A modified proximity approach in the fusion of heavy-ions

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By using a suitable set of the surface energy coefficient, nuclear radius, and universal function, the original proximity potential 1977 is modified. The overestimate of the data by 4% reported in the literature is significantly reduced. Our modified proximity potential reproduces the experimental data nicely compared to its older versions.

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I. INTRODUCTION

Recently, great theoretical and experimental efforts are taken to studying the fusion of heavy nuclei leading to several new phenomena including the understanding of the formation of neutron -rich and super heavy elements [1, 2]. The precise knowledge of the interaction potential between two nuclei is a difficult task and continuing efforts are needed in this direction. This problem has been of very active research over the last three decades and remains one of the most widely studied subject in low-energy heavy-ion physics [1–10].

The total interaction potential is sum of the long range Coulomb repulsive force and short range nuclear attractive force. The Coulomb part of the interaction potential is well-known, whereas nuclear part is not clearly understood. A large number of efforts have been made to giving simple and accurate forms of the nuclear interaction potentials [1–10]. Among such efforts, proximity potential is well known for its simplicity and numerous applications. Based upon the proximity force theorem [4, 5], a simple formula for ion-ion interaction potential as a function of the separation between the surfaces of two approaching nuclei was presented [4, 5].

As pointed out by many authors [7], original form of the proximity potential 1977 overestimates the experimental data by 4% for fusion barrier heights. In a recent study involving the comparison of 16 proximity potentials, one of us and collaborators pointed out that proximity potential 1977 overestimates the experimental data by 6.7% for symmetric colliding nuclei [1]. Similar results were obtained for asymmetric colliding nuclei [1].

With the passage of time, several improvement/ modifications were made over the original proximity potential 1977 to remove the gray part of the potential. It includes either the better form of the surface energy coefficient [6] or the universal function and/or nuclear radius [7]. A careful look reveals that these modifications/improvements are not able to explain the experimental data [1, 8]. A deep survey also pointed out that these technical parameters (i.e. surface energy coefficient, nuclear radius, and universal function) were chosen quite arbitrarily in the literature. Among them, the surface energy coefficient is available in a large variety of forms from time to time [1, 2]. It affects the fusion barrier heights and cross sections significantly [1, 2]. Also, nuclear radius is available in large variety of forms [1, 2]. These forms varies either in terms of its coefficients or either different mass or isospin dependence. The third technical parameter i.e, the universal function, is also parametrized in different forms [1, 4, 5, 7]. Unfortunately, no systematic study is available in the literature, where one can explore the role of these technical parameters in fusion barrier positions, heights, and cross sections. Alternatively, a best set of the above-mentioned parameters is still missing.

In the present study, our aim is to modify the original proximity potential 1977 by using a suitable set of the above-stated technical parameters available in the literature. In addition, to compare the final outcome with the huge amount of experimental data available since last three decades. The choice of the potential and its form to be adopted is one of the most challenging task when one wants to compare the experimental data with theory. The present systematic study includes the reactions with combine mass between A = 19 and A = 294 units. In total, 390 experimentally studied reactions with symmetric as well as asymmetric colliding partners are taken into consideration. Section II describes the Model in brief, Section III depicts the Results and Summary is presented in Section IV.

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II. THE MODEL

The total ion-ion interaction potential $V_T(r)$ between two colliding nuclei with charges Z_1 and Z_2, center separation r, and density distribution assumed spherical, and frozen, is approximated as [7]

$$V_T(r) = V_N(r) + \frac{Z_1 Z_2 e^2}{r}, \tag{1}$$

where e is the charge unit. The above form of the Coulomb potential is suitable when two approaching nuclei are well separated. The nuclear part of the potential $V_N(r)$ is calculated in the framework of the proximity potential 1977 [4] as

$$V_N\left(r\right) = 4\pi\overline{R}\gamma b\Phi\left(\frac{r - C_1 - C_2}{b}\right) \text{ MeV}, \tag{2}$$

where $\overline{R} = \frac{C_1 C_2}{C_1 + C_2}$ is the reduced radius. Here C_i denotes the matter radius and is calculated using relation [7]

$$C_i = c_i + \frac{N_i}{A_i} t_i \quad (i = 1, 2), \tag{3}$$

where c_i denotes the half-density radii of the charge distribution and t_i is the neutron skin of the nucleus. To calculate c_i, we used the relation given in Ref. [7] as

$$c_i = R_{00i}\left(1 - \frac{7}{2}\frac{b^2}{R_{00i}^2} - \frac{49}{8}\frac{b^4}{R_{00i}^4} + \cdots\cdots\right) \qquad (i = 1, 2). \tag{4}$$

Here, R_{00} is the nuclear charge radius read as

$$R_{00i} = 1.2332 A_i^{1/3}\left\{1 + \frac{2.348443}{A_i} - 0.151541\left(\frac{N_i - Z_i}{A_i}\right)\right\} \text{ fm}, \tag{5}$$

where N_i and Z_i refer to neutron and proton contents of target/projectile nuclei. This form of radius is taken from the recent work of Royer and Rousseau [11] and is obtained by analyzing as many as 2027 masses with N, Z ≥ 8 and a mass uncertainty ≤ 150 keV. The neutron skin t_i used in Eq. (3) is calculated according to Ref. [7].

The surface energy coefficient γ was taken from the work of Myers and Świątecki [12] and has the form

$$\gamma = \gamma_0\left[1 - k_s\left(\frac{N - Z}{A}\right)^2\right], \tag{6}$$

where N and Z refer to the total neutrons and protons content. It is clear from Eqs. (5) and (6) that both nuclear radius as well as surface energy coefficient depend on the relative neutron excess. In the above formula, γ_0 is the surface energy constant and k_s is the surface-asymmetry constant. Both constants were first parameterized by Myers and Świątecki [12] by fitting the experimental binding energies. The first set of these constants yielded values γ_0 and $k_s = 1.01734$ MeV/fm^2 and 1.79, respectively. In original proximity version, γ_0 and k_s were taken to be 0.9517 MeV/fm^2 and 1.7826 [13], respectively. Later on, these values were revised in a large variety of forms depending upon the advancement in the theory as well in experiments [1, 2]. In total, 14 such coefficients are highlighted in Ref. [2] and the role of extreme 4 sets is analyzed deeply. Out of them, two best sets of surface energy coefficients are stressed. In the present study, we shall restrict to the latest set of γ values i.e. $\gamma_0 = 1.25284$ MeV/fm^2 and $k_s = 2.345$ presented in Ref [2]. This particular set of values were obtained directly from a least-squares adjustment to the ground-state masses of 1654 nuclei ranging from ^{16}O to ^{263}106 and fission-barrier heights [14].

The universal function $\Phi(\frac{r - C_1 - C_2}{b})$ used in Eq. (1) has been derived by several authors in different forms [4, 5, 7]. In original proximity potential, $\Phi(\frac{r - C_1 - C_2}{b})$ was parametrized in the cubic-exponential form [4]

$$\Phi\left(\xi\right) = \begin{cases} -\frac{1}{2}\left(\xi - 2.54\right)^2 - 0.0852\left(\xi - 2.54\right)^3, & \text{for } \xi \leq 1.2511, \\ -3.437\exp\left(-\frac{\xi}{0.75}\right), & \text{for } \xi \geq 1.2511, \end{cases} \tag{7}$$

with $\xi = (r - C_1 - C_2)/b$. The surface width b (i.e. $b = \frac{\pi}{\sqrt{3}}a$ with $a = 0.55$ fm) has been evaluated close to unity. We labeled this universal function as Φ-1977.

Later on, Blocki et al., [5] modified the above form as

$$\Phi(\xi) = \begin{cases} -1.7817 + 0.9270\xi + 0.143\xi^2 - 0.09\xi^3, & \text{for } \xi \leq 0.0 \ , \\ -1.7817 + 0.9270\xi + 0.01696\xi^2 - 0.05148\xi^3, & \\ & \text{for } 0.0 \leq \xi \leq 1.9475 \ , \\ -4.41 \exp\left(-\frac{\xi}{0.7176}\right), & \text{for } \xi \geq 1.9475 \ . \end{cases} \tag{8}$$

In the present study, we use this form of universal function and marked it as Φ-1981. By using the above stated parameters, we construct a new proximity potential and labeled as Prox 2010. Along with the above modified form, we shall also use the original proximity potential 1977 [4] and its recently modified form proximity potential 2000 [7]. We labeled them as Prox 1977 and Prox 2000, respectively.

III. RESULTS AND DISCUSSIONS

By using the above new version of the proximity potential (Prox 2010) along with its older versions (i.e. Prox 1977 and Prox 2000), fusion barriers are calculated for 390 reactions by using the conditions:

$$\frac{dV_T(r)}{dr}|_{r=R_B} = 0, \quad \text{and} \quad \frac{d^2V_T(r)}{dr^2}|_{r=R_B} \leq 0. \tag{9}$$

The height of the barrier and position is marked, respectively, as V_B and R_B.

As one see from the preceding section, three factors govern the success of proximity potential are (i) the surface energy coefficient, (ii) the universal function, and (iii) nuclear radius. We analyzed the literature very carefully and found that the latest information on these three factors can shape the new proximity potential. Recently, the role of surface energy coefficient stated above is studied in detail in Ref [2]. As for as radius is concern, we shall restrict to its latest form given in Ref. [11]. However, the role of the third parameter i.e., the universal function in fusion barriers is analyzed in Fig. 1. Here, we display ΔV_B (%) and ΔR_B (%) defined as

$$\Delta V_B\ (\%) = \frac{V_B^{theor} - V_B^{expt}}{V_B^{expt}} \times 100, \tag{10}$$

and

$$\Delta R_B\ (\%) = \frac{R_B^{theor} - R_B^{expt}}{R_B^{expt}} \times 100, \tag{11}$$

as a function of Z_1Z_2 using two sets of above mentioned universal functions [Eqs. (7) and (8)]. It is clear from the figure that deviations are significantly reduced by using Φ-1981 compared to its original form Φ-1977. The universal function Φ-1981 reduces the average deviation over 390 reactions by 1 % for fusion barriers. The experimental values are taken directly from the literature [1, 2, 7]. Actually, it is clear from the literature that no experiment can extract information about the fusion barriers directly. All experiments measure the fusion differential cross sections and then with the help of a theoretical model, one can extract the fusion barriers.

In Fig. 2, we display the theoretical fusion barrier heights V_B^{theor} (MeV) and positions R_B^{theor} (fm) verses the corresponding experimental values. We note from the figure that Prox 2010 potential reproduces the experimental fusion barrier heights within 1.4%. This result is in close agreement with other recently parametrized potentials presented in Ref. [1]. However, the original form of the proximity potential 1977 presented in Ref. [1] overestimates the data by 6.7% for symmetric colliding nuclei. However, the fusion barrier positions show some scattering from the central line (marked by shaded area). This scattering may be due to the variation in the experimental setups and theoretical method one used to extract these values [15, 16].

We quantify our outcome in Figs. 3 and 4. In Fig. 3, the percentage deviations between the theoretical and experimental values are presented. The original proximity potential 1977 (Prox 1977) along with its recently modified form (Prox 2000) are also displayed. We note from the upper panel of Fig. 3 that Prox 2010 potential on average gives better results compared to its older versions for fusion barrier heights. However, slight deviations are visible for fusion barrier positions. This may be due to the fact that in the proximity potential Prox 2010, we use the value of surface energy coefficient that gives stronger attraction compared to one used in Prox 1977 and Prox 2000 potentials. Therefore, in Prox 2010 potential, the counterbalance between the repulsive Coulomb and attractive nuclear part of the interaction potential occurs at larger distances, and hence it pushes the barrier outwards. The fusion barrier heights are reproduced within ± 5% on average. On the other hand, fusion barrier positions reproduced the experimental values within ± 10%. Especially for the heavier colliding nuclei, we see that Prox 2010 potential reproduces the data much better on the average compared to other versions. For lighter nuclei, however, small scattering is visible. This could also be due to the uncertainty in the radius of the lighter colliding nuclei. In Figs. 1-4, only 155 reactions are displayed to maintain the clarity. The average deviation for the fusion barrier heights over 390 reactions is 0.77 % using our modified potential Prox 2010, whereas Prox 1977, and Prox 2000 give 3.99 %, and 4.45 %, respectively. This shows that our modified proximity explains the experimental data nicely.

In Fig. 4, we display the difference between the theoretical and experimentally extracted fusion barriers. We further note that Prox 2010 potential gives better results. The difference especially for the heavy systems is significantly reduced. This was the problem with original as well as its recently modified form as pointed out by several authors [7, 8]. It is clear from Figs. 3 and 4, that Prox 2010 potential is able to reproduce the experimental data much better than its older versions. The small difference is not significant because of the uncertainties in the analysis of the experimental data.

Finally, we test our newly modified proximity potential Prox 2010 on fusion probabilities. In Fig. 5, we display the fusion cross sections σ_{fus} (in mb) as a function of the center-of-mass energy $E_{c.m.}$ (MeV) for the reactions of $^{26}Mg +^{30} Si$ [17], $^{16}O +^{46} Ti$ [18], $^{48}Ca +^{48} Ca$ [19], $^{12}C +^{92} Zr$ [20], $^{40}Ca +^{58} Ni$ [21], and $^{16}O +^{144} Sm$ [22]. The fusion cross sections are calculated using well known Wong model [23]. The older versions of proximity potentials that is, Prox 1977 and Prox 2000 are also displayed. It is clearly visible from the figure that Prox 2010 potential is in good agreement, whereas, its older forms are far from the experimental data. We further note that Prox 1977 and Prox 2000 potentials show similar results. It means that no improvements is seen in Prox 2000 potential as was claimed in Ref. [7].

IV. SUMMARY

In the present study, we present a best set of the surface energy coefficient, the nuclear radius, and the universal function available in the literature. We find that these parameters which were used quite arbitrarily in past years affect the fusion barrier heights, positions, and cross sections significantly. By using the above set of parameters, a new proximity potential is constructed. Our newly constructed proximity potential Prox 2010 reproduces the fusion barriers and cross sections better than its earlier versions.

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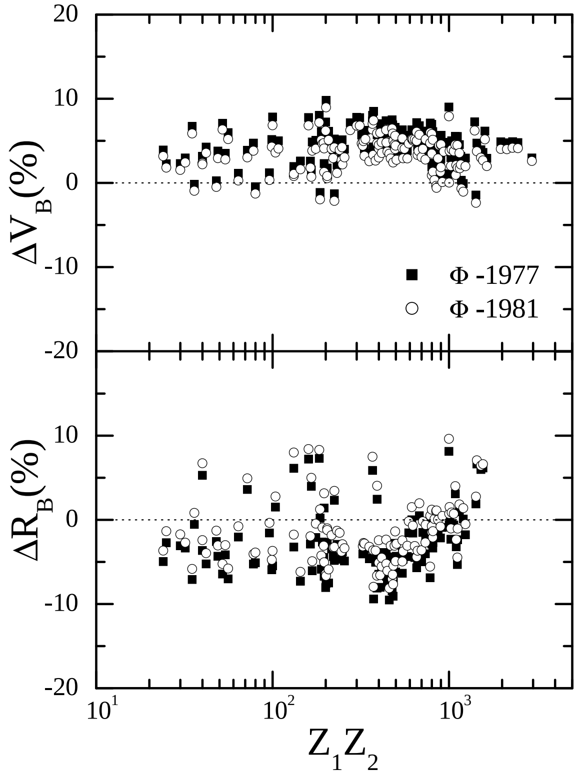

FIG. 1: The percentage deviation ΔV_B (%) and ΔR_B (%) as a function of Z_1Z_2 using two different sets of universal functions [Eqs. (7) and (8)] implemented in the original proximity potential Prox 1977. The experimental values are taken from Refs. [1, 2, 7].

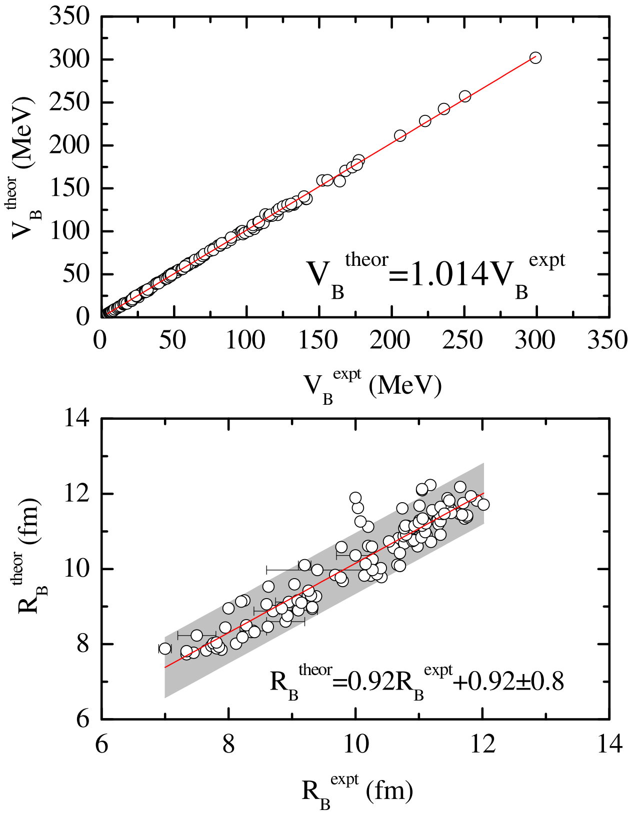

FIG. 2: The fusion barrier heights V_B (MeV) and positions barriers R_B (fm) as a function of the corresponding experimental values using our modified proximity potentials Prox 2010. The experimental values are taken from Refs. [1, 2, 7]

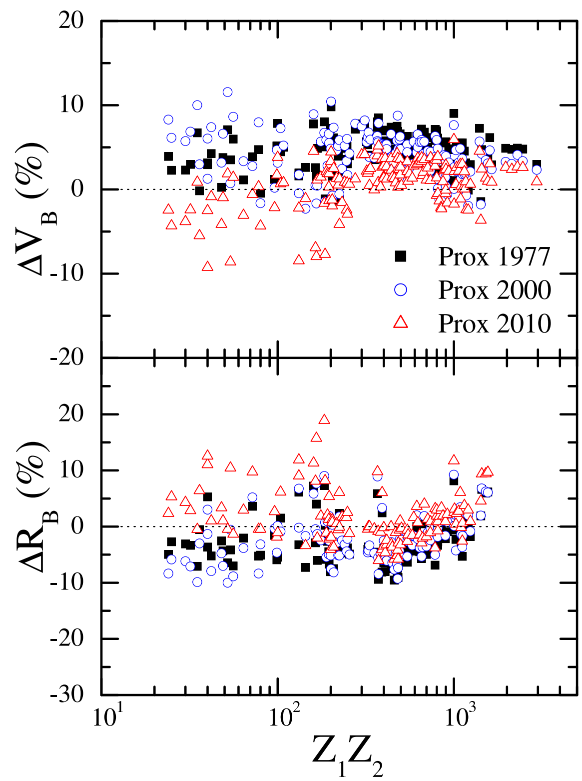

FIG. 3: The same as Fig.1, but for different older proximity potentials along with our modified form i.e., Prox 1977, Prox 2000, and Prox 2010, respectively.

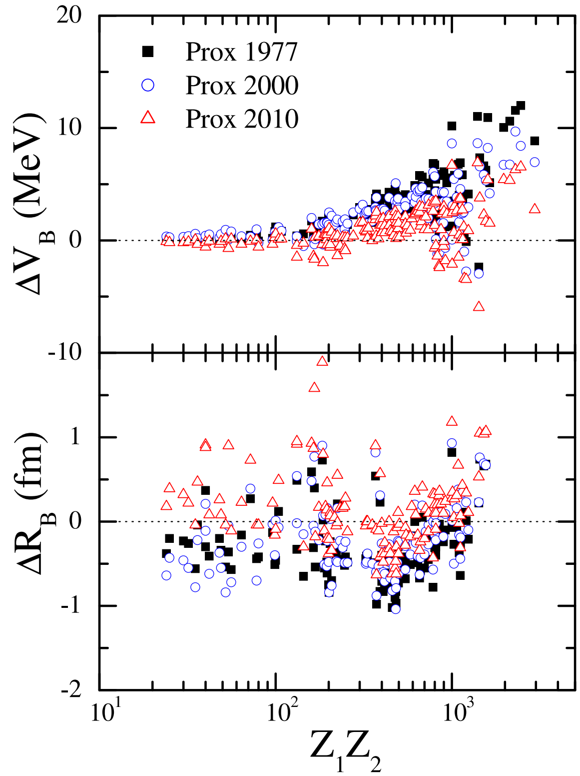

FIG. 4: The variation of ΔV_B ($= V_B^{theor} - V_B^{expt}$) and ΔR_B ($= R_B^{theor} - R_B^{expt}$) as a function of $Z_1 Z_2$ using Prox 1977, Prox 2000, and Prox 2010 potentials. The experimental values are taken from Refs. [1, 2, 7].

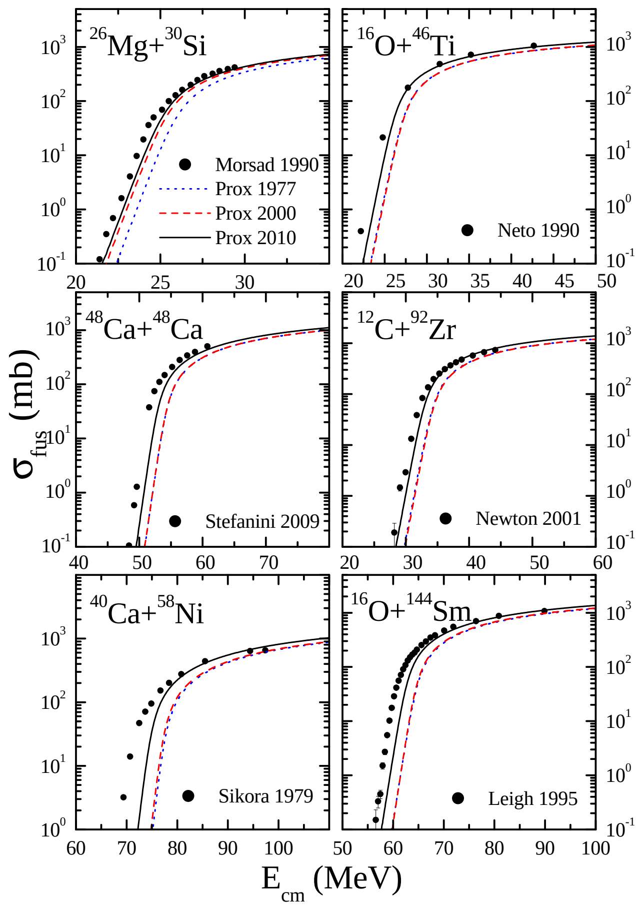

FIG. 5: (Color online) The fusion cross sections σ_{fus} (mb) as a function of center-of-mass energy $E_{c.m.}$ using older versions of proximity potential (Prox 1977 and Prox 2000) along with new version (Prox 2010). The experimental data are taken from Morsad 1990 [17], Neto 1990 [18], Stefanini [19], Newton 2001 [20], Sikora 1979 [21], and Leigh 1995 [22].