Propagation of Voltage in a Neuron

The Cable Equation

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Overview

1. Motivation
2. Neuronal Cable Equation
3. Passive Membrane (Linear Cable Equation)
4. Bi-stable Ion Channels

How Do Neurons Communicate?

Within one cell

- Electrochemical signals
- Membrane Potential: $\Delta V_m = V_i - V_e$
- Ions: charge-carriers
- Ion Channels in Membrane

Between cells

- Neurotransmitters

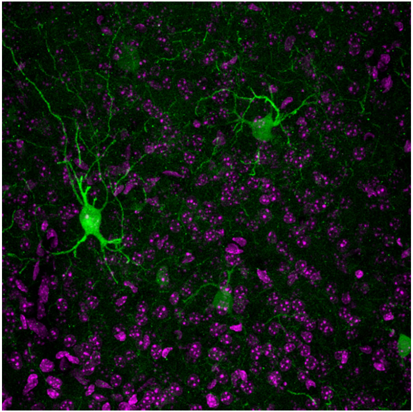

Figure: Mouse neurons, 40X. Bosch Institute Advanced Microscopy Facility, The University of Sydney

Action Potentials

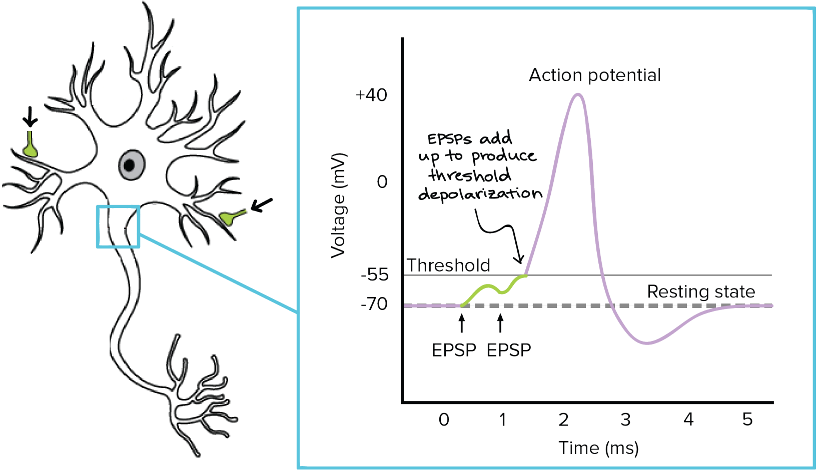

Figure: Changes in axonal membrane voltage due to an action potential. Image from Khan Academy

Hodgkin-Huxley's Neuronal Cable Model

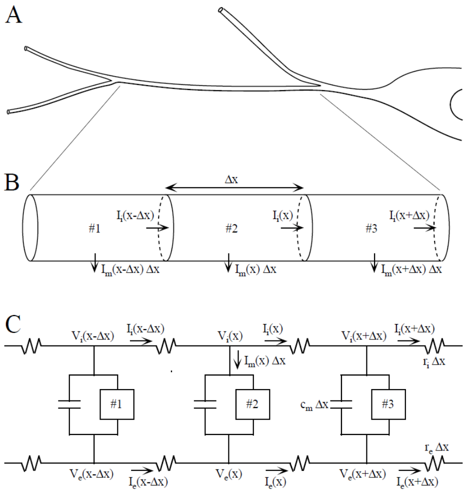

Figure: Differential membrane patches as circuit. Image from jh.edu/motn

- 1-D & Ohmic assumption
- Intracellular current
- Extracellular current
- Membrane current
- Membrane as capacitor
- Ion channels as conductances
- Length Constant:
  $$\lambda = \sqrt{\frac{r_m}{r_i + r_e}}$$
- Time Constant: $r_m C_m$

Cable Equation

$$\frac{\partial v(x,t)}{\partial t} = \frac{\partial^2 v(x,t)}{\partial x^2} + f(v(x,t)) + J_{ext}(x,t)$$

- $\frac{\partial^2 v(x,t)}{\partial x^2}$ represents current coming in from adjacent segments

Passive Membrane

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Green's Functions

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Numerical Solutions

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Traveling Wave Solutions

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Speed of Traveling Wave

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Stability of Traveling Wave

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Numerical Solutions for Traveling Wave

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Conclusion

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