PROPOSITIONAL LOGIC REFERENCE SHEET

Truth Tables for the Logical Operators

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	$\sim p$
T	F
F	T

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Logical Equivalences

1. Commutative Laws: $p \wedge q \equiv q \wedge p$
   $p \vee q \equiv q \vee p$

2. Associative Laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
   $(p \vee q) \vee r \equiv p \vee (q \vee r)$

3. Distributive Laws: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
   $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

4. Identity Laws: $p \wedge \mathbf{t} \equiv p$
   $p \vee \mathbf{c} \equiv p$

5. Negation Laws: $p \vee \sim p \equiv \mathbf{t}$
   $p \wedge \sim p \equiv \mathbf{c}$

6. Double Negation Law: $\sim(\sim p) \equiv p$

7. Idempotent Laws: $p \wedge p \equiv p$
   $p \vee p \equiv p$

8. Universal Bound Laws: $p \vee \mathbf{t} \equiv \mathbf{t}$
   $p \wedge \mathbf{c} \equiv \mathbf{c}$

9. De Morgan's Laws: $\sim(p \wedge q) \equiv \sim p \vee \sim q$
   $\sim(p \vee q) \equiv \sim p \wedge \sim q$

10. Negations of **t** and **c**: $\sim\mathbf{t} \equiv \mathbf{c}$
    $\sim\mathbf{c} \equiv \mathbf{t}$

More Equivalences for Conditionals

- Conditional written as "or" statement: $p \to q \equiv \sim p \vee q$
- Biconditional: $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

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Forms of Conditional Statements

- Contrapositive of $p \rightarrow q$: $\sim q \rightarrow \sim p$ (logically equivalent)
- Converse of $p \rightarrow q$: $q \rightarrow p$ (not logically equivalent)
- Inverse of $p \rightarrow q$: $\sim p \rightarrow \sim q$ (not logically equivalent)

Necessary and Sufficient Conditions

- "r is a **sufficient condition** for s" means "$r \rightarrow s$"
- "r is a **necessary condition** for s" means "$\sim r \rightarrow \sim s$" (or "$s \rightarrow r$" by contraposition)
- "r is a **necessary and sufficient condition** for s" means "$r \leftrightarrow s$"

Rules of Inference

Modus Ponens

$p \rightarrow q$
p
$\therefore\ q$

Modus Tollens

$p \rightarrow q$
$\sim q$
$\therefore\ \sim p$

Generalization (a.k.a. $\vee$ Introduction)

p
$\therefore\ p \vee q$

q
$\therefore\ p \vee q$

Specialization (a.k.a. $\wedge$ Elimination)

$p \wedge q$
$\therefore\ p$

$p \wedge q$
$\therefore\ q$

Elimination (better, $\vee$ Elimination)

$p \vee q$
$\sim q$
$\therefore\ p$

$p \vee q$
$\sim p$
$\therefore\ q$

Transitivity

$p \rightarrow q$
$q \rightarrow r$
$\therefore\ p \rightarrow r$

Cases

$p \vee q$
$p \rightarrow r$
$q \rightarrow r$
$\therefore\ r$

Contradiction

$\sim p \rightarrow \mathbf{c}$
$\therefore\ p$