Digital Communications and Laboratory
Fourth Homework

Faccin Dario, Santi Giovanni

Transmission system

In the following, two different scenarios are developed: a single carrier transmission using Receiver (b) of homework 3 and an OFDM transmission with M=512 sub-channels. Both implementations are firstly simulated using coding to improve the error correction capability of the system; in particular, a *Low-density parity check* LDPC encoding is applied to the initial data stream in order to allow the receiver to detect codewords which do not belong to the code $\mathcal{C}$ and eventually to correct them with the use of an LDPC decoder. These functions are provided by the Matlab toolbox. Finally, the same transmission is simulated without coding to appreciate the difference in terms of P_{bit} for varying SNR.
In this report we decided to treat only the implementation of the encoded transmission, since the uncoded one follow by simply removing the LDPC encoder and decoder.

Symbols generation

A data stream of bits b_l is generated using a PN sequence of length $2^{20}-1$ repeated once. This is encoded by an LDPC encoder provided by the Matlab toolbox, which reads sequences of 32400 bits and generates codewords c_m of length 64800 bits. An interleaver is used in order to reduce the probability of having *burst* errors, then a bitmap maps couples of bits into a QPSK constellation with *Gray coding.* This scenario is given in Figure [9].

Single Carrier

The channel for the single carrier transmission is that implemented in homework 3.
The receiver filter consists of a filter g_M matched to the transmission filter q_c followed by a *Decision Feedback Equalizer* (DFE) filter. The matched filter is simply computed as $g_m(t) = q_c^*(t_0 - t)$, where t_0 is the timing phase. Since the global impulse response of the system $h = g_c * g_M$ at the input of c is defined @$\frac{T}{4}$, a downsampling of a factor 4 is required between the output of h and the input of c. The equations used to compute both c and b filter have already been described in homework 3. The scenario just described is given in Figure [1].

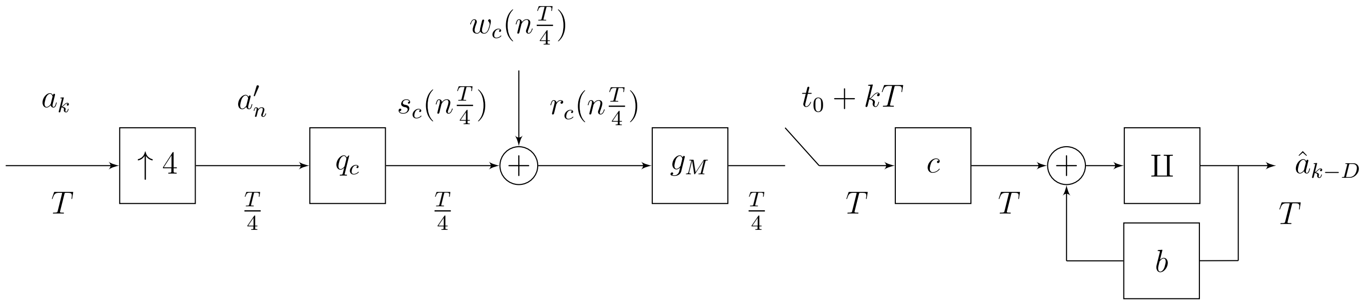

Figure 1. Model for the SC channel.

OFDM

Orthogonal frequency division multiplexing is an efficient modulation technique by which blocks of $\mathcal{M}$ symbols are transmitted in parallel over $\mathcal{M}$ subchannels, using $\mathcal{M}$ modulation filters with frequency resonses $\mathcal{H}_i(f)$, $i = 1, 2, \ldots, \mathcal{M}-1$. The $\mathcal{M}$ input symbols at the k_{th} modulation interval are represented by the vector

$$\mathbf{a}_k = [a_k[0], \ldots, a_k[\mathcal{M}-1]]^T \tag{1}$$

where $a_k[i] \in \mathcal{A}[i]$, $i = 0, \ldots, \mathcal{M}-1$.
We implemented the *Discrete Multitone* DMT version in which the transmit and receiver filter backs use a prototipe filter with impulse response given by

$$h_n = \begin{cases} 1 & \text{if } 0 \leq n \leq \mathcal{M}-1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

In this way the impulse response of the polyphase components of the prototype filter are simply $\{h_n^(l)\} = \{\delta_n\}$ for $l = 0, 1, \ldots, \mathcal{M}$. As a consequence, because the frequency responses are constants, we obtain directly the transmit signal by applying a P/S conversion at the output of the IDFT. To equalise the channel, we implemented the baseband equivalent system given in Figure [2]. The channel is given in Figure [3] .

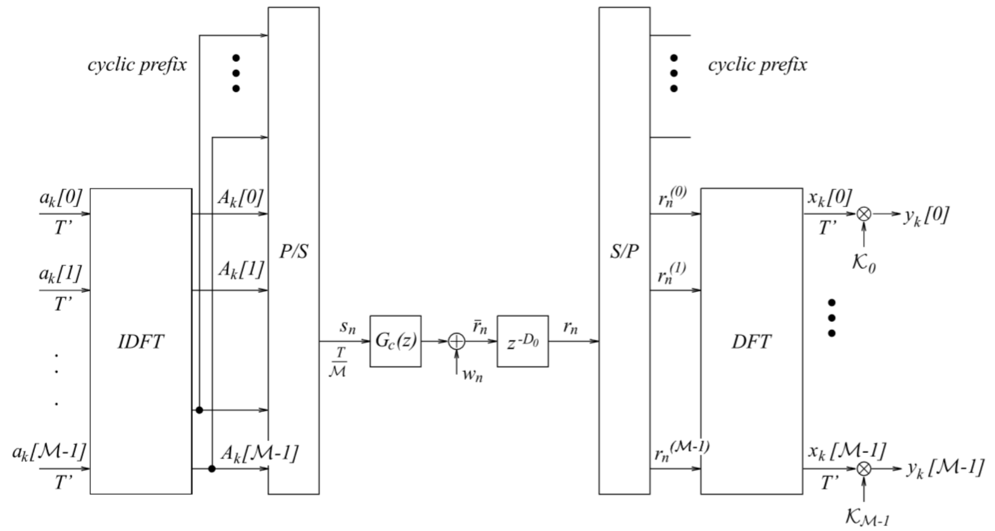

Figure 2. Block diagram of a DMT system with cyclic prefix and frequency-domain equalizer.

We exploited the concept of circular convolution to obtain a convolution in the time domain as a product of finite length vectors in the frequency domain. To do so, we extended the block of samples $\mathbf{A}_k$ by repeating $N_{px} = 18$ elements before transmitting through the channel. The value of N_{px} was chosen equal to the support of the equivalent channel impulse response $h(mT_{OFDM})$. In particular, after the modulation, each block of samples is cyclically extended by copying the N_{px} sample $A_k[\mathcal{M} - N_{px}], \ldots, A_k[\mathcal{M}-1]$ in front of the block. After the P/S conversion, where the N_{px} samples of the cyclic extension are the first to be sent, the $N_{px} + \mathcal{M}$ samples are transmitted over the channel. At the receiver, blocks of samples of length $N_{px} + \mathcal{M}$ are taken; the boundaries between blocks are set such that the last $\mathcal{M}$ samples depend only on the elements of only one cyclically extended block of samples. The first N_{px} samples of a block are thus discarded.

The resulting vector $\mathbf{r}_k$ can be expressed as

$$\mathbf{r}_k = \mathbf{\Xi}_k \mathbf{g}_C + \mathbf{w}_k \tag{3}$$

where

- $\mathbf{g}_C$ is the $\mathcal{M}$-component vector of the channel impulse response extended with $\mathcal{M} - N_{px} - 1$ zeros;
- $\mathbf{\Xi}_k$ is an $\mathcal{M}\ x\ \mathcal{M}$ circulant matrix, given by

$$\mathbf{\Xi}_k = \begin{bmatrix} A_k[0] & A_k[\mathcal{M}-1] & \cdots & A_k[1] \\ A_k[1] & A_k[0] & \cdots & A_k[2] \\ \vdots & \vdots & \ddots & \vdots \\ A_k[\mathcal{M}-1] & A_k[\mathcal{M}-2] & \cdots & A_k[0] \end{bmatrix}$$

- $\mathbf{w}_k$ is a vector of additive noise.

Moreover, since $\mathbf{\Xi}$ is a circulant matrix, it satisfies the property

$$\mathbf{F}_{\mathcal{M}} \mathbf{\Xi}_k \mathbf{F}_{\mathcal{M}}^{-1} = \mathrm{diag}\{\mathbf{a}_k\} \tag{4}$$

Defining the DFT of the vector $\mathbf{g}_C$ as $\mathbf{G}_C = \mathbf{F}_{\mathcal{M}} \mathbf{g}_C$ and using equation 4, we can express the demodulator output as

$$\mathbf{x}_k = \mathbf{F}_{\mathcal{M}} \mathbf{r}_k = \mathrm{diag}\{\mathbf{a}_k\} \mathbf{G}_C + \mathbf{W}_k \tag{5}$$

Finally, to equalize the channel using the zero-forcing criterion, $\mathbf{x}_k$ is multiplied by the diagonal matrix $\mathbf{K}$ which elements are simply given by

$$K_i = [\mathbf{F}]_{i,i} = \frac{1}{\mathcal{G}_{C,i}} \qquad\qquad i = 0, \ldots \mathcal{M} - 1 \tag{6}$$

Therefore the input to the data detector is given by

$$\mathbf{y}_k = \mathbf{K}\mathbf{x}_k = \mathbf{a}_k + \mathbf{K}\mathbf{W}_k \tag{7}$$

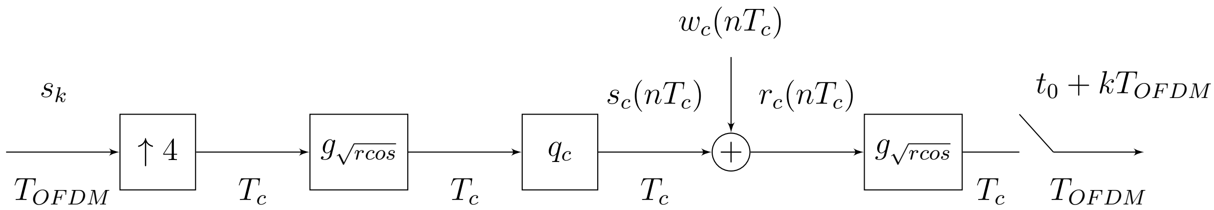

Figure 3. Channel model for the OFDM transmission.

Detection

At the detection point, we implemented the following system

For the single carrier transmission, the output of the DFE filter is a sequence of QPSK symbols y_k. These are decoded exploiting the LLR representation given by

$$l'_{2k} = -2\frac{\mathcal{R}[y_k]}{\sigma_w^2} \tag{8}$$

$$l'_{2k+1} = -2\frac{\mathcal{I}[y_k]}{\sigma_w^2} \tag{9}$$

The generated data stream, which length is twice that of y_k, is *deinterlead* and decoded by the LDPC decoder provided. The final output $\hat{b}_l$ is compared with the initial data b_l to count the number of errors introduced by the channel, and the P_{bit} is finally computed as

$$P_{bit} = \frac{\text{number of errors}}{\text{number of sent bits}} \tag{10}$$

For the OFDM transmission y_k is a matrix with $\mathcal{M} = 512$ rows corresponding to the output of the $\mathcal{M}$ subchannels. Each LLR element is computed using equations 8 and 9, then the final LLR vector is generated in such a way that two consecutive values at position i and *i+1* corresponded to the real and imaginary part of the same symbol in y_k. This operation was computed for all columns. The resulting vector was deinterlived and decoded with the same procedure of the single carrier transmission. Finally the *bit error probability* was computed using equation 10.

Bit Error Probabilities

The filters we used are given as follow.

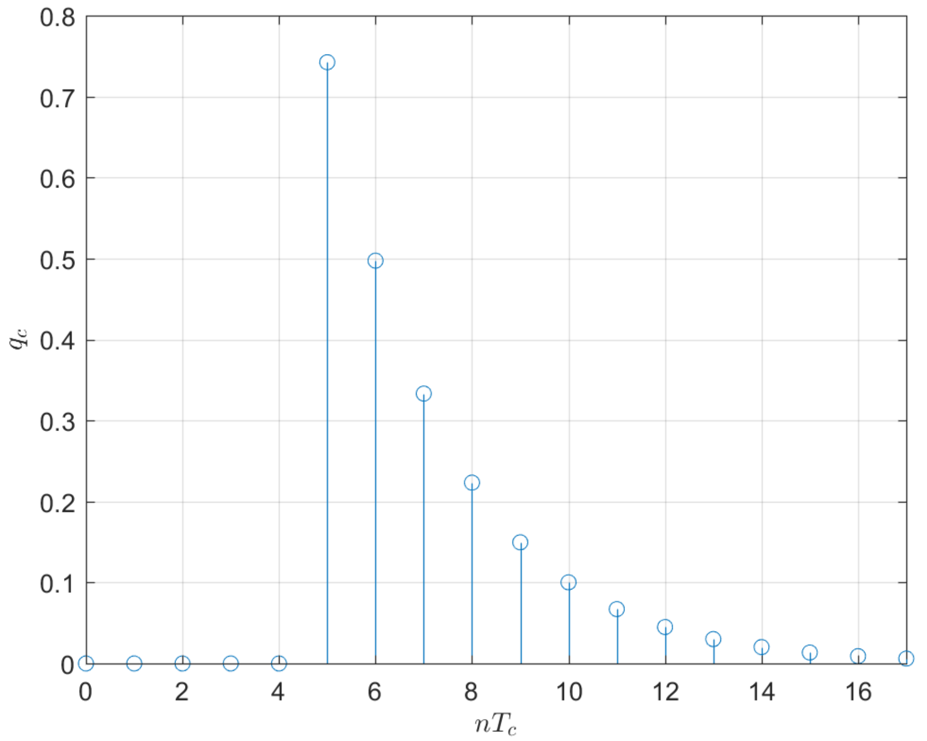

Figure 4. Impulse response of the filter q_c.

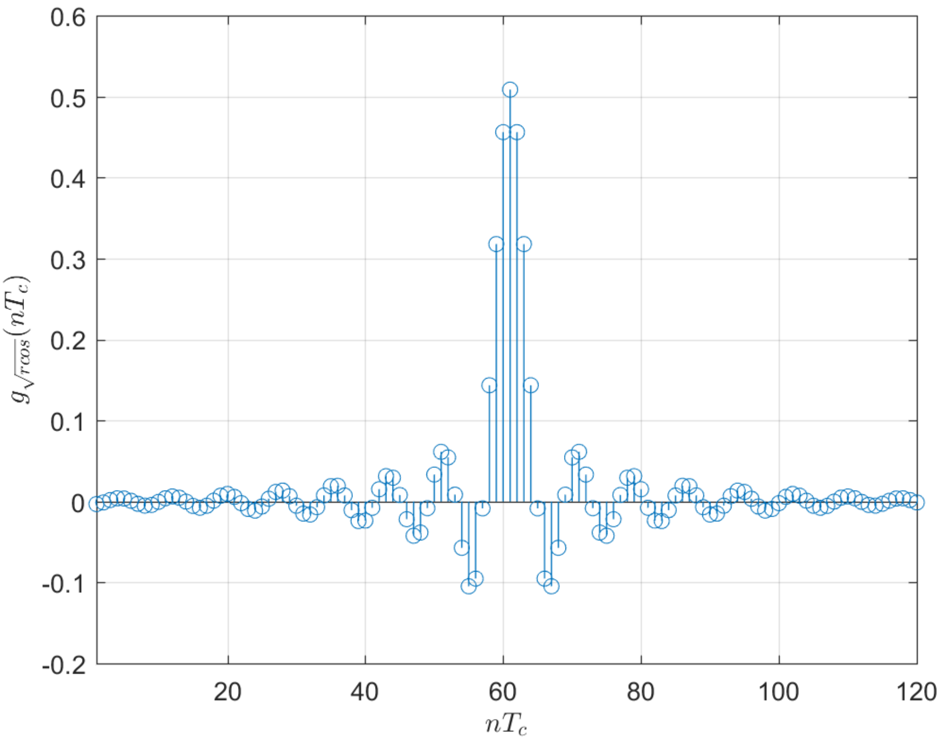

Figure 5. Impulse response of the square root rised-cosine filter.

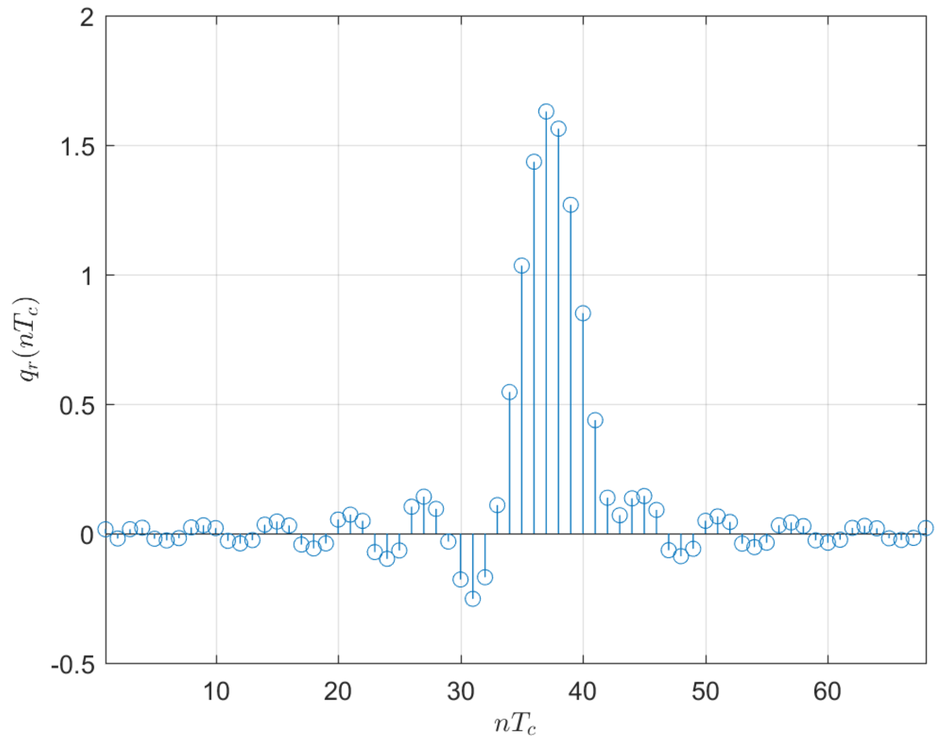

Figure 6. Equivalent channel impulse response $q_r(nT_c) = g_{\sqrt{srrc}}(nT_c) * q_c(nT_c) * g_{\sqrt{srrc}}(nT_c)$.

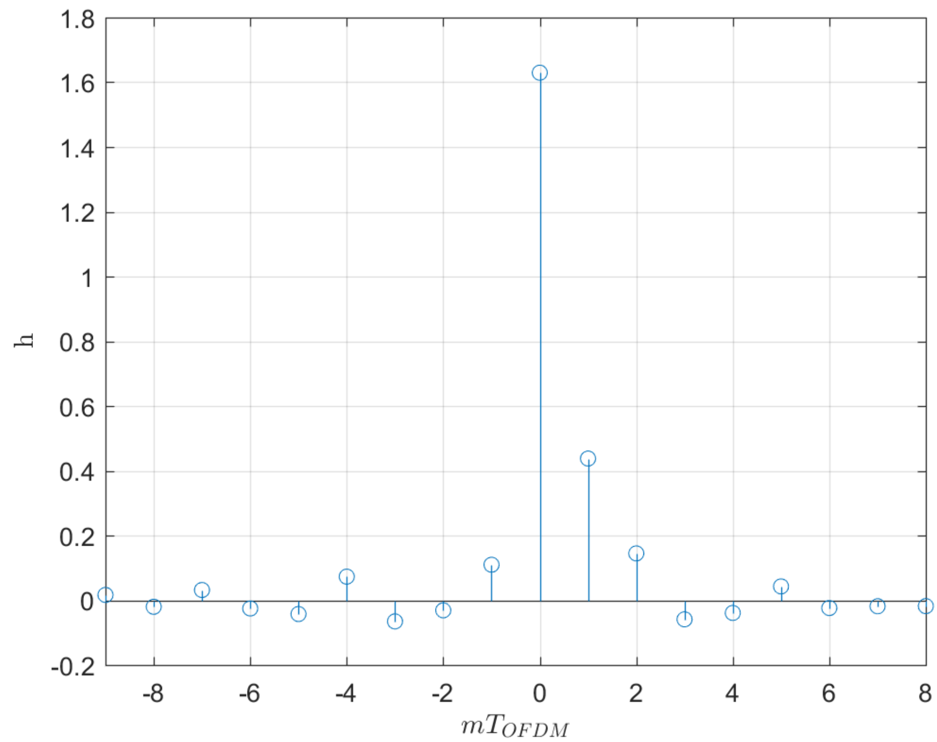

Figure 7. Equivalent channel impulse response after the downsampling, given by $h(mT_{OFDM}) = q_R(t_0 + mT_{OFDM})$. Since h has length 18, we chose $N_{px} = 18$ as the *prefix length*.

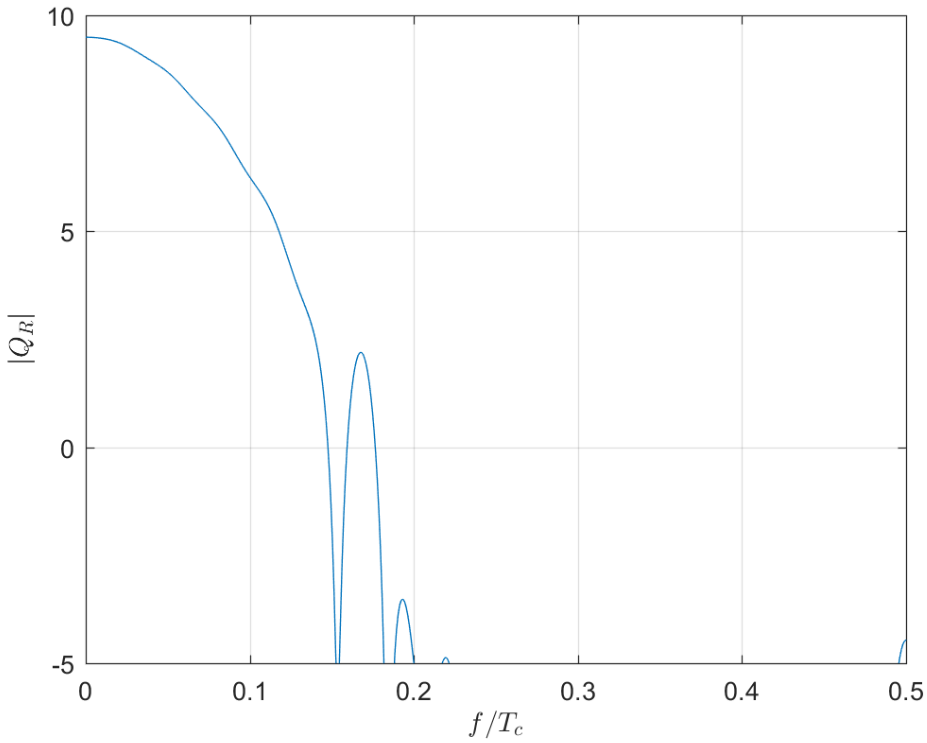

Figure 8. Frequency response of the *square root rised cosine* filter.

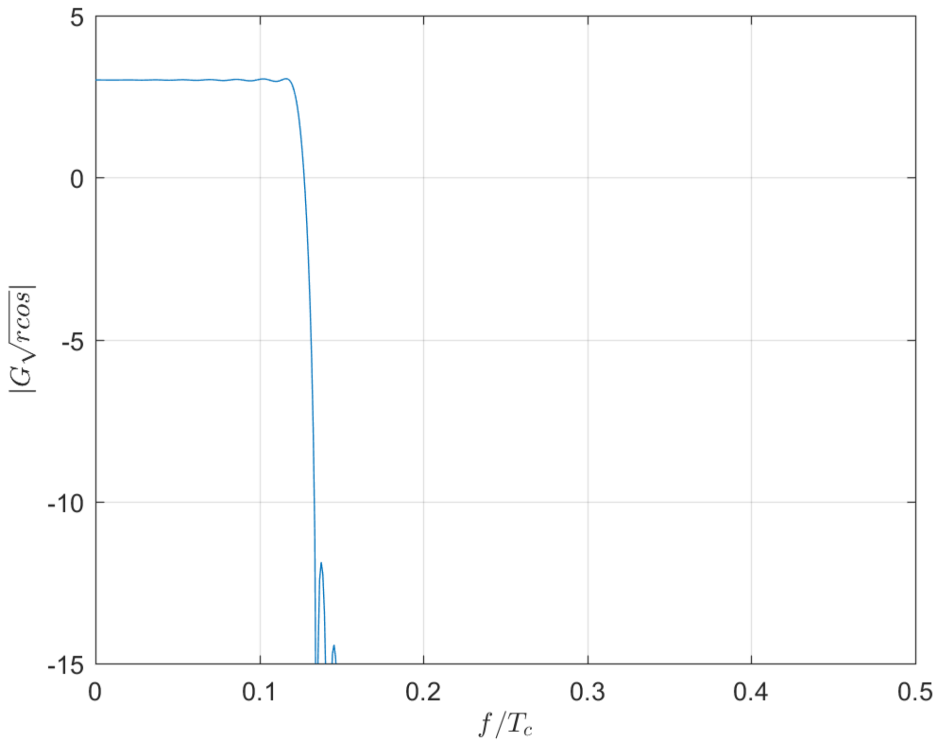

Figure 9

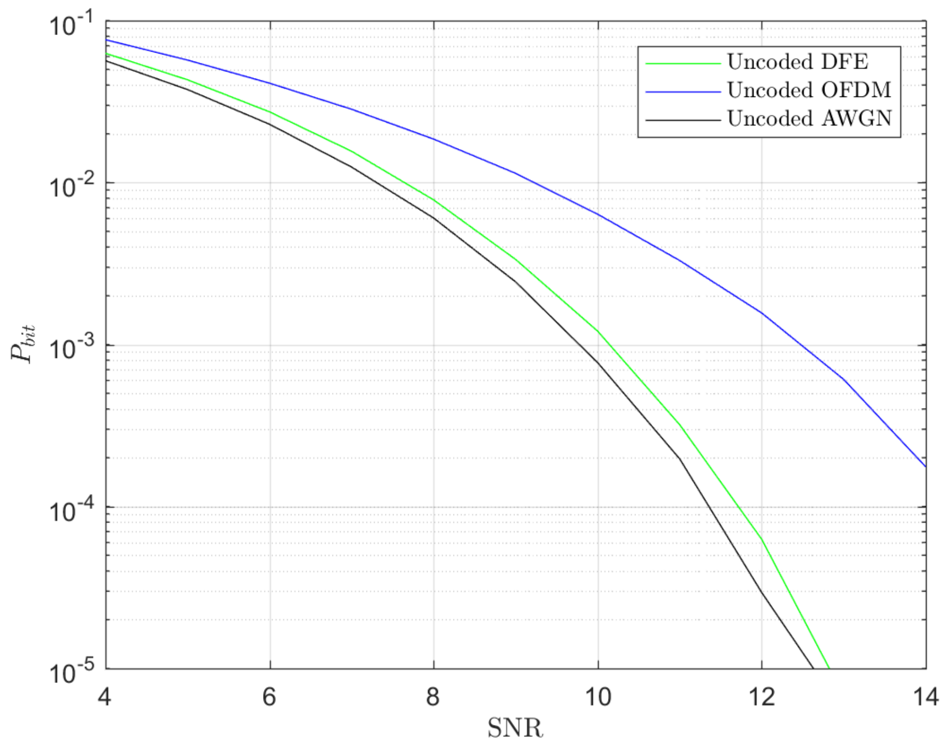

Figure 10. Simulated P_{bit} for the uncoded transmission.

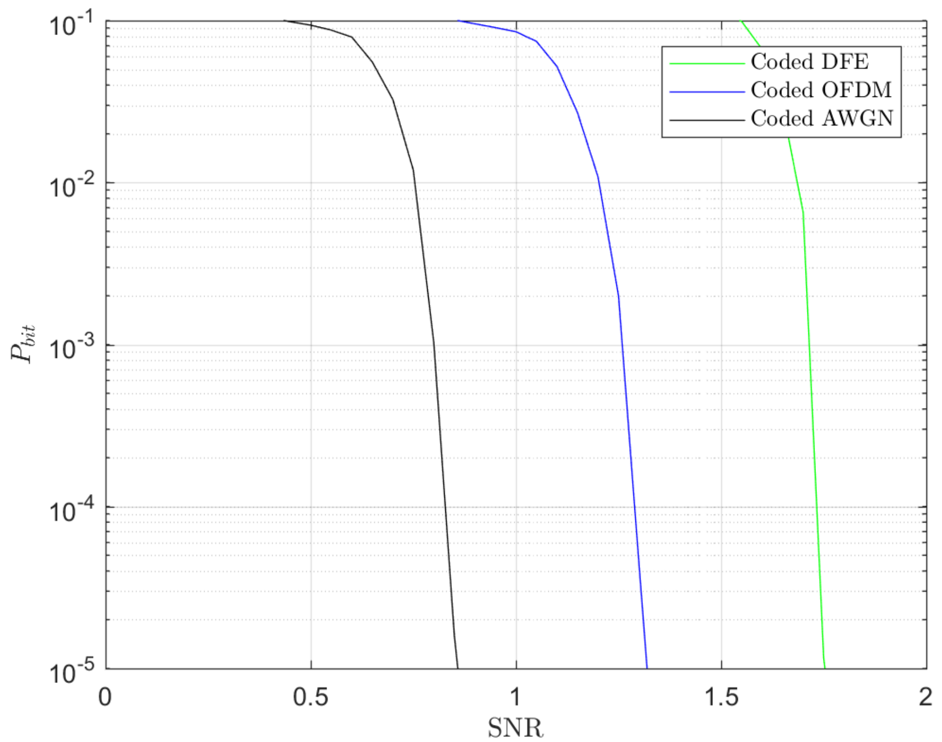

Figure 11. Simulated P_{bit} for the coded transmission.