Task 1

1) $A = \begin{pmatrix} 3 & -5 \\ -2 & 7 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -3 & 4 \\ -5 & 6 & 7 \\ -8 & 9 & 1 \end{pmatrix}$

$A^t = \begin{pmatrix} 3 & -2 \\ -5 & 7 \end{pmatrix}$ $B^t = \begin{pmatrix} 2 & -5 & -8 \\ -3 & 6 & 9 \\ 4 & 7 & 1 \end{pmatrix}$

2) a) $\begin{pmatrix} 3 & 2 & -1 \\ -2 & 7 & 4 \\ 1 & 6 & 8 \end{pmatrix} \odot \begin{pmatrix} 2 & -3 & -4 \\ -5 & -6 & 7 \\ -8 & 9 & 1 \end{pmatrix} = \begin{pmatrix} 6 & -6 & 4 \\ 10 & -42 & 28 \\ -8 & 54 & 8 \end{pmatrix}$

b) $\begin{pmatrix} 3 & 2 & -1 \\ -2 & 7 & 4 \\ 1 & 6 & 8 \end{pmatrix} \begin{pmatrix} 2 & -3 & -4 \\ -5 & -6 & 7 \\ -8 & 9 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -30 & 1 \\ -71 & 0 & 61 \\ -92 & -33 & 46 \end{pmatrix}$

$6 + (-10) + 8 = 4$
$-9 + (-12) - 9 = -30$
$-12 + 14 - 1 = 1$
$-4 - 35 - 32 = -71$
$6 - 42 + 36 = 0$

$8 + 49 + 4 = 61$
$2 - 30 - 64 = -92$
$-3 - 36 + 72 = -33$
$-4 + 42 + 8 = 46$

56 - 19 = 37

-24 -18 1 = -31

-16 63 6

c) One conclusion that could be drawn is that the matrix product has higher numbers than the element-wise product.

d) $\begin{pmatrix} 2 & -3 & -4 \\ -5 & -6 & 7 \\ -8 & 9 & 1 \end{pmatrix} \odot \begin{pmatrix} 3 & 2 & -1 \\ -2 & 7 & 4 \\ 1 & 6 & 8 \end{pmatrix} = \begin{pmatrix} 6 & -6 & 4 \\ 10 & -42 & 28 \\ -8 & 54 & 8 \end{pmatrix}$

$\begin{pmatrix} 2 & -3 & -4 \\ -5 & -6 & 7 \\ -8 & 9 & 1 \end{pmatrix} \begin{pmatrix} 3 & 2 & -1 \\ -2 & 7 & 4 \\ 1 & 6 & 8 \end{pmatrix} = \begin{pmatrix} 8 & -41 & -46 \\ 4 & -10 & 37 \\ -41 & 53 & 52 \end{pmatrix}$

e) the element-wise product remains the same, meaning that it is a more flexibile ~~a~~ operation. (commutative)

3) a) ~~det (A)~~ $= \cancel{168 + 8 + 12 - 12} + 7$

$\det(A) = 168 + 8 + 12 + 7 + 32 - 72 =$

$= 168 + 59 - 72 =$

$= 227 - 72 =$

$= 155$

~~det (C) = (1·6·8) + (4·3·7) + (2·5·12) − (4·6·5) − (2·3·8) − (1·12·7) = 48 + 84 + 120 − 120 − 48 − 84~~

$\det(B) = 168 + 12 + 8 + 7 + 32 - 72 = 155$

$\det(C) = 48 + 84 + 120 - \cancel{120}\, 48 - 84 = 0$

b) $\left.\begin{matrix}\det(A) = \det(A^t) \\ A = B^t\end{matrix}\right\} \Rightarrow \det(A) = \det(B)$

c) $\Rightarrow$ The second column is the first column $\times 2$ $\Rightarrow$ The determinant is 0 $\Rightarrow$ The matrix is not invertible

4) a) $A^{-1} = \dfrac{1}{\det(A)} A^t = \dfrac{1}{25} \cdot \begin{pmatrix} 3 & +2 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} \frac{3}{25} & \frac{+2}{25} \\ \frac{-2}{25} & \frac{7}{25} \end{pmatrix}$

$\det(A) = 21 + 4 = 25$

$B^{-1} = \dfrac{1}{\det(B)} \cdot B^t = \dfrac{1}{20} \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$

$\det(B) = 20$

b) $\begin{pmatrix} \frac{3}{25} & \frac{+2}{25} \\ \frac{-2}{25} & \frac{7}{25} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{3}{125} & \frac{1}{50} \\ \frac{-2}{125} & \frac{7}{100} \end{pmatrix}$

$\frac{3}{25} \cdot \frac{1}{5} = \frac{3}{125}$

$\frac{-2}{25} \cdot \frac{1}{4} = \frac{-1}{50}$

$B \cdot A = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 3 & +2 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 12 & -8 \\ 10 & 35 \end{pmatrix}$

$$\det(B \cdot A) = 35 \cdot 12 + 80 = 5(7 \cdot 12 + 16) = 5(84+16) = 5 \cdot 100$$

$$= \cancel{480}\ 500$$

$$(B \cdot A)^{-1} = \frac{1}{\det(B \cdot A)} \cdot (B \cdot A)^t = \frac{1}{500} \cdot \begin{pmatrix} 12 & 10 \\ -8 & 35 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{12}{500} & \frac{10}{500} \\ \frac{-8}{500} & \frac{35}{500} \end{pmatrix} = \begin{pmatrix} \frac{3}{125} & \frac{1}{50} \\ \frac{-2}{125} & \frac{7}{100} \end{pmatrix}$$

c) - $B \cdot B^{-1} = I_2$

- They are diagonal matrices