Network Analysis and Simulation

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EXERCISE 1

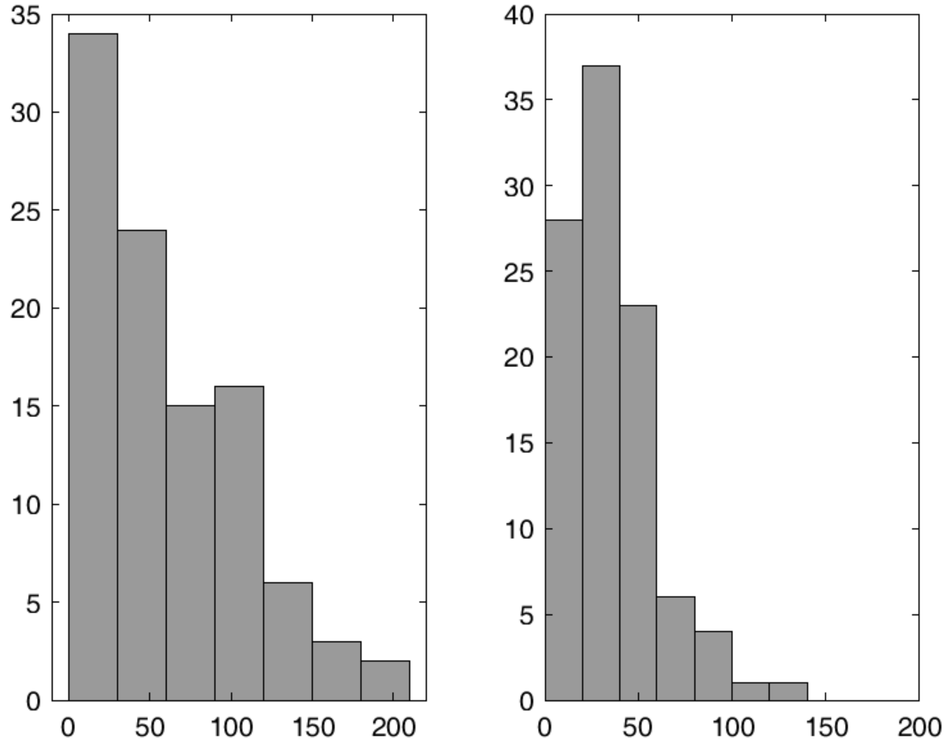


Figura 2.1

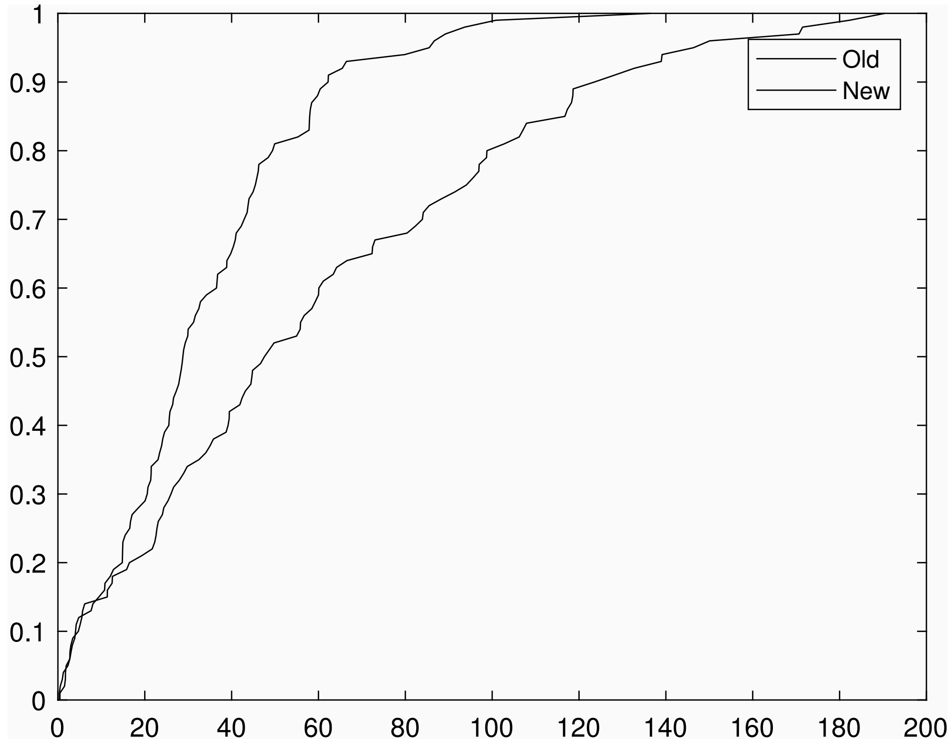


Figura 2.2

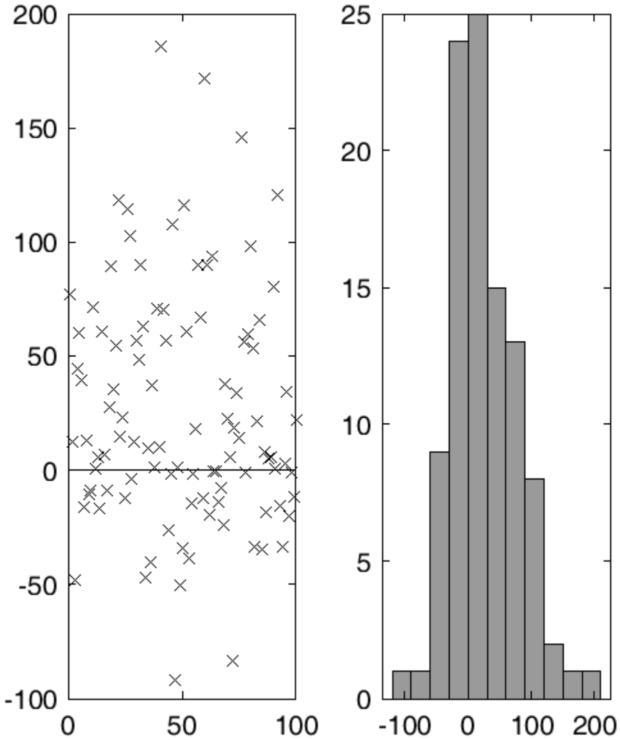


Figura 2.7

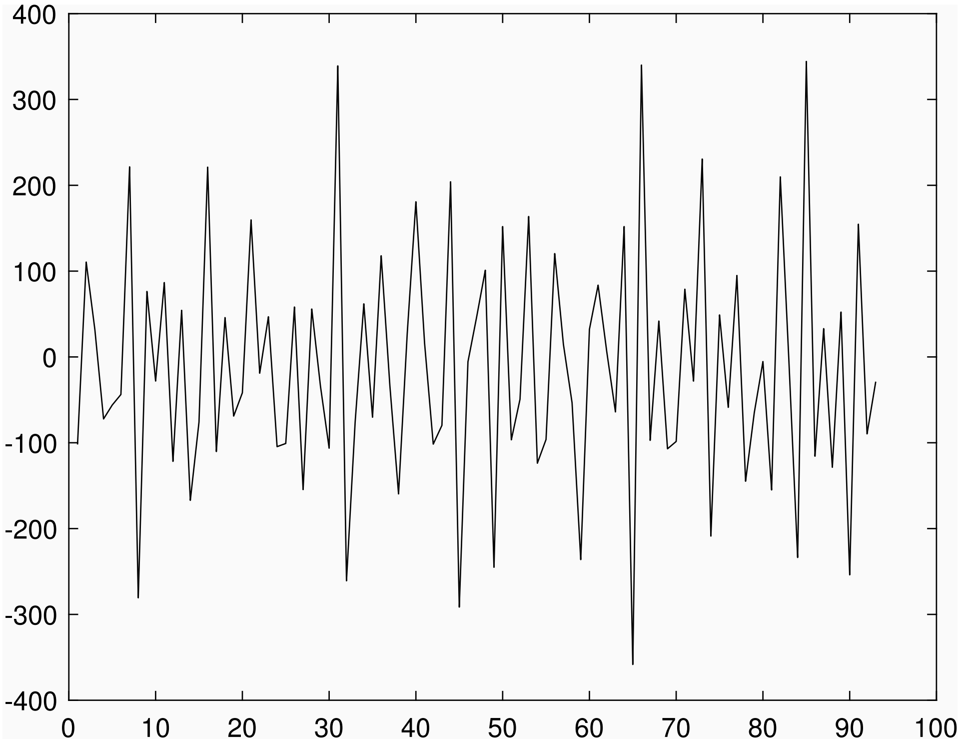
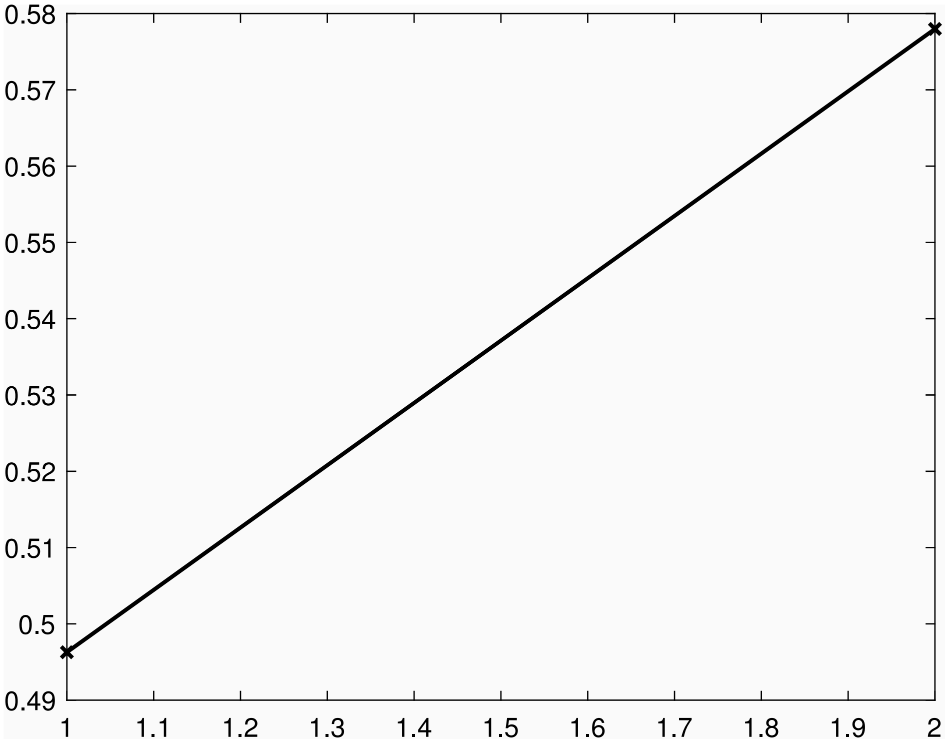


Figura 2.10

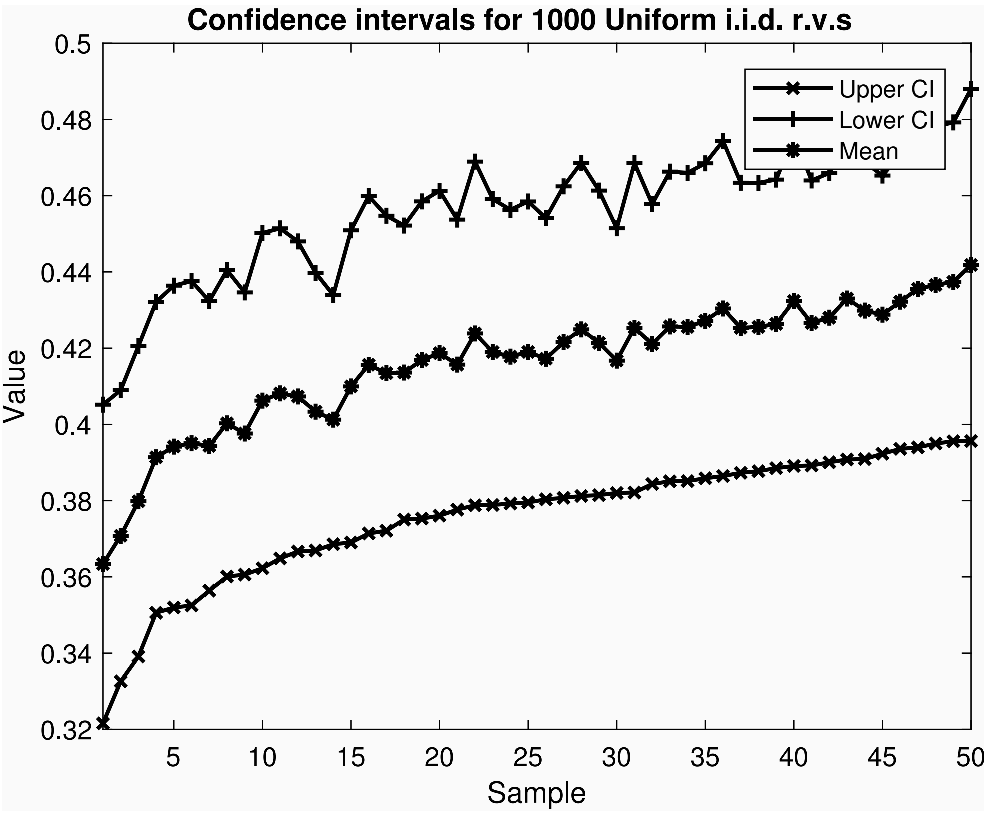
EXERCISE 3

- Confidence interval for 48 i.i.d. U(0,1) random values



CI for Uniform Distribuition

- repetition o the experiment for 1000 times



CI for 1000 times

EXERCISE 3

Prove that, for n $U(0,1)$ random variables, we have $\mathbb{E}\left(U_{(j)}\right) = \frac{j}{n+1}$.

Consider the beta distribution with the following probability density function:
$f_X(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ defined for $x \in [0,1]$.
For $\alpha = 1$ and $\beta = 1$, it can be shown that the beta distribution coincides with the uniform distribution:

$$\begin{aligned}
f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \\
&= \frac{\Gamma(2)}{\Gamma(1)\Gamma(1)} x^0(1-x)^0 \\
&= \frac{0!}{0!0!} x^0(1-x)^0 \\
&= 1
\end{aligned}$$

Therefore we have $f_X(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & x \in [0,1] \end{cases}$, which is the uniform distribution.

For any positive integer, it holds $\Gamma(m) = (m-1)!$.

$$\begin{aligned}
E(U_{(j)}) &= \frac{n!}{(j-1)!(n-j)!} \int_0^1 x^j [1-x]^{n-j} dx \\
&= \frac{\Gamma(n+1)}{\Gamma(j)\Gamma(n-j+1)} \int_0^1 x^j [1-x]^{n-j} dx \\
&= \frac{\Gamma(n+1)}{\Gamma(j)\Gamma(n-j+1)} \cdot \frac{\Gamma(j+1)\Gamma(n-j+1)}{\Gamma(n+2)} \overbrace{\int_0^1 \frac{\Gamma(n+2)}{\Gamma(j+1)\Gamma(n-j+1)} x^j [1-x]^{n-j} dx}^{=1 \text{ by normalization condition}} \\
&= \frac{\Gamma(n+1)}{\Gamma(j)\Gamma(n-j+1)} \cdot \frac{\Gamma(j+1)\Gamma(n-j+1)}{\Gamma(n+2)} \\
&= \frac{\Gamma(j+1)\Gamma(n+1)}{\Gamma(j)\Gamma(n+2)} = \frac{j}{n+1}.
\end{aligned}$$

1 EXERCISE 4