

$Q$  is the charge of the magnet  
 $H$  is the magnetizing field  
 $B_r$  is the strength of the magnet  
 $V$  is the volume of the magnet  
 $A$  is the surface area of the magnet  
 $\ell$  is the length of the magnet  
 $x$  is the position of the incoming ball  
 $x_m$  is the position of the magnet  
 $x_i$  is the initial position of the incoming ball  
 $x_f$  is the final position of the incoming ball  
 $m$  is the mass of the ejected ball  
 $f$  is the force applied to the incoming ball by the magnet  
 $W$  is the total work done by the magnet to the incoming ball  
 $v$  is the velocity of the ejected ball  
 $\mu_0$  is the constant of permeability

Assuming that friction doesn't exist and there is no magnetic force between the ball and magnet (it is almost negligible—this can be experimentally determined), we can find the final velocity using an energy approach. Concretely, the change in the magnetic potential energy between the incoming ball and magnet is equal to the kinetic energy of the ejected ball. Since the change in magnetic potential energy is equal to work done on the ball by the magnet, we can write a few equations

$$KE = \frac{1}{2}mv^2 = PE_i - PE_f = W$$

$$W = \int_{x_m - x_i}^{x_m - x_f} f dx$$

From our equations where we found the velocity of the incoming ball, we know that  $f = \frac{AV^2(B_r)^2}{32\pi^2\ell^2\mu_0} \cdot \frac{1}{(x_m - x)^4}$ . Using this, we get

$$mv^2 = W = \int_{x_m - x_i}^{x_m - x_f} \left[ \frac{AV^2(B_r)^2}{32\pi^2\ell^2\mu_0} \cdot \frac{1}{(x_m - x)^4} \right] dx = \frac{AV^2(B_r)^2}{32\pi^2\ell^2\mu_0} \int_{x_m - x_i}^{x_m - x_f} \left[ \frac{1}{(x_m - x)^4} \right] dx$$

$$\frac{1}{2}mv^2 = \frac{AV^2(B_r)^2}{32\pi^2\ell^2\mu_0} \cdot \left( \frac{1}{3(x_f)^3} - \frac{1}{3(x_i)^3} \right)$$

Isolating for  $v$ , we get

$$v = \sqrt{\frac{AV^2(B_r)^2}{16m\pi^2\ell^2\mu_0} \cdot \left( \frac{1}{3(x_f)^3} - \frac{1}{3(x_i)^3} \right)}$$

NOTE: we don't need to consider rotational energy as we assume friction is negligible. We can substitute in all of the values found to get

$$v = 0.2993 \text{ m/s}$$

We can find the difference of energy (using the integral) to be 0.000202J.