Referee Report

Title: Multiline queues with spectral parameters, by E. Aas, D. Grinberg and T. Scrimshaw

The authors study multiline queues by relating them to various combinatorial objects such as multispecies exclusion processes, pseudo-partitions, lattice paths and tableaux.

In the last twenty years, there has been a very large amount of articles that explored stochastic interacting particle processes using very different techniques (such as the Bethe Ansatz, operator algebras, queuing algorithms, KZ equations, Askey-Wilson polynomials...). The present manuscript appears to me as a culminating point in which the combinatorial essence of the multispecies exclusion process is unveiled and the number of free parameters is taken to its maximum, allowing the authors to prove various conjectures and to give a bird's eye view of a whole class of problems.

The manuscript is written in a very clear manner. Step by step, the authors introduce new objects through formal definitions. These definitions are illustrated by well-chosen examples and by various diagrams. They are also shown to be 'minimal'. The proofs are written very carefully and are sound. In the end, the whole construction elaborated by the authors leads almost naturally to theorems 3.1 and 3.9, which are, in my opinion, the principal results of the paper.

The strategy followed by the authors is extremely powerful by its generality and by its synthetic character. The results obtained in the manuscript are very impressive and are likely to have a strong impact on the community. To my taste, the link between combinatorics and integrability could have been explored in more details, or at least deserves future research.