

# Systematic Study On Alpha Decay In $^{184-216}\text{Bi}$ Nuclei

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**Abstract.** A systematic study on the  $\alpha$ -decay half lives of the isotopes of Bi ( $Z = 83$ ) nuclei in the region  $184 \leq A \leq 216$  has been done using the Coulomb and proximity potential model (CPPM). The computed half lives are compared with the experimental data and they are in good agreement. We have modified the assault frequency and re-determined the half lives and they show a better agreement with the experimental value. The standard deviation of the logarithm of half life with the former assault frequency is found to be 1.323 and with the modified assault frequency, it is found to be 0.223. This reveals that the Coulomb and proximity potential model (CPPM), with the modified deformation dependent assault frequency is more apt for the alpha decay studies. Using our model we could also demonstrate the influence of the 126 neutron shell closure in both parent and daughter nuclei on the alpha decay half lives.

**Keywords:** Alpha decay, Coulomb and proximity potential model.

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## INTRODUCTION

Alpha decay is the important mode of decay for the unstable medium, heavy and superheavy nuclei. Since the observation by Rutherford in 1909 [1, 2],  $\alpha$  decay has been one of the widely discussed topics of nuclear physics as it can provide some reliable knowledge on nuclear structure [3-9] and is used to identify new isotopes mainly in the superheavy region. A detailed analysis of the proton, alpha and cluster radioactivity can give valuable information on the structure and nuclear mass of some exotic nuclei which are not yet experimentally observed [10]. It is considered that  $\alpha$  decay process can provide peculiar information about the spectroscopy of very neutron deficient nuclei in the region  $Z > 82$  and from the decay data of ground and isomeric states detailed structure information on energy levels can be determined.

A large number of artificially produced radioactive isotopes of Bi ( $Z = 83$ ) with the mass number in the range  $184 \leq A \leq 216$  are known today, of which most of the isotopes are unstable against alpha particle, electron capture and  $\beta^-$  decays [11]. Attempts have been made since the late forties to detect a possible alpha radioactivity from the  $^{209}\text{Bi}$  isotope, and results indicated half-life values in the range  $(2.0-2.7) \times 10^{17}\text{y}$  [12–14], or lower limits of  $3 \times 10^{15}\text{y}$  [15],  $2.0 \times 10^{18}\text{y}$  [16, 17], and  $1.0 \times 10^{19}\text{y}$  [18] for the alpha decay half life. The spontaneous alpha particle emission from natural bismuth was detected unequivocally for the first time by de Marcillac *et al* [19]. It was this spectacular finding, the lowest alpha activity ever detected ( $\sim 12$  disintegrations  $\text{h}^{-1} \text{kg}^{-1}$ ),

motivated us to study the possible alpha radioactivity from all the isotopes of Bi known till now.

The  $\alpha$ -decay theory was formulated by Gamow [20] and independently by Gurney and Condon [21] in 1928 on the basis of quantum tunneling and this explanation of nuclear phenomena was one of the early triumphs in quantum mechanics. Later on, as a consequence of this pioneering work, several theoretical studies were performed to understand the phenomenon of alpha decay. These theoretical studies are based on various microscopic [6, 22–24], macroscopic cluster [25–33] and fission [34–36] approaches to the explanation of  $\alpha$ -decay. Usually alpha and cluster radioactivity leads to doubly magic  $^{208}\text{Pb}$  which is highly stable against the alpha and cluster decays. The intention of this paper is to study how Bi ( $Z = 83$ ), near to proton shell closure behaves against alpha decay and thereby study the role of neutron shell closure in these isotopes.

Most of the theoretical alpha decay studies on the isotopes of bismuth has been confined to those isotopes whose half lives are experimentally known, and otherwise taking the angular momentum transferred by the alpha particle as  $\ell = 5$  [10, 37]. In the present work, the alpha decay half lives of Bi isotopes in the region  $184 \leq A \leq 216$  have been calculated for their ground state to ground state transition within the Coulomb and proximity potential model (CPPM) [38, 39, 40, 41] proposed by Santhosh *et al.*, in 2000. The angular momentum values for all the isotopes have been calculated using the spin parity selection rules and we have been able to calculate and systematize the alpha decay half lives of ground-state to ground-state transitions for all alpha-emitter

bismuth isotopes with minimum possible value of angular momentum.

The details of our model CPPM are presented in section 2, results and discussions on the alpha decay of the nuclei under study are given in section 3 and a conclusion on the entire work is given in section 4.

## THE COULOMB AND PROXIMITY POTENTIAL MODEL (CPPM)

In Coulomb and proximity potential model (CPPM) the potential energy barrier is taken as the sum of Coulomb potential, proximity potential and centrifugal potential for the touching configuration and for the separated fragments. For the pre-scission (overlap) region, simple power law interpolation as done by Shi and Swiatecki [42] is used. The inclusion of proximity potential reduces the height of the potential barrier, which closely agrees with the experimental result [43]. The proximity potential was first used by Shi and Swiatecki [42] in an empirical manner and has been quite extensively used by Gupta et al [44] in the preformed cluster model (PCM) which is based on pocket formula of Blocki et al. [45] given as:

$$\Phi(\varepsilon) = -\left(\frac{1}{2}\right)(\varepsilon - 2.54)^2 - 0.0852(\varepsilon - 2.54)^3, \quad \text{for } \varepsilon \leq 1.2511 \quad (1)$$

$$\Phi(\varepsilon) = -3.437 \exp\left(\frac{-\varepsilon}{0.75}\right), \quad \text{for } \varepsilon \geq 1.2511 \quad (2)$$

where  $\Phi$  is the universal proximity potential. In the present model, another formulation of proximity potential [46] is used as given by Eqs. 6 and 7. In this model cluster formation probability is taken as unity for all clusters irrespective of their masses, so the present model differs from PCM by a factor  $P_0$ , the cluster formation probability. In the present model assault frequency,  $\nu$  is calculated for each parent-cluster combination which is associated with vibration energy. But Shi and Swiatecki [47] get  $\nu$  empirically, unrealistic values  $10^{22}$  for even A parent and  $10^{20}$  for odd A parent.

The interacting potential barrier for a parent nucleus exhibiting exotic decay is given by,

$$V = \frac{Z_1 Z_2 e^2}{r} + V_p(z) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2}, \quad \text{for } z > 0 \quad (3)$$

Here  $Z_1$  and  $Z_2$  are the atomic numbers of the daughter and emitted cluster, 'z' is the distance between the near surfaces of the fragments, 'r' is the distance between fragment centers. The term  $\ell$  represents the angular momentum,  $\mu$  the reduced

mass and  $V_p$  is the proximity potential. The proximity potential  $V_p$  is given by Blocki et al. [45] as,

$$V_p(z) = 4\pi\gamma b \left[ \frac{C_1 C_2}{(C_1 + C_2)} \right] \Phi\left(\frac{z}{b}\right) \quad (4)$$

with the nuclear surface tension coefficient,

$$\gamma = 0.9517[1 - 1.7826(N - Z)^2 / A^2] \text{ MeV/fm} \quad (5)$$

where N, Z and A represent neutron, proton and mass number of parent respectively,  $\Phi$  represents the universal the proximity potential [46] given as

$$\Phi(\varepsilon) = -4.41e^{-\varepsilon/0.7176}, \quad \text{for } \varepsilon \geq 1.9475 \quad (6)$$

$$\Phi(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.0169\varepsilon^2 - 0.05148\varepsilon^3, \quad \text{for } 0 \leq \varepsilon \leq 1.9475 \quad (7)$$

With  $\varepsilon = z/b$ , where the width (diffuseness) of the nuclear surface  $b \approx 1$  and Süsmann central radii  $C_i$  of fragments related to sharp radii  $R_i$  as,

$$C_i = R_i - \left(\frac{b^2}{R_i}\right) \quad (8)$$

For  $R_i$  we use semi empirical formula in terms of mass number  $A_i$  as [45],

$$R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3} \quad (9)$$

The potential for the internal part (overlap region) of the barrier is given as,

$$V = a_0 (L - L_0)^n, \quad \text{for } z < 0 \quad (10)$$

Here  $L = z + 2C_1 + 2C_2$  and  $L_0 = 2C$ , the diameter of the parent nuclei. The constants  $a_0$  and  $n$  are determined by the smooth matching of the two potentials at the touching point.

Using one dimensional WKB approximation, the barrier penetrability P is given as,

$$P = \exp\left\{-\frac{2}{\hbar} \int_a^b \sqrt{2\mu(V - Q)} dz\right\} \quad (11)$$

Here the mass parameter is replaced by  $\mu = mA_1 A_2 / A$ , where 'm' is the nucleon mass and  $A_1, A_2$  are the mass numbers of daughter and emitted cluster respectively. The turning points "a" and "b" are determined from the equation  $V(a) = V(b) = Q$ . The above integral can be evaluated numerically or analytically, and the half life time is given by

$$T_{1/2} = \left(\frac{\ln 2}{\lambda}\right) = \left(\frac{\ln 2}{\nu P}\right) \quad (12)$$

where,  $\nu = \left(\frac{\omega}{2\pi}\right) = \left(\frac{2E_v}{h}\right)$  represent the number of

assaults on the barrier per second and  $\lambda$  the decay constant.  $E_v$ , the empirical vibration energy is given as [48],

$$E_v = Q \left\{ 0.056 + 0.039 \exp \left[ \frac{(4 - A_2)}{2.5} \right] \right\}, \text{ for } A_2 \geq 4 \quad (13)$$

In the classical method, the alpha particle is assumed to move back and forth in the nucleus and the usual way of determining the assault frequency is through the expression given by  $\nu = \text{velocity} / (2R)$ , where  $R$  is the radius of the parent nuclei. But the alpha particle has wave properties; therefore a quantum mechanical treatment is more accurate. Thus, assuming that the cluster vibrates in a harmonic oscillator potential with a frequency  $\omega$ , which depends on the vibration energy  $E_v$ , we can identify this frequency as the assault frequency  $\nu$  given in eqns. (12)-(13).

## RESULTS AND DISCUSSIONS

Using the Coulomb and Proximity Potential Model (CPPM) we have calculated the alpha decay half lives of the isotopes of Bi ( $Z = 83$ ) nuclei within the range  $184 \leq A \leq 216$ . In CPPM the external drifting potential barrier is obtained as the sum of the Coulomb potential, proximity potential and centrifugal potential. In Figure 1 we have plotted the potential energy barrier for the emission of alpha particle from the  $^{213}\text{Bi}$  isotopes with  $\ell = 0$  and  $\ell = 5$ . The energy released in the alpha transitions between the ground state energy levels of the parent nuclei and the ground state energy levels of the daughter nuclei is given as

$$Q_{gs \rightarrow gs} = \Delta M_p - (\Delta M_\alpha + \Delta M_d) + k(Z_p^\varepsilon - Z_d^\varepsilon) \quad (14)$$

where  $\Delta M_p, \Delta M_d, \Delta M_\alpha$  are the mass excess of the parent, daughter and alpha particle respectively. The  $Q$  values for alpha decay between the ground state of the parent and daughter nucleus is extracted from the experimental mass excess values of Audi *et al.*, [49]. The effect of atomic electrons on the energy of the alpha particle has not been included in the mass excess given in Ref. [49]. So for a more accurate calculation of  $Q$  value, we have included the electron screening effect [50] in equation (14). The term  $k(Z_p^\varepsilon - Z_d^\varepsilon)$  represents this correction, where the quantity  $kZ^\varepsilon$  represents the total binding energy of the  $Z$  electrons in the atom. Here the values of  $k = 8.7\text{eV}$  and  $\varepsilon = 2.517$  for nuclei with  $Z \geq 60$ ; and  $k = 13.6\text{eV}$  and  $\varepsilon = 2.408$  for nuclei with  $Z < 60$ , have been derived from data reported by Huang *et al* [51]. The value of angular

momentum transferred by  $\alpha$  particle is obtained from the spin-parity selection rule:

$$|J_i - J_j| \leq \ell_\alpha \leq J_i + J_j, \text{ and } \frac{\pi_i}{\pi_j} = (-1)^{\ell_\alpha} \quad (15)$$

where  $J_i, J_j, \pi_i$  and  $\pi_j$  are the spin and parity of parent nucleus and daughter nucleus respectively.

The entire transitions along with various parameters are evaluated and are comprehended in Table 1. The elements and its transitions are arranged in column 1. In this study, we have done the calculations with the minimum value of angular momentum, which follow the spin-parity selection rule as given in eqn. (15). Columns 2 and 3 represent respectively the spin and parity of the parent and daughter nuclei taken from [52]. The brackets are used to specify the uncertainty in spin and/or parity and '?' symbol signifies those spin parity states which are not yet observed. Although  $\alpha$ -particle can transfer any momentum which follows equation (15), in our study we have evaluated only that potential which has the minimum possible value of angular momentum. The minimum values of possible angular momenta are arranged in column 4. The  $Q$  values of these alpha decays are arranged in column 5. The column 7 of Table 1 is tabulated with the calculated partial half lives using the CPPM and they are explicitly denoted as  $T_{1/2}^{cal.(1)}$ . While checking the agreement with the corresponding experimental data given in column 6, we have found that the calculated values show only a deviation of order one with respect to the corresponding experimental data; even though the experimental values are spanned over a wide range from  $10^2$  seconds to  $10^{26}$  seconds.

Thus by comparing with the experimental values, we have seen that, in most of the transitions the calculated values are less than the experimental data. To get a better match, we have parameterized the assault frequency using the prescription given by Denisov *et al* [53] given as,

$$\log_{10} \nu = a_0 + a_1((-1)^\ell - 1) + a_2 I + a_3 \beta_2 + a_4 \beta_4 + a_5 \ell(\ell + 1) A^{-1/6} \quad (16)$$

where,  $I = \frac{N-Z}{N+Z}$  is the proton-neutron symmetry,  $A$ ,

$N$  and  $Z$  are respectively the number of nucleons, number of neutrons and number of protons in the daughter nucleus,  $\beta_2$  and  $\beta_4$  are the quadruple and hexadecapole deformation values of the nuclei which interact with  $\alpha$ -particle. The constants in equation (16) are taken from our earlier work [54] and we would like to mention that these values were evaluated by minimizing the term;

$\sum_{\text{odd-odd}} (\log_{10} T_{1/2}^{\text{cal.}} - \log_{10} T_{1/2}^{\text{exp.}})^2$ , the sum of the square of deviation of partial calculated half life values of odd-odd nuclei in the range  $83 \leq Z \leq 101$  from the ground state of parent nucleus to the ground and excited state of daughter nucleus, with the corresponding experimental one. For this evaluation, only the transitions between the states having specified angular momentum were taken. The value of the coefficients ' $a_i$ ' are given in table 2. The dependence of experimental half life and calculated penetrability against  $Z^{1/2} A^{-1/6}$  and  $ZQ^{-1/2}$  were studied in Ref [54] and it was found that the functional dependence of half life and penetrability are same on both these quantities. Thus it was concluded that the assault frequency is nearly independent of the terms  $Z^{1/2} A^{-1/6}$  and  $ZQ^{-1/2}$  and hence these quantities were not considered for the computation of assault frequency. The advantage of this expression for assault frequency is that, the assault frequency,  $\nu$  can be found as a function of variables like quadruple ( $\beta_2$ ) and hexadecapole ( $\beta_4$ ) deformation, proton-neutron symmetry  $I$ , and the angular momentum transfer  $\ell$ . A comparison of the coefficients given in table 2 reveals that the assault frequency is highly dependent on  $I$  and is least dependent on the angular momentum transfer, whereas deformation made a considerable contribution in this expression. Using this modified deformation dependent assault frequency, we have recalculated the alpha half life values for all the transitions and are tabulated as  $T_{1/2}^{\text{cal.}(2)}$  in column 7 of table 1.

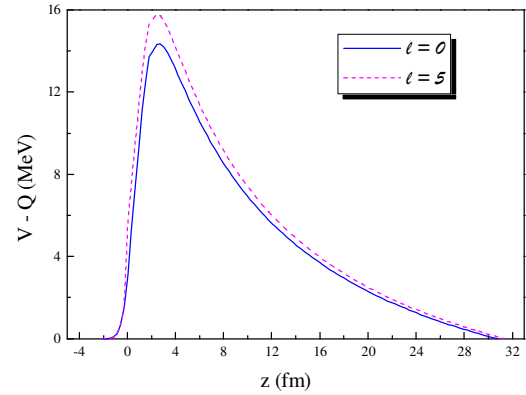
Now by comparing the half life values ( $T_{1/2}^{\text{cal.}(2)}$ ) with the experimental values, it is clear that all of the transitions are in good agreement with the experimental values. In column 8, we have tabulated the alpha decay half life values reported by Tavares et al., [37] within the framework of the semi-empirical approach to alpha decay by tunneling. It can be seen that our calculated alpha half lives matches well with these values. Figure 2 represents the comparison of the  $\log_{10}(T_{1/2})$  values, evaluated using both the procedures, with the experimental half life values. In this figure we have also plotted the alpha decay half life values given by Tavares et al [37]. From this, it is clear that our calculated values with modified assault frequency show a better match with the experimental values. The  $\log_{10}(T_{1/2})$  values and the Q values plotted against the neutron numbers of the daughter nuclei in the corresponding alpha decays are given in Figure 3 which gives an insight related to the results for the bismuth isotopes. It is evident that when both these quantities are plotted against the neutron number of the daughter (Tl) isotopes, the behavior of the alpha decay half lives (figure 3(a)) is a perfect inverse

reflection of the Q values (figure 3(b)). From figures 2 and 3 the influence of the neutron shell closure ( $N = 126$ ), particularly on the  $^{209}\text{Bi}$  isotope, which exhibits practically the highest half life, is clearly evidenced. From figure 3 it is also clear that the  $^{211}\text{Bi}$  isotope with least alpha half life is more alpha active than its neighbouring isotopes. The influence of the neutron shell closure ( $N = 126$ ), on the daughter nucleus  $^{207}\text{Tl}$  is also seen from these figures.

As a conclusion for the discussion, and in order to check the agreement between the experimental and calculated values, we have evaluated the standard deviation  $\sigma$ , for the half lives. The standard deviation,  $\sigma$  for half life is given by

$$\sigma = \left\{ \frac{1}{(n-1)} \sum_{i=1}^n (\log_{10} T_i^{\text{cal.}} - \log_{10} T_i^{\text{exp.}})^2 \right\}^{1/2} \quad (17)$$

The standard deviation of the logarithm of half life is found to be 1.323 for calculation 1 and that with the modified assault frequency, the standard deviation of the logarithm of half life is found to be reduced to 0.223. We have also calculated the standard deviation of the logarithm of half life for the calculations done by Tavares et al., and it is found to be 0.228. This reveals that the Coulomb and proximity potential model (CPPM), with the modified deformation dependent assault frequency is more apt for the alpha decay studies.



**FIGURE 1.** Potential energy barrier for the emission of  $^4\text{He}$  from  $^{213}\text{Bi}$  with  $\ell = 0$  and  $\ell = 5$ .

## CONCLUSIONS

The alpha half lives of the bismuth isotopes in the range  $184 \leq A \leq 216$  from ground state to ground state have been computed and analyzed with the observed alpha decay half lives within the Coulomb and proximity potential model (CPPM). The computed half lives on comparison with the experimental data show

good agreement. In particular, the alpha decay half-life value of the unique naturally occurring  $^{209}\text{Bi}$  isotope has been reproduced successfully with our approach as  $1.45 \times 10^{19}$  y, which was very recently measured to be  $1.9 \times 10^{19}$  y. We have also predicted the alpha decay half lives for a number of alpha transitions with minimum possible angular momentum values in bismuth isotopes using our model and thereby made it possible to demonstrate the influence of the 126 neutron shell closure on the alpha decay half lives. On

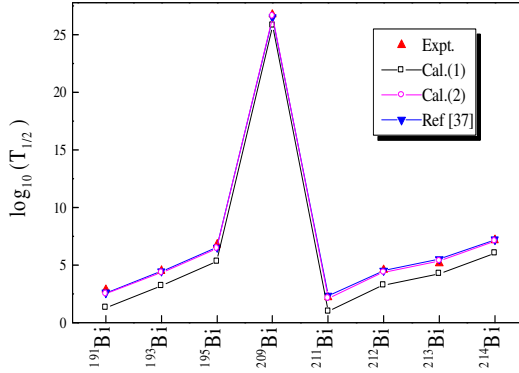
modifying the assault frequency, we have re-determined the half lives and they show a better agreement with the experimental values. The standard deviation is found to be 1.323 with the former assault frequency and with the modified assault frequency, it is found to be reduced as 0.223. This reveals that CPPM, with the modified assault frequency is more apt for the alpha decay studies and it is clear that the approach can be successfully applied to other isotopic sequences of alpha emitter nuclides.

**TABLE 1.** Comparison of computed alpha decay half lives for Bi isotopes in the range  $184 \leq A \leq 216$  with the corresponding experimental values. Here Q-values are in MeV and half-lives in seconds.

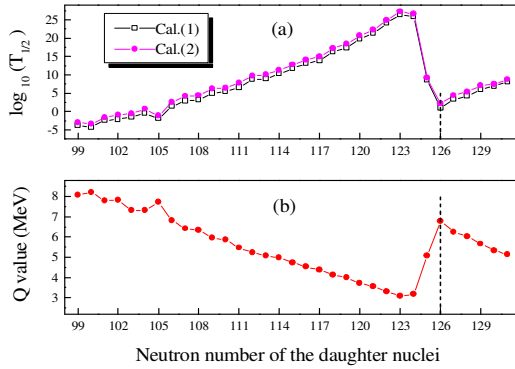
Transitions	$J_i^\pi$	$J_f^\pi$	$\ell_{min}$	Q	$T_{1/2}^{exp.}$	CPPM $T_{1/2}^{cal.(1)}$	$T_{1/2}^{cal.(2)}$	$T_{1/2}$ [37]
$^{184}\text{Bi} \rightarrow ^{180}\text{Tl}$	?	$(4^-, 5^-)$	0	8.060		$1.441 \times 10^{-4}$	$1.029 \times 10^{-3}$	
$^{185}\text{Bi} \rightarrow ^{181}\text{Tl}$	$1/2^+$	$(1/2^+)$	0	8.201		$5.079 \times 10^{-5}$	$3.666 \times 10^{-4}$	
$^{186}\text{Bi} \rightarrow ^{182}\text{Tl}$	$(3^+)$	$(7^+)$	4	7.790		$4.165 \times 10^{-3}$	$2.388 \times 10^{-2}$	
$^{187}\text{Bi} \rightarrow ^{183}\text{Tl}$	$(9/2^-)$	$(1/2^+)$	5	7.824		$6.501 \times 10^{-3}$	$1.183 \times 10^{-1}$	$1.87 \times 10^{-1}$
$^{188}\text{Bi} \rightarrow ^{184}\text{Tl}$	$(10^-)$	?	0	7.300		$3.834 \times 10^{-2}$	$2.415 \times 10^{-1}$	
$^{189}\text{Bi} \rightarrow ^{185}\text{Tl}$	$(9/2^-)$	$(1/2^+)$	5	7.310		$3.038 \times 10^{-1}$	$5.103 \times 10^0$	$7.50 \times 10^0$
$^{190}\text{Bi} \rightarrow ^{186}\text{Tl}$	$(3^+)$	$(7^+)$	4	7.710		$5.982 \times 10^{-3}$	$6.760 \times 10^{-2}$	
$^{191}\text{Bi} \rightarrow ^{187}\text{Tl}$	$(9/2^-)$	$(1/2^+)$	5	6.814	$7.07 \times 10^2$	$2.070 \times 10^1$	$3.200 \times 10^2$	$4.04 \times 10^2$
$^{192}\text{Bi} \rightarrow ^{188}\text{Tl}$	$(3^+)$	$(2^-)$	1	6.410		$1.151 \times 10^2$	$1.542 \times 10^4$	
$^{193}\text{Bi} \rightarrow ^{189}\text{Tl}$	$(9/2^-)$	$(1/2^+)$	5	6.339	$3.19 \times 10^4$	$1.614 \times 10^3$	$2.295 \times 10^4$	$2.87 \times 10^4$
$^{194}\text{Bi} \rightarrow ^{190}\text{Tl}$	$(3^+)$	$2^{(-)}$	1	5.950		$1.336 \times 10^4$	$1.584 \times 10^6$	
$^{195}\text{Bi} \rightarrow ^{191}\text{Tl}$	$(9/2^-)$	$(1/2^+)$	5	5.867	$6.10 \times 10^6$	$2.203 \times 10^5$	$2.866 \times 10^6$	$3.38 \times 10^6$
$^{196}\text{Bi} \rightarrow ^{192}\text{Tl}$	$(3^+)$	$(2^-)$	1	5.471		$3.614 \times 10^6$	$5.490 \times 10^7$	
$^{197}\text{Bi} \rightarrow ^{193}\text{Tl}$	$(9/2^-)$	$1/2^{(+)}$	5	5.242		$4.651 \times 10^8$	$5.340 \times 10^9$	$5.27 \times 10^9$
$^{198}\text{Bi} \rightarrow ^{194}\text{Tl}$	$(2^+, 3^+)$	$2^-$	1	5.071		$6.905 \times 10^8$	$9.697 \times 10^9$	
$^{199}\text{Bi} \rightarrow ^{195}\text{Tl}$	$9/2^-$	$1/2^+$	5	4.967		$1.815 \times 10^{10}$	$1.957 \times 10^{11}$	$2.00 \times 10^{11}$
$^{200}\text{Bi} \rightarrow ^{196}\text{Tl}$	$7^+$	$2^-$	5	4.737		$5.482 \times 10^{11}$	$5.607 \times 10^{12}$	$5.55 \times 10^{12}$
$^{201}\text{Bi} \rightarrow ^{197}\text{Tl}$	$9/2^-$	$1/2^+$	5	4.535		$1.302 \times 10^{13}$	$1.282 \times 10^{14}$	$1.26 \times 10^{14}$
$^{202}\text{Bi} \rightarrow ^{198}\text{Tl}$	$5^+$	$2^-$	3	4.367		$7.132 \times 10^{13}$	$7.831 \times 10^{14}$	$2.00 \times 10^{15}$
$^{203}\text{Bi} \rightarrow ^{199}\text{Tl}$	$9/2^-$	$1/2^+$	5	4.129		$1.763 \times 10^{16}$	$1.562 \times 10^{17}$	$1.36 \times 10^{17}$
$^{204}\text{Bi} \rightarrow ^{200}\text{Tl}$	$6^+$	$2^-$	5	3.991		$2.629 \times 10^{17}$	$2.237 \times 10^{18}$	$1.85 \times 10^{18}$
$^{205}\text{Bi} \rightarrow ^{201}\text{Tl}$	$9/2^-$	$1/2^+$	5	3.730		$6.184 \times 10^{19}$	$4.896 \times 10^{20}$	$3.98 \times 10^{20}$
$^{206}\text{Bi} \rightarrow ^{202}\text{Tl}$	$6^+$	$2^-$	5	3.565		$2.301 \times 10^{21}$	$1.735 \times 10^{22}$	$1.59 \times 10^{22}$
$^{207}\text{Bi} \rightarrow ^{203}\text{Tl}$	$9/2^-$	$1/2^+$	5	3.317		$1.196 \times 10^{24}$	$8.429 \times 10^{24}$	$7.00 \times 10^{24}$
$^{208}\text{Bi} \rightarrow ^{204}\text{Tl}$	$5^+$	$2^-$	3	3.086		$2.389 \times 10^{26}$	$1.800 \times 10^{27}$	$3.91 \times 10^{27}$
$^{209}\text{Bi} \rightarrow ^{205}\text{Tl}$	$9/2^-$	$1/2^+$	5	3.172	$6.00 \times 10^{26}$	$6.745 \times 10^{25}$	$4.492 \times 10^{26}$	$3.22 \times 10^{26}$
$^{210}\text{Bi} \rightarrow ^{206}\text{Tl}$	$1^-$	$0^-$	0	5.071		$3.666 \times 10^8$	$1.470 \times 10^9$	
$^{211}\text{Bi} \rightarrow ^{207}\text{Tl}$	$9/2^-$	$1/2^+$	5	6.786	$1.54 \times 10^2$	$9.248 \times 10^0$	$1.343 \times 10^2$	$2.27 \times 10^2$
$^{212}\text{Bi} \rightarrow ^{208}\text{Tl}$	$1^{(-)}$	$5^+$	5	6.242	$3.73 \times 10^4$	$1.852 \times 10^3$	$2.428 \times 10^4$	$3.40 \times 10^4$
$^{213}\text{Bi} \rightarrow ^{209}\text{Tl}$	$9/2^-$	$(1/2^+)$	5	6.017	$1.41 \times 10^5$	$1.842 \times 10^4$	$2.345 \times 10^5$	$3.25 \times 10^5$
$^{214}\text{Bi} \rightarrow ^{210}\text{Tl}$	$1^-$	$5^+$	5	5.656	$1.45 \times 10^7$	$1.080 \times 10^6$	$1.272 \times 10^7$	$1.66 \times 10^7$
$^{215}\text{Bi} \rightarrow ^{211}\text{Tl}$	$(9/2^-)$	?	0	5.339		$7.819 \times 10^6$	$3.208 \times 10^7$	
$^{216}\text{Bi} \rightarrow ^{212}\text{Tl}$	$(6^-, 7^-)$	?	0	5.134		$1.288 \times 10^8$	$5.060 \times 10^8$	

**TABLE 2.** Coefficient of various terms in the expression for assault frequency.

$a_0$	19.63
$a_1$	0.27
$a_2$	0.59
$a_3$	-0.497
$a_4$	$9.27 \times 10^{-2}$
$a_5$	$8.9 \times 10^{-3}$



**FIGURE 2.** The comparison of the  $\log_{10}(T_{1/2})$  values, evaluated using both the procedures, with the experimental half life values.



**FIGURE 3.** The alpha decay half lives (a) and the Q values (b) plotted against the neutron number of the daughter nuclei.

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