rule Construction

- Clause exploration from general to specific
  - Closed and safe rules with negation
    $livesIn(X, Y) \leftarrow marriedTo(X, Z), livesIn(Z, Y), not\ isA(X, researcher)$



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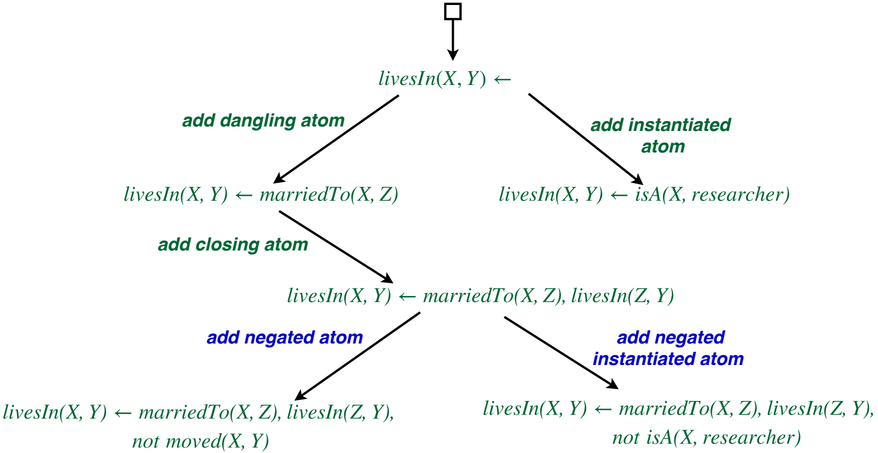
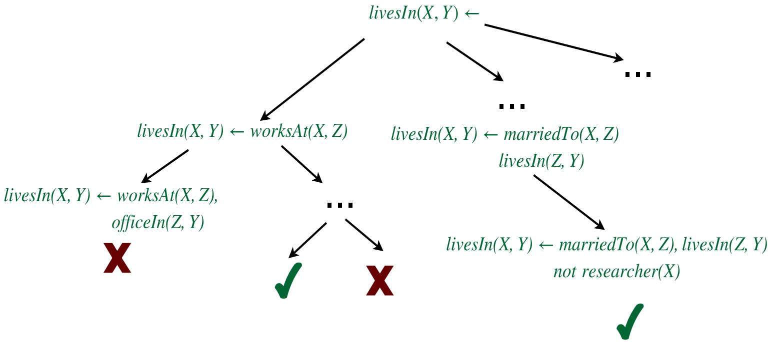


Rule Prunning



Prune rule search space relying on

- novel hybrid embedding-based rule measure

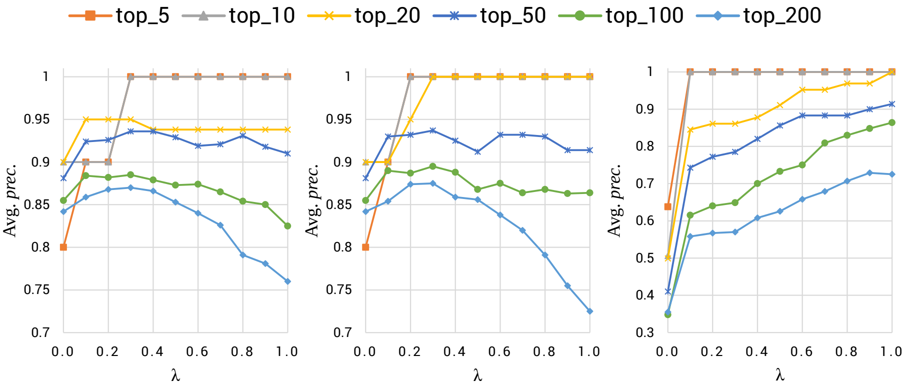
Evaluation Setup

- Datasets:
  - FB15K: 592K facts, 15K entities and 1345 relations
  - Wiki44K: 250K facts, 44K entities and 100 relations
- Training graph $\mathcal{G}$: remove 20% from the available KG
- Embedding models $\mathcal{G}_p^i$:
  - TransE [Bordes *et al.*, 2013], HolE [Nickel *et al.*, 2016]
  - With text: SSP [Xiao *et al.*, 2017]
- Goals:
  - Evaluate effectiveness of our hybrid rule measure

$$\mu(r, \mathcal{G}_p^i) = (1 - \lambda) \times \mu_1(r, \mathcal{G}) + \lambda \times \mu_2(r, \mathcal{G}_p^i)$$

  - Compare against state-of-the-art rule learning systems

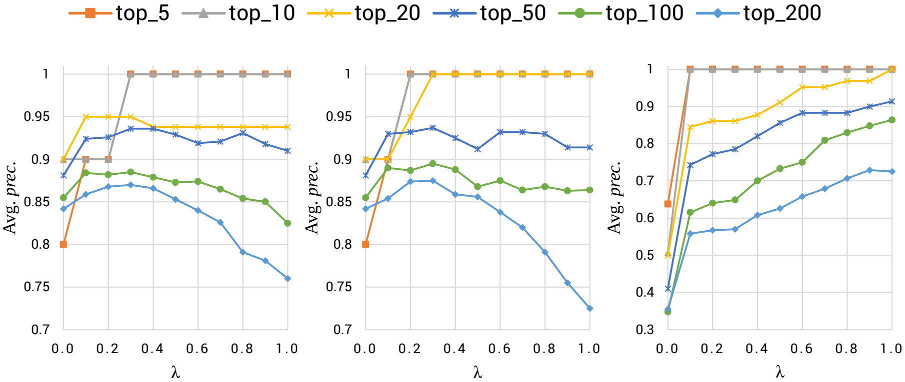
Evaluation of Hybrid Rule Measure



(a) Conf-HolE (b) Conf-SSP (c) PCA-SSP

Precision of *top-k* rules ranked using variations of μ on FB15K.

Evaluation of Hybrid Rule Measure



(a) Conf-HolE (b) Conf-SSP (c) PCA-SSP

Precision of *top-k* rules ranked using variations of μ on FB15K.

- Positive impact of embeddings in all cases for $\lambda = 0.3$
- **Note:** in (c) comparison to AMIE [Galarraga *et al.*, 2015] ($\lambda = 0$)

Examples of Learned Rules

Rules learned from Wikidata and IMDB

Nobles are typically married to nobles, but not in the case of Chinese dynasties

r_1 : $\mathit{nobleFamily}(X, Y) \leftarrow \mathit{spouse}(X, Z), \mathit{nobleFamily}(Z, Y), \mathit{not}\ \mathit{isA}(Y, \mathit{chineseDynasty})$

Plots of films in a sequel are written by the same writer, unless a film is American

r_2 : $\mathit{writtenBy}(X, Z) \leftarrow \mathit{hasPredecessor}(X, Y), \mathit{writtenBy}(Y, Z), \mathit{not}\ \mathit{american\_film}(X)$

Spouses of film directors appear on the cast, unless they are silent film actors

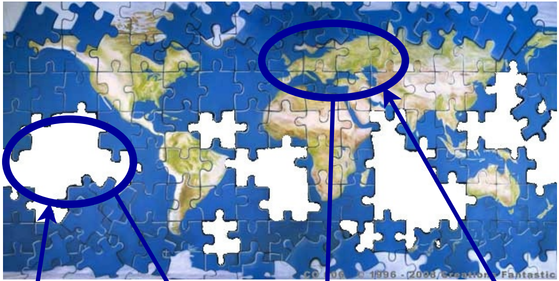
r_3 : $\mathit{actedIn}(X, Z) \leftarrow \mathit{isMarriedTo}(X, Y), \mathit{directed}(Y, Z), \mathit{not}\ \mathit{silent\_film\_actor}(X)$

Meta-data about Missing Facts in the KG

- Mining cardinality assertions from the Web [Mirza *et al.*, 2016]
  - *"... Albert Einstein had **3** children ..."*
- Estimating recall of KGs by crowd sourcing [Razniewski *et al.*, 2016]
  - *20 % of Nobel laureates in physics are missing*
- Predicting completeness in KGs [Galárraga *et al.*, 2017]
  - $complete(X, hasChild) \leftarrow child(X)$

Exploiting Cardinality Meta-data in Rule Learning

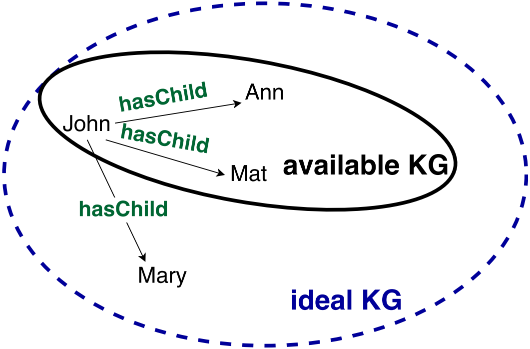
Goal: make use of cardinality constraints on edges of the KG to improve rule learning.



T. Pellissier-Tanon, D. Stepanova, S. Razniewski, P. Mirza, G. Weikum. Completeness-aware rule learning from KGs. ISWC2017.
`https://github.com/Tpt/CARL.git`

Cardinality Statements

- $num(p, s)$: Number of outgoing p-edges from s in the ideal KG
- $miss(p, s)$: Number of missing p-edges from s in the available KG
- If $miss(p, s) = 0$, then $complete(p, s)$, otherwise $incomplete(p, s)$



$num(hasChild, john) = 3$
$miss(hasChild, john) = 1$
$incomplete(hasChild, john)$

Completeness Confidence

$conf_{comp}$: do not penalize rules that predict new facts in incomplete areas

$$conf_{comp}(r) = \frac{|\blacktriangle|}{|\blacktriangle| + |\triangle| - npi(r)}$$

- $npi(r)$: number of facts added to incomplete areas by r
- Generalizes standard confidence ($miss(r) = 0$)
- Generalizes PCA confidence ($miss(r) \in \{0, +\infty\}$)

Other Completeness-aware Measures

$precision_{comp}$: penalize r that predict facts in complete areas

$$precision_{comp}(r) = 1 - \frac{npc(r)}{|\blacktriangle| + |\triangle|}$$

$recall_{comp}$: ratio of missing facts filled by r

$$recall_{comp}(r) = \frac{npi(r)}{\sum_s miss(h, s)}$$

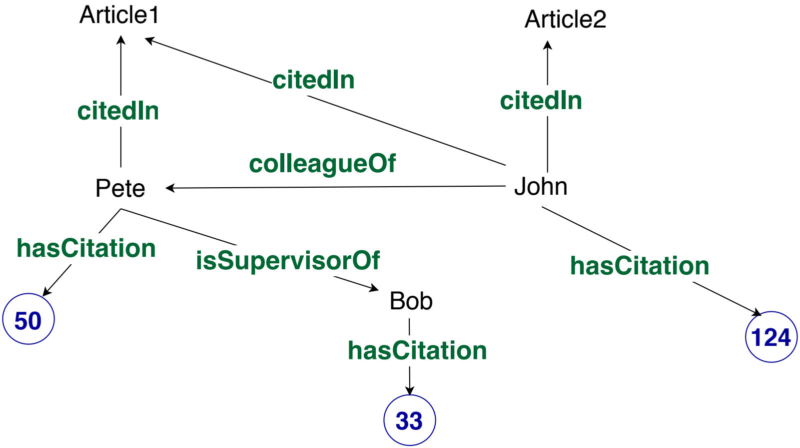
dir_metric : proportion of predictions in complete and incomplete parts

$$dir\_metric(r) = \frac{npi(r) - npc(r)}{2 \cdot (npi(r) + npc(r))} + 0.5$$

wdm : weighted combination of confidence and directional metric

$$wdm(r) = \beta \cdot conf(r) + (1 - \beta) \cdot dir\_metric(r)$$

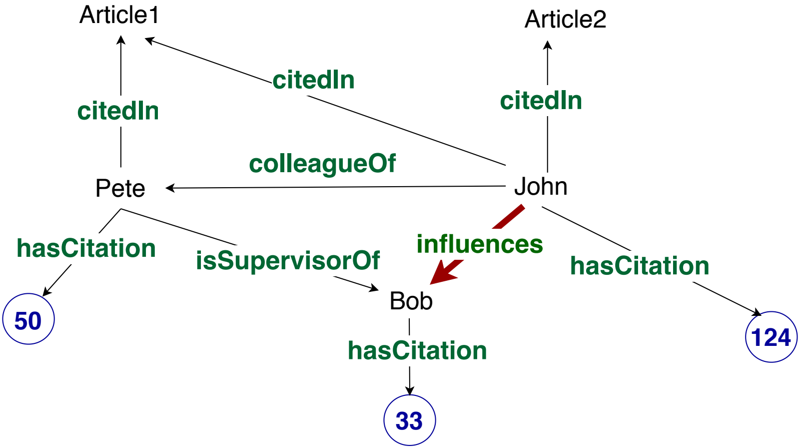
Numerical Rules



$$\mathit{influences}(X, Y) \leftarrow \mathit{colleagueOf}(X, Z), \mathit{supervisorOf}(Z, Y),$$
$$X.\mathit{hasCitation} > Z.\mathit{hasCitation}$$

Numerical KG construction: *V. Thinh Ho, D. Stepanova, D. Milchevski, J. Strötgen, G. Weikum: Enhancing Knowledge Bases with Quantity Facts. WWW 2022*

Numerical Rules



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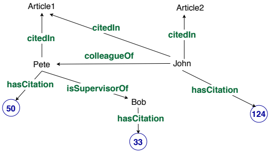
NeuralLP

NeuralLP [Yang *et al.*, 2017]: Differentiable rule learning via (sparse) matrix-vector multiplication

$$M_{\text{citedIn}} = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{ccccc|c} \text{john} & \text{pete} & \text{bob} & \text{article1} & \text{article2} & \\ \end{array}$$

$$M_{\text{citedIn}} = \begin{array}{cc} & \begin{array}{ccccc} \text{john} & \text{pete} & \text{bob} & \text{article1} & \text{article2} \end{array} \\ & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \text{john} \\ \text{pete} \\ \text{bob} \\ \text{article1} \\ \text{article2} \end{array} \end{array}$$



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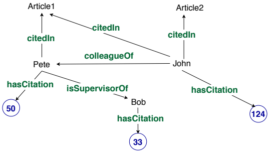

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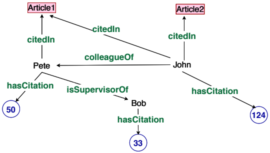

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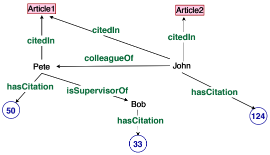
(columns: john, pete, bob, article1, article2)

$$v_{\text{john}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{l} \text{john} \\ \text{pete} \\ \text{bob} \\ \text{article1} \\ \text{article2} \end{array} \qquad M_{\text{citedIn}} v_{\text{john}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$



Apply rules (*path counting*) by sparse matrix-vector multiplication

$$\text{influences}(\mathbf{X},\mathbf{Z}) \leftarrow \text{colleagueOf}(\mathbf{X},\mathbf{Y}), \text{supervisorOf}(\mathbf{Y},\mathbf{Z})$$

$$\text{influences}(\textbf{john},\mathbf{Z}) = \text{one\_hot}(\textbf{ john})M^T_{colleagueOf} M^T_{supervisorOf}$$

Numerical Rule Learning

$p = hasCitation$

$$f = \begin{array}{c} \begin{matrix} john & pete & bob & article1 & article2 \end{matrix} \\ \begin{bmatrix} 124 & 50 & 33 & \mathtt{NaN} & \mathtt{NaN} \end{bmatrix} \end{array}$$

Numerical Rule Learning

$$p = hasCitation \qquad f = \begin{array}{ccccc} john & pete & bob & article1 & article2 \\ [124 & 50 & 33 & \mathtt{NaN} & \mathtt{NaN}] \end{array}$$

Comparison matrix:

$$M_{r_p^{\leq}} = \begin{array}{c} \begin{array}{ccccc} 124 & 50 & 33 & \mathtt{NaN} & \mathtt{NaN} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 124 \\ 50 \\ 33 \\ \mathtt{NaN} \\ \mathtt{NaN} \end{array}$$

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Problem: may be a dense matrix $\Rightarrow$ cannot be treated efficiently

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Problem: may be a dense matrix $\Rightarrow$ cannot be treated efficiently
Trick: Sort values by permutation matrices to allow for efficient computation

Numerical Rule Learning

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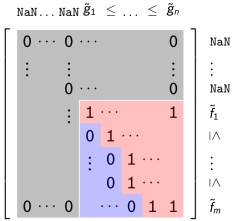
Comparison matrix:

$$M_{r_p^{\leq}} = \begin{array}{c} \begin{array}{ccccc} 124 & 50 & 33 & \mathtt{NaN} & \mathtt{NaN} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 124 \\ 50 \\ 33 \\ \mathtt{NaN} \\ \mathtt{NaN} \end{array}$$

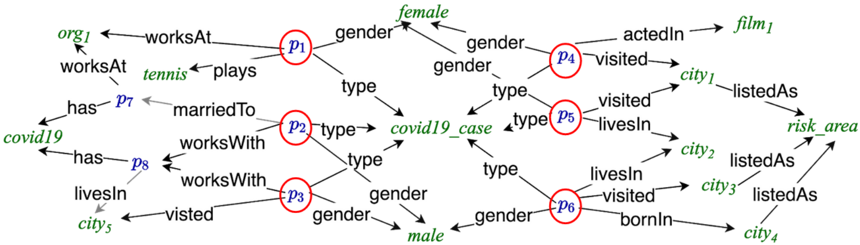


Problem: may be a dense matrix ⇒ cannot be treated efficiently
Trick: Sort values by permutation matrices to allow for efficient computation

P. Wang, D. Stepanova, C. Domokos, Z. Kolter. Differentiable Learning of Numerical Rules from KGs. ICLR 2020

Explainable Clustering

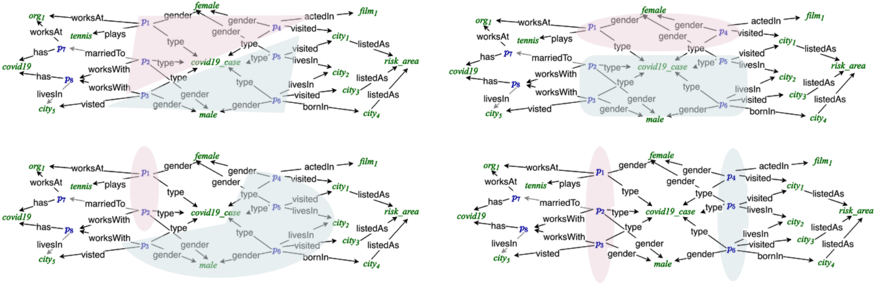
Huge Knowledge Graphs ⇨ Hard to Explore ⇨ Requires Summarization
E.g. Clustering



Which is the best division for $T=\{p_1 \ldots p_6\}$?

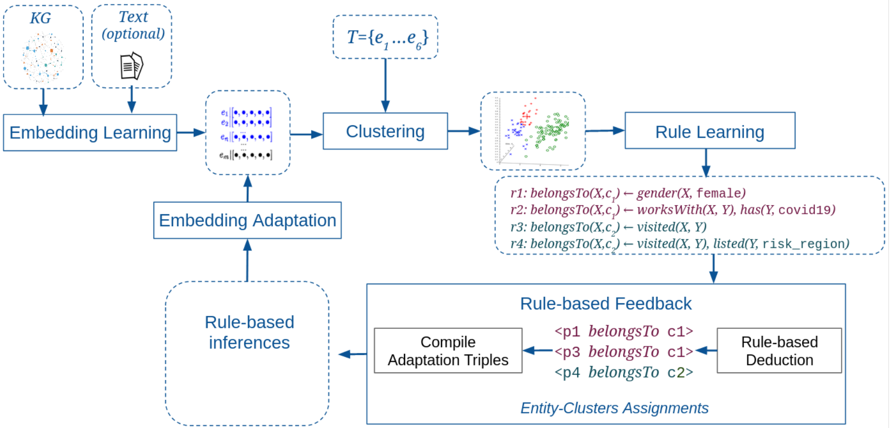
Explainable Clustering

Huge Knowledge Graphs → Hard to Explore → Requires Summarization *E.g. Clustering*



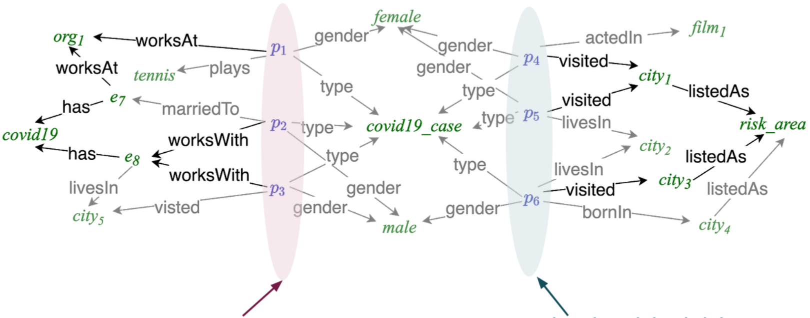
Which is the best division for $T=\{p_1 \dots p_6\}$?

Explainable Clustering



M. Gad-Elrab, D. Stepanova, T. Kien Trung, H. Adel, G. Weikum: Explainable Embedding-based Clustering in KGs. ISWC 2020.
`https://github.com/mhmgad/ExCut`

Explainable Clustering



"People got infected from coworkers"

r1: belongsTo(X,C$_1$) ← worksWith(X, Y), has(Y, `covid19`*)*

"People who visited risk area"

r3: belongsTo(X,C$_2$) ← visited(X, Y), listed(Y, `risk_region`*)*

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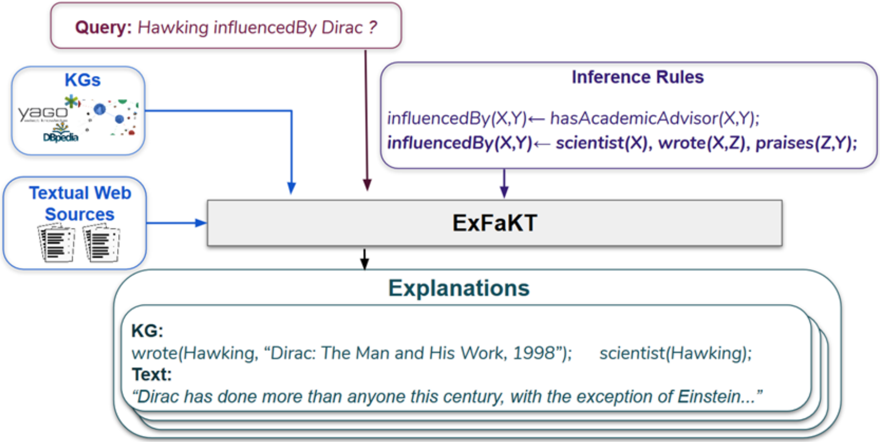
`https://github.com/mhmgad/ExCut`

Rule-based Fact Checking



M. Gad-Elrab, D. Stepanova, J. Urbani, G. Weikum. ExFakt: A Framework for Explaining Facts over KGs and Text. WSDM 2019.
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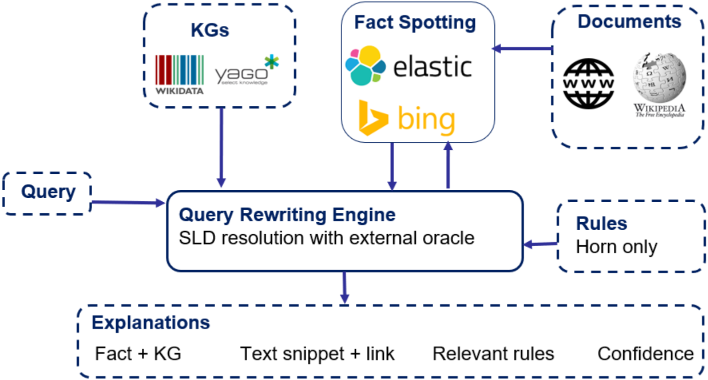
`https://github.com/mhmgad/ExFaKT`

Rule-based Fact Checking



M. Gad-Elrab, D. Stepanova, J. Urbani, G. Weikum. ExFakt: A Framework for Explaining Facts over KGs and Text. WSDM 2019.
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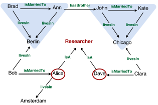
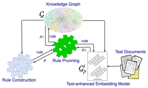
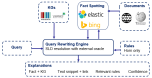
`https://github.com/mhmgad/ExFaKT`

Summary

- Rule learning is a helpful tool for knowledge graph completion, summarization, analytics
- Exploiting KG embedding models to guide rule learning is beneficial
- Rules can be learned from KGs that contain numerical values
- Applications:
  - Explainable clustering
  - Rule-based fact checking







Outlook

  - Learning rules from text, images, etc.
  - Make use of rules for explaining ML models
  - Rules for KG cleaning...

For internship opportunities please contact me at daria.stepanova@de.bosch.com

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