

# Propagation of Voltage in a Neuron

## The Cable Equation

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# Overview

1. Motivation
2. Neuronal Cable Equation
3. Passive Membrane
4. Bi-stable Ion Channels

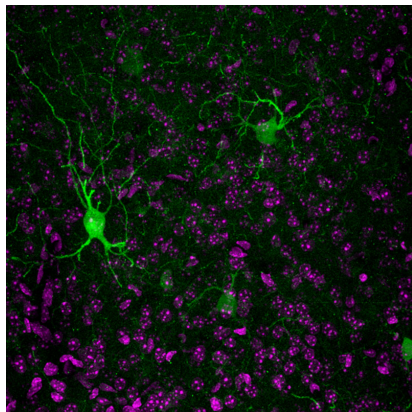
# How Do Neurons Communicate?

## Within one cell

- ▶ Electrochemical signals
- ▶ Membrane Potential:  
 $\Delta V_m = V_i - V_e$
- ▶ Ions: charge-carriers
- ▶ Ion Channels in Membrane

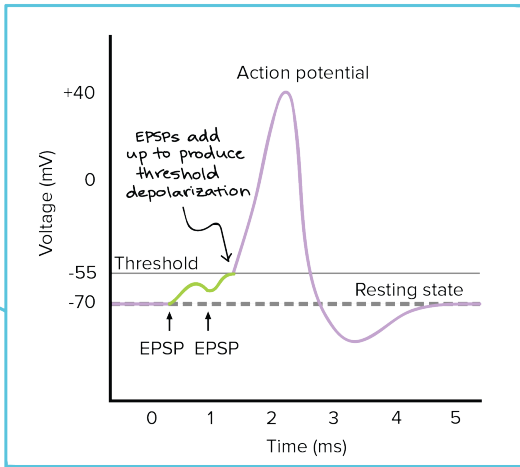
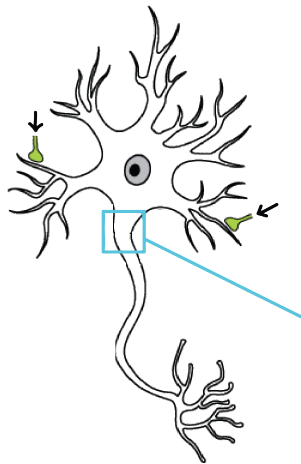
## Between cells

- ▶ Neurotransmitters



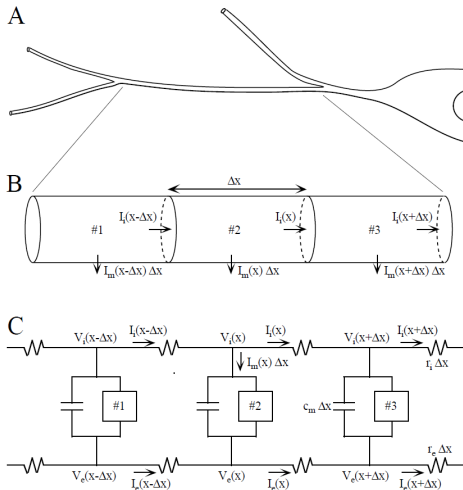
**Figure:** Mouse neurons, 40X. Bosch Institute Advanced Microscopy Facility, The University of Sydney

# Action Potentials



**Figure:** Changes in axonal membrane voltage due to an action potential.  
Image from Khan Academy

# Hodgkin-Huxley's Neuronal Cable Model



- ▶ 1-D & Ohmic assumption
- ▶ Intracellular current
- ▶ Extracellular current
- ▶ Membrane current
- ▶ Membrane as capacitor
- ▶ Ion channels as conductances
- ▶ Length Constant:
 
$$\lambda = \sqrt{\frac{r_m}{r_i + r_e}}$$
- ▶ Time Constant:  $r_m C_m$

**Figure:** Differential membrane patches as circuit. Image from [jh.edu/motn](http://jh.edu/motn)

# Passive Membrane

Linear Cable Equation with Impulse Current Injection:

$$\frac{\partial v(x, t)}{\partial t} - \frac{\partial^2 v(x, t)}{\partial x^2} + v(x, t) = \delta(t - t_0)\delta(x - x_0)$$

$$x \in (-\infty, \infty), \quad t \in (0, \infty), \quad x_0 = 0, \quad t_0 = 0$$

Boundary Conditions:

$$\lim_{x \rightarrow \pm\infty} v(x, t) = 0$$

# Green's Function for Linear Infinite Cable Equation

Fourier transform  $\rightarrow$  2 ODE's  $\rightarrow$  Inverse Fourier transform

$$v(x, t) = \frac{\mathcal{H}(x, t)}{\sqrt{4\pi t}} e^{-t - \frac{x^2}{4t}}$$

$$G_{\infty}(x - x_0, t - t_0) = \frac{\mathcal{H}(x - x_0, t - t_0)}{\sqrt{4\pi(t - t_0)}} e^{-(t - t_0) - \frac{(x - x_0)^2}{4(t - t_0)}}$$

Fundamental Solution for Arbitrary Forcing Term,  $J_{\text{ext}}(x, t)$ :

$$\hat{v}(x, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_{\infty}(x - x_0, t - t_0) J_{\text{ext}}(x_0, t_0) dx_0 dt_0$$

## Multiple Current Injections:

$$(x^*, t^*, A^*) = (1, 0.3, 0.3), (10, 1.1, 1), (30, 0, 0.5)$$



# animation of traveling wave solution

# animation of perturbed wave solution

# Numerical Solutions

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# Traveling Wave Solutions

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# Speed of Traveling Wave

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# Stability of Traveling Wave

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# Numerical Solutions for Traveling Wave

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# Conclusion

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