# The cable project

Elise Niedringhaus, Sarah Liddle and Darice Guittet

December 4, 2017

## 1 2

### 1.1 2a

### 1.2 2b

Boundary conditions:

$$\lim_{x \to \infty} \frac{\partial v(x,t)}{\partial x} = 0$$

and

$$\lim_{x \to \infty} \frac{\partial v(x,t)}{\partial x} = 0$$

Application of the change of variables $v(x,t) = V(\xi)$, where $\xi = x - ct$ to $\frac{\partial v(x,t)}{\partial t} = \frac{\partial^2 v(x,t)}{\partial x^2} - v(x,t) + H(v - \theta) + J_{ext}(x,t)$ yields the following second order ordinary differential equation:

$$-cV'(\xi) = V''(\xi) - V(\xi) + H(V(\xi) - \theta) + J_{ext}(x,t) \tag{1}$$

### 1.3 2c

The two ordinary differential equations are:

$$-cV_1'(\xi) = V_1''(\xi) - V_1(\xi) \tag{2}$$

for $\xi \in (0, \infty)$ and

$$-cV_2'(\xi) = V_2''(\xi) - V_2(\xi) + 1 \tag{3}$$

for $\xi \in (-\infty, 0)$

The boundary conditions are

$$\lim_{\xi \to -\infty} \frac{dV_1(\xi)}{d\xi} = 0$$

$$\lim_{\xi \to \infty} \frac{dV_2(\xi)}{d\xi} = 0$$

$$\lim_{\xi \to -\infty} V_1(\xi) = 1$$

$$\lim_{\xi \to \infty} V_2(\xi) = 0$$

$$V_1(0) = V_2(0)$$

$$\frac{dV_1}{d\xi}(0) = \frac{dV_2}{d\xi}(0)$$

### 1.4 2d

Equation 2 is a homogeneous ordinary differential equation. The corresponding characteristic equation, $r^2 + cr - 1 = 0$, has roots of $r = \frac{-c \pm \sqrt{c^2+4}}{2}$. Thus,

$$V_1 = c_1 e^{\frac{1}{2}(-c+\sqrt{c^2+4})\xi} + c_2 e^{-\frac{1}{2}(c+\sqrt{c^2+4})\xi}, \xi \in (0, \infty) \tag{4}$$

In order to solve equation 3, use the method of undetermined coefficients. First, determine the homogeneous solution of the differential equation by solving $cV_2'(\xi) = V_2''(\xi) - V_2(\xi)$ for $V_2$. The characteristic equation, $r^2 + cr - 1 = 0$, has roots of $r = \frac{-c \pm \sqrt{c^2+4}}{2}$. Thus, the homogeneous solution is

$$V_{2,h} = c_3 e^{\frac{1}{2}(-c+\sqrt{c^2+4})\xi} + c_4 e^{-\frac{1}{2}(c+\sqrt{c^2+4})\xi} \tag{5}$$

Guess a particular solution of the form $V_{2,p} = A$. Plugging $V_{2,p}$ into equation 3 yields

$$-c(0) = 0 - A + 1$$

$$A = 1$$

$$V_{2,p} = 1$$

Thus, the solution to equation 3 is

$$V_2 = c_3 e^{\frac{1}{2}(-c+\sqrt{c^2+4})\xi} + c_4 e^{-\frac{1}{2}(c+\sqrt{c^2+4})\xi} + 1, \xi \in (-\infty, 0) \tag{6}$$

Next, use the boundary conditions to eliminate the arbitrary coefficients.

$$\lim_{\xi \to \infty} \frac{dV_1(\xi)}{d\xi} = 0 \Rightarrow c_1 = 0$$

$$\lim_{\xi \to -\infty} \frac{dV_2(\xi)}{d\xi} = 0 \Rightarrow c_4 = 0$$

$$V_1(0) = V_2(0) \Rightarrow c_2 = c_3 + 1$$

$$\frac{dV_1}{d\xi}(0) = \frac{dV_2}{d\xi}(0) \Rightarrow -\frac{1}{2}(c + \sqrt{c^2+4})c_2 = \frac{1}{2}(-c + \sqrt{c^2+4})c_3$$

The solution to this linear system of equations is $c_1 = 0, c_2 = \frac{\frac{1}{2}(-c+\sqrt{c^2+4})}{\frac{1}{2}(-c+\sqrt{c^2+4})+\frac{1}{2}(c+\sqrt{c^2+4})} = \frac{-c+\sqrt{c^2+4}}{2\sqrt{c^2+4}}, c_3 = \frac{-\frac{1}{2}(c+\sqrt{c^2+4})}{\frac{1}{2}(-c+\sqrt{c^2+4})+\frac{1}{2}(c+\sqrt{c^2+4})} = \frac{-c-\sqrt{c^2+4}}{2\sqrt{c^2+4}}$, and $c_4 = 0$.
The solution to equation 2 is

$$V_1(\xi) = \frac{-c + \sqrt{c^2+4}}{2\sqrt{c^2+4}} e^{-\frac{1}{2}(c+\sqrt{c^2+4})\xi}, \xi \in (0, \infty). \tag{7}$$

The solution to equation 3 is

$$V_2 = \frac{-c - \sqrt{c^2+4}}{2\sqrt{c^2+4}} e^{\frac{1}{2}(-c+\sqrt{c^2+4})\xi} + 1, \xi \in (-\infty, 0). \tag{8}$$