Digital Communications and Laboratory
Third Homework

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PROBLEM

The following system was considered. A stream of QPSK symbols is upsampled with period T/4 and filtered with a filter q_c which output is $s_c\left(n\frac{T}{4}\right) = \alpha s_c\left((n-1)\frac{T}{4}\right) + \beta a'_{n-5}$. This signal is transmitted through the channel, which introduces the noise component $w_c\left(n\frac{T}{4}\right)$ with PSD $\mathcal{P}_{w_c}(f) = N_0$. Note that noise components are iid with $pmd \sim \mathcal{CN}(0, \sigma^2_{w_c})$. The SNR at the output of the system is therefore

$$\Gamma = \frac{M_{s_c}}{N_0 \frac{1}{T}} = \frac{\sigma_a^2 E_{q_c}}{\sigma^2_{w_c}}$$

with $\sigma_a^2 = 2$ and $E_{q_c} = \sum_m |q_c\left(m\frac{T}{4}\right)|^2$.

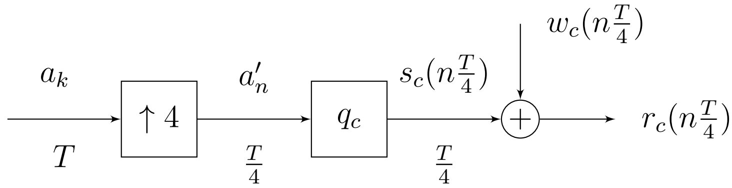

Figure 1. Model for the transmission system of Problem 1.

The QPSK symbols are generated with a PN sequence of length $L = 2^{20} - 1$ in order to provide a stream of bits with spectral characteristics similar to those of a white noise signal. Two consecutive bits are then coupled and mapped into one of the possible constellations symbols, associating the first and second bit to the real and imaginary part respectively.
The q_c filter in linear and frequency domain is given in Figures [2].

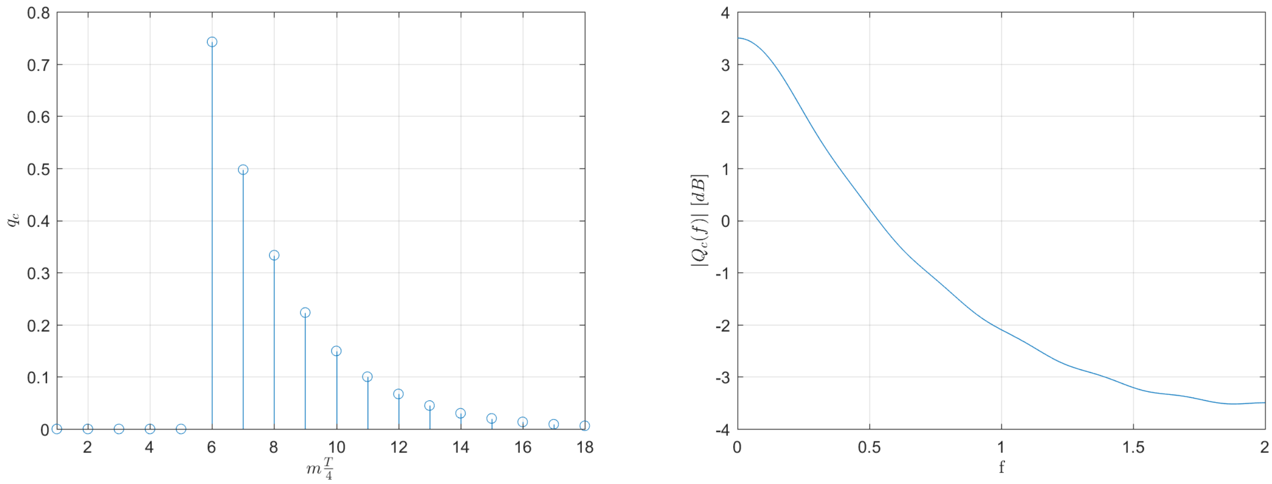

Figure 2. Impulsive response (left) and Frequency response (rigth) of the filter q_c.

In the following, 6 different receiver configurations are studied. For each of this, an SNR value of $\Gamma = 10\ dB$ was assumed.

Receiver a

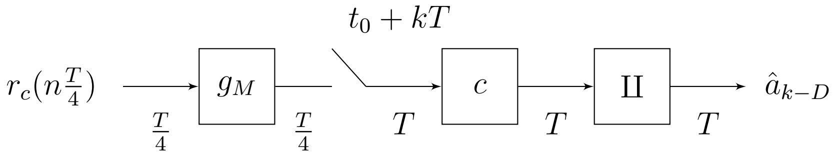

Figure 3. Model for the receiver (a).

The receiver filter consist of a match filter g_M matched to the transmission filter q_c. From now on, we may refer to the global impulse response of the system at the input of the *linear equalizer* c as $h = g_c * g_M$. Since it is defined @$\frac{T}{4}$, a downsampling of a factor 4 is required between the output of h and the input of c.

The filter c attempts to find the optimum trade-off between removing the ISI and enhancing the noise at the decision point. Since the LE can be seen as a particular case of a DFE, we evaluated the coefficients of the filters c and b (will be introduced from Receiver b) using the same algorithm which exploits the Wiener filter theory to determine the optimum coefficients. Let the filter c and b have length M_1 and M_2, with a delay D introduced by c. Then the LE can be seen as a DFE with $M_2 = 0$. The coefficients are computed using the MSE applied to the cost function

$$J = E\left[|a_{k-D} - y_k|^2\right] \tag{1}$$

The optimum FF filter is given by

$$\mathbf{c}_{opt} = \mathbf{R}^{-1}\mathbf{p}$$

where the matrices $\mathbf{R}$ and $\mathbf{p}$ are computed using

$$[\mathbf{R}]_{p,q} = \sigma_a^2 \left(\sum_{j=-N_1}^{N_2} h_j h^*_{j-(p-q)} - \sum_{j=1}^{M_2} h_{j+D-q} h^*_{j+d-p} \right) + r_{\tilde{w}}(p-q) \tag{2}$$

$$[\mathbf{p}]_p = \sigma_a^2 h^*_{D-p}, \qquad p, q = 0, 1, \ldots, M_1 - 1 \tag{3}$$

The optimum FB coefficients are given by

$$b_i = -\sum_{l=0}^{M_1-1} c_{opt,l} h_{i+D-l} \qquad i = 1, 2, \ldots, M_2 \tag{4}$$

The minimum-cost function can be expressed in close form:

$$J_{min} = \sigma_a^2 \left(1 - \sum_{l=0}^{M_1-1} c_{opt,l} h_{D-l} \right) \tag{5}$$

Receiver b

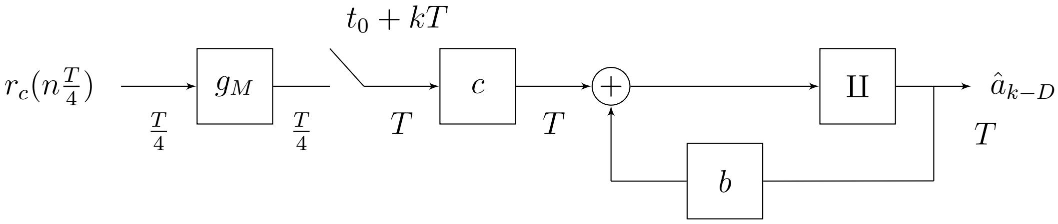

Figure 4. Model for the receiver (b).

Receiver c

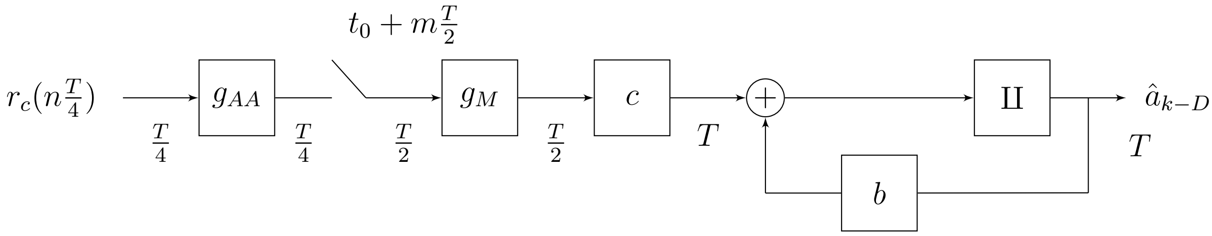

Figure 5. Model for the receiver (c).

Receiver d

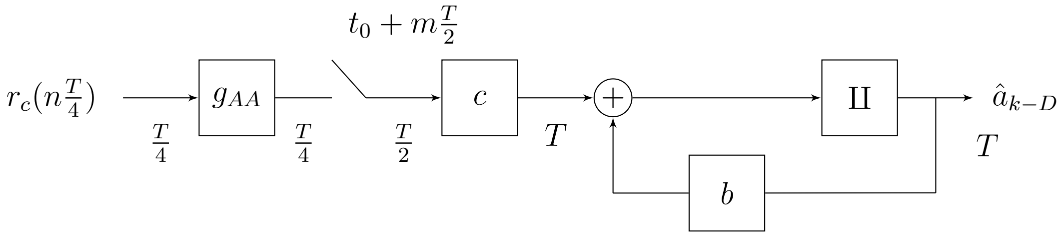

Figure 6. Model for the receiver (d).

Receiver e

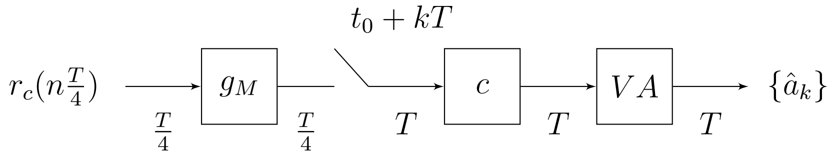

Figure 7. Model for the receiver (e).

Receiver f

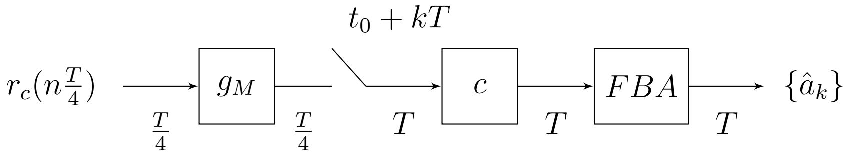

Figure 8. Model for the receiver (f).