STATISTICAL MODELS OF FRAGMENTATION PROCESSES

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Received 12 November 1974

A model with minimal correlations among nucleon momenta agrees well with refined data on fragment momentum distributions resulting from peripheral nucleus-nucleus collisions. The postulates are identical to those of Feshbach and Huang, but are applied to compute a quantity more relevant to experiment: the dependence of the momentum distribution on fragment mass.

Over a year ago, Heckman et al. [1] reported early results of experiments in which relativistic heavy ions (^{12}C, ^{16}O) impinged on various nuclear targets, and fragments of the projectile nucleus were observed. In the rest frame of the projectile, the fragments were found to have a momentum distribution $\exp(-p^2/2\sigma^2)$, with $\sigma \approx m_\pi c$, regardless of the mass of the fragment. Feshbach and Huang [2] attempted to understand this result in a model which described the projectile disintegration as a fast process governed by the distribution of nucleon momenta in the projectile before collision. Since then, more refined data [3] have shown that the quantity σ^2 does vary with fragment mass number K, in a manner well fit by the expression

$$\sigma^2 = \sigma_0{}^2\, K(A-K)/(A-1) \tag{1}$$

with $\sigma_0 \approx 90$ MeV/c.[‡1]

We wish to show here that

1) Eq. (1) is an immediate consequence of the FH statistical hypothesis. Even the constant σ_0 is determined, and is only 10% higher than given by experiment.

2) The FH theory superficially agrees with the *older* experimental results because it yields a quantity not directly comparable to experiment. If this is taken into account, one additional approximation in the FH theory makes a direct comparison possible. That approximation yields eq. (1) again!

3) If, instead of assuming a sudden liberation of virtual clusters, one supposes that the projectile has come to thermal equilibrium at excitation temperature T, eq. (1) follows once more, this time with $\sigma_0{}^2$ proportional to the temperature.

4) The above conclusions hold provided that the momentum transfer to the projectile nucleus from collision with the target is small compared to σ_0.

1. Suppose that A nucleons are assembled with zero net three momentum, $p_A = 0$. If K of these nucleons chosen at random, should go off together as a single fragment, what would be the mean square total momentum $p_K{}^2$? The A nucleons have mean square momentum $\langle p^2\rangle$ and the momenta of different nucleons are correlated by the requirement $p_A = 0$,

$$A\langle p^2\rangle + \sum_{i\neq j} \langle p_i \cdot p_j\rangle = 0, \tag{2}$$

or

$$\langle\langle p_i \cdot p_j\rangle\rangle = -\langle p^2\rangle/(A-1) \tag{3}$$

where the double bracket denotes an average over all $i \neq j$.

This gives immediately

$$\langle p_K{}^2\rangle = \langle\langle(\Sigma_{i=1}^{K}\, p_i)^2\rangle\rangle = K(A-K)\langle p^2\rangle/(A-1). \tag{4}$$

where the double bracket here indicates an average over all possible choices of the K nucleons making up the fragment. (As the charge and perhaps the spin of the fragment are not arbitrary, some approximation is

* Supported by NSF GP-32998X.

‡1 The parabolic form eq. (1), was suggested a year ago by W.A. Wenzel, using reasoning very similar to that given in the present paper section 1, below, but he was discouraged from pursuing the matter by the apparent disagreement. The same form was adopted by the experimenters in plotting their data, following work by J.V. Lepore and R.J. Riddell, who found the parabola in a simple model satisfying the assumptions of the present section 1.

involved in identifying $\langle\langle p_K{}^2\rangle\rangle$ with the mean square momentum of any type of fragment with mass number K.)

Since the σ in eq. (1) corresponds to a single Cartesian component of the three momentum p_K, we get

$$\sigma_0{}^2 = \langle p^2\rangle/3. \qquad (5)$$

FH quote, as a reasonable first approximation,

$$\langle p^2\rangle = 3\, p_F{}^2/5 \qquad (6)$$

where the Fermi momentum is taken as $p_F \approx 230$ MeV/c for ^{16}O [2]. This yields $\sigma_0 \approx 100$ MeV/c, 10% above the experimental value. There are qualitative arguments to suggest that the theoretical σ_0 could have a lower value because, on the one hand, the very light or very heavy fragments should exhibit the lower p_F of the nuclear surface, and, on the other hand, medium mass fragments are likely to have larger-than-average negative values of $\langle p_i \cdot p_j\rangle$ for their constituent nculeons. However, such claims require the existence of a far more complete theory for their evaluation.

2. Feshbach and Huang [2], using a formalism developed earlier by Huang [4], solved (in the large A limit) the following problem: Given that A nucleons are distributed among n fragments, each fragment labeled only by its three-momentum, what is the momentum distribution of any fragment?‡2 From the nature of their formulation, it follows at once that each of the fragments has the same distribution because there is nothing except momentum to label the fragments. Therefore, the suggestion, that they had confirmed the universal σ_0 of a year ago, was misleading. To proceed further, it is necessary to add a further approximation, that the n fragments all have the same mass number K. The approximation is accurate provided A is large, since the probability distribution for A nucleons among n fragments is strongly peaked for near equal size of all fragments. We may then substitute $K \approx A/n$ in eq. (1), obtaining the Feshbach-Huang formula

$$\sigma_0{}^2 = \langle p^2\rangle A(n-1)/3n^2 \qquad (7)$$

albeit with a further factor $A/(A-1)$ on the right hand side. This latter difference is not important, in view of the large A approximations in their calculation and in our approximation. One should note that, if a single fragment of mass number K is observed, the most probable value of n is not necessarily A/K. Indeed, for K or $A-K$ small compared to A, n cannot possibly be peaked at A/K. However, the calculation in section 1 shows that the momentum distribution of a fragment is not sensitive to the manner of fragmentation of the remainder of the nucleus. Thus, the validity of our approximation is sufficient to derive the Feshbach-Huang result with the simpler procedure employed here, even though the converse of our approximation ($n \approx A/K$ for a given fragment) is not always valid.

3. Suppose that the nucleus, after excitation, comes to equilibrium at temperature T. Then, each Cartesian component p_{Ka} of momentum of a given fragment K can be associated with a single degree of freedom, except that the center of mass motion of the remainder of the nucleus is perfectly correlated, since p_A is zero. Therefore, we have

$$kT/2 = \langle p_{K\alpha}{}^2\rangle/2m_N K + \langle p_{K\alpha}{}^2\rangle/2m_N(A-K) \qquad (8)$$

or

$$\sigma^2 = m_N kTK(A-K)/A \qquad (9)$$

where k is Boltzmann's constant and m_N is the nucleon mass (we neglect binding corrections to fragment masses). This would correspond to an excitation temperature of about 9 MeV/k, remarkably close to the mean nucleon binding energy.

4. If the projectile nucleus received a mean squared momentum transfer $p_x{}^2$ in some direction, σ^2 in that direction would be changed to

$$\sigma'^2 = \sigma^2 + p_x{}^2(K/A)^2 \qquad (10)$$

This means that the momentum distribution of the heaviest fragments is most sensitive to collisional momentum transfer.

We conclude that the momentum distribution of projectile fragments is not sensitive to the speed of the fragmentation process. Whether fragmentation ocurred almost immediately after collision, with insignificant final state fragment-fragment interaction (sudden hypothesis), or whether it occurred only after a rough equilibrium was established (thermodynamic hypoth-

‡2 They compute a mean value of the number of ^{16}O fragment $\langle n\rangle \approx 7$. There are some misprints in their paper, but their result quoted below in eq. (7) is correct. Experiment gives $\langle n\rangle \approx 7$, in agreement with FH theory.

esis) cannot be established by momentum alone. However, fragment mass distributions, as well as correlations among fragments, may be more informative.‡3 On physics grounds, one is disinclined to believe that a compound nuclear state of such high temperature could emit intact heavy fragments with appreciable probability. On the other side, we don't yet know the mechanism for distribution of energy to the fragments in an impulse picture. These questions and doubts have still to be settled, but one may hope that a theory which does so will not alter the results on momentum distributions, which depend on so little.‡4

‡3 Related to this, H. Feshbach (private communication), following remarks by T.E.O. Ericson, has shown that kinetic energies derived from Coulomb repulsion of initially tangent spherical fragment nuclei are comparable with the observed kinetic energies, when only two fragments are produced. However, for the more typical case of many fragments, the Coulomb repulsion is an order of magnitude smaller than the total kinetic energy. It seems likely to the present author that Coulomb forces are not a major influence in the fragmentation of light and medium nuclei.

‡4 The extremes required to get away from the momentum distribution, eq. (1), are demonstrated by Bhaduri [5]. In order to explain the old result of constant σ, independent of K, Bhaduri is forced to the explicit assumtion that the mean square momentum $\langle p^2 \rangle$, of the nucleons which end up in a fragment of mass number K, is inversely proportional to K! Despite statements to the contrary, this work is not consistent with FH.

I thank H. Feshbach, D. Greiner, H. Heckman, A. Jackson, J. Lepore, P. Lindstrom, R. Riddell, W. Swiatecki and W. Wenzel for helpful conversations.

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