

## Homework #7: Invariants and Introduction to Z

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### Part 1: Invariants

Consider the Diverging Counter example of Chapter 10 of GWC09. Prove that  $x + y = 0$  is an invariant of the *DivergingCounter* state machine.

DivergingCounter = (  
 $[x, y : \mathbb{Z}]$ ,  
 $\{s : [x, y : \mathbb{Z}] \mid s(x) = -s(y)\}$ ,  
 $\{poke(i : \mathbb{Z})\}$ ,  
 $\delta ==$

$$\begin{array}{l}
 poke(i : \mathbb{Z}) \\
 \mathbf{pre} \ i > 0 \\
 \mathbf{post} \ x' = x + i \wedge y' = y - i
 \end{array}$$

).

1. Base case: show that  $\theta$  holds in the initial state.

Here there's only one initial state:

Proof:

(NOTE: In your proof use style C (in Section 10.1.1) of reasoning about invariants and a similar degree of formalism as in the lecture on this topic.)

### Part 2: Z

NOTE: For this part of the assignment you must format your answers using L<sup>A</sup>T<sub>E</sub>X and typecheck the answers using *fuzz*, Z-EVES, or the Community Z tools.

Write a Z specification of the following system. Your specification should include sufficient explanatory prose to make it easily understandable. (The prose is important—answers with little or no prose will receive a low grade.)

A teacher wants to keep a register of students in the class, and to record which students have completed their homework.

Let the given set *Student* represent the set of all students who might ever be enrolled in a class:

$[Student]$

Specify each of the following:

1. The state space for a register.

HINT: use two sets of students:

<i>Register</i>
<i>enrolled</i> : $\mathbb{P} \textit{Student}$
<i>completed</i> : $\mathbb{P} \textit{Student}$
...

2. An operation to enroll a new student.
3. The initial state(s) for your state space.
4. An operation to record that a student (already enrolled in class) has completed the homework.
5. An operation to inquire whether a student (who must be enrolled) has completed the homework (the answer is to be either 'Yes' or 'No').
6. A robust version of the system. (Be sure to use the schema calculus, as illustrated by the class lecture and the paper by Spivey on Z.)