Contents

Distributed Systems

First Lecture

DA RECUPERARE !!!

A Space-Time diagram shows the evolution of a distributed system over time, it is composed of a number of timelines, each associated to a process p_i. Events, occur across this timelines, and are represented by points on the diagram. If an event e_1 is causally related to another event e_2, then e_1 is to the left of e_2 and there is a line connecting them.

Second Lecture

Definitions

We now give a set of formal definitions for very natural concepts in distributed systems.

We define the *history* of a process p_i as the sequence of all events that occur in the timeline of p_i.

$$h_i = \left\langle e_i^1, e_i^2, \dots, e_i^n \right\rangle$$

We can then have the history of the computation, which is a collection of all the histories of all the processes.

$$H = \langle h_1, h_2, \dots, h_n \rangle$$

A *prefix history* is a sequence of events that occur in the timeline of a process up to a certain point.

$$h_i^k = \left\langle e_i^1, e_i^2, \dots, e_i^k \right\rangle$$

A *local state* of process i after event e_i^k is the result of the computation up to that point, and is denoted as σ_i^k.

A *global state* is the collection of all local states of all processes at a certain point in time.

$$\sigma^k = \left\langle \sigma_1^k, \sigma_2^k, \ldots, \sigma_n^k \right\rangle$$

A *run* k' is a total re-ordering of the events such that the internal order of every process is preserved. This means that a *run* allows the interleaving of the events of different processes, but does not break the order of events in the history of a single process.

A *cut* C is a collection of the prefixes of every processes' history up to a certain point.

Consistency

A cut C is said to be *consistent* iff:

$$e \rightarrow e' \land e' \in C \Rightarrow e \in C$$

Or, in other words, if an event is in the cut, then all the events that causally precede it are also in the cut.

By sequentializing the events in a run and extracting a prefix, we can trivially obtain a consistent cut.

Consistency of Intersection

A possible course exercise is, given two cuts C_1 and C_2, to prove that their intersection is also a consistent cut.

Given the definition of consistency accomodated for the intersection of two cuts:

$$e \rightarrow e' \land e' \in C_1 \cap C_2 \Rightarrow e \in C_1 \cap C_2$$

We know that if e' is in the intersection, then it is in both C_1 and C_2. By the definition of consistency, we know that e must be in both C_1 and C_2, and therefore in their intersection.

Formally:

$$e \rightarrow e' \land e' \in C_1 \rightarrow e \in C_1$$

$$e \rightarrow e' \land e' \in C_2 \rightarrow e \in C_2$$

Therefore:

$$e \to e' \land e' \in C_1 \cap C_2 \to e \in C_1 \land e \in C_2 \to e \in C_1 \cap C_2$$

Consistency of Union

Given two consistent cuts C_1 and C_2, we can prove that their union is also a consistent cut.

Given the definition of consistency accomodated for the union of two cuts:

$$e \to e' \land e' \in C_1 \cup C_2 \Rightarrow e \in C_1 \cup C_2$$

Then, formally:

$$e \to e' \land e' \in C_1 \to e \in C_1$$

$$e \to e' \land e' \in C_2 \to e \in C_2$$

Therefore:

$$e \to e' \land e' \in C_1 \cup C_2 \to e \in C_1 \lor e \in C_2 \to e \in C_1 \cup C_2$$

Run Reconstruction

Assume that we have a set of processes $P = \{p_1, p_2, p_3, \}$, and a monitor process p_0 that is responsible for detecting deadlocks.

Everytime that an event is generated, its responsible process sends a message to the monitor, which then updates its local state.

The monitor process can then reconstruct the run by aggregating the local process states into a global state.

Assuming asynchrony and unbounded message delays, the monitor process does not reconstruct a run, since the order of the individual processes can be inverted.

If we assume a FIFO message delivery, then the monitor process can reconstruct a run. This can be trivially achieved by having a sequence number, in a system such as TCP.

This does *not* recover consistency, since the order of separate processes can still be inverted. However, this level of *local consistency* is sufficient for deadlock detection.

Simplifying Assumptions for Consistency

Assume that I have:

1. A finite δ_i notification delay on each process p_i.
2. A global clock C that is synchronized with all processes. We call an idealized version of this clock *Real Clock* (RC).

Then, the monitor process can offline-reconstruct a run by using the global clock to order the events.

Online-reconstruction can be achieved (and is preferred) by exploiting the additional δ_i information. Once we receive a notification, we await time $\max \delta_i$ before committing the event to the global state, in the meantime, we store the event in a buffer that is ordered by the global clock.

Real Clock Property

The clock property relates the *happens before* relation of messages to the real time of the system.

$$e \to e' \Rightarrow TS(e) < TS(e')$$

We can obtain a trivial clock by designing a system as such:

1. Each process has associates a sequence number $k \in \mathbb{N}$ to each event.
2. Each event i, which receives a message m from event j, has a sequence number given by the formula $\max(k_i, k_j) + 1$.

This gives us the clock property *by construction*, since each event has a sequence number that is always greater than the sequence number of the event that caused it and of any event that it is casually related to.

This device is called a *Lamport Clock*, or a *Logical Clock*.

History of an event

Given an event e, we can define its history as the sequence of events that causally precede it. These are all the events that the monitor process must have received before the event e is committed to the global state.

$$\Theta(e) = \{e' \mid e' \to e\}$$

This is a *consistent* cut.

Opaque detection of consistent cuts

Given a notification about an event, for example, e_5^4, the monitor process can ensure that e_1^4 can be committed to global state by checking that for every process, we have received a notification with lamport clock ≥ 4.

We might have some processes that consistently lag behind, and therefore do not reach the lamport clock in a timely manner. We can trivially solve this by sending a liveness check to all processes for which a sufficiently high lamport number has not been received. If the process is alive but not working, it will re-adjust its lamport clock and send a notification.

This will not impact throughput, since the monitor process is not a bottleneck, and the liveness check is a very lightweight operation, that is performed only when necessary.

Building a Better Clock

We can build a *better clock*, that is, one with the enhanced property:

$$e \to e' \iff TS(e) < TS(e')$$

This is called a *Vector Clock*.

It works by communicating the history of the current event in a compact way, we do this by sending a vector of length n where n is the number of processes in the system. The vector's entries are updated to be the highest lamport number detected for that relative process, either from a message that has been received, or from the local lamport clock (in the case of the current process).

This allows us to check whether:

$$VC(e) \subseteq VC(e')$$

$$e \to e' \iff VC(e) \subseteq VC(e')$$

When messages are sent, the vector clock is attached to the message, and the receiving process updates its vector clock by taking the maximum of the two vectors, and summing 1 to the current process.

The monitor process p_0 keeps track of its own vector, where each entry is the highest lamport number that it has received from each process. When it receives a notification, if the vector clock is consistent with the monitor's vector clock, then the event is committed to the global state.

Since vectors are ***not*** sets, we have to use component-wise comparison to check for causality.

$$VC(e) < VC(e') \iff \forall i \in \{1, 2, \dots, n\} \quad VC(e)_i < VC(e')_i$$

So we are back to the original definition of the happens before relation.

Third Lecture

We now discuss some more advanced vector clock properties.

Vector Clock Properties

Strong Clock Property

Assume we have two events e and e' . Then, we have that:

$$e \rightarrow e' \iff VC(e) \leq^{\star} VC(e')$$

Where $\leq^s tar$ is a ***partial order*** relation, such as:

$$\forall VC(e_i)[k] \leq VC(e_j)[k] \quad \wedge \quad \exists k' VC(e_i)[k'] < VC(e_j)[k']$$

Meaning that all the elements of the vector clock of e are less than or equal to the corresponding elements of the vector clock of e', and at least one element is strictly less, so that we have a *strict* partial order.

Retrieving History Cardinality

Given a vector clock $VC(e)$, we can retrieve the number of events that causally precede it by taking the sum of all the elements of the vector clock.

$$|\Theta(e)| = \sum_{i=1}^{n} VC(e)[i]$$

Retrieving Causality

Given two events e_i and e_j, we can check whether they are causally related by checking the vector clocks.

$$e_i \rightarrow e_j \iff VC(e_i)[i] \leq VC(e_j)[i]$$

Existance of a Causal Event

We want to check whether

$$\exists e_k : \neg(e_k \rightarrow e_i) \wedge \left(e_k \rightarrow e_j\right)$$

This is true if and only if:

$$\exists k : VC(e_i)[k] < VC(e_j)[k]$$

Meaning that there is a certain event that ticked the clock of e_j but not the clock of e_i.

Snapshots

Now we assume that the processes do ***not*** send notifications, rather, the monitoring process p_0 polls processes with a snapshot request.

This was already discussed in the first lecture, and we know that what we retrieve cannot retrieve a consistent cut.

We can change the protocol to improve this method. Now p_0 sends a snapshot request to all processes, however, when a process receives a snapshot request, it has to broadcast it to any other process.

Only requests from p_0 are broadcasted, so that we do not flood the network with snapshot requests.

When a process receives a snapshot request, it sends a snapshot of its local state to p_0.

With this behavior, under the reasonable FIFO-channel assumption, we can reconstruct a consistent cut.

Proof of Consistency

Assume that we have e_i and e_j such that $e_i \to e_j$, and that e_i is in the snapshot C, we want to prove that e_j is also in the snapshot.

By contradiction, assume that $e_i \notin C$, then p_i received the snapshot request before e_i, but then, since the channels are FIFO, and e_j was spawned by a received message from p_i, then the broadcasted snapshot request sent to p_j reached p_j before e_j was spawned, and therefore e_j is also ***not*** in the snapshot.

This protocol is known as the *Chandy-Lamport* protocol.