Homework #1: Formal Models and Formal Systems

Garlan   Due: August 31, 2016

NOTE 1: This homework must be completed using a LaTex document compiler. So first you will need to install LaTeX and an editor for it. The course web site has instructions on how to do this. It also has a template that you should use to create your answers, as well as examples of LaTex usage that you might find useful when creating your answers.

NOTE 2: For some of these problems (and in general for the homework in this class), there may be more than one correct answer.

1. Consider the game described in Chapter 2 of GWC10. Suppose that the container starts out with N balls.

   (a) List five aspects of the real world that were *not* represented in our formal model.

   (b) How many "turns" will it take for the game to stop? Briefly explain why. (Hint: use the $\lfloor \ \rfloor$ notation to express your solution, if required.)

   (c) What is the largest number of extra black balls needed, and what configuration of the container causes this number to be required? Assume that when two black balls are taken out of the container one is put back into the container and the other into the stock of extra balls.

   (d) Argue (informally) that the game stops.

2. Consider a language with alphabet $\{\lambda, \bullet, (,), x, y, z\}$ and syntax

   $$expression \quad = variable\ name \mid expression, expression$$
   $$\mid\ \text{``}\lambda\text{''}, variable\ name, \text{``}\bullet\text{''}, expression$$
   $$\mid\ \text{``(''}, expression, \text{``)''};$$
   $$variable\ name = \text{``}x\text{''}\mid\text{``}y\text{''}\mid\text{``}z\text{''};$$

   Are the following wffs of the language? For those that are not briefly explain why.

   (a) $\lambda\, x \bullet yz$

   (b) $\lambda \bullet x\, \lambda \bullet y$

   (c) $\lambda\, y \bullet x \bullet z$

   (d) $\lambda\, x \bullet x(yz)$

   (e) $\lambda\, x \bullet \lambda\, y \bullet xyz$

3. Using the *Stars* formal system of Example 3.4 from Chapter 3 of GWC10 formally show that

   $$*\diamond**\circ***** \vdash *\diamond****\circ*******$$

4. In Chapter 3 of GWC10 the *Stars* formal system of Example 3.4 was interpreted as a system for adding certain positive integers. For example, $1 + 3 = 4$ could be proved a theorem of *Stars*.

(a) Augment *Stars* so that you can prove statements such as $3 + 4 = 7$ and $15 + 2 = 17$. You need to handle only expressions involving the addition of positive integers.

NOTE: Your answer should include the alphabet, syntax, inference system, and interpretation.

(b) Show that that $3 + 4 = 7$ is a theorem of the augmented system. (Note: do not forget to provide an interpretation of your result at the end of the derivation process.)