Propagation of Voltage in a Neuron

The Cable Equation

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Overview

1. Motivation
2. Neuronal Cable Equation
3. Passive Membrane (Linear Cable Equation)
4. Bi-stable Ion Channels

How Do Neurons Communicate?

Within one cell

- Electrochemical signals
- Membrane Potential:
  $\Delta V_m = V_i - V_e$
- Ions: charge-carriers
- Ion Channels in Membrane

Between cells

- Neurotransmitters

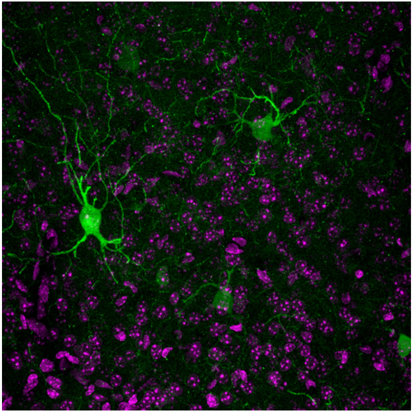

Figure: Mouse neurons, 40X. Bosch Institute Advanced Microscopy Facility, The University of Sydney

Action Potentials

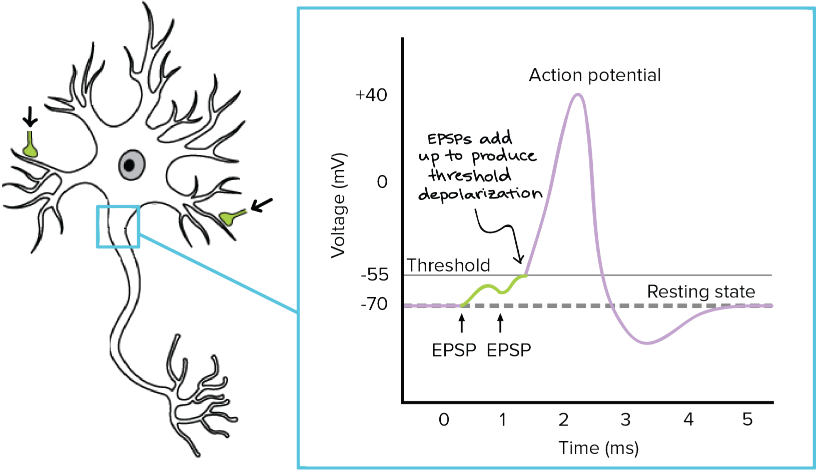

Figure: Changes in axonal membrane voltage due to an action potential. Image from Khan Academy

Hodgkin-Huxley's Neuronal Cable Model

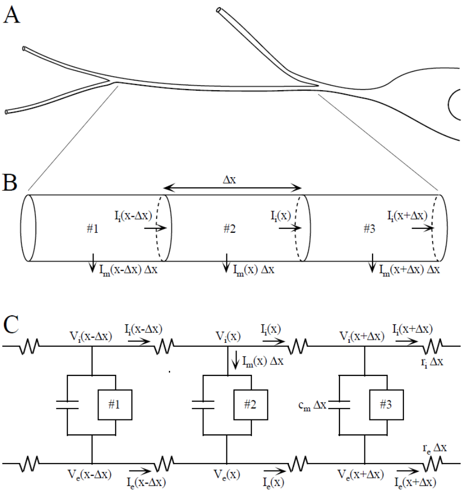

Figure: Differential membrane patches as circuit. Image from jh.edu/motn

- 1-D & Ohmic assumption
- Intracellular current
- Extracellular current
- Membrane current
- Membrane as capacitor
- Ion channels as conductances
- Length Constant: $\lambda = \sqrt{\frac{r_m}{r_i + r_e}}$
- Time Constant: $r_m C_m$

Passive Membrane

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Green's Functions

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Numerical Solutions

- Finite Difference Method
- Plugging approximations for the partial derivatives into the original PDE gives:

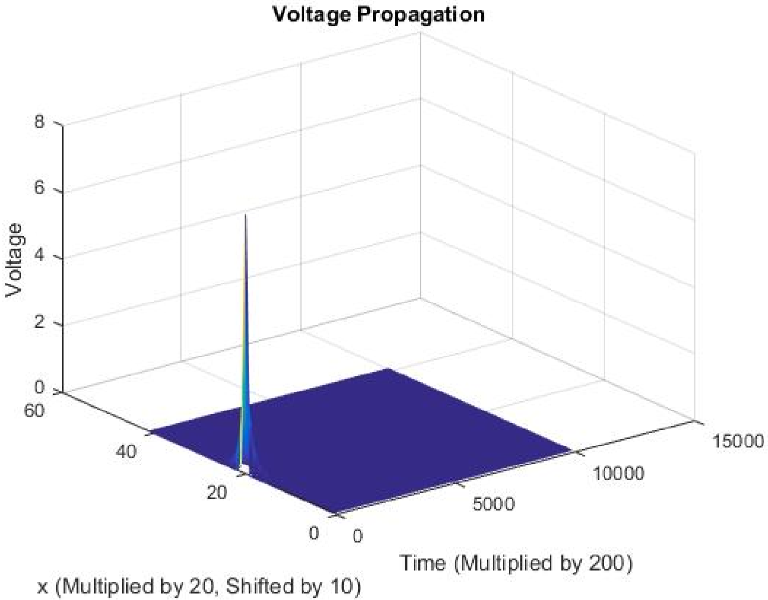
$$\frac{v_i^{j+1} - v_i^j}{\Delta t} = \frac{v_{i+1}^j - 2v_i^j + v_{i-1}^j}{(\Delta x)^2} - v_i^j + J_{ext}(x,t)$$

- Solving for the unknown term:

$$v_i^{j+1} = \frac{\Delta t}{(\Delta x)^2}(v_{i+1}^j + v_{i-1}^j) + (1 - \frac{\Delta t(2 + (\Delta x)^2)}{(\Delta x)^2}) v_i^j + \Delta t * J_{ext}(x,t)$$

- Stability

Numerical Solutions Results



Traveling Wave Solutions

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Speed of Traveling Wave

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Stability of Traveling Wave

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Numerical Solutions for Traveling Wave

- Finite Difference Method–Similar to Previous Numerical Solution
- Resultant Equation (solved for the unknown term):

$$v_i^{j+1} = \frac{\Delta t}{(\Delta x)^2}(v_{i+1}^j + v_{i-1}^j) + (1 - \frac{\Delta t(2 + (\Delta x)^2)}{(\Delta x)^2})v_i^j + \Delta t * H(v_i^j - \theta)$$

Resuts

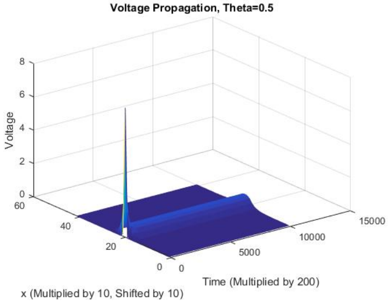

Figure: $\theta = 0.5$

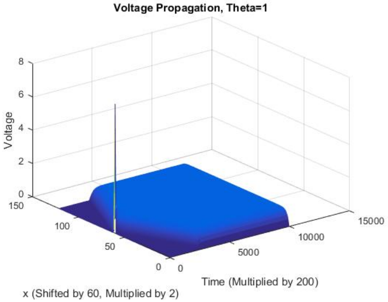

Figure: $\theta = 0.1$

- Numerically Solving for Speeds of the Waves
- Comparison with Analytical Results

Periodically Varying Threshold, Numerical Methods

- What is a periodically varying threshold?
- Governing Equation:

$$\frac{\partial v(x,t)}{\partial t} = \frac{\partial^2 v(x,t)}{\partial x^2} - v + H(v - \theta(1 + 0.5\cos(x))) + J_{ext}(x,t)$$

- Numerical Solution:

$$v_i^{j+1} = \frac{\Delta t}{(\Delta x)^2}(v_{i+1}^j + v_{i-1}^j) + (1 - \frac{\Delta t(2 + (\Delta x)^2)}{(\Delta x)^2})v_i^j$$

$$+\Delta t * H(v_i^j - \theta(1 + C\cos(x)))$$

Results

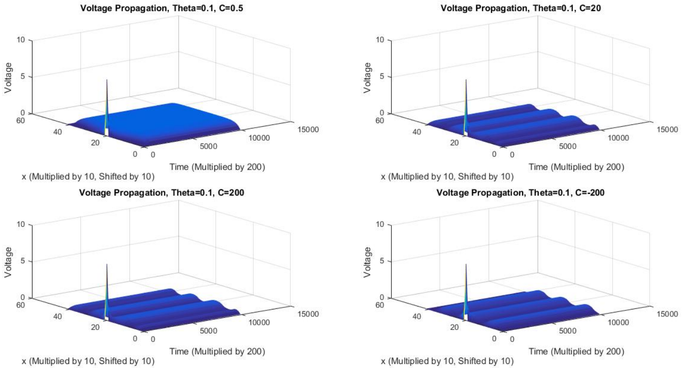

Figure: Varying C, $\theta = 0.1$

Conclusion

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