Digital Communications and Laboratory
Fourth Homework

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Transmission system

In the following, two different scenarios are developed: a single carrier transmission using Receiver (b) of homework 3 and an OFDM transmission with $\mathcal{M}=512$ sub-channels. Both implementations are firstly simulated using channel coding to improve the error correction capability of the system; in particular, a *Low-density parity check* LDPC encoder is applied to the initial data stream in order to encode the original data and eventually to correct them with the use of an LDPC decoder. These functions are provided by the Matlab toolbox. Finally, the same transmission is simulated without coding to appreciate the difference in terms of P_{bit} for varying SNR.

In this report we decided to treat only the implementation of the encoded transmission; the uncoded one, indeed, follows by simply removing the LDPC and the interleaver blocks.

Symbols generation

A data stream of bits b_l is generated using a PN sequence of length $2^{20} - 1$ repeated once. This is encoded by an LDPC encoder provided by the Matlab toolbox, which reads sequences of 32400 bits and generates codewords c_m of length 64800 bits. An interleaver is used in order to reduce the probability of having *burst* errors, then a bitmap maps pairs of bits into a QPSK constellation with *Gray coding*. This scenario is given in Figure [1].
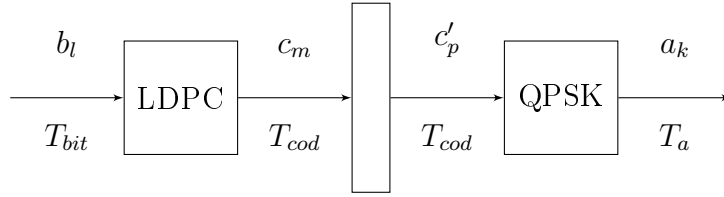

Figure 1. Generation of the encoded QPSK symbols.

Single Carrier

The channel for the single carrier transmission is that implemented in homework 3.

The receiver filter consists of a filter g_M matched to the transmission filter q_c followed by a *Decision Feedback Equalizer* (DFE) filter. The matched filter is simply computed as $g_m(t) = q_c^*(t_0 - t)$, where t_0 is the timing phase. Since the global impulse response of the system $h = g_c * g_M$ at the input of c is defined @$\frac{T}{4}$, a downsampling of a factor 4 is required between the output of h and the input of c. The equations used to compute both filters c and b have already been discussed in homework 3. The scenario just described is given in Figure [2].
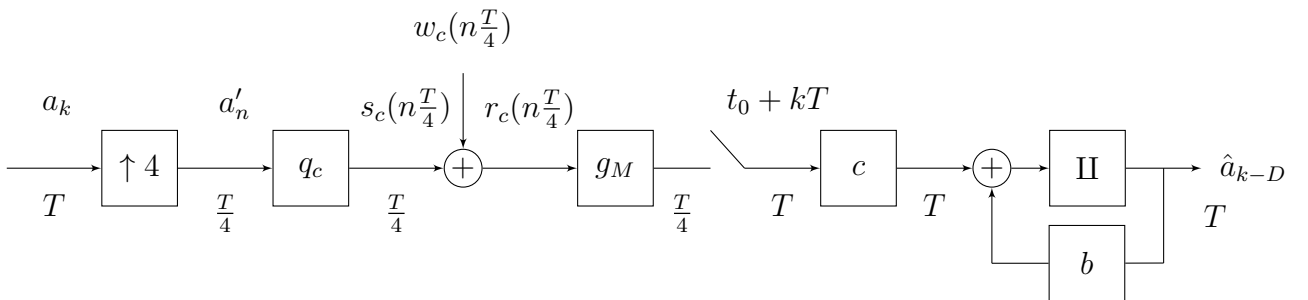

Figure 2. Model for the SC channel.

OFDM

Orthogonal frequency division multiplexing is an efficient modulation technique by which blocks of $\mathcal{M}$ symbols are transmitted in parallel over $\mathcal{M}$ sub-channels, using $\mathcal{M}$ modulation filters with frequency responses $\mathcal{H}_i(f)$, $i = 1, 2, \ldots, \mathcal{M} - 1$. The $\mathcal{M}$ input symbols at the k_{th} modulation interval are represented by the vector

$$\mathbf{a}_k = [a_k[0], \ldots, a_k[\mathcal{M} - 1]]^T \tag{1}$$

where $a_k[i] \in \mathcal{A}[i]$, $i = 0, \ldots, \mathcal{M} - 1$.

We implemented the *Discrete Multitone* DMT version in which the transmit and receiver filter backs use a prototipe filter with impulse response given by

$$h_n = \begin{cases} 1 & \text{if } 0 \leq n \leq \mathcal{M} - 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

In this way the impulse response of the polyphase components of the prototype filter are simply $\{h_n^{(l)}\} = \{\delta_n\}$ for $l = 0, 1, \ldots, \mathcal{M}$. As a consequence, because the frequency responses are constants, we obtain directly the transmit signal by applying a P/S conversion at the output of the IDFT. To equalise the channel, we implemented the baseband equivalent system given in Figure [3]. The channel is given in Figure [4] .
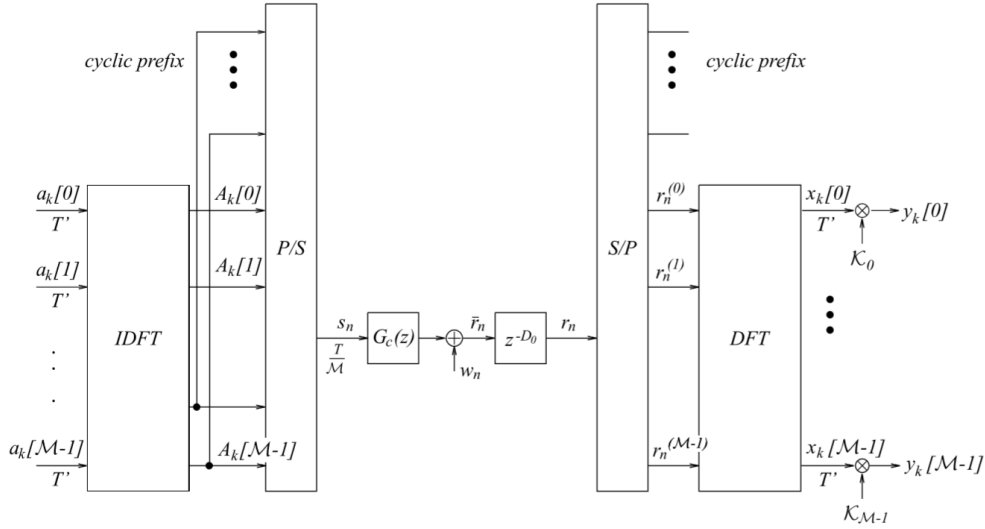

Figure 3. Block diagram of a DMT system with cyclic prefix and frequency-domain equalizer.
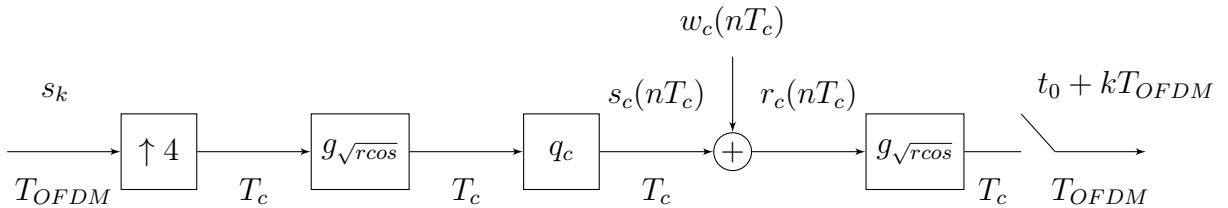

Figure 4. Channel model for the OFDM transmission.

The OFDM channel receives sequences of $\mathcal{M} + N_{px}$ symbols and generates a noisy output with period T_{OFDM}. In the simulation process, the input signal s_k is firstly oversampled of a factor 4, then filtered with the cascade of a *square root raised cosine* filter with roll-off factor equal to $\rho = 0.0625$ (Figure [7]) and the q_c filter introduced for the single-carrier transmission (Figure 6). Note that the system is now working at $T_c = T_{OFDM}/4$. Here the useful signal sums up with the noise component $w_c(nT_c) \sim \mathcal{CN}(0, \sigma_{w_c}^2)$, where respectively

$$\sigma_{w_c}^2 = \frac{\sigma_a^2}{\mathcal{M}} \frac{E_{g_{\sqrt{rcos}} * q_c}}{\Gamma} \tag{3}$$

where $\sigma_a = 2/\mathcal{M}$, E is the energy of the filter given by the convolution of $g_{\sqrt{rcos}}$ and q_c and Γ is the SNR expressed in linear units. The new signal $r_c(nT_c)$ is filtered again with the *square root raised cosine* filter, then the resulting signal is sampled @T_{OFDM} starting from the *timing phase* $t_0 = \bar{t}_0 T_c$. This is chosen as the index at which the peak of the overall impulse response $q_r(nT_c) = g_{\sqrt{srrc}}(nT_c) * q_c(nT_c) * g_{\sqrt{srrc}}(nT_c)$ occurs; by looking at Figure [8], we obtained a value of $\mathbf{\bar{t}_0 = 37}$. The final impulse response sampled @T_{OFDM} starting at t_0 is $h(mT_{OFDM}) = q_R(t_0 + mT_{OFDM})$ and it is given in Figure [9].

We exploited the concept of circular convolution to obtain a convolution in the time domain as a product of finite length vectors in the frequency domain. To do so, we extended the block of samples $\mathbf{A}_k$ by repeating $\mathbf{N_{px}} = \mathbf{18}$ elements before transmitting through the channel. The value of N_{px} was chosen equal to the support of the equivalent channel impulse response $h(mT_{OFDM})$ given in Figure [9]. In particular, after the modulation, each block of samples is cyclically extended by copying the N_{px} sample $A_k[\mathcal{M} - N_{px}], \ldots, A_k[\mathcal{M} - 1]$ in front of the block. After the P/S conversion, where the N_{px} samples of the cyclic extension are the first to be sent, the $N_{px} + \mathcal{M}$ samples are transmitted over the channel. At the receiver, blocks of samples of length $N_{px} + \mathcal{M}$ are taken; the boundaries between blocks are set such that the last $\mathcal{M}$ samples depend only on the elements of only one cyclically extended block of samples. The first N_{px} samples of a block are thus discarded.
The resulting vector $\mathbf{r}_k$ can be expressed as

$$\mathbf{r}_k = \mathbf{\Xi}_k \mathbf{g}_C + \mathbf{w}_k \tag{4}$$

where

- $\mathbf{g}_C$ is the $\mathcal{M}$-component vector of the channel impulse response extended with $\mathcal{M} - N_{px} - 1$ zeros;

- $\mathbf{\Xi}_k$ is an $\mathcal{M} \times \mathcal{M}$ circulant matrix, given by

$$\mathbf{\Xi}_k = \begin{bmatrix} A_k[0] & A_k[\mathcal{M} - 1] & \cdots & A_k[1] \\ A_k[1] & A_k[0] & \cdots & A_k[2] \\ \vdots & \vdots & \ddots & \vdots \\ A_k[\mathcal{M} - 1] & A_k[\mathcal{M} - 2] & \cdots & A_k[0] \end{bmatrix}$$

- $\mathbf{w}_k$ is a vector of additive noise.

Moreover, since $\mathbf{\Xi}$ is a circulant matrix, it satisfies the property

$$\mathbf{F}_{\mathcal{M}} \mathbf{\Xi}_k \mathbf{F}_{\mathcal{M}}^{-1} = \text{diag}\{\mathbf{a}_k\} \tag{5}$$

Defining the DFT of the vector $\mathbf{g}_C$ as $\mathbf{G}_C = \mathbf{F}_{\mathcal{M}}\mathbf{g}_C$ and using equation (5), we can express the demodulator output as

$$\mathbf{x}_k = \mathbf{F}_{\mathcal{M}}\mathbf{r}_k = \text{diag}\{\mathbf{a}_k\}\mathbf{G}_C + \mathbf{W}_k \tag{6}$$

Finally, to equalize the channel using the zero-forcing criterion, $\mathbf{x}_k$ is multiplied by the diagonal matrix $\mathbf{K}$ which elements are simply given by

$$K_i = [\mathbf{F}]_{i,i} = \frac{1}{\mathcal{G}_{C,i}} \qquad i = 0, \dots \mathcal{M} - 1 \tag{7}$$

Therefore the input to the data detector is given by

$$\mathbf{y}_k = \mathbf{K}\mathbf{x}_k = \mathbf{a}_k + \mathbf{K}\mathbf{W}_k \tag{8}$$

Detection

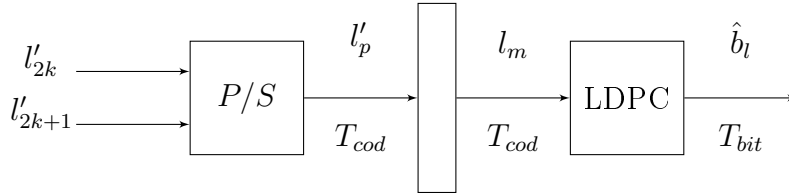At the detection point, we implemented the following system

Figure 5. Decoding scheme

For the single carrier transmission, the output of the DFE filter is a sequence of QPSK symbols y_k. These are decoded exploiting the LLR representation given by

$$l'_{2k} = -2\frac{\mathcal{R}[y_k]}{\sigma_i^2/2} \tag{9}$$

$$l'_{2k+1} = -2\frac{\mathcal{I}[y_k]}{\sigma_i^2/2} \tag{10}$$

where $\sigma_i^2 = \dfrac{J_{min} - \sigma_a^2|1 - \psi_D|^2}{|\psi_D|^2}$.

The generated data stream, which length is twice that of y_k, is *deinterleaved* and decoded by the LDPC decoder provided. The final output $\hat{b}_l$ is compared with the initial data b_l to count the number of errors introduced by the channel, and then P_{bit} is finally computed as

$$P_{bit} = \frac{\text{number of errors}}{\text{number of sent bits}} \tag{11}$$

For the OFDM transmission y_k is a matrix with $\mathcal{M} = 512$ rows corresponding to the output of the $\mathcal{M}$ subchannels. Each LLR element is computed using equations (9) and (10), with the only difference that now $\sigma_i^2 = \frac{1}{2}\sigma_{w_c}^2 \cdot \mathcal{M} \cdot |K_i|^2$. The final LLR vector is generated in such a way that two consecutive values at position i and $i+1$ corresponds to the real and imaginary part of the same symbol in y_k. This operation is computed for all columns. The resulting vector is deinterleaved and decoded with the same procedure of the single carrier transmission. Finally the *bit error probability* was computed using equation (11).

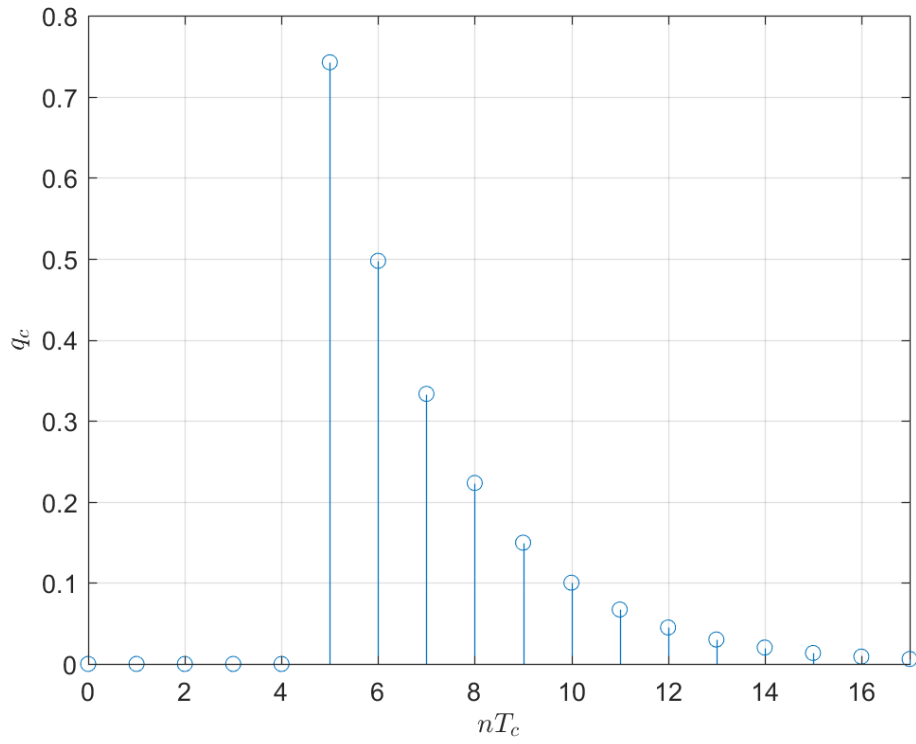The filters we used are given as follow.
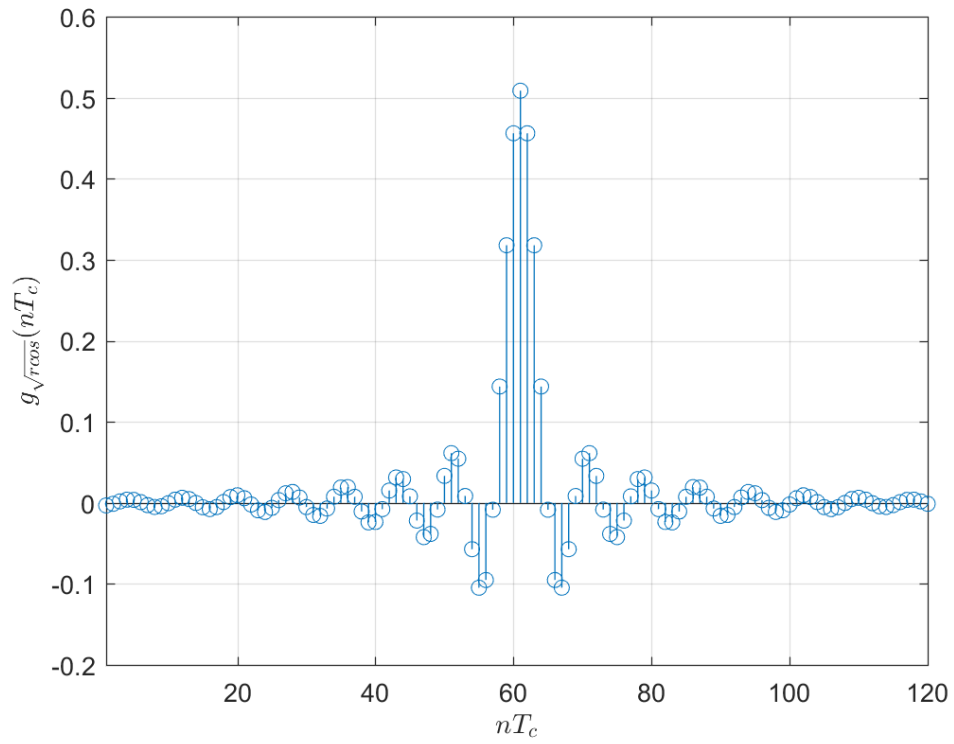

Figure 6. Impulse response of the filter q_c.

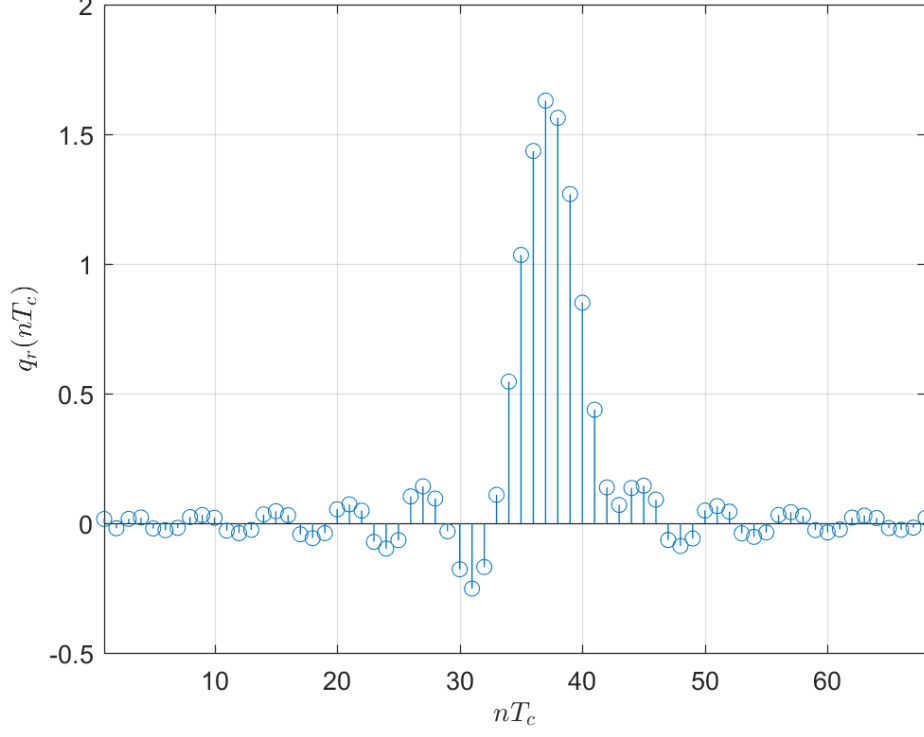**Figure 7.** Impulse response of the square root raised-cosine filter.

Figure 8. Equivalent channel impulse response $q_r(nT_c) = g_{\sqrt{srrc}}(nT_c) * q_c(nT_c) * g_{\sqrt{srrc}}(nT_c)$.
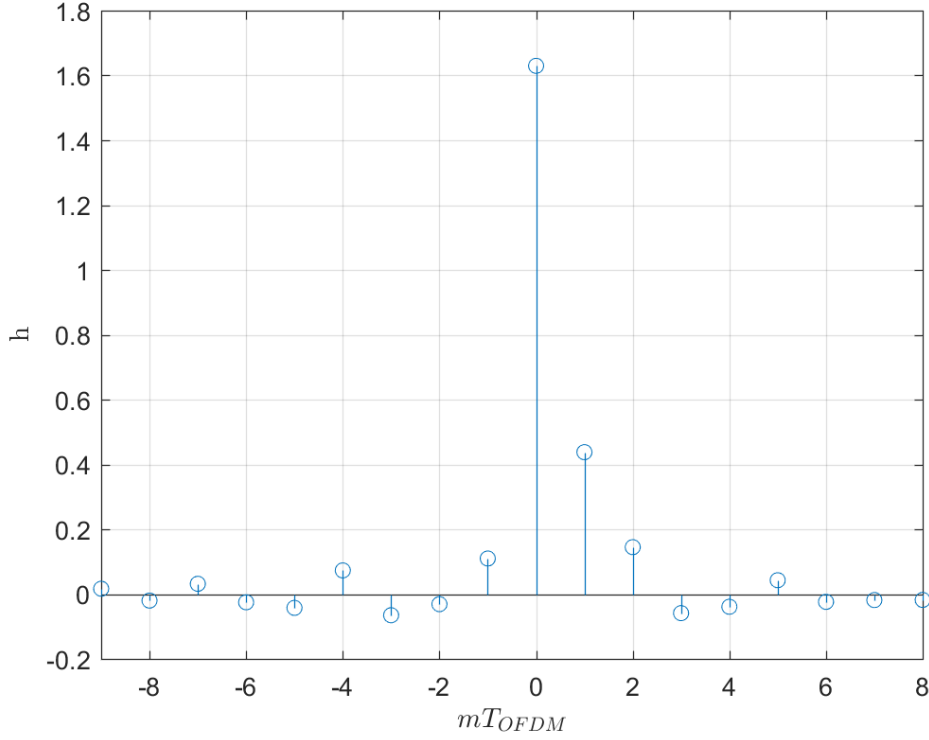

Figure 9. Equivalent channel impulse response after the downsampling, given by $h(mT_{OFDM}) = q_R(t_0 + mT_{OFDM})$. Since h has length 18, we chose $N_{px} = 18$ as the *prefix length*.
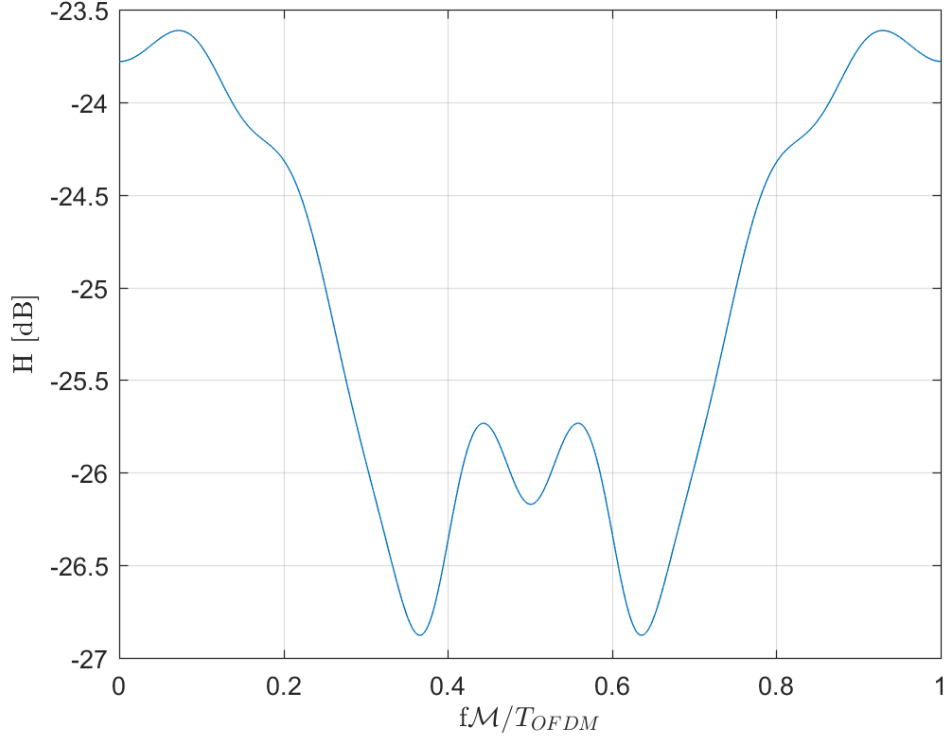
Figure 10. FFT of the equivalent channel impulse response h computed over $\mathcal{M}$ samples. Because of how the Matlab function implements the FFT algorithm, the function is plotted after dividing H by its length. However, this just results in a different scale range while the overall shape remains the same.
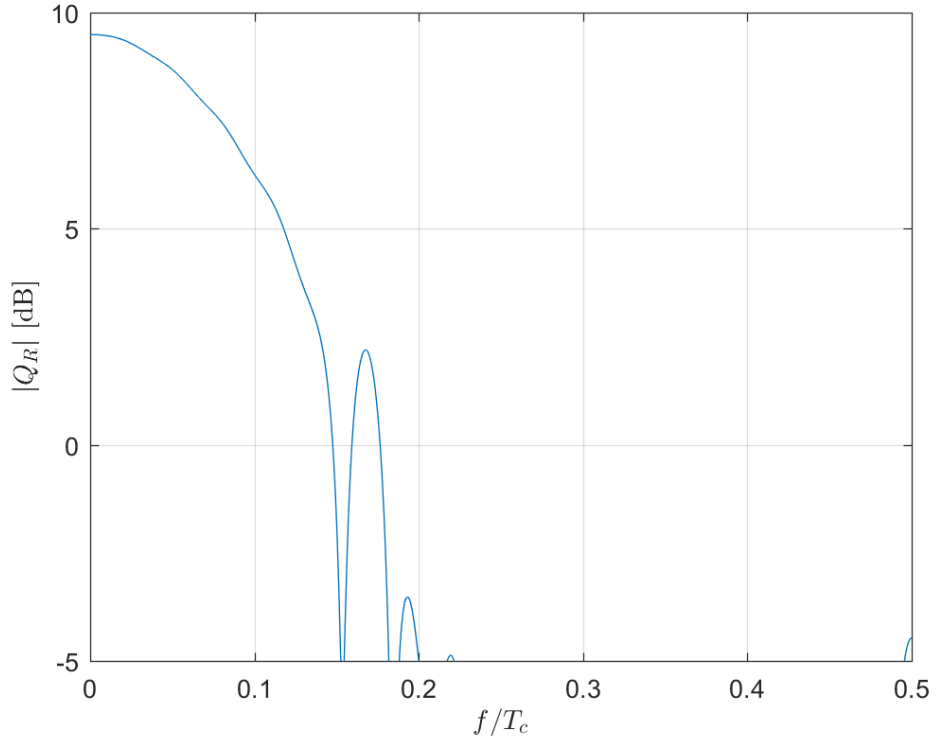

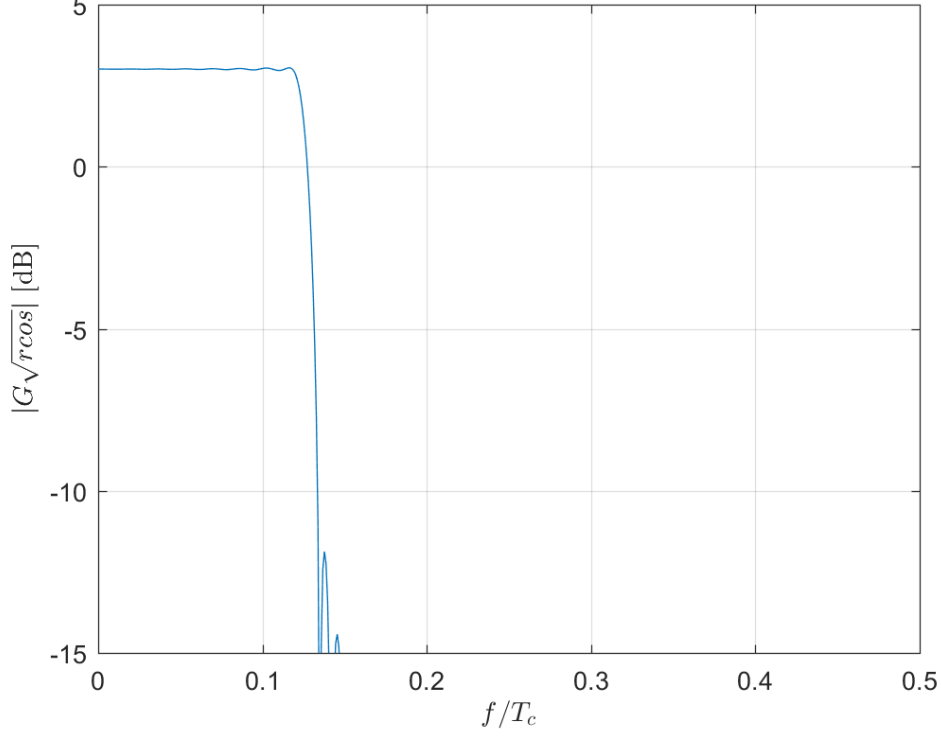**Figure 11.** Frequency response of the *square root rised cosine* filter.

Figure 12

Bit Error Probabilities

Here the P_{bit} obtained for the different configurations are given as function of the SNR Γ.

For the uncoded data transmission, the DFE receiver shows better performances. This is to be expected since the channel is well equalized by the *decision feedback equalizer* filter; this can be noted by observing Figure [9] of homework 3, where the overall impulse response ψ presents no precursors. On the other hand, the OFDM transmission system shows worse performances due to the fact that the zero-forcing criterion amplifies the noise signal components, as given in equation (8).

For the coded data transmission, the DFE still shows good performances being approximately one dB over the AWGN bound. The OFDM system, however, thanks to the LDPC encoding scheme, performs even better having a gap of less then 0.5 dB. From this result, we decided to not introduce further redundancy in the OFDM transmission system by exploiting the so called *virtual carriers*; typically, indeed, their number in a DMT transmission may be a non-negligible percentage of $\mathcal{M}$. However, being the performances of our system very close to the AWGN bound, we kept the scenario described in the previous paragraphs.
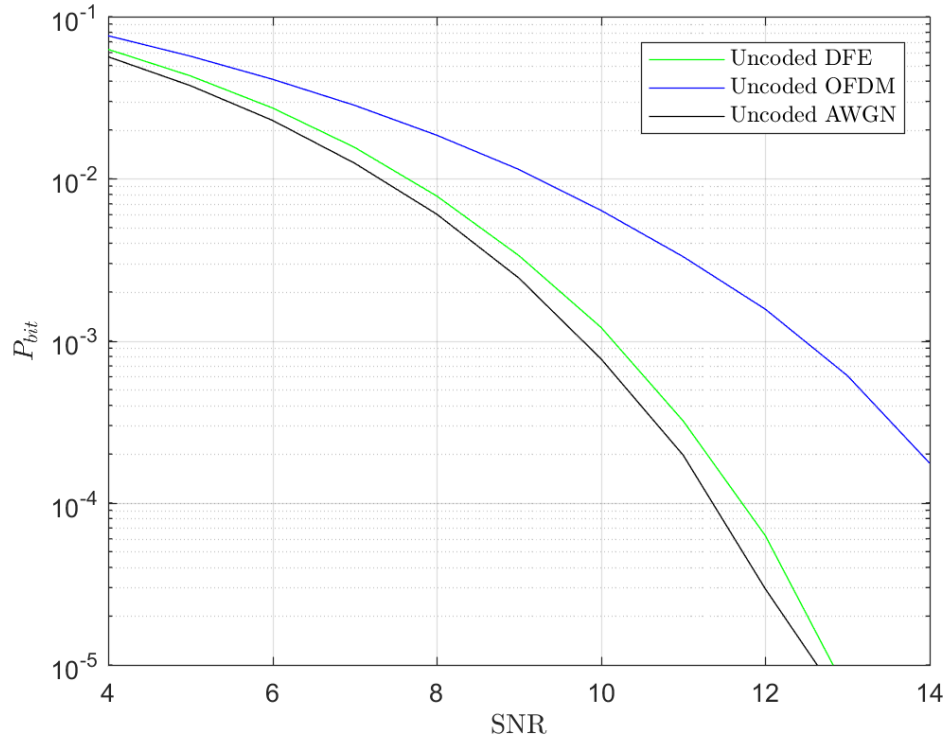
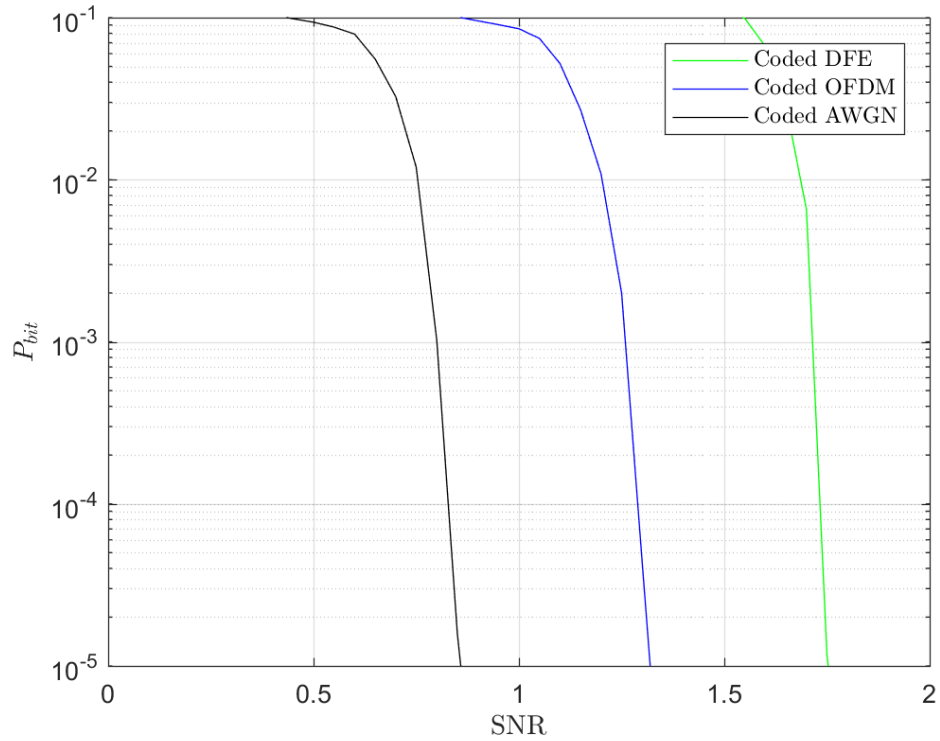**Figure 13.** Simulated P_{bit} for the uncoded transmission.

Figure 14. Simulated P_{bit} for the coded transmission.

Bibliography

[1] Nevio Benvenuto, Giovanni Cherubini, *Algorithms for Communication Systems and their Applications*. Wiley, 2002.