

Homework #3: Proof

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INSTRUCTIONS: before completing your homework, make sure you have considered the following:

- You are allowed to use only primitive inference rules.
- Remember to include vertical lines to represent the scope any assumptions.
- Remember that all assumptions in your proofs must be discharged.
- Double-check that your line references are correct when applying inference rules.

I. Proof: Propositional Logic

In this section *use only the primitive inference rules* of propositional calculus.

1. Provide derivations for each of the following, using natural deduction:

- a. $p \wedge q, p \Rightarrow s, q \Rightarrow t \vdash s \wedge t$
1. $p \wedge q$ premise
 2. $p \Rightarrow s$ premise
 3. $q \Rightarrow t$ premise
 4. p \wedge -elim, 1
 5. s modus ponens, 2,4
 6. q \wedge -elim, 1
 7. t modus ponens, 3,6
 8. $s \wedge t$ \wedge -intro, 5,7

- b. $q \Rightarrow \neg p, p \wedge q \vdash r$
1. $q \Rightarrow \neg p$ premise
 2. $p \wedge q$ premise
 3. q \wedge -elim, 2
 4. $\neg p$ modus ponens, 1
 5. p \wedge -elim, 2
 6. $\neg r$ assumption 1
 7. $\neg p$ Copy from 4 1
 8. p Copy from 5 1
 4. r \neg -elim, 6-8

- c. $p \wedge q \vdash p \Rightarrow q$
1. $p \wedge q$ premise
 2. p \wedge -elim, 1
 3. q \wedge -elim, 1
 4. $p \Rightarrow q$ \Rightarrow -intro, 2,3

- d. $\neg\neg q \vdash q \vee r$
1. $\neg\neg q$ premise
 2. $\neg q$ $\neg - elim, 1$
 3. q $\neg - elim, 2$
 4. $q \vee r$ $\vee - intro, 3$
- e. $p \Rightarrow (q \wedge r), \neg p \Rightarrow r, p \vee \neg p \vdash r$
1. $p \Rightarrow (q \wedge r)$ premise
 2. $\neg p \Rightarrow r$ premise
 3. $p \vee \neg p$ premise
 4. p assumption 1
 5. $q \wedge r$ \Rightarrow -elim, 1, 4 1
 6. r $\wedge - elim, 5$

II. Proof: Predicate Logic

In this section *use only the primitive inference rules* of predicate calculus.

2. Show using natural deduction:

- a. $\forall x : T \bullet P(x) \wedge Q(x) \dashv\vdash (\forall x : T \bullet P(x)) \wedge (\forall y : T \bullet Q(y))$
- 1.
 - 2.
 - 3.
 - 4.
- b. $\exists x : T \bullet P(x) \vee Q(x) \dashv\vdash (\exists x : T \bullet P(x)) \vee (\exists x : T \bullet Q(x))$
- 1.
 - 2.
 - 3.
 - 4.

(NOTE: $p \dashv\vdash q$ is a shorthand for “ $p \vdash q$ and $q \vdash p$.” That is, for $p \dashv\vdash q$ you need to show two separate derivations: one for $p \vdash q$ and another for $q \vdash p$.)