Models of SW Systems	**Name: NAME**
Assignment NUMBER	**Due: DATE**

1 Exercise 1

Consider the game described in Chapter 2 of GWC10. Suppose that the container starts out with N balls.

(a) List five aspects of the real world that were *not* represented in our formal model.

(Answer here)

(b) How many "turns" will it take for the game to stop? Briefly explain why. (Hint: use the $\lfloor\ \rfloor$ notation to express your solution, if required.)

(Answer here)

(c) What is the largest number of extra black balls needed, and what configuration of the container causes this number to be required? Assume that when two black balls are taken out of the container one is put back into the container and the other into the stock of extra balls.

(Answer here)

(d) Argue that the game stops.

(Answer here)

2 Lambda Language

Consider a language with alphabet $\{\lambda, \bullet, (,), x, y, z\}$ and syntax

$$\begin{aligned} \textit{expression} &= \textit{variable name} \mid \textit{expression}, \textit{expression} \\ &\mid \text{“}\lambda\text{”}, \textit{variable name}, \text{“}\bullet\text{”}, \textit{expression} \\ &\mid \text{“(”}, \textit{expression}, \text{“)”}; \end{aligned}$$

$$\textit{variable name} = \text{“}x\text{”} \mid \text{“}y\text{”} \mid \text{“}z\text{”};$$

Are the following wffs of the language? For those that are not briefly explain why.

(a) $\lambda\, x \bullet yz$
(Answer here)

(b) $\lambda \bullet x\, \lambda \bullet y$
(Answer here)

(c) $\lambda\, y \bullet x \bullet z$
(Answer here)

(d) $\lambda\, x \bullet x(yz)$
(Answer here)

(e) $\lambda\, x \bullet \lambda\, y \bullet xyz$
(Answer here)

3 Stars, Derivation

Using the *Stars* formal system of Example 3.4 from Chapter 3 of GWC09 formally show that

$*\diamond{**}\circ{*****} \vdash *\diamond{****}\circ{*******}$

(Answer here)

4 Stars, Incompleteness

In Chapter 3 of GWC09 the *Stars* formal system of Example 3.4 was interpreted as a system for adding certain positive integers. For example, $1 + 3 = 4$ could be proved a theorem of *Stars*.

(a) Augment *Stars* so that you can prove statements such as $3 + 4 = 7$ and $15 + 2 = 17$ to be theorems. You need to handle only expressions involving the addition of positive integers. NOTE: Your answer should include the alphabet, syntax, inference system, and interpretation.

(Answer here)

(b) Prove that $3 + 4 = 7$ is a theorem of the augmented system. (Note: do not forget to provide an interpretation of your result at the end of the derivation process.)

(Answer here)