Digital Communications and Laboratory
Third Homework

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PROBLEM

The following system was considered. A stream of QPSK symbols is upsampled with period T/4 and filtered with a filter q_c which output is $s_c\left(n\frac{T}{4}\right) = \alpha s_c\left((n-1)\frac{T}{4}\right) + \beta a'_{n-5}$. This signal is transmitted through the channel, which introduces the noise component $w_c\left(n\frac{T}{4}\right)$ with PSD $\mathcal{P}_{w_c}(f) = N_0$. Note that noise components are iid with $pmd \sim \mathcal{CN}(0, \sigma^2_{w_c})$. The SNR at the output of the system is therefore

$$\Gamma = \frac{M_{s_c}}{N_0 \frac{1}{T}} = \frac{\sigma_a^2 E_{q_c}}{\sigma^2_{w_c}}$$

with $\sigma_a^2 = 2$ and $E_{q_c} = \sum_m |q_c\left(m\frac{T}{4}\right)|^2$.

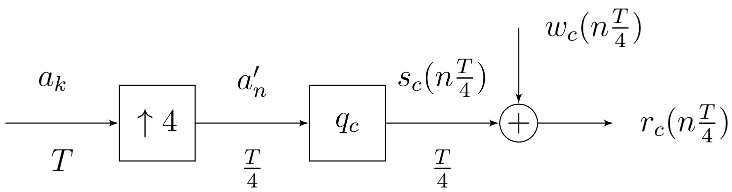

Figure 1. Model for the transmission system of Problem 1.

The QPSK symbols are generated with a PN sequence of length $L = 2^{20} - 1$ in order to provide a stream of bits with spectral characteristics similar to those of a white noise signal. Two consecutive bits are then coupled and mapped into one of the possible constellations symbols, associating the first and second bit to the real and imaginary part respectively.
The q_c filter in linear and frequency domain is given in Figures [2].

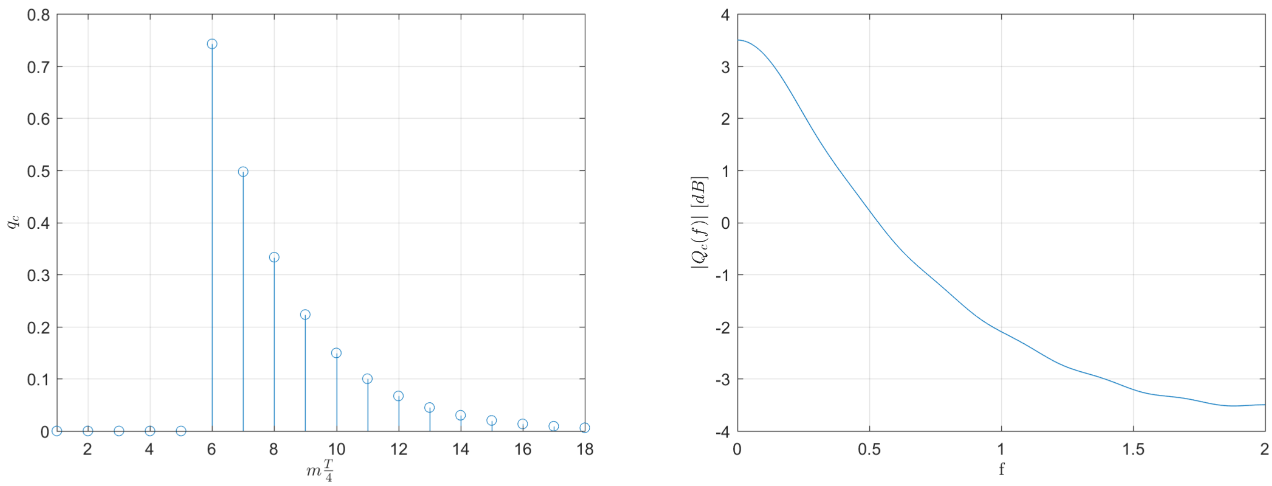

Figure 2. Impulse response (left) and Frequency response (right) of the filter q_c.

In the following, 6 different receiver configurations are studied. For each of this, an SNR value of $\Gamma = 10\ dB$ was assumed.

Receiver a

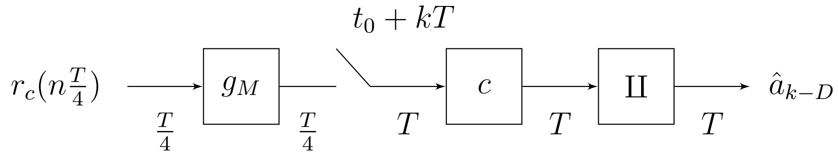

Figure 3. Model for the receiver (a).

The receiver filter consists of a filter g_M matched to the transmission filter q_c followed by a *linear equalizer filter* c. The matched filter is simply computed as $g_m(t) = q_c^*(t_0 + t)$, where t_0 is the timing phase. It is given in Figure [4]. From now on, we may refer to the global impulse response of the system at the input of c as $h = g_c * g_M$. Since it is defined @$\frac{T}{4}$, a downsampling of a factor 4 is required between the output of h and the input of c.
The filter c attempts to find the optimum trade-off between removing the ISI and enhancing the noise at the decision point. Since the LE can be seen as a particular case of a DFE, we evaluated the coefficients of the filters c and b (will be introduced from Receiver (b)) using the same algorithm which exploits the Wiener filter theory to determine the optimum coefficients. Let the filter c and b have length M_1 and M_2, with a delay D introduced by c. Then the LE can be seen as a DFE with $M_2 = 0$. The coefficients are computed using the MSE applied to the cost function

$$J = E\left[|a_{k-D} - y_k|^2\right] \tag{1}$$

The optimum FF filter is given by

$$\mathbf{c}_{opt} = \mathbf{R}^{-1}\mathbf{p} \tag{2}$$

where the matrices $\mathbf{R}$ and $\mathbf{p}$ are computed using

$$[\mathbf{R}]_{p,q} = \sigma_a^2 \left(\sum_{j=-N_1}^{N_2} h_j h_{j-(p-q)}^* - \sum_{j=1}^{M_2} h_{j+D-q} h_{j+d-p}^* \right) + r_{\tilde{w}}(p-q) \tag{3}$$

$$[\mathbf{p}]_p = \sigma_a^2 h_{D-p}^*, \qquad p, q = 0, 1, \ldots, M_1 - 1 \tag{4}$$

The optimum FB coefficients are given by

$$b_i = -\sum_{l=0}^{M_1-1} c_{opt,l} h_{i+D-l} \qquad i = 1, 2, \ldots, M_2 \tag{5}$$

The minimum-cost function can be expressed in close form as:

$$J_{min} = \sigma_a^2 \left(1 - \sum_{l=0}^{M_1-1} c_{opt,l} h_{D-l} \right) \tag{6}$$

The output signal is

$$\begin{aligned} y_k &= x_{FF,k} + x_{FB,k} \\ &= \sum_{i=0}^{M_1-1} c_i x_{k-i} + \sum_{j=1}^{M_2} b_j a_{k-D-j} \end{aligned} \tag{7}$$

The equalizing filter c and the overall impulse response $\psi_i = h * c_i$ for an SNR of $\Gamma = 10$ dB are given in Figure [5].

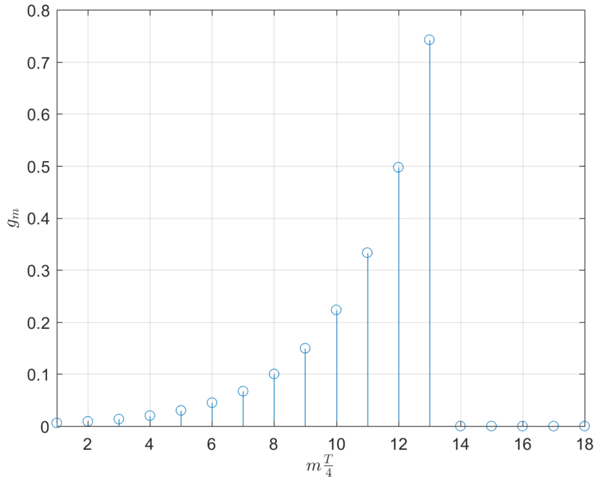

Figure 4. Matched filter g_m of the receiver filter.

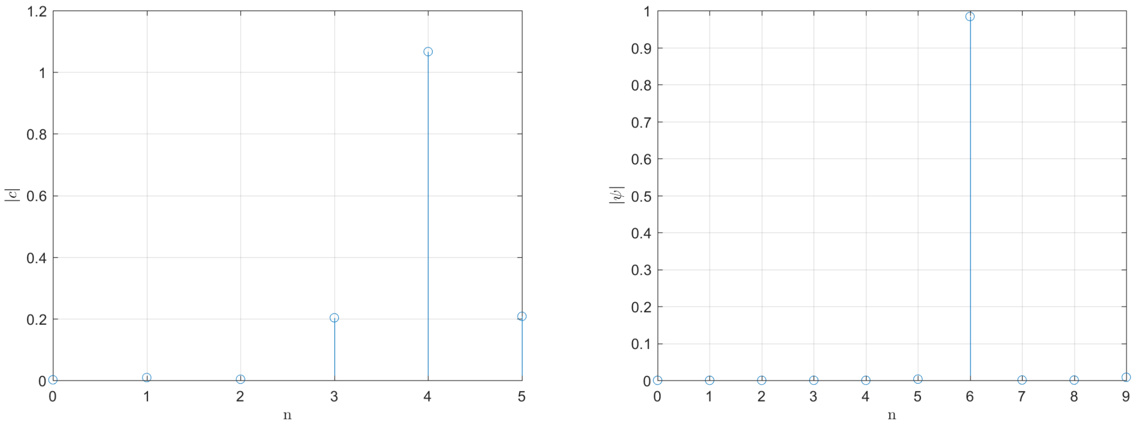

Figure 5. Coefficients of the Equalizer filter c (left) and of the overall impulse response ψ_i (right).

The parameters we obtained are reported in Table [3].

$\bar{\mathbf{t}}_0$	$\mathbf{M_1}$	$\mathbf{M_2}$	**D**
1	7	0	6

Table 1. Parameters of the Receiver (a).

The final signal at the output of the equalizer filter is processed by a threshold detector, which maps each received symbol according to the following rule:

$$\tilde{a}_k \mapsto \hat{a}_k = \mathrm{sgn}(\Re[\tilde{a}_k]) + i \cdot \mathrm{sgn}(\Im[\tilde{a}_k])$$

Note that the actual transmitted symbol at time k is detected at time $k + D$ because of the delay introduced by the filter c. This detection configuration will be used also for receivers b, c and d.

Receiver b

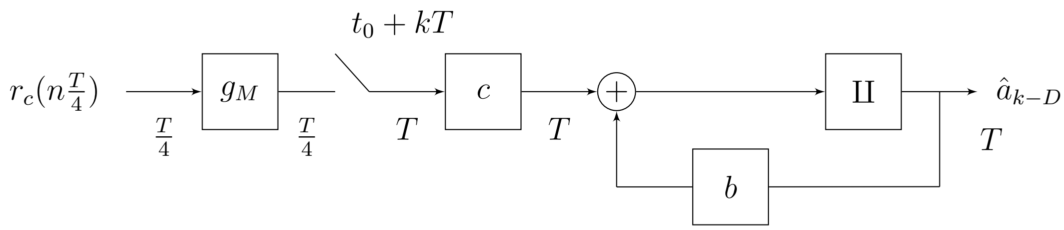

Figure 6. Model for the receiver (b).

The only difference with Receiver (a) is that here the *Decision Filter Equalizer* introduces a feedback filter to remove the postcursors of the impulse response h at the input of the FF filter. Using the algorithm described in the previous section, we obtained the filters given as follow.

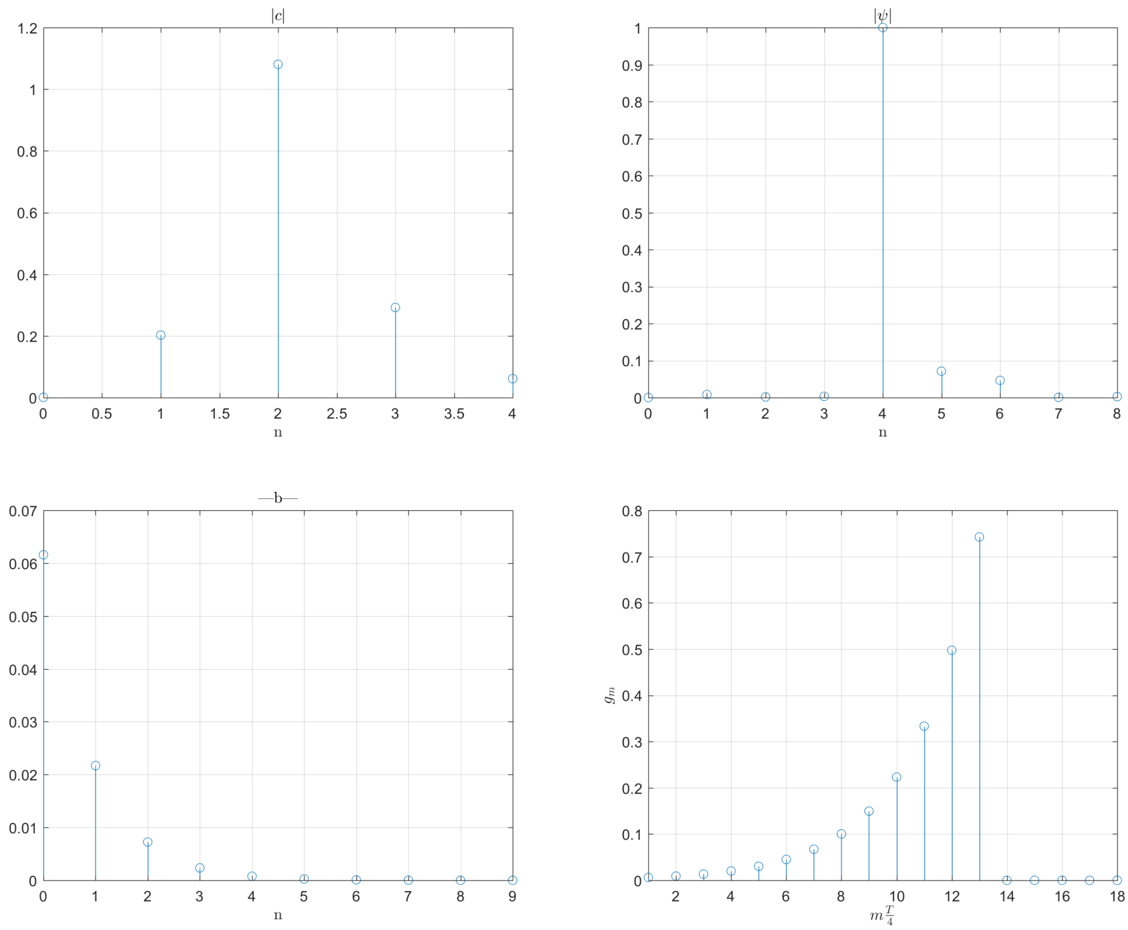

Figure 7. Coefficients of the FF filter c, of the overall impulse response ψ_i, of the FB filter b and of the matched filter g_m. Note that g_m is the same as for receiver (a).

$\bar{\mathbf{t}}_0$	$\mathbf{M_1}$	$\mathbf{M_2}$	$\mathbf{D}$
1	1	1	1

Table 2. Parameters of the Receiver (b).

Receiver c

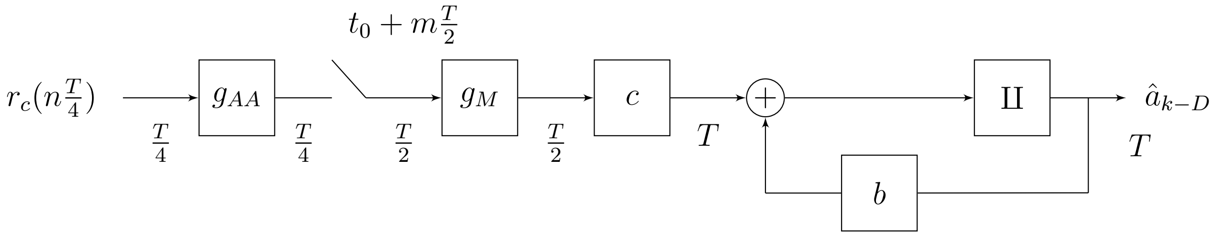

Figure 8. Model for the receiver (c).

In this case the analog filter is replaced by a digital *anti-aliasing* filter g_{AA}. The signal at the output is downsampled of a factor 2, then a cascade of a matched filter and a DFE filter processes the signal and produces the output symbols $\hat{a}_{k-D}$. The filter g_m is needed in order to allow the equalizing filter defined in equation 2 to act as a match filter. Note also that the filter c is followed by a downsampling of a factor 2.
The anti-aliasing and the matched filters are given in Figure [9].

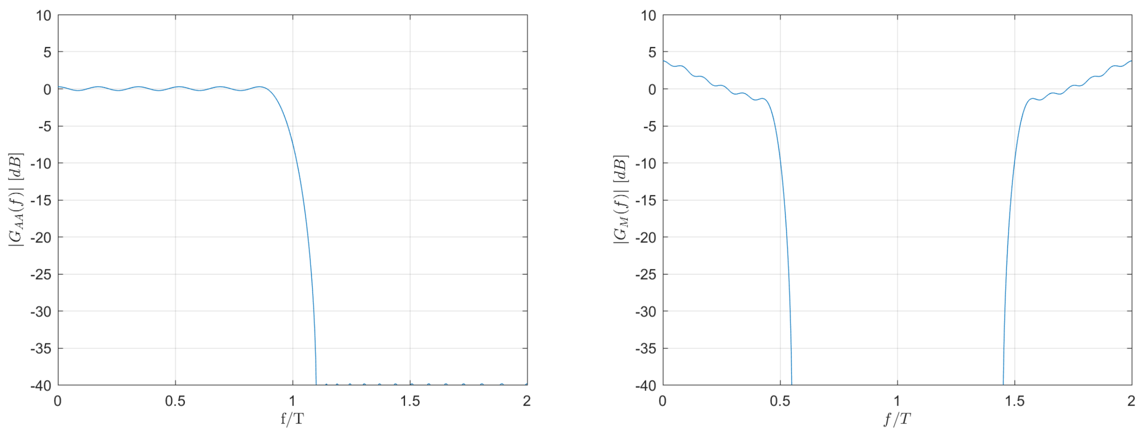

Figure 9. Anti-aliasing and matched filters of the Receiver (c).

The remaining filters are given in Figure [10].

$\bar{t}_0$	M_1	M_2	D
1	1	1	1

Table 3. Parameters of the Receiver (c).

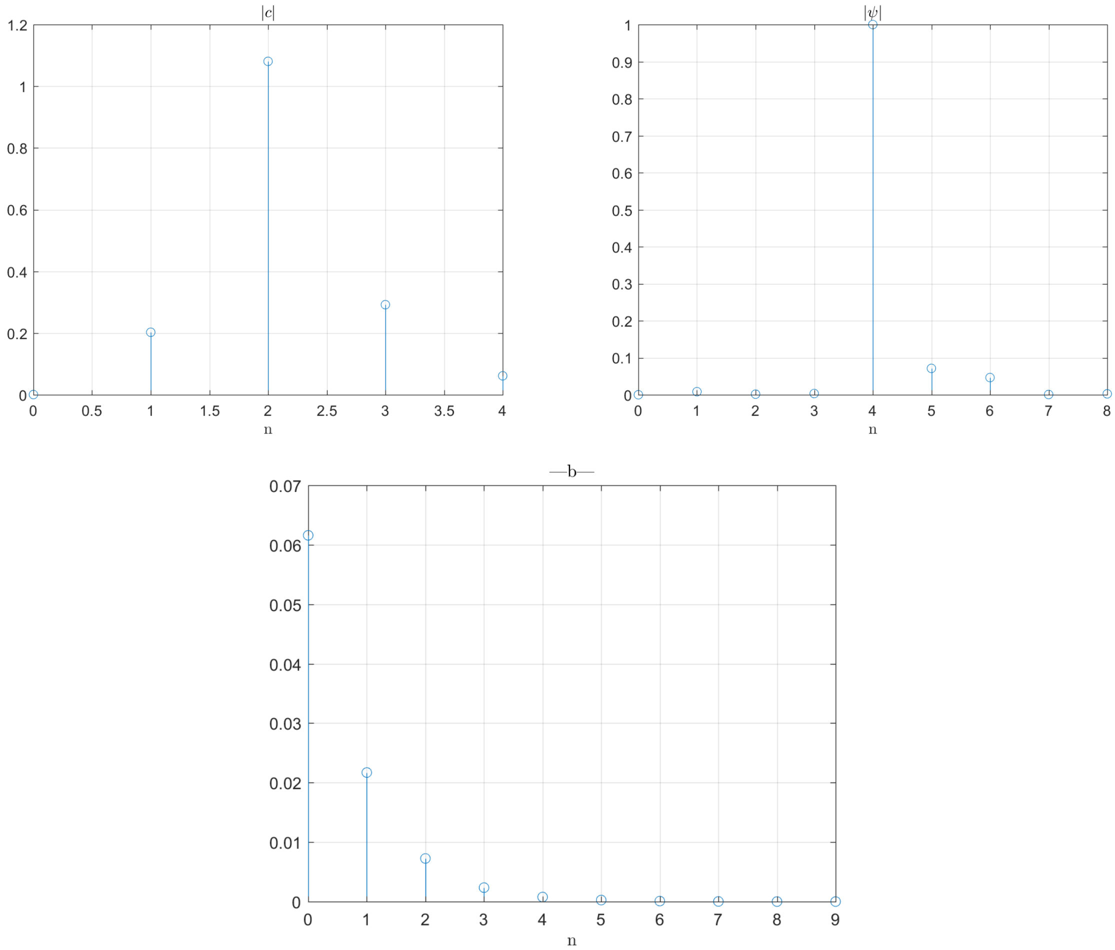

Figure 10. Coefficients of the FF filter c, of the overall impulse response ψ_i and of the FB filter b.

Receiver d

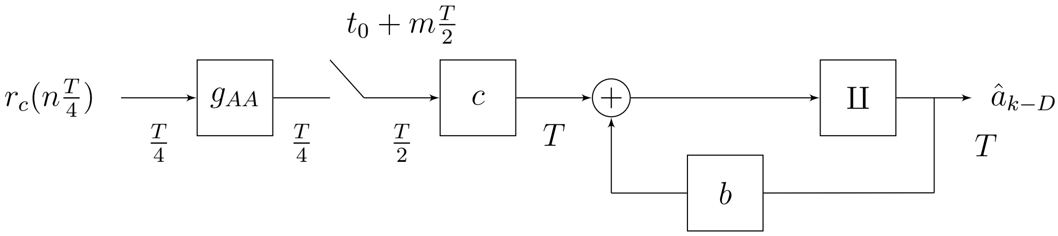

Figure 11. Model for the receiver (d).

Receiver e

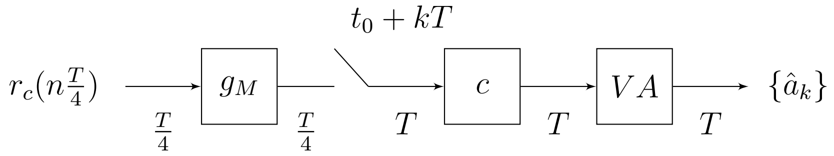

Figure 12. Model for the receiver (e).

Receiver f

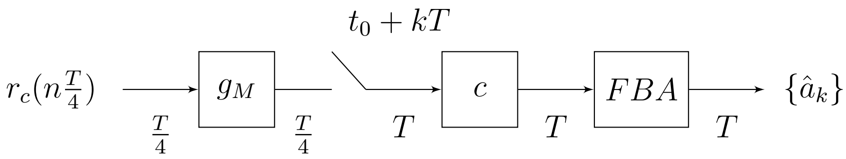

Figure 13. Model for the receiver (f).

Still using the channel as given in the previous section, the signal at the output of the c filter is now processed by a *Forward-Backward algorithm* exploiting the Max-Log-MAP to detect the received symbols $\hat{a}_k$. For each instants $k = 0, 1, \ldots, K-1$, the algorithm performs the computation of five metrics, respectively:

1. The channel transition metric, $c_k(i,j)$, $i, j = 1, \ldots, N_s$;
2. The Backward metric, $\tilde{b}_k(i)$, $i = 1, \ldots, N_s$;
3. The Forward metric, $\tilde{f}_k(j)$, $j = 1, \ldots, N_s$;
4. The State metric, $\tilde{v}_k(i)$, $i = 1, \ldots, N_s$;
5. The Log-Likelyhood function of the isolated symbol $\tilde{l}_k(\beta) = \max\limits_{\substack{i \in [1, \ldots, N_s], \\ [\sigma_i]_1 = \beta}} = \tilde{v}_k(i)$.

The detected symbol is then evaluated using the following decision rule:

$$\hat{a}_{k+L_1} = \underset{\beta \in \mathcal{A}}{\operatorname{argmax}} \, \tilde{l}_k(\beta) \tag{8}$$

BER comparison

For each of the different receiver configurations previously described, the symbol error probability is evaluated by simulations with different values of the SNR Γ.

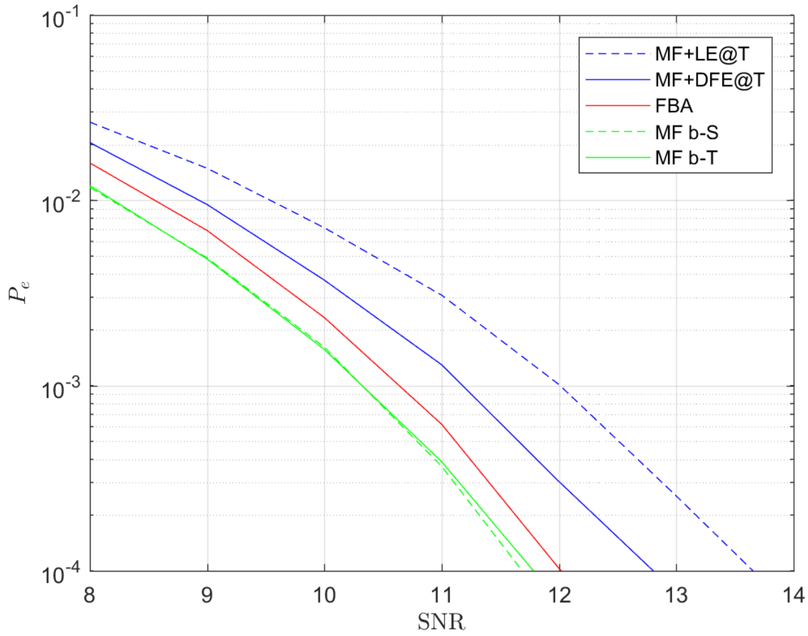

Figure 14. Siimulated BER for the different receivers at different SNR values.