Q is the charge of the magnet
H is the magnetizing field
B_r is the strength of the magnet
V is the volume of the magnet
A is the surface area of the magnet
ℓ is the length of the magnet
r is the distance between the magnet and ball
x is the position of the ball
x_1 is the initial position of the ball
x_2 is the position of the magnet
μ_0 is the permeability of free space
m is the mass of the ball
f is the force applied to the ball
a is the acceleration of the ball
v is the velocity of the ball
v_0 is the initial velocity of the ball
μ is the coefficient of rolling friction
x_0 is the initial position of the ball
$g = -9.8m/s^2$

$$Q = \frac{B_r V}{\mu_0 \ell}$$

$$H = \frac{Q}{4\pi r^2}$$

$$f = \frac{\mu_0 H^2 A}{2}$$

$$\frac{f}{m} = a$$

$$r = |x_2 - x|$$

Substituting $\frac{B_r V}{\mu_0 \ell}$ for Q, and $|x_2 - x|$ for r we get

$$H = \frac{\frac{B_r V}{\mu_0 \ell}}{4\pi(x_2 - x)^2}$$

Substituting this value of H in the equation for f, we get

$$f = \frac{\mu_0 (\frac{\frac{B_r V}{\mu_0 \ell}}{4\pi(x_2 - x)^2})^2 A}{2}$$

Lastly, using the equation $f = ma$,

$$a = \frac{\mu_0 (\frac{\frac{B_r V}{\mu_0 \ell}}{4\pi(x_2 - x)^2})^2 A}{2m}$$

Simplifying, we get

$$a = \frac{AV^2(B_r)^2}{32\pi^2 \ell^2 \mu_0 m} \cdot \frac{1}{(x_2 - x)^4}$$

Noting that this function gives us acceleration as a function of position, we can use an interesting derivation to find velocity as a function of position. We know that

$$\frac{dv}{dt} = a$$

Using this, we can do some interesting manipulations.

$$a = \frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{dx}{dx} = \frac{dx}{dt} \cdot \frac{dv}{dx} = v \cdot \frac{dv}{dx}$$

Manipulating the differential equation $a = v \cdot \frac{dv}{dx}$, we get

$$adx = vdv$$

$$\int adx = \int vdv$$

$$\int (\frac{AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m} \cdot \frac{1}{(x_2 - x)^4})dx = \frac{v^2}{2} + c_1$$

Since $\frac{AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m}$ is constant, we get

$$\frac{AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m} \cdot \int \frac{1}{(x_2 - x)^4}dx = \frac{v^2}{2} + c_1$$

$$\frac{AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m} \cdot \frac{1}{3\left(x_2 - x\right)^3} + C = \frac{v^2}{2}$$

Isolating for v, we get

$$v = \sqrt{\frac{2AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m} \cdot \frac{1}{3\left(x_2 - x\right)^3} + C}$$

When $v = v_0$, $x = x_1$, so we can write the equation

$$v_0 = \sqrt{\frac{2AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m} \cdot \frac{1}{3\left(x_2 - x_1\right)^3} + C}$$

Solving for C, we get

$$C = v_0^2 - \frac{2AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m} \cdot \frac{1}{3\left(x_2 - x_1\right)^3}$$

Substituting this into the original equation for velocity, we get

$$v = \sqrt{\frac{2AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m} \cdot \frac{1}{3\left(x_2 - x\right)^3} + v_0^2 - \frac{2AV^2(B_r)^2}{32\pi^2\ell^2\mu_0 m} \cdot \frac{1}{3\left(x_2 - x_1\right)^3}}$$

With these values for the velocity, we can use Euler's method to estimate position (it is in fact impossible to find position as a function of time, the differential equation has no solution).

NOTE: We assume that the position of the magnet is unchanging for these equations, when in fact, it does change. We can experimentally determine that the position of the magnet moves by a negligible amount, and accounting for the change in position of the magnet would unnecessarily complicate the equations and involve double-integrals.

If we want to consider magnetic attraction to be negligible, we can write an equation for velocity before the incoming ball gets close enough to the magnet such that the magnetic attraction is not negligible.

$$v^2 = v_0^2 + a(x - x_1))$$

$$a = \frac{F}{m} = \frac{-\mu mg}{m} = -\mu g$$

$$v = \sqrt{v_0{}^2 - \mu g(x - x_1)}$$

We can now try to find the specific equations for the collision in the video. From the graphs, we can experimentally determine $v_0 = 0.0767\text{m/s}$, and $\mu = 0.0052$, to get the equation

$$v = \sqrt{0.0767^2 - 0.0052 \cdot 9.8(x - x_1)} = \sqrt{0.00588 - 0.051(x - x_1)}$$

. Since x_1 is 0m in our collision, we can write $v = \sqrt{0.00588 - 0.051x}$

Including magnetic attraction, we now need to try to find the value of the constants. We already know that $v_0 = 0.0767$ m/s. We can measure and get $\ell = 0.002$ m, $A = 0.000233\text{m}^2$, $V = 1.9 \cdot 10^{-7}$ m^3, $m = 0.0142$ g. $x_1 = 0$m, and $x_2 = 0.0773$m. We can determine B_r experimentally, getting $B_r = 1.034$. Substituting all of these values in, we get the equation for velocity to be

$$v = \sqrt{\frac{1.064 \cdot 10^{-7}}{(0.0773 - x)^3} - 0.62}$$