CMU 17-651 | Models of Software Systems | Fall 2018

Homework #2: Logic

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I. Propositional Logic

1. Construct a truth table for each of the following. Include in your tables intermediate expressions needed to construct the final truth tables column.

   a. $p \wedge (p \vee q)$

p	q	$(p \vee q)$	$p \wedge (p \vee q)$
false	false	false	false
true	false	true	true
false	true	true	false
true	true	true	true

   b. $\neg p \wedge (p \vee (q \Rightarrow p))$

p	q	$(q \Rightarrow p)$	$(p \vee (q \Rightarrow p))$	$\neg p \wedge (p \vee (q \Rightarrow p))$
false	false	true	true	true
true	false	true	true	false
false	true	false	false	false
true	true	true	true	false

   c. $(p \Rightarrow q) \Rightarrow (\neg p \vee q)$

p	q	$(p \Rightarrow q)$	$(\neg p \vee q)$	$(p \Rightarrow q) \Rightarrow (\neg p \vee q)$
false	false	true	true	true
true	false	false	false	true
false	true	true	true	true
true	true	true	true	true

2. Which of the above sentences are:

   i. valid?: c
   Because c is true for all values of p and q.

   ii. satisfiable?: a, b, c
   Because are true under at least one interpretation of atomic propositions.

   iii. contingent?: a, b
   Because they are neither tautologies nor contradictions.

   iv. inconsistent?: none
   Because we have no sentences that are false for all propositions of q and p.

   Briefly explain why.

3. Demonstrate using truth tables that the following sentences have the same meaning. Include intermediate expressions, as above.

   - $p \Rightarrow q$
   - $\neg(p \wedge \neg q)$

col1	col2	col3	col4	col5
p	q	$(p \wedge \neg q)$	$\neg(p \wedge \neg q)$	$p \Rightarrow q$
false	false	false	true	true
true	false	true	false	false
false	true	false	true	true
true	true	false	true	true

The truth table for col4 and col5 are equal, hence, we demonstrated that $\neg(p \wedge \neg q)$ has the same meaning than $p \Rightarrow q$ for all values of p and q.

II. Predicate Logic

4. Which occurrences of the variables x and y are free and which are bound in each of the following? Briefly explain why.
   NOTE: Recall that a variable may be both bound and free in the same sentence. In such cases, explain where in the sentence the variable is bound, and where it is free.
   a. $(\exists\, y : N \bullet y > 2) \wedge (\forall\, x : N \bullet x + 1 > x)$
      Both y and x are bounded, y by $\exists\, y : N$ and x by $\forall\, x : N$
   b. $x = 2 * y$
      x and y are free because there are no quantifiers.
   c. $(\exists\, y : N \bullet y > 2) \wedge (\forall\, x : N \bullet x > y)$
      y is bounded in $(\exists\, y : N \bullet y > 2)$ but is not in $(\forall\, x : N \bullet x > y)$.
      x is bounded where it appears.
   d. $\forall\, x : N \bullet ((\exists\, y : N \bullet y > x) \wedge x = 2 * y)$
      x is bounded by the initial $\forall\, x : N$ for the whole expression, but y is free in $x = 2 * y$.
5. Translate the following sentences into predicate logic (with equality), using the translation key provided.
   NOTE: You may only use the standard universal and existential quantifiers ($\forall$ and $\exists$). Do *not* use the unique existential quantifier ($\exists!$).

E:	the set of elephants
A:	the set of animals
$G(x)$:	x is green
$E(x)$:	x is an elephant
$N(x, y)$:	the name of x is y

   a. Some elephants are green.
      $\exists\, x : E \bullet G(x)$
   b. All elephants are green.
      $\forall\, x : E \bullet G(x)$
   c. If an animal is green, it is an elephant.
      $\forall\, x : A \bullet G(x) \Rightarrow E(x)$
   d. No green animal is an elephant.
      $\forall\, x : A \bullet G(x) \Rightarrow \neg E(x)$
   e. There is exactly one green elephant.
      $\exists\, x : E \bullet G(x) \wedge \forall\, y : E \bullet G(y) \Rightarrow x = y$
   f. There is *exactly one* green elephant, and his name is James.
      $\exists\, x : E \bullet G(x) \wedge \forall\, y : E \bullet G(y) \Rightarrow x = y \wedge N(x, James)$
6. Translate the following sentences into predicate logic (with equality), using the translation key provided.
   NOTE: You may only use the standard universal and existential quantifiers ($\forall$ and $\exists$). Do *not* use the unique existential quantifier ($\exists!$).

S:	the set of students
T:	the set of topics, which has *logic* and *models* as elements
$MSE(s)$:	s is an MSE student
$Likes(s,t)$:	student s likes topic t

a. Some MSE students like logic.
$\exists\, s : S \bullet MSE(s) \land Likes(s, logic)$

b. MSE students like logic.
$\forall\, s : S \bullet MSE(s) \Rightarrow Likes(s, logic)$

c. MSE students like logic, and only logic.
$\forall\, s : S \bullet (MSE(s) \Rightarrow (Likes(s, logic))) \land (\forall\, t : T \bullet (t \neq logic \Rightarrow \neg Likes(s,t)))$

d. No MSE student likes logic.
$\forall\, s : S \bullet MSE(s) \Rightarrow \neg Likes(s, logic)$

e. If an MSE student likes logic then he/she likes Models.
$\exists\, s : S \bullet (MSE(s) \land Likes(s, logic)) \Rightarrow Likes(s, Models)$

f. Exactly one MSE student likes Models.
$\exists\, s : S \bullet MSE(x) \land \forall\, y : S \bullet (MSE(y) \land Likes(y, Models)) \Rightarrow s = y$

7. In this class we will be creating various models of an infusion pump. An infusion pump is a device used in hospitals to feed fluids intravenously to patients through one of several "infusion lines." Each line is a physical tube connected to a patient.

   Consider the following excerpt from a description of a typical pump provided to us by the Food and Drug Administration:

   A. An infusion line may become pinched causing the flow to be blocked. This will be recognized by the pump as an occlusion and will cause the pump to alarm.
      i. The mitigation is to straighten the line and re-start the pump.
      ii. Caregiver may silence the alarm during the procedure.

   B. The infusion line may become plugged. The pump will recognize an occlusion and alarm.
      i. The mitigation is to clear the line and re-start the pump.
      ii. Caregiver may silence the alarm during the procedure.

   C. Electrical failure may occur causing the pump to switch to battery operation.
      i. Pump will switch over to battery power and notify the caregiver visually.
      ii. Switch may not occur if the battery is not properly charged.

   Questions:

   a. Define some sets and predicates appropriate to this domain (similar to the elephant problem above).
   P: the set of pumps .
   $Pinched(x)$: x is pinched.
   $Plugged(x)$: x is plugged.
   $BatteryCharged(x)$: x battery is charged.
   $AlarmOn(x)$: x is alarm on.
   $PoweredByGrid(x)$: x is the device being powered by the electrical grid.
   $AlarmSilenced(x)$: x if the nurse overrides the alarm manually.
   $On(x)$: x is the device is powered or not.

   b. Using the sets and predicates you defined express the following statements in predicate logic:

i.. An alarm will sound whenever the line is "pinched" or "plugged."
$\forall\, p : P \bullet (\mathit{Pinched}(p) \vee \mathit{Plugged}(p)) \Rightarrow \mathit{AlarmOn}(p)$

ii.. If there is an electrical failure the battery power will be on unless the battery is not properly charged.
$\forall\, p : P \bullet (\mathit{PoweredByGrid}(p) \vee \mathit{BatteryCharged}(p)) \Rightarrow \mathit{On}(p)$

c. Does your collection of predicates allow you to say "The alarm will continue to sound until the care giver turns it off." Why or why not?
Yes because I added the $\mathit{AlarmSilenced}(x)$ predicate to express if the care giver has turned the alarm off:
$\forall\, p : P \bullet ((\mathit{Pinched}(p) \vee \mathit{Plugged}(p) \wedge \neg \mathit{AlarmSilenced}(p)) \Rightarrow \mathit{AlarmOn}(p)$