CMU 17-651 Models of Software Systems Fall 2016

Homework #3: Proof

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INSTRUCTIONS: before completing your homework, make sure you have considered the following:

- You are allowed to use only primitive inference rules.
- Remember to include vertical lines to represent the scope any assumptions.
- Remember that all assumptions in your proofs must be discharged.
- Double-check that your line references are correct when applying inference rules.

I. Proof: Propositional Logic

In this section *use only the primitive inference rules* of propositional calculus.

1. Provide derivations for each of the following, using natural deduction:

a. $(p \land q) \land r \vdash p \land (q \land r)$

b. $p \land q, p \Rightarrow s, q \Rightarrow t \vdash s \land t$

1.	$p \land q$	premise
2.	$p \Rightarrow s$	premise
3.	$q \Rightarrow t$	premise
4.	p	$\land$-elim, 1
5.	s	modus ponens, 2,4
6.	q	$\land$-elim, 1
7.	t	modus ponens, 3,6
8.	$s \land r$	$\land$-intro, 5,7

c. $q \Rightarrow \neg p, p \land q \vdash r$

1.	$q \Rightarrow \neg p$	premise	
2.	$p \land q$	premise	
3.	q	$\land -elim, 2$	
4.	$\neg p$	modus ponens, 1	
5.	p	$\land -elim, 2$	
6.	$\neg r$	assumption	l
7.	$\neg p$	Copy from 4	l
8.	p	Copy from 5	l
4.	r	$\neg -$ *elim*, 6-8	

d. $p \land q \vdash p \Rightarrow q$

1.	$p \land q$	premise
2.	p	$\land -elim$,1
3.	q	$\land -elim$,1
4.	$p \Rightarrow q$	$\Rightarrow -intro$,2,3

e. $\neg\neg q \vdash q \vee r$

1.	$\neg\neg q$	premise
2.	$\neg q$	$\neg - elim, 1$
3.	q	$\neg - elim, 2$
4.	$q \vee r$	$\vee - intro, 3$

f. $p \Rightarrow (q \wedge r), \neg p \Rightarrow r, p \vee \neg p \vdash r$

II. Proof: Predicate Logic

In this section *use only the primitive inference rules* of predicate calculus.

2. Show using natural deduction:

   a. $\forall\, x : T \bullet P(x) \wedge Q(x) \dashv\vdash (\forall\, x : T \bullet P(x)) \wedge (\forall\, y : T \bullet Q(y))$
   b. $\exists\, x : T \bullet P(x) \vee Q(x) \dashv\vdash (\exists\, x : T \bullet P(x)) \vee (\exists\, x : T \bullet Q(x))$

   (NOTE: $p \dashv\vdash q$ is a shorthand for "$p \vdash q$ and $q \vdash p$." That is, for $p \dashv\vdash q$ you need to show two separate derivations: one for $p \vdash q$ and another for $q \vdash p$.)