Spin-Orbit Interaction (SOI)

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Motivation

Although the SOI is a weak interaction (larger for heavier atoms), it is important around band degeneracies (at the Fermi level) and in low energy limits. This has a few consequences (Galitski):

1. SOI is essential to several phenomena such as topological insulators.
2. SOI increases the energy scale at which quantum effects play a large role (i.e., no need for large magnetic fields or ultracold settings).

Toy-Model

Tight-binding (non-interacting) model on **square lattice**:

$$\hat{H}_{TB} = -t\sum_{\langle i,j\rangle} c^{\dagger}_{i,\sigma} R^{\sigma,\sigma'}_{i,j} c_{j,\sigma'} + h.c.$$

with **Rashba spin-orbit coupling**

$$R^{\hat{x}}_{i,j} = \cos\alpha \pm i\sin\alpha\sigma_y \quad \text{(Zhang)}$$

Fourier Transform to obtain

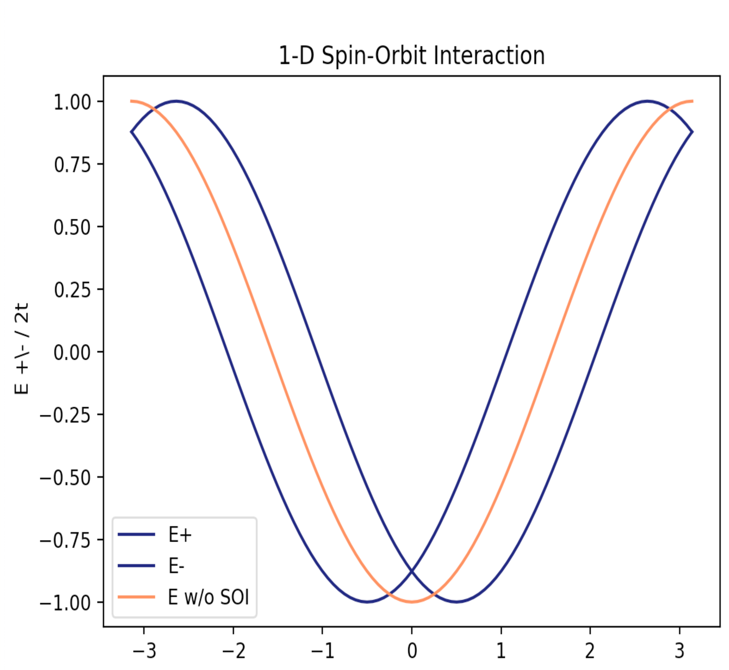
$$\sum_k \psi^{\dagger}_k H(k)\psi_k$$

H(k) can be written as a linear combination of Pauli-matrices

$$H(k) = d_0 + \boldsymbol{d}(\vec{\boldsymbol{k}}) \rightarrow E_{\pm}(k) = d_0 \pm |\boldsymbol{d}(\vec{\boldsymbol{k}})|$$

<u>1-D Chain</u>

$$-2t\cos(k\pm\alpha)$$

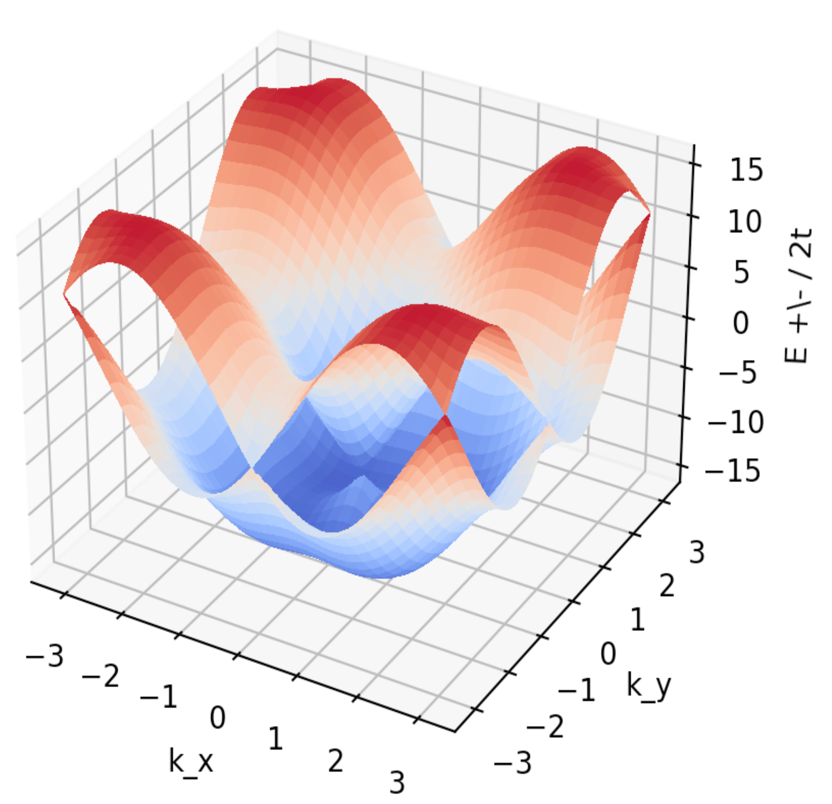


<u>2-D Square Lattice</u>

$$d_0(\boldsymbol{k}) = -2t(cos\alpha cosk_x + cos\beta cosk_y)$$

$$\boldsymbol{d}(\vec{\boldsymbol{k}}) = -2t(sin\beta sink_y, sin\alpha sink_x, 0)$$

2D Spin-Orbit Coupling



Spin texture from eigenvectors:

$$\chi_{\pm}(\boldsymbol{k}) = \begin{bmatrix} 1 \\ \mp i e^{i\phi_k} \end{bmatrix}, \phi_k = arctan(d_x/d_y)$$

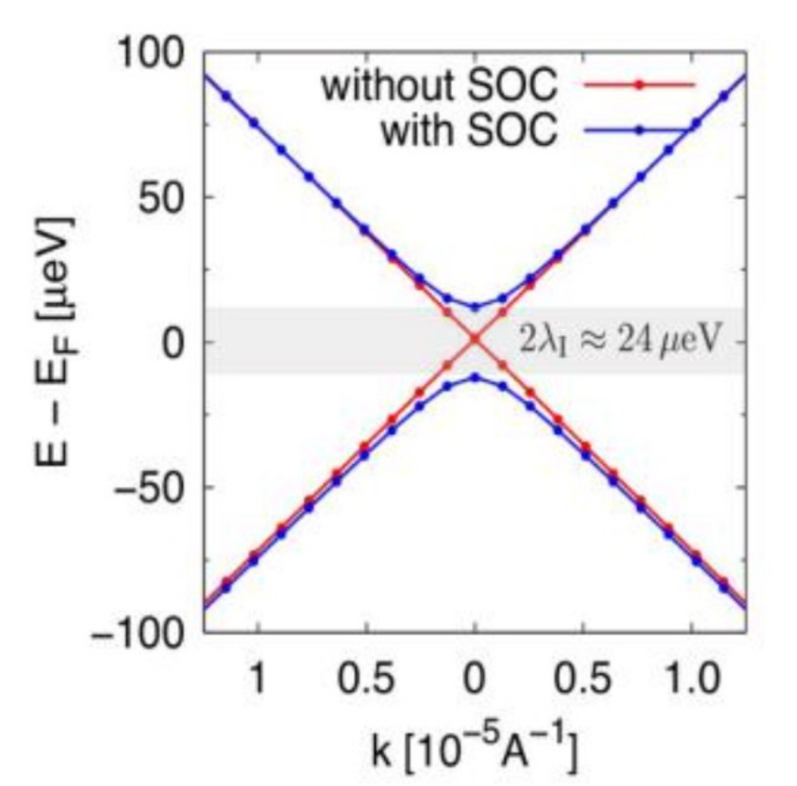
Theory

Rashba spin-orbit coupling (RSOI) couples spin degree of freedom with orbital motion. The electric field of the nucleus is a magnetic field in the comoving frame for the electron (Zeeman effect), which leads to the momentum-dependent Zeeman interaction (Galitski)

$$\mu B = \sigma_x k_y - \sigma_y k_x$$

The SOI splits degeneracies as it does in single-atom energy levels. Now, one band sits above and the other below.

Comparing with graphene:

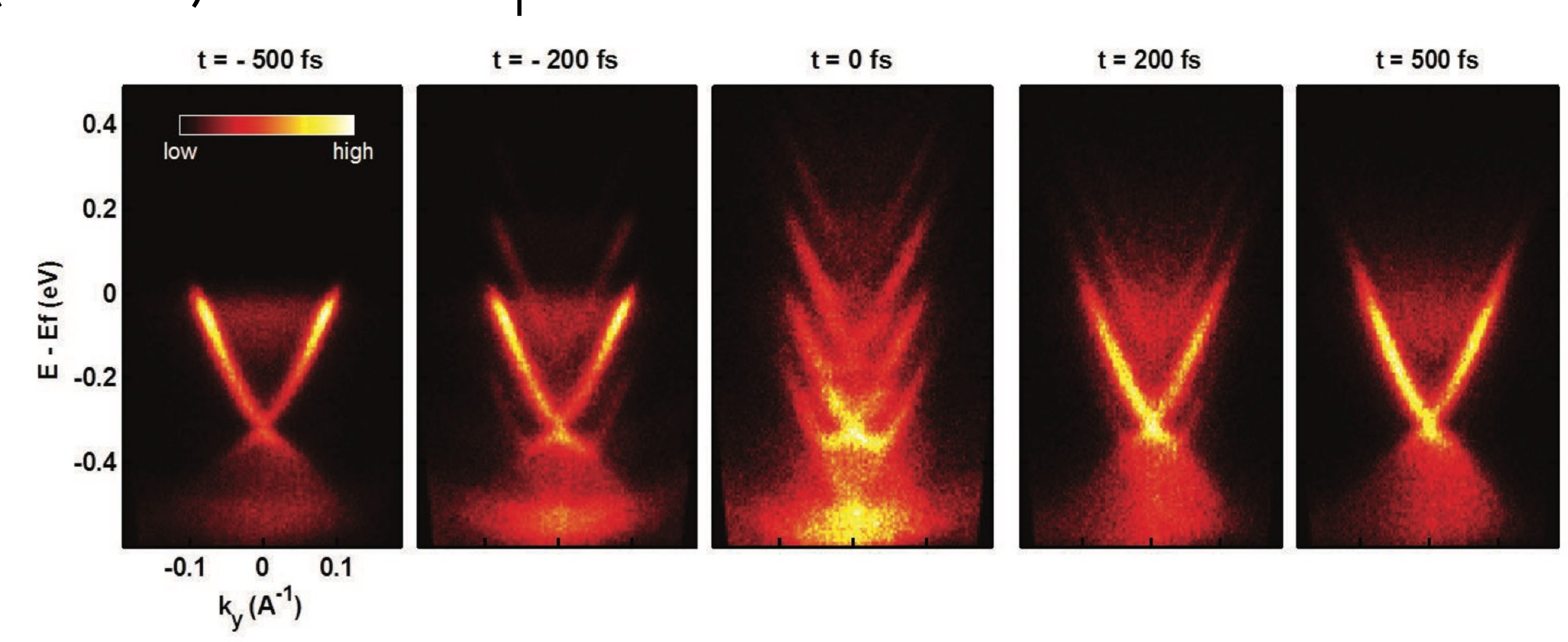


Gmitra et al., PRB **80** 235431 (2009)

- Changes topology and introduces band gap near the Fermi level (topologically protected)
- In 2-D the relation between the energy of topological electrons and their momentum form a Dirac cone (compare with result from 2-D square lattice), typical of topological insulator

Experimental Aspects

Energy vs. momentum graph of topological electrons in 2-D (two directions of momentum) forms a Dirac cone whose crossing is protected by time reversal symmetry (Ashoori). Experimentally Angle-Resolved Photoemission Spectra (ARPES) is used to probe band structure.



Throughout the excitation due to the pulses the crossing at the tip of the Dirac cone is conserved, which is a sign of time reversal symmetry (Ashoori).
SOI is important near the Fermi level, so exploring physics in this regime doesn't necessarily necessitate an external magnetic field, such as when measuring the Quantum Spin Hall Effect (Ashoori).

References

Ashoori , Ray, et al. “Graphene and Topological Insulators Meet the Novel Materials Pushing the Physics Frontier.” *MIT Physics Annual 2015*, 2015.
Gmitra, M., et al. “Band-Structure Topologies of Graphene: Spin-Orbit Coupling Effects from First Principles.” *Physical Review B*, vol. 80, no. 23, 2009, https://doi.org/10.1103/physrevb.80.235431.
Zhang, Shizhong, et al. “Spin-Orbit Coupling in Optical Lattices.” *Annual Review of Cold Atoms and Molecules*, 2015, pp. 135–179., https://doi.org/10.1142/9789814667746_0003.
Galitski, Victor, and Ian B. Spielman. “Spin–Orbit Coupling in Quantum Gases.” *Nature*, vol. 494, no. 7435, 2013, pp. 49–54., https://doi.org/10.1038/nature11841.